Two-Loop Fermionic Corrections to Bhabha Scattering

Stefano Actis

DESY, Zeuthen

in collaboration with M.Czakon, J.Gluza and T.Riemann

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The Bhabha scattering at the ILC

- Luminosity at e^-/e^+ colliders $\Leftrightarrow$ $\mathcal{L} = \mathcal{N}_{bh}/\sigma_{bh-th}$

$\ast$ two regions $\Rightarrow$ σ_{bh} large

- ILC $\Rightarrow$ small angle (SABS)
- $1 - 10$ GeV $\Rightarrow$ large angle (LABS)

σ_{bh-th} (SA and LA) is essential for the ILC

- SABS $\longrightarrow$ luminosity
- LABS $\longrightarrow$ low-energy luminosity $\longrightarrow$ hadronic corrections to $\alpha(p^2)$

QED massive corrections

σ_{bh} SA (high E) / LA (low E) QED dominated $\rightarrow$ 2L QED corrections (massive)

– electron-loop $\Rightarrow$ analytic
[Bonciani-Ferroglia-Mastrolia-Remiddi-van der Bij] (2004)

– photonic $\Rightarrow$ analytic
[Bonciani-Ferroglia] (2005)

$*$ except for two-loop boxes $\Rightarrow$ approximated

$$\frac{d\sigma_2}{d\sigma_0} = \delta_2^{(2)} \ln^2\left(\frac{s}{m_e^2}\right) + \delta_2^{(1)} \ln\left(\frac{s}{m_e^2}\right) + \delta_2^{(0)} + \mathcal{O}\left(\frac{m_e^2}{s}\right)$$

[Glover-Tausk-van der Bij] (2001) $\Rightarrow$ $\delta_2^{(1)}$ $\delta_2^{(2)}$

[Penin] (2005) $\Rightarrow$ $\delta_2^{(0)}$

$*$ electron mass $\neq 0$ $\rightarrow$ MC event generators (BHLUMI, BABAYAGA,...)

Three goals

1. cross-check of the fermionic corrections

   $\Rightarrow$ **[Bonciani-Ferroglia-Mastrolia-Remiddi-van der Bij]** (2004)

   diagrammatic approach: reduction to MI $+$ differential equations
   $+$ Mellin-Barnes

2. allow for other flavours

3. cross-check of the two-loop photonic box corrections

   $\Rightarrow$ **[Glover-Tausk-van der Bij]** (2001), **[Penin]** (2005)

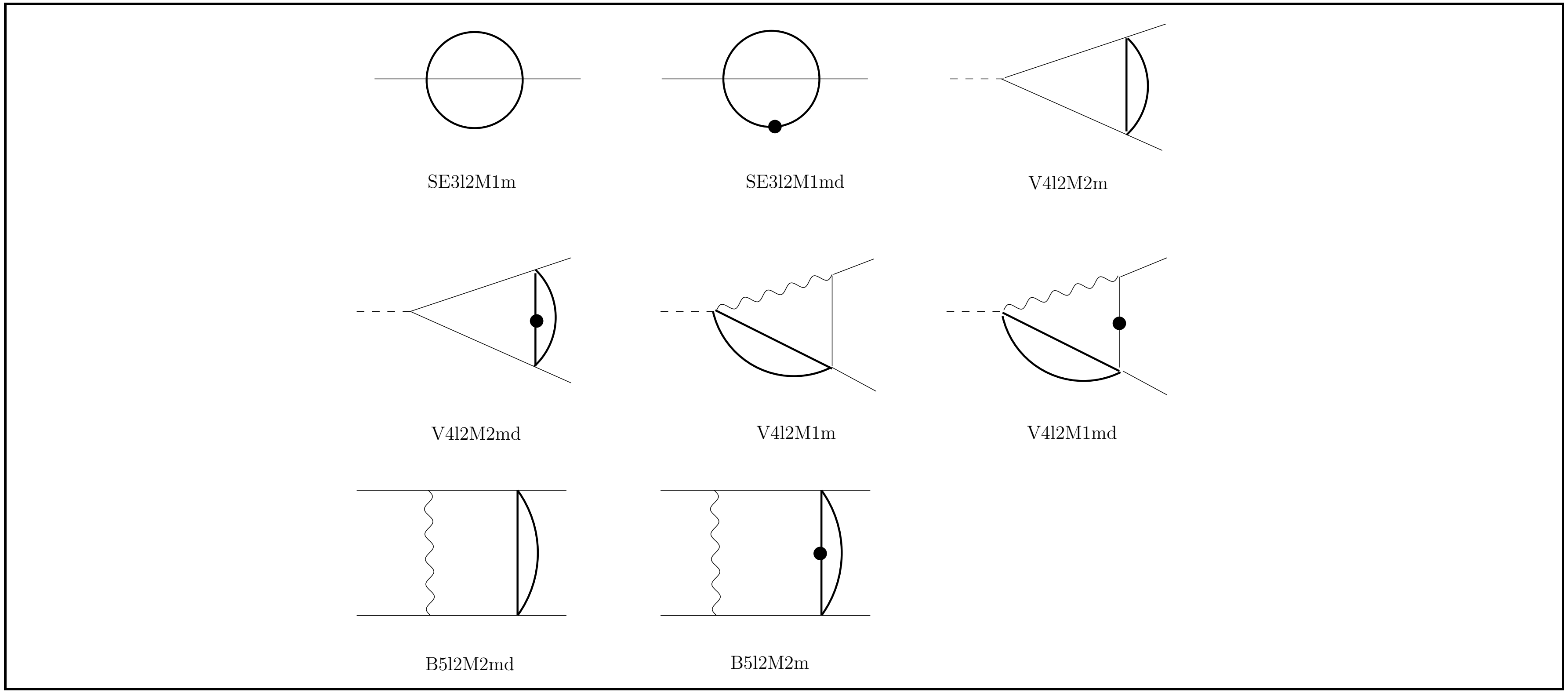
   logarithmic corrections

   massless result **[Bern-Dixon-Ghinculov]** (2000) $\Rightarrow \ln(m_e^2/s) \Leftrightarrow$ IF poles in DR

