Complete Higgs mass dependence of $t\bar{t}$ threshold production to order $\alpha\alpha_s$

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Top quarks at threshold

- $e^+e^- \to t\bar{t}$ in the limit $\sqrt{s} \sim 2m_t$
  ➪ 2 small scales: v and α_s
- effective theory necessary:

$$\mathcal{L}_{\rm QCD} \stackrel{\rm threshold}{\longrightarrow} \mathcal{L}^{\rm eff}_{\rm (p)NRQCD}$$

- 2 steps:
  1. Compute couplings of $\mathcal{L}^{\rm eff}_{\rm (p)NRQCD}$
  2. Compute quantum corrections within $\mathcal{L}^{\rm eff}_{\rm (p)NRQCD}$
- Here: consider vector currect in full and effective theory

$$j^i = c_v \tilde{j}^i + \ldots$$

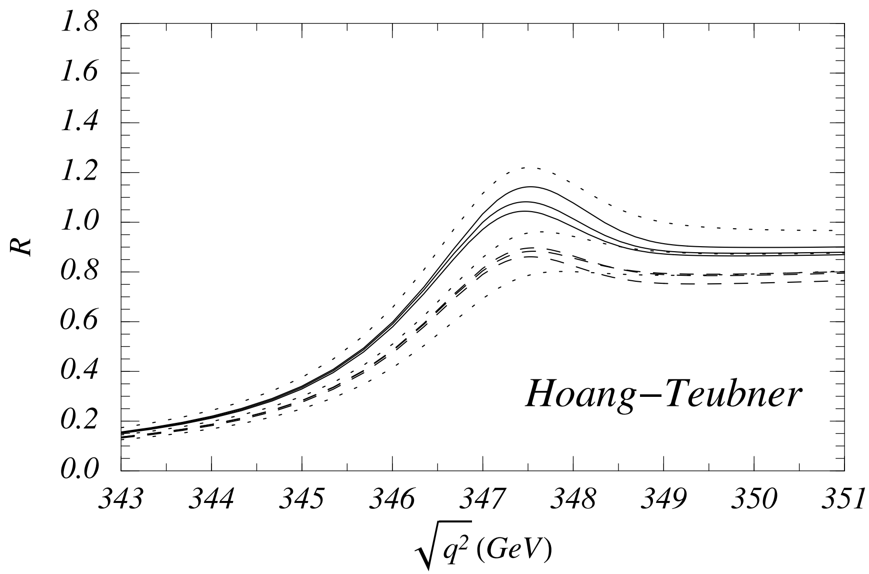
$c_v =$ coupling $=$ matching coefficient

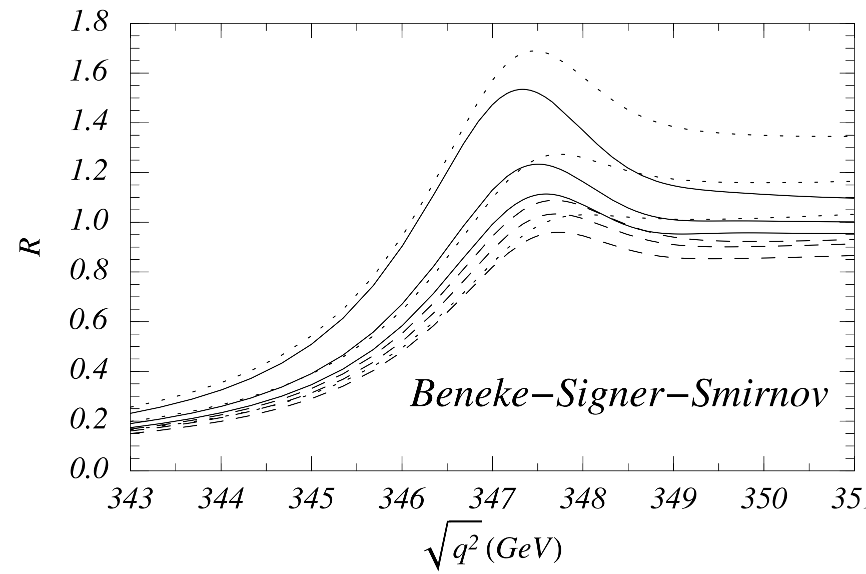
$$\sigma(e^+e^- \to t\bar{t}) \sim \text{Im}\,[G] \sim c_v^2 \text{Im}\langle\tilde{j}\tilde{j}\rangle$$

Motivation

- since 2000: $\sigma(e^+e^- \to t\bar{t})$ complete to NNLO

[Hoang,Beneke,Melnikov,Nagano,Ota,Penin,Pivovarov, Signer,Smirnov,Sumino,Teubner,Yakovlev,Yelkhovsky'00]





- short-distance mass ➪ stabilization of peak position
- normalization ??
- large differences between different groups ??

Motivation (2)

➪ N^3LO calculation necessary
NNLL calculation necessary

Important steps:

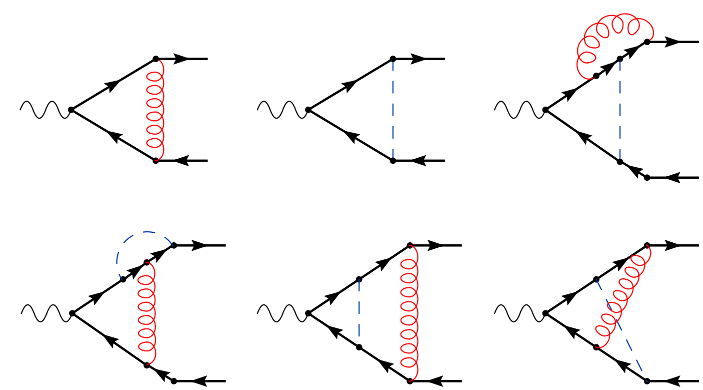
- N^3LO Hamiltonian: [Kniehl,Penin,Smirnov,MS'02]
- $\delta E_n^{(3)}$: [Penin,MS'02],[Penin,Smirnov,MS'05],[Beneke,Kiyo,Schuller'05]
- $\delta \psi_n^{(3)}$: [Penin,Smirnov,MS'05],[Beneke,Kiyo,Schuller'05]
- resummation of logarithms (NNLL)

  [Hoang,Manohar,Stewart,Teubner'01],[Hoang'04], [Penin,Pineda,Smirnov,MS'04], [Pineda,Signer'06]
- ...

