Dominant two-loop electroweak corrections to W-pair production at ILC

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W-pair production at ILC

- Search for anomalous gauge boson couplings

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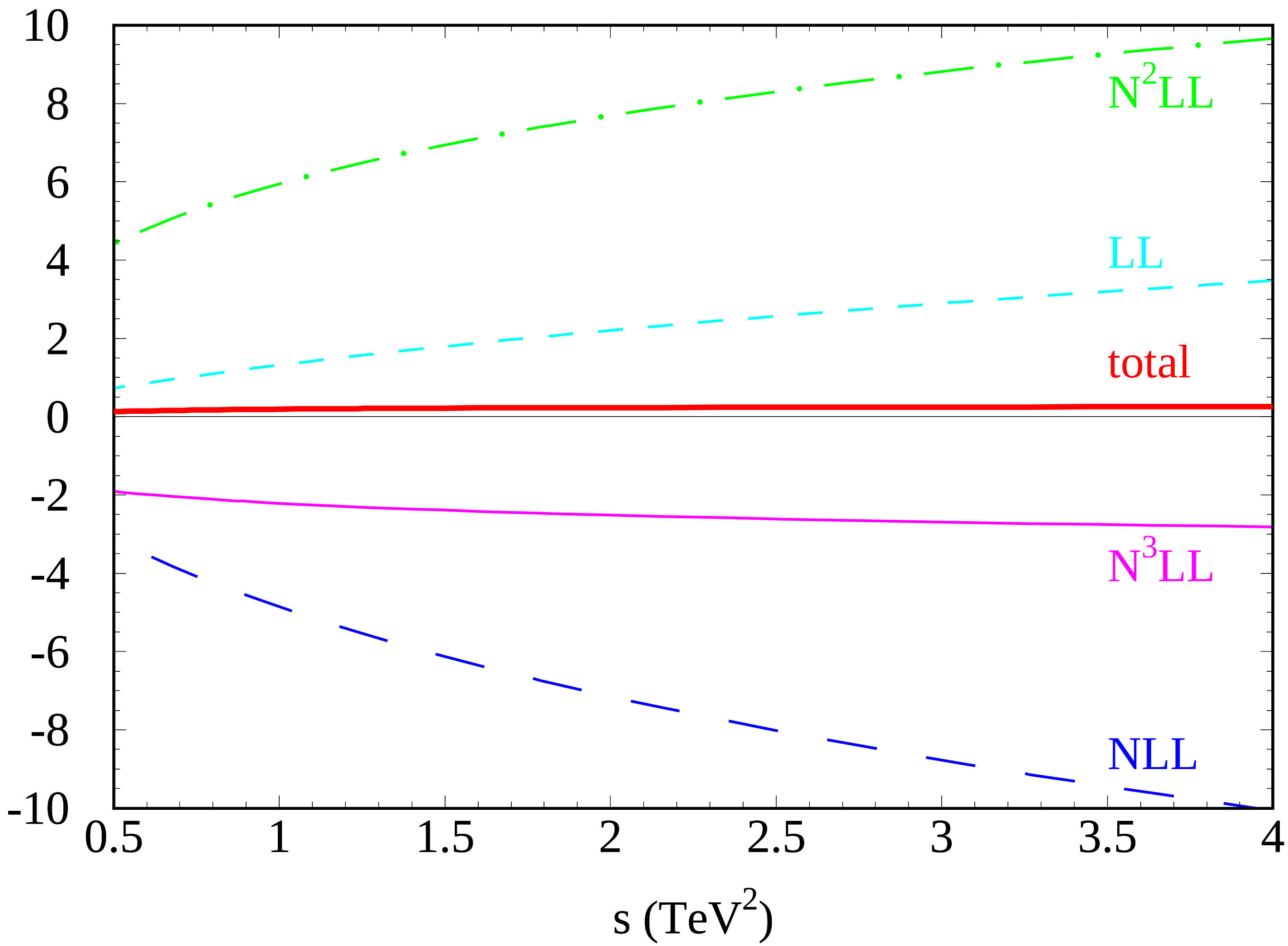
- *Subleading logs are equally important!*

Two-loop corrections to $\sigma/\sigma_{\rm Born}(e^+e^- \to d\bar{d})$

(Jantzen, Kühn, Penin, Smirnov)



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  - N^{2}LL: $\alpha_{ew}^2 \ln^2(s/M_{Z,W}^2)$ ⇨ *This talk*

Based on

- General approach (Jantzen, Kühn, Moch, Penin, Smirnov)
- Preliminary results (Kühn, Metzler, Penin)

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hard evolution equation → s (in both logs)

infrared evolution equation → λ^2

Hard evolution

(Mueller; Collins; Sen; Sterman,...)