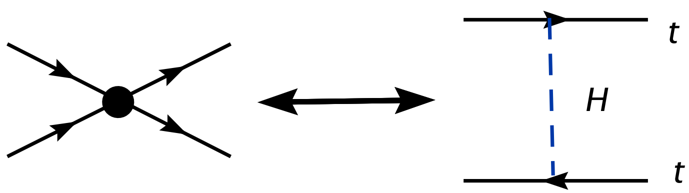
EW corrections

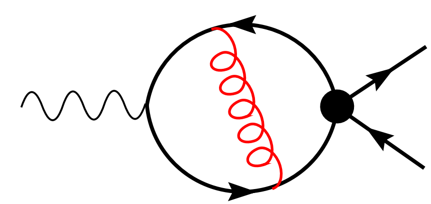
- consider: $\alpha \sim \alpha_s^2$ ➪ N^{3}LO QCD corrections
  $\leftrightarrow$ mixed ew/QCD corrections, $\mathcal{O}(\alpha\alpha_s)$
- NLO-ew: only matching coefficient affected

[Kühn,Guth'92]



- NNLO ($\mathcal{O}(\alpha\alpha_s)$)
  - matching coefficient to 2 loops
  - new operator in effective theory



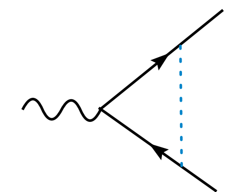


- Here: consider diagrams involving a Higgs boson

NLO ($\mathcal{O}(\alpha)$) matching coefficient

- Consider $\gamma - t - \bar{t}$ Green function and require equality — up to $\mathcal{O}(1/m_t)$:

$$Z_2 \Gamma_v = c_v \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v + \ldots$$

⇨ $c_v = \delta\Gamma^{(1)} + \delta Z_2^{(1)}$ $\qquad \delta\Gamma^{(1)} \sim$ 

- Threshold expansion [Beneke,Smirnov'98]

- Result: ($y_H = m_t/M_h$)

$$c_v^{H,\mathrm{ew}} = \frac{m_t^2}{M_W^2} \left[\frac{3y_H^2 - 1}{12 y_H^2} - \frac{2 - 9y_H^2 + 12 y_H^4}{48 y_H^4} \ln y_H^2 - \frac{(-2 + 5y_H^2 - 6y_H^4)}{24 y_H^2} \Psi(y_H) \right]$$

$$\Psi(x) = \begin{cases} \frac{\sqrt{4x^2-1}}{x^2} \arctan \sqrt{4x^2 - 1} & \text{for } x \geq \frac{1}{2} \\ \frac{\sqrt{1-4x^2}}{2x^2} \ln \frac{1-\sqrt{1-4x^2}}{1+\sqrt{1-4x^2}} & \text{for } x < \frac{1}{2} \end{cases}$$

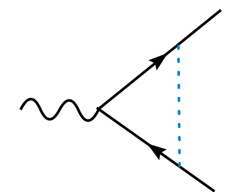
(full result: [Guth,Kühn'92],[Hoang,Reißer'06])

NLO ($\mathcal{O}(\alpha)$) matching coefficient

- Consider $\gamma - t - \bar{t}$ Green function and require equality — up to $\mathcal{O}(1/m_t)$:

$$Z_2 \Gamma_v = c_v \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v + \ldots$$

$\Rightarrow$ $c_v = \delta\Gamma^{(1)} + \delta Z_2^{(1)}$

$\delta\Gamma^{(1)} \sim$ 

- Threshold expansion [Beneke,Smirnov'98]
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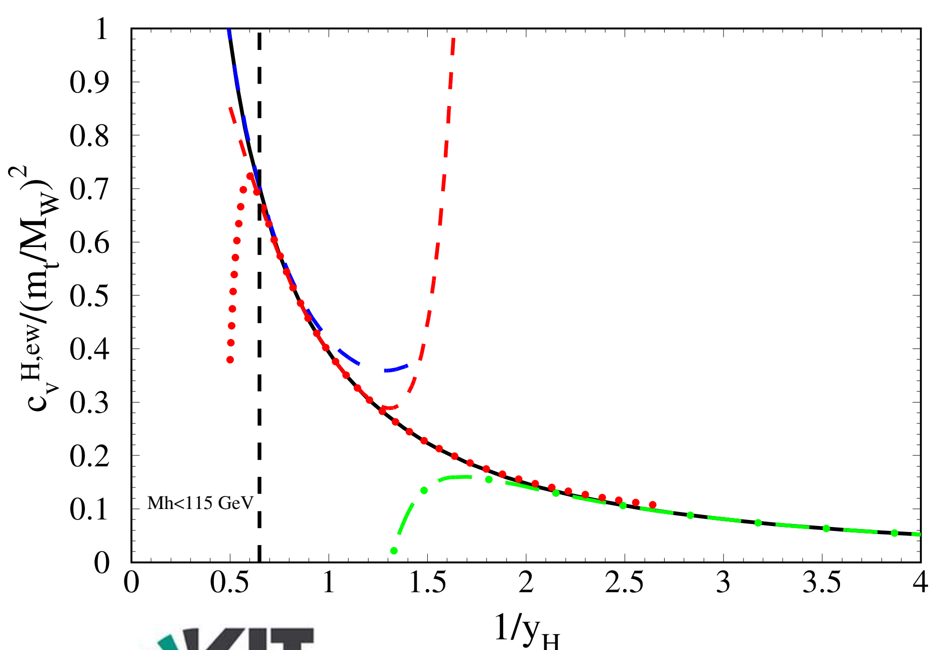


exact

$m_t \gg M_h$

$m_t \ll M_h$

$m_t \approx M_h$

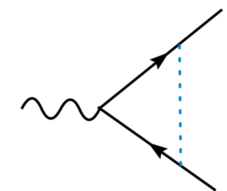
$1 - \frac{M_h^2}{m_t^2}$ (dashed)

$1 - \frac{m_t^2}{M_h^2}$ (dotted)

NLO ($\mathcal{O}(\alpha)$) matching coefficient

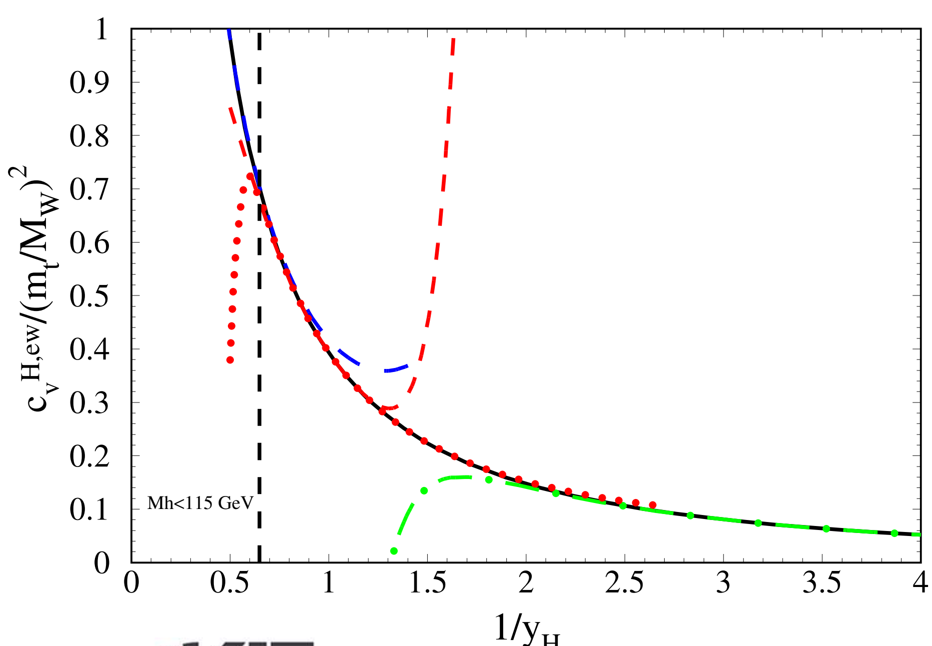
- Interesting:

$$\lim_{m_t\to\infty} c_v^{H,\mathrm{ew}} = \frac{m_t^2}{M_W^2}\left(\frac{m_t}{M_h}\frac{\pi}{4} - \ln\frac{m_t}{M_h} + \ldots\right)$$