Infrared field renormalization

$$\frac{\partial}{\partial \ln Q^2}\mathcal{Z}_i = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\gamma_i(\alpha(x)) + \zeta_i(\alpha(Q^2)) + \xi_i(\alpha(M^2))\right]\mathcal{Z}_i$$

Reduced amplitude

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi(\alpha(Q^2))\tilde{\mathcal{A}}$$

Amplitude decomposition ⇨ $\mathcal{A}(e^+e^- \to W^+W^-) = \mathcal{Z}_e\mathcal{Z}_W\tilde{\mathcal{A}}$

Hard evolution

Solution

$$\mathcal{Z}_i = \exp\left\{\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x} \frac{\mathrm{d}x'}{x'}\gamma_i(\alpha(x')) + \zeta_i(\alpha(x)) + \xi_(\alpha(M^2))\right]\right\}$$

$$\tilde{\mathcal{A}} = \mathcal{A}_0(\alpha(M^2))\mathrm{Pexp}\left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\chi(\alpha(x))\right]$$

Anomalous dimensions ⇨ $\gamma, \quad \zeta, \quad \chi$

Initial conditions ⇨ $\xi, \quad \mathcal{A}_0$

Two-loop corrections to $\sigma(e^+e^- \to W^+W^-)$

$$\alpha^2 \ln^2(s/M^2)$$

- NNLL evolution:
  - $\zeta^{(1)},\ \chi^{(1)}$ ⇨ *1-loop QCD*
  - $\gamma^{(2)}$ ⇨ *2-loop QCD* (Kodaira, Trentedue)
  - $\xi^{(1)},\ \mathcal{A}_0^{(1)}$ ⇨ *1-loop EW* (Beenakker *et al.*)

- Equivalence theorem:
  - $\sigma(e^+e^- \to W_L^+W_L^-)$ ⇨ $\sigma(e^+e^- \to \phi^+\phi^-)$

Two-loop corrections to $\sigma(e^+e^- \to W^+W^-)$

Massive $SU(2)$ *model,* $M_H = M$, 12 *massless left-handed doublets*

$$\left[\frac{\delta\sigma}{\sigma}\right]_L = \left(\frac{\alpha}{4\pi}\right)^2 \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{145}{3}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{5453}{36} - \frac{335\sqrt{3}}{12} + \frac{31\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right.$$

$$\approx \left(\frac{\alpha}{4\pi}\right)^2 \left[4.50\,\ln^4\left(\frac{s}{M^2}\right) - 48.33\,\ln^3\left(\frac{s}{M^2}\right) + 101.55\,\ln^2\left(\frac{s}{M^2}\right)\right]$$

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$$\left[\frac{\delta \mathrm{d}\sigma/\mathrm{d}\Omega}{\mathrm{d}\sigma/\mathrm{d}\Omega}\right]_T = \left(\frac{\alpha}{4\pi}\right)^2 \left[\frac{121}{8}\ln^4\left(\frac{s}{M^2}\right) + \left(-\frac{341}{18} + \ldots\right)\ln^3\left(\frac{s}{M^2}\right) + \left(-\frac{863}{24} + \frac{143\sqrt{3}}{18} + \frac{209\pi^2}{36} + \ldots\right)\ln^2\left(\frac{s}{M^2}\right)\right]$$

$$\approx \left(\frac{\alpha}{4\pi}\right)^2 \left[15.13\,\ln^4\left(\frac{s}{M^2}\right) - (18.94 + \ldots)\ln^3\left(\frac{s}{M^2}\right) + (64.57 + \ldots)\,\ln^2\left(\frac{s}{M^2}\right)\right]$$

$SU(2) \times U(1)$ model

- **QED logs**

$$\mathcal{A}_{f\bar{f}\to f'\bar{f}'} = \exp\left[-\frac{\alpha_e}{4\pi}(Q_e^2 + Q_W^2)\ln^2\left(\frac{s}{\lambda^2}\right) + \ldots\right]\bar{\mathcal{A}}(M^2/s) + \mathcal{O}(\lambda/M)$$