⇨ Coulomb-like singularity

- Result: ($y_H = m_t/M_h$)



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$1 - \frac{M_h^2}{m_t^2}$ (dashed)

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NNLO matching coefficient

- $$Z_2\Gamma_v = c_v\tilde{Z}_2\tilde{Z}_v^{-1}\tilde{\Gamma}_v + \ldots$$

➪ $c_v = \delta\Gamma^{(2)} + \delta Z_2^{(2)}$ + 1-loop terms

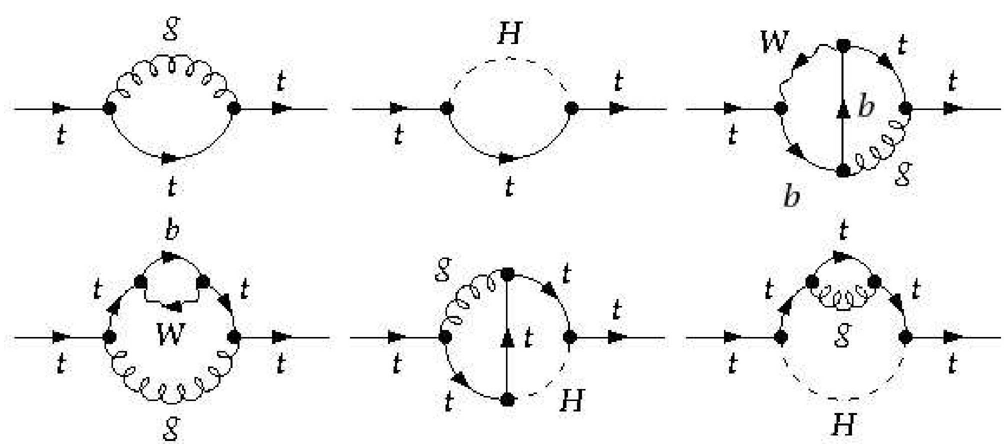
- NEW: IR divergences

(➪ off-shell calaculation and limit $\sqrt{s} \to 2m_t$ not possible)

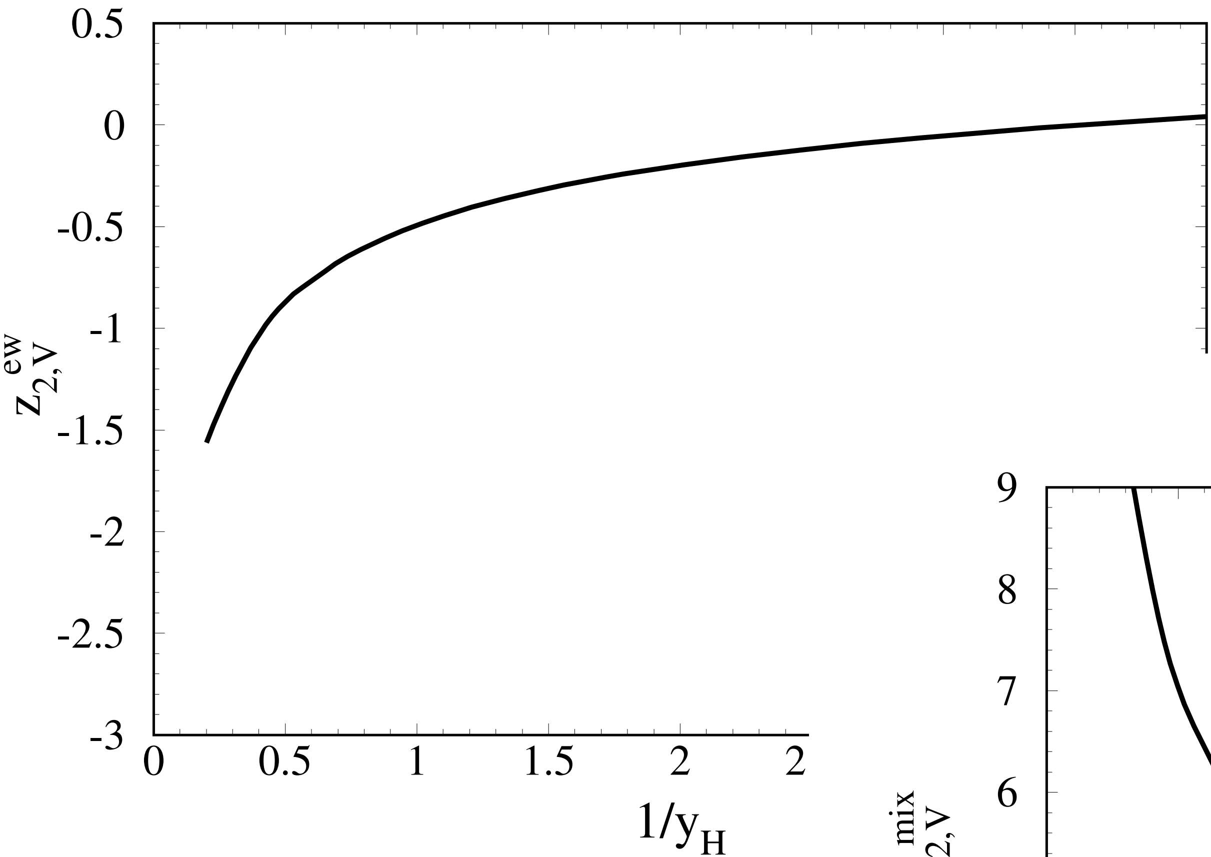
- exact calculation quite involved

idea: $m_t \ll M_h$
$m_t \approx M_h$
($m_t \gg M_h$)