  - *Compute in symmetric phase with* $\lambda = M$
  - *Factorize* $\exp\left[-\frac{\alpha_e}{4\pi}(Q_e^2 + Q_W^2)\ln^2\left(\frac{s}{M^2}\right) + \ldots\right]$

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- Yukawa enhanced logs $\quad \propto (m_t^2/M_W^2)^{1,2}$
  - *Work in progress*

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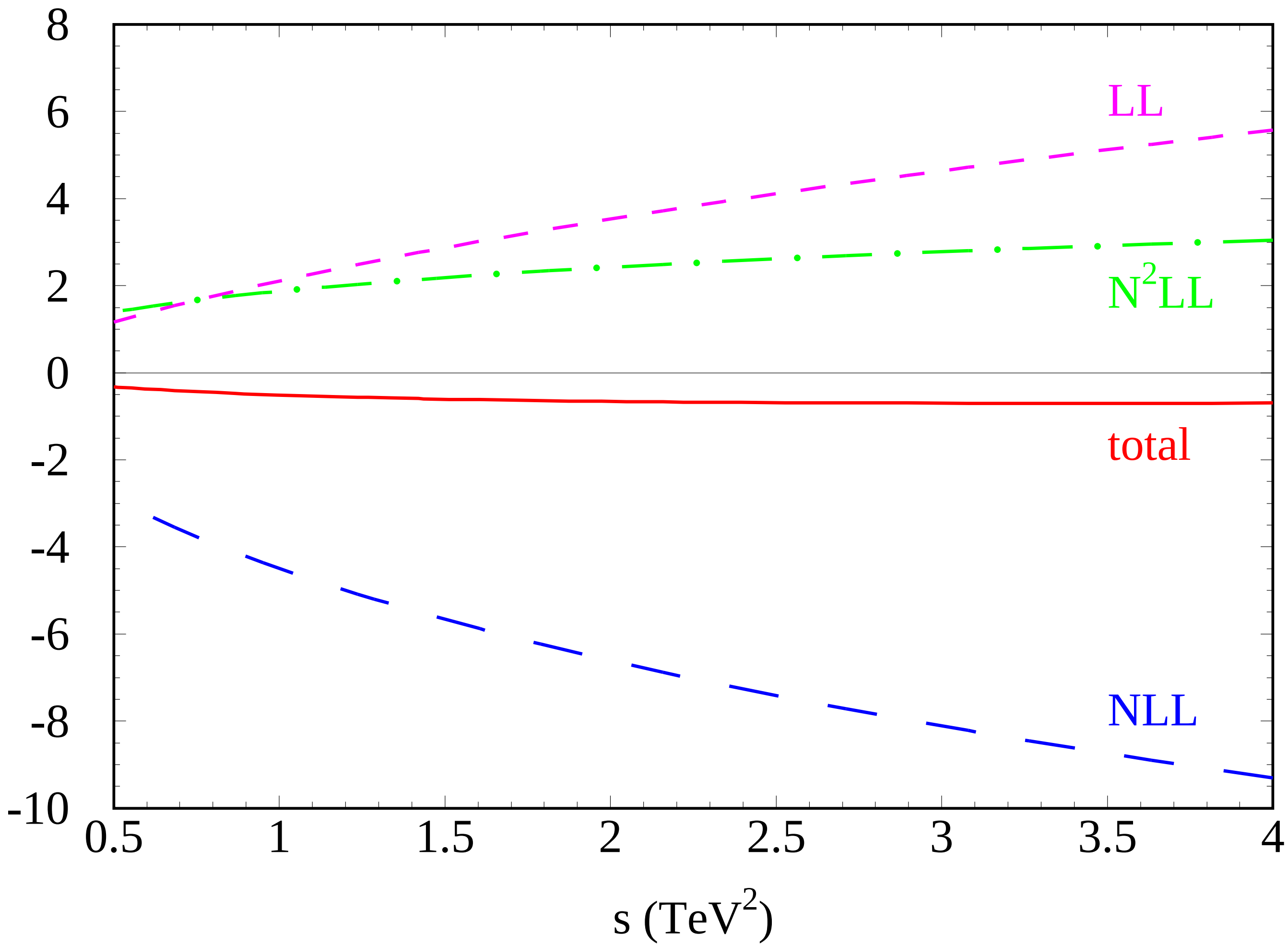
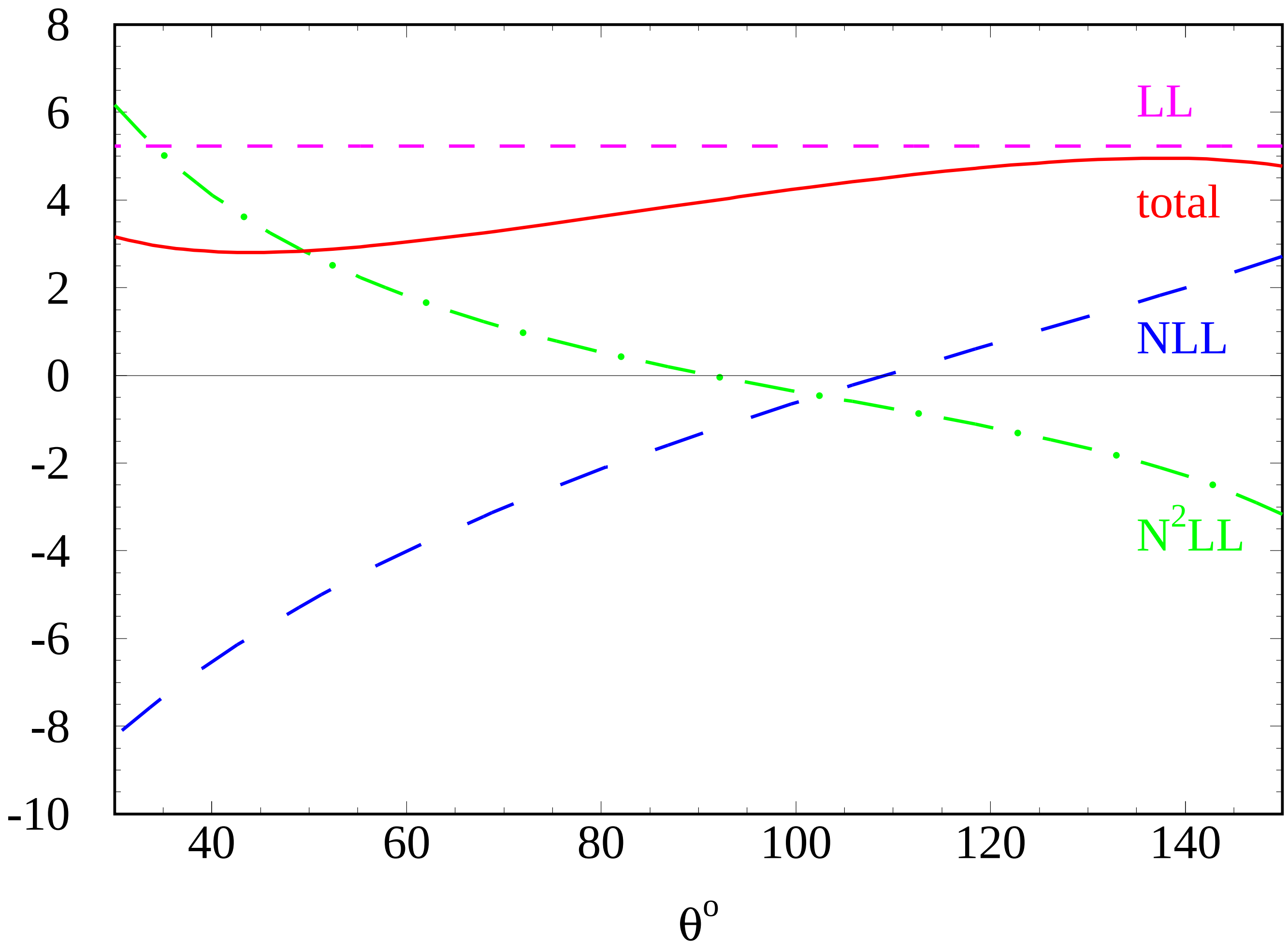


Two-loop corrections to $\mathrm{d}\sigma/\mathrm{d}\sigma_{\mathrm{Born}}(e^+e^- \to W_T^+ W_T^-)$



$$s = 1\ \mathrm{TeV}^2$$

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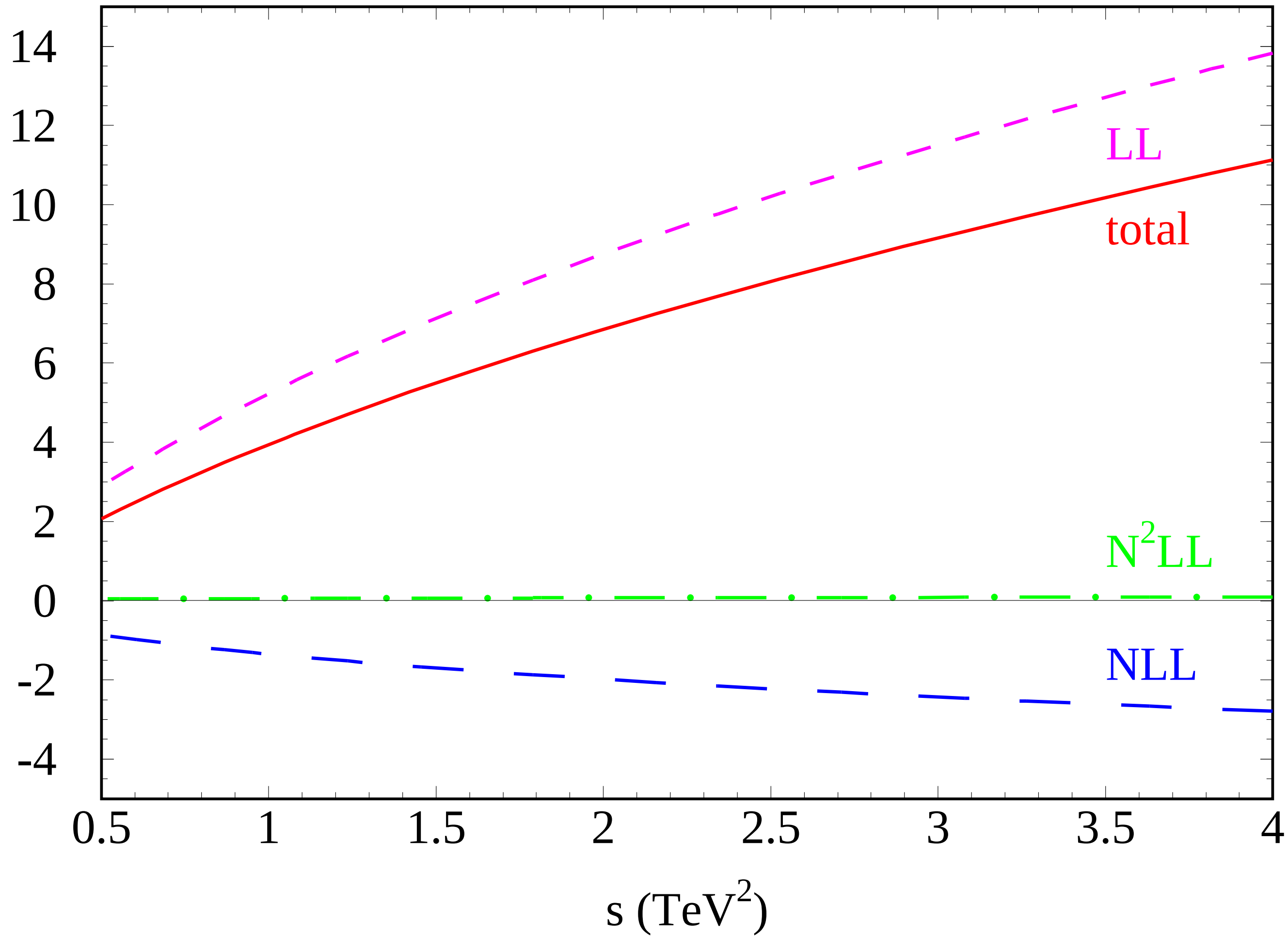

$$\theta = 90^o$$

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- Dominant two-loop electroweak corrections to W-pair production at ILC have been evaluated
- Theoretical uncertainty of the cross sections $\sigma(e^+e^- \to W^+W^-)$ is reduced to $1-2\%$
- Problems to solve:
  - *Yukawa enhanced terms*
  - *Linear logs*
  - *Small angle W_T production*