- 1. step: Z_2 to 2 loops



Higgs mass dep. of (complete) Z_2^{OS}

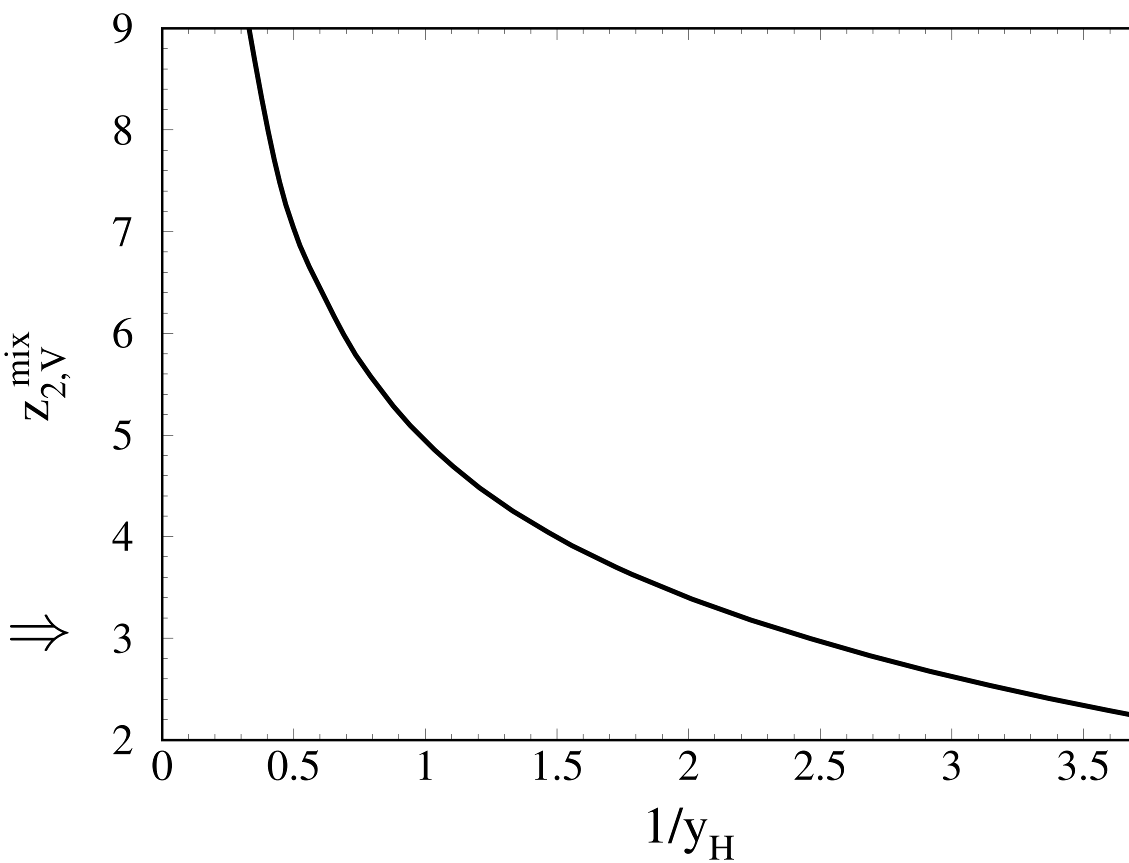


exact $\quad (y_H = \frac{m_t}{M_h})$

$\Uparrow$

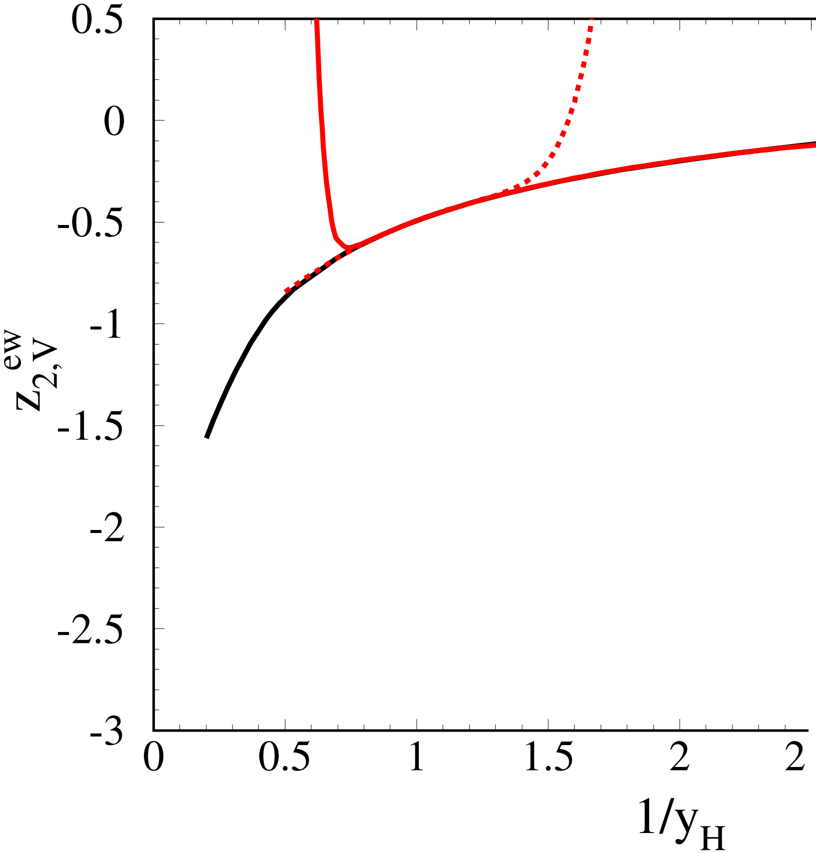
$\mathcal{O}(\alpha)$

$\mathcal{O}(\alpha\alpha_s) \Rightarrow$



[Eiras,MS'05]

Higgs mass dep. of (complete) Z_2^{OS}



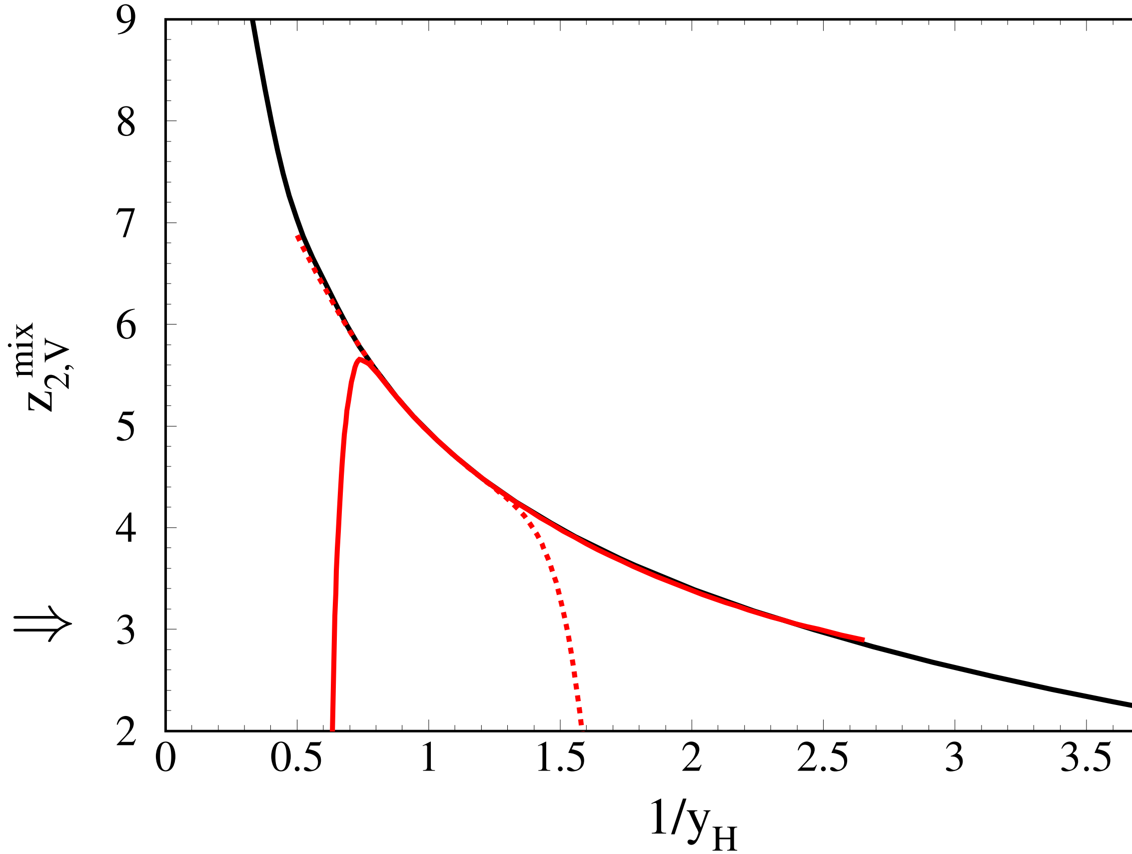
exact $\left(y_H = \frac{m_t}{M_h}\right)$

$m_t \approx M_h$

$\Uparrow$

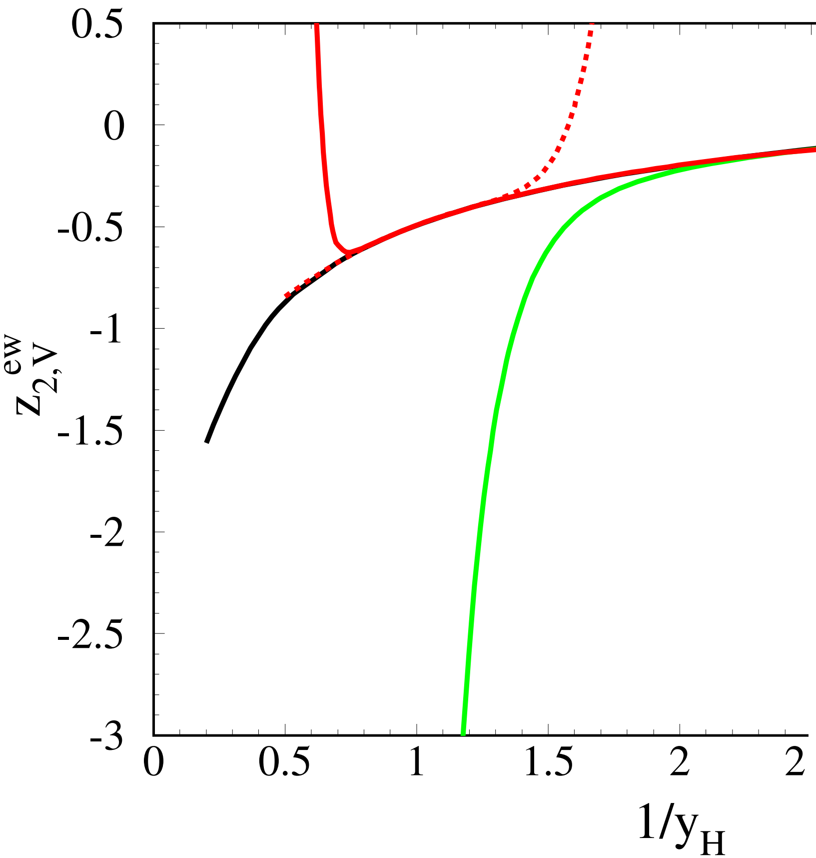
$\mathcal{O}(\alpha)$

$\mathcal{O}(\alpha\alpha_s) \Rightarrow$



[Eiras,MS'05]

Higgs mass dep. of (complete) Z_2^{OS}



exact $\quad (y_H = \frac{m_t}{M_h})$

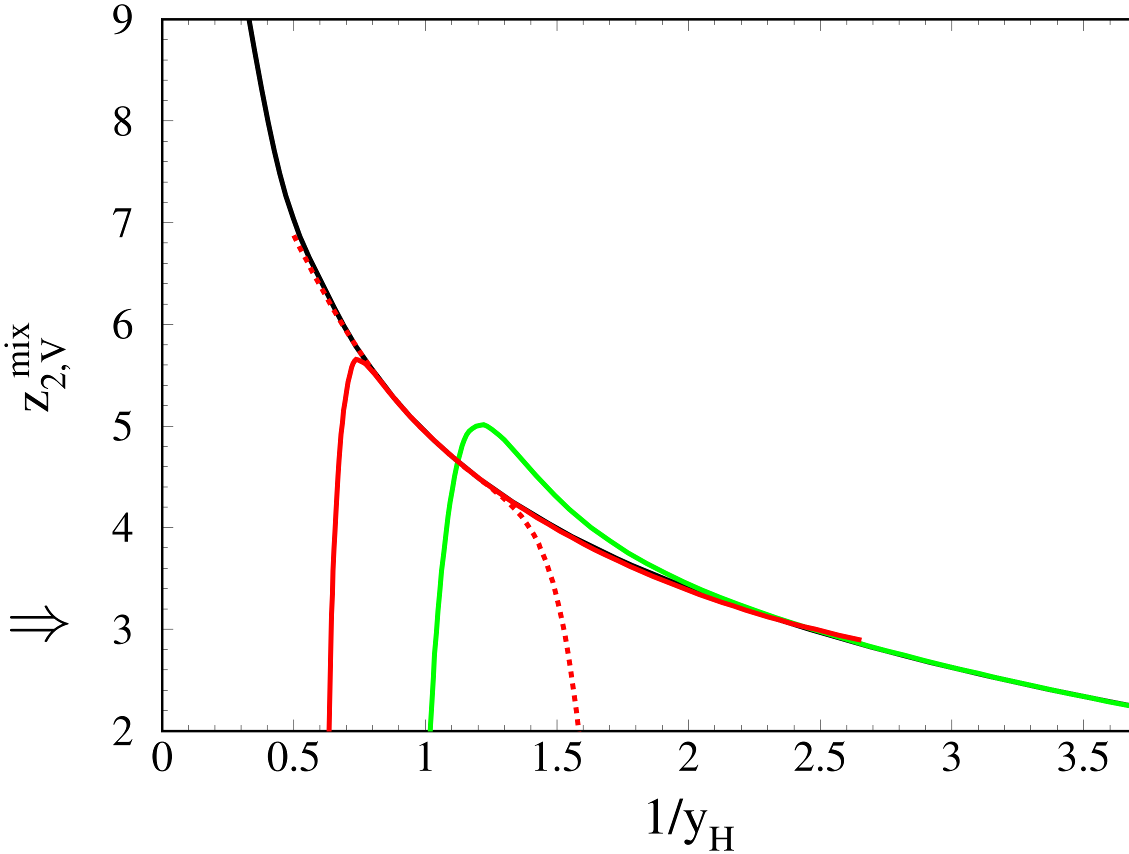
$m_t \approx M_h$

$m_t \ll M_h$

$\Uparrow$

$\mathcal{O}(\alpha)$

$\mathcal{O}(\alpha\alpha_s) \Rightarrow$



[Eiras,MS'05]

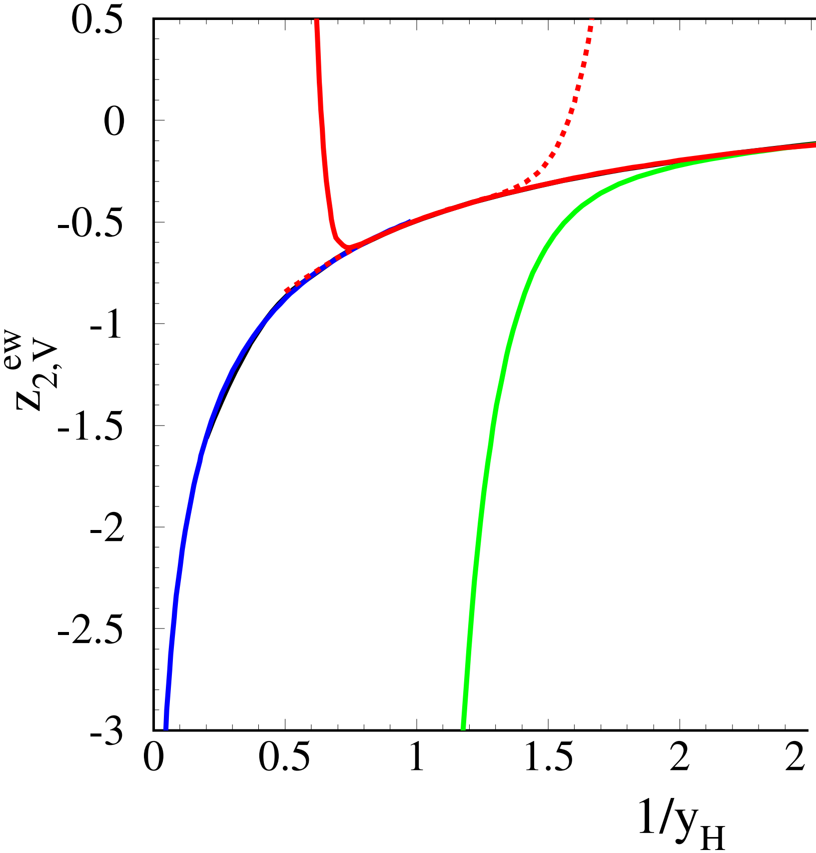
Higgs mass dep. of (complete) Z_2^{OS}

exact $\quad (y_H = \frac{m_t}{M_h})$

$m_t \approx M_h$

$m_t \ll M_h$

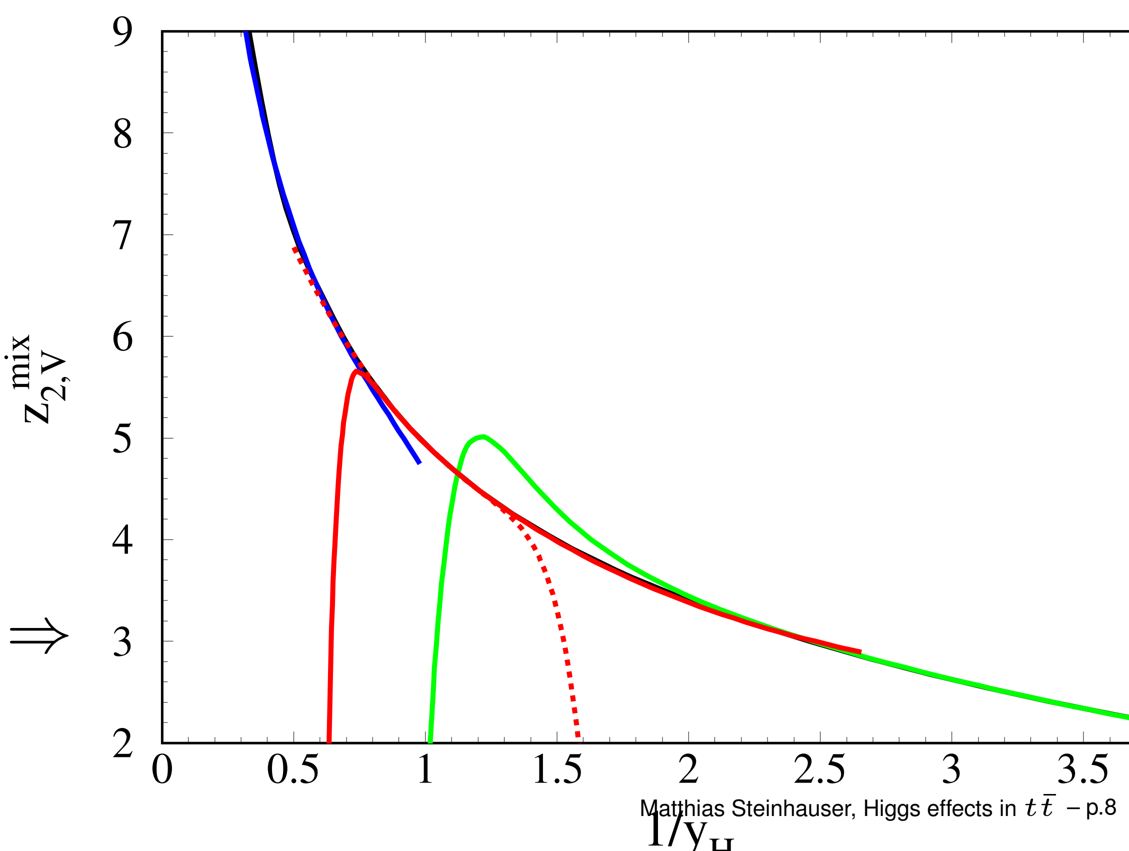
$m_t \gg M_h$



$\Uparrow$

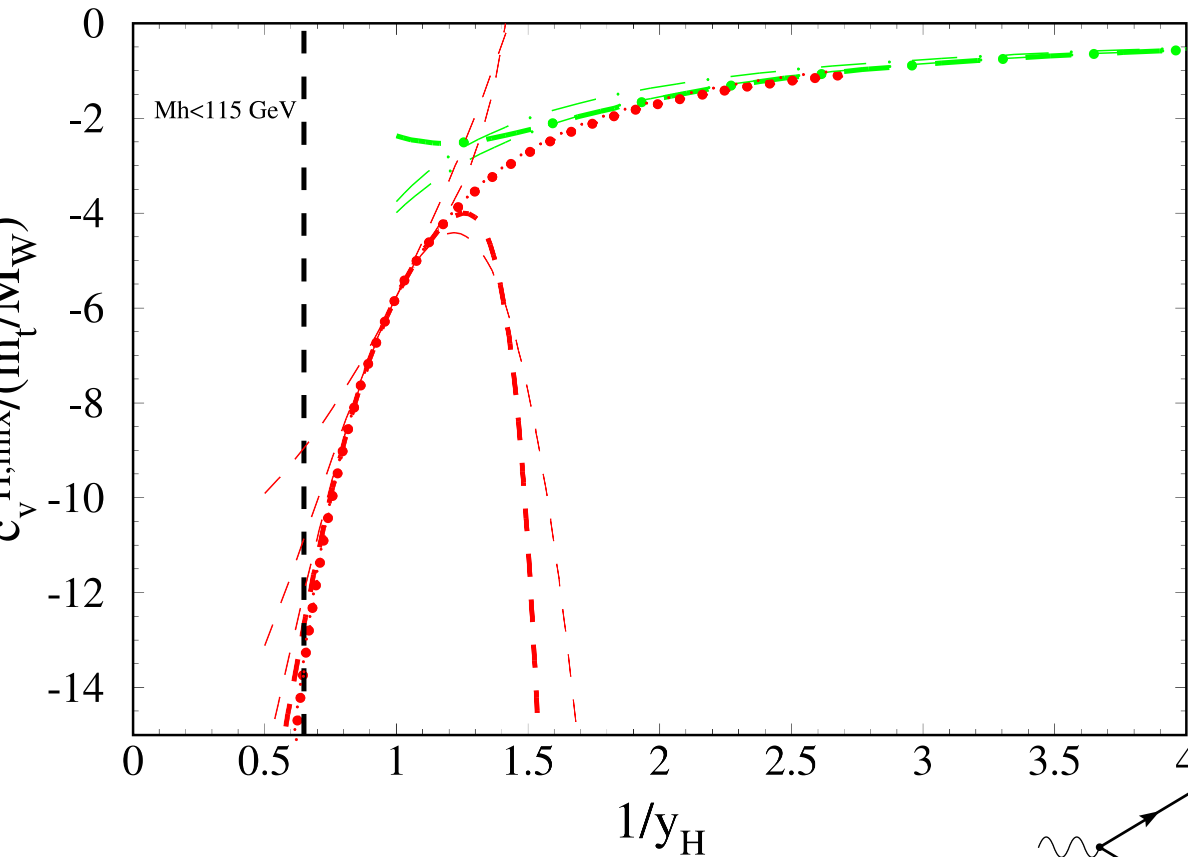
$\mathcal{O}(\alpha)$

$\mathcal{O}(\alpha\alpha_s) \Rightarrow$



[Eiras,MS'05]

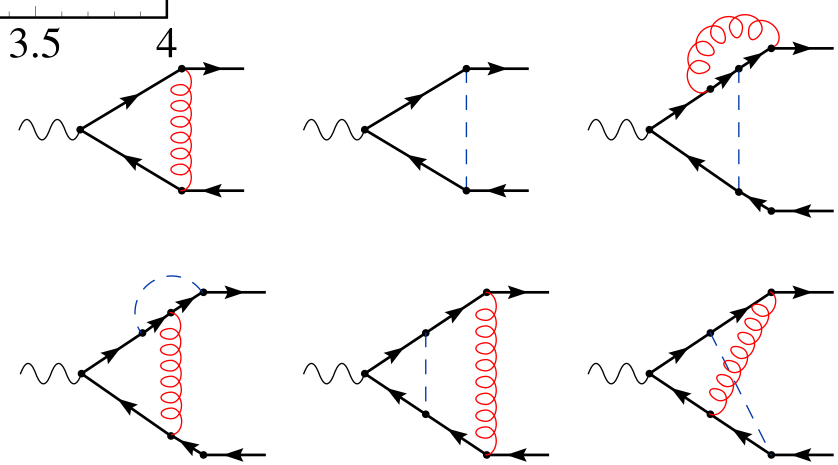
2. step: vertices ⇨ c_v



$\mathcal{O}(\alpha\alpha_s) \qquad (y_H = \frac{m_t}{M_h})$

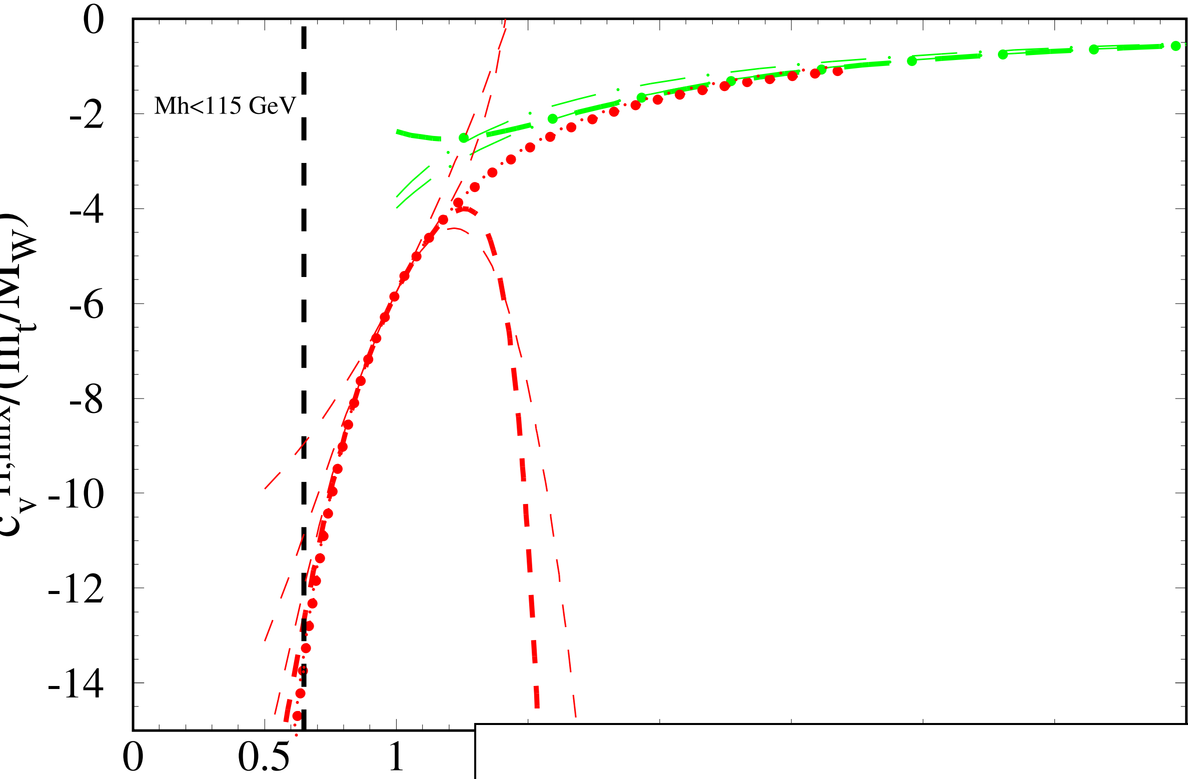
$m_t \approx M_h$

$m_t \ll M_h$



[Eiras,MS'06]

2. step: vertices ⇨ c_v



[Eiras,MS'06]

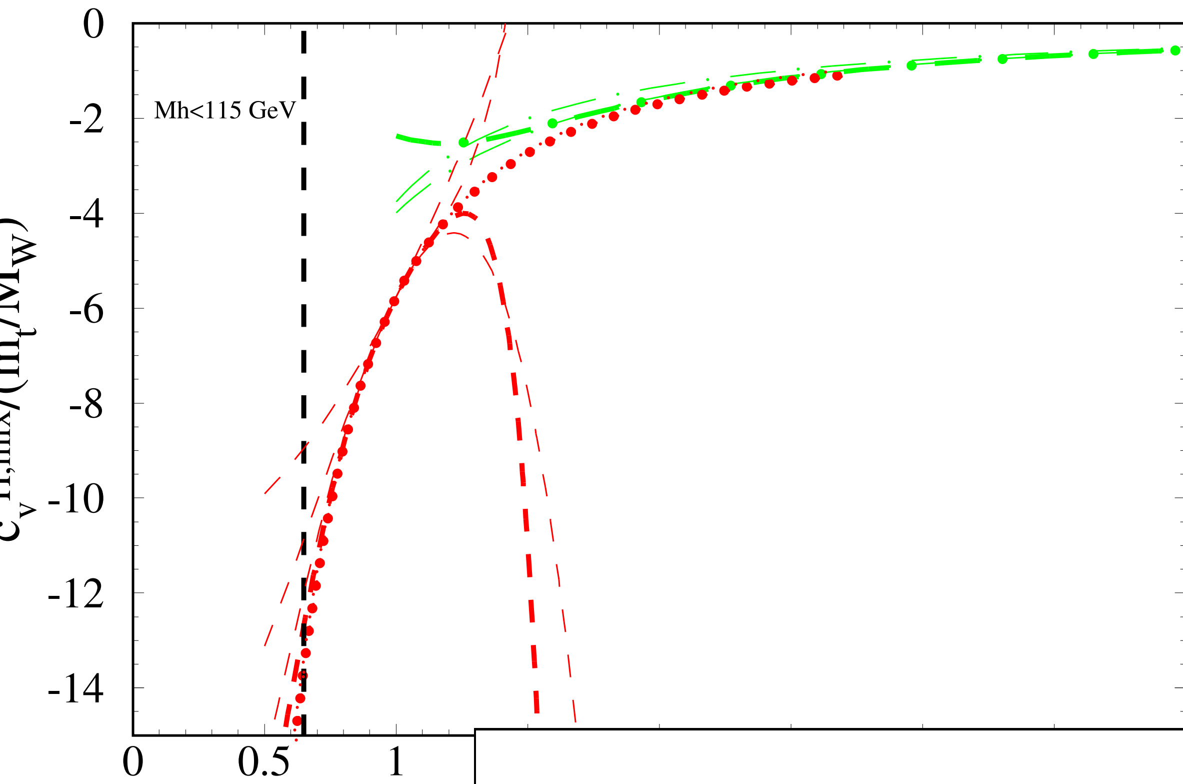
$\mathcal{O}(\alpha\alpha_s) \qquad (y_H = \frac{m_t}{M_h})$

$m_t \approx M_h$

$m_t \ll M_h$

$$c_v = \frac{m_t^2}{M_W^2}\Bigg[\left(\frac{\pi^2}{8}\ln\frac{m_t^2}{\mu^2} - \frac{29}{216} - \frac{277\pi^2}{2304} - \frac{\pi^2\ln 2}{8} - \frac{21\zeta_3}{16} + \frac{139}{216}\ln y_H^2 - \frac{103}{288}\ln^2 y_H^2\right) y_H^2 + \left(\frac{583}{576} - \frac{875\pi^2}{6912} + \frac{151}{192}\ln y_H^2 - \frac{17}{16}\ln^2 y_H^2\right) y_H^4 + \left(\frac{1533691}{432000} - \frac{27103\pi^2}{138240} - \frac{66647}{43200}\ln y_H^2 - \frac{2251}{720}\ln^2 y_H^2\right) y_H^6 + \ldots\Bigg]$$

2. step: vertices ⇨ c_v



$\mathcal{O}(\alpha\alpha_s) \qquad (y_H = \frac{m_t}{M_h})$

$m_t \approx M_h$

$m_t \ll M_h$

[Eiras,MS'06]

$$y_{H,1a} = (1 - \frac{M_h^2}{m_t^2})$$

$$y_{H,1b} = (1 - \frac{m_t^2}{M_h^2})$$

$$c_v^{\text{dashed}} = \frac{m_t^2}{M_W^2}\left[\frac{\pi^2}{8}\frac{1}{1-y_{H,1a}}\ln\frac{m_t^2}{\mu^2} - 5.760 - 5.533y_{H,1a} - 5.704y_{H,1a}^2 - 5.888y_{H,1a}^3 - 6.053y_{H,1a}^4 - 6.200y_{H,1a}^5 + \ldots\right],$$

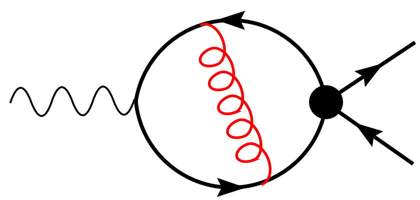
$$c_v^{\text{dotted}} = \frac{m_t^2}{M_W^2}\left[\frac{\pi^2}{8}(1-y_{H,1b})\ln\frac{m_t^2}{\mu^2} - 5.760 + 5.533y_{H,1b} - 0.171y_{H,1b}^2 + 0.0124y_{H,1b}^3 + 0.0304y_{H,1b}^4 + 0.0296y_{H,1b}^5 + \ldots\right]$$

Effective theory: new operator

- new operator in effective theory:

$$\delta\mathcal{H}_H = -\frac{\alpha\pi m_t^2}{s_W^2 M_W^2 M_h^2}$$

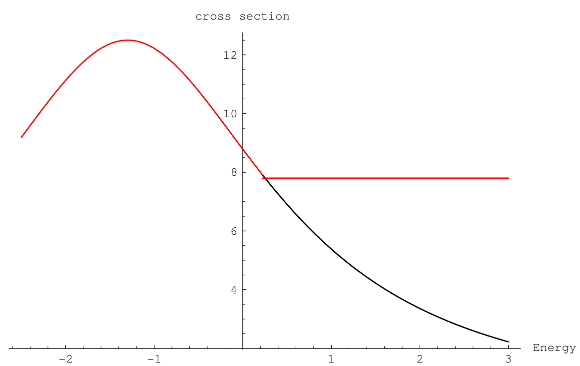
- inserted in



➪ cancellation of divergences form c_v

Application I: top quark mass

- $\delta E_H = \langle \psi_C | \delta \mathcal{H}_H | \psi_C \rangle = E_n^C \frac{\alpha \alpha_s C_F m_t^4}{2 s_W^2 n M_W^2 M_h^2}$
- $E_{\text{res}} = 2m_t + E_1^{\text{p.t.}} + \delta E^{\Gamma}$ ⇨ $\delta m_t \approx -\frac{\delta E_H}{2}$



M_h (GeV)	δm_t (MeV)
120	26
200	9
500	1

Application II: peak of cross section

- $\sigma(e^+e^- \to t\bar{t})\Big|_{\rm peak} \propto c_v^2|\psi_1(0)|^2$

  $\propto |\psi_n^C(0)|^2 \left(1 + \delta c_v^{(1)} + \delta c_v^{(2)} + \ldots\right) \times \left(1 + \delta\psi_n^{(1)} + \delta\psi_n^{(2)} + \ldots\right)$

- $\delta\psi_n^{(2),H}(0) = \psi_n^C(0)\frac{\alpha\alpha_s m_t^4}{s_W^2 M_W^2 M_h^2} C_F\left[-\frac{1}{4}\ln\left(\frac{\alpha_s C_F m_t}{\mu}\right) + \frac{3}{8}\right]$

- ⇨ $\frac{1}{\epsilon}$ pole cancels

- $\sigma(e^+e^- \to t\bar{t})\Big|_{\rm peak} \approx \sigma(e^+e^- \to t\bar{t})\Big|_{\rm peak,LO} \times$

  $(1 - 0.243_{\rm NLO} + 0.435_{\rm NNLO} - 0.268_{\rm N^3LO'} + \delta_H^{(1)} + \delta_H^{(2)} \ldots)$

M_h (GeV)	$\delta_H^{(1)}$	$\delta_H^{(2)}$
120	0.067	0.036
200	0.034	0.011
500	0.009	0.002
	[Guth,Kühn'92]	[Eiras,MS'06]

Conclusions

- $\mathcal{O}(\alpha\alpha_s)$, M_h dependent part to $\sigma(e^+e^- \to t\bar{t})$ threshold
- $Z_2^{\rm OS}$ to $\mathcal{O}(\alpha\alpha_s)$ (complete!)
- 2-loop matching coefficient ("M_h")
- $\delta m_t \lesssim 30$ MeV
- $\delta\sigma_{\rm peak}^{\rm H,\alpha\alpha_s}$ can be of order 3%