   $\neq$ diagrammatic approach $\Rightarrow$ exact

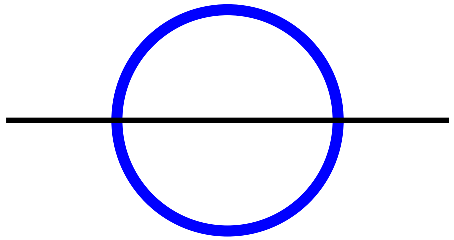
Reduction to Master Integrals

- $\mathcal{M}_2 \Rightarrow \mathcal{M}_0^\star \mathcal{M}_2 \Rightarrow \sum_s \mathcal{M}_0^\star \mathcal{M}_2 \Rightarrow$ scalar integrals
- **IdSolver** (Czakon) implementation of the Laporta algorithm
  $\Rightarrow$ reduction to 8 MI **[Czakon-Gluza-Riemann]** (2004)



[hep-ph:0412164] $\longrightarrow$ two methods

Method I: analytical results through Mellin-Barnes representation



$\longrightarrow$ **wave-function renormalization**

1. MB representation

$$\int_{c-i\infty}^{c+i\infty} dz \;\; R^{z+\epsilon} \, \frac{\prod \Gamma(\ldots)}{\prod \Gamma(\ldots)} \qquad d = 4 - 2\epsilon \qquad R = m_e^2/m_\mu^2$$

2. $\epsilon = \epsilon_0 \;\; \longrightarrow$ analytic continuation $\longrightarrow \; \epsilon = 0$ **[Smirnov/Tausk]** (1999)
   automatization by [**Anastasiou-Daleo** + **Czakon**] (2005)

3. loop integral $\Leftrightarrow$ sum over residua

MB: exact results

- harmonic sums (generalized)

  `Summer` [Vermaseren] - **`XSummer`** [Moch-Uwer] - **`nestedsums`** [Weinzierl]

- more scales ($m_f \neq m_e$) $\Rightarrow$ inverse binomial sums

$$\sum_{i=1}^{\infty} \frac{1}{\binom{2i}{i}} \frac{u^i}{(i+1/2)} S_1(i) \qquad u = m_e^2/m_f^2$$

  $*$ **`[Davydychev-Kalmykov]`** (2004)

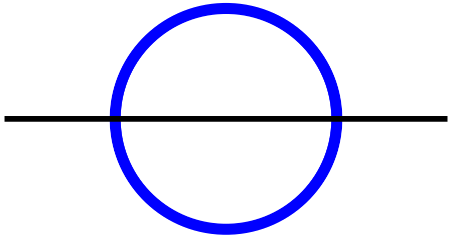
  $*$ **`[Weinzierl]`** (2004)

$\longrightarrow$ extract analytically all the UV and IF residues

$\longrightarrow$ cross-check all the exact electron-loop corrections

finite parts $m_f \neq m_e \Rightarrow$ asymptotic expansions

MB: asymptotic expansions



- MB representation (after resolution of ϵ singularities)

$$\int_{c-i\infty}^{c+i\infty} dz \;\; R^z \, \frac{\prod \Gamma(\ldots)}{\prod \Gamma(\ldots)} \qquad R = m_e^2/m_\mu^2$$

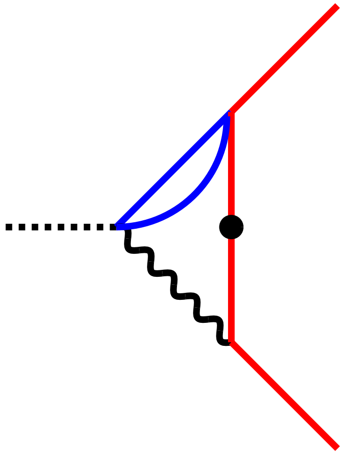
- [**Roth-Denner**] (1996) $\Rightarrow$ asymptotic expansions for one-loop integrals

close to the right $\Rightarrow$ poles $z = n$ $+$ residue 1st pole ($n = 0$) $\to R^0$

$+$ residue 2nd pole ($n = 1$) $\to R^1$

$\ldots$

Method II: numerical evaluation through dispersion relations



[**Bauberger-Berends-Böhm-Buza**] (1994)

$$I \to \underbrace{B_0(s:M^2,M^2)B_0(m^2;\underline{m^2},0)}_{\text{sing.}} + \int dk_1 \frac{B_0[(p_1+k_1)^2;M^2,M^2] - B_0(s;M^2,M^2)}{(k_1^2-m^2)^2(k_1-p_2)^2}$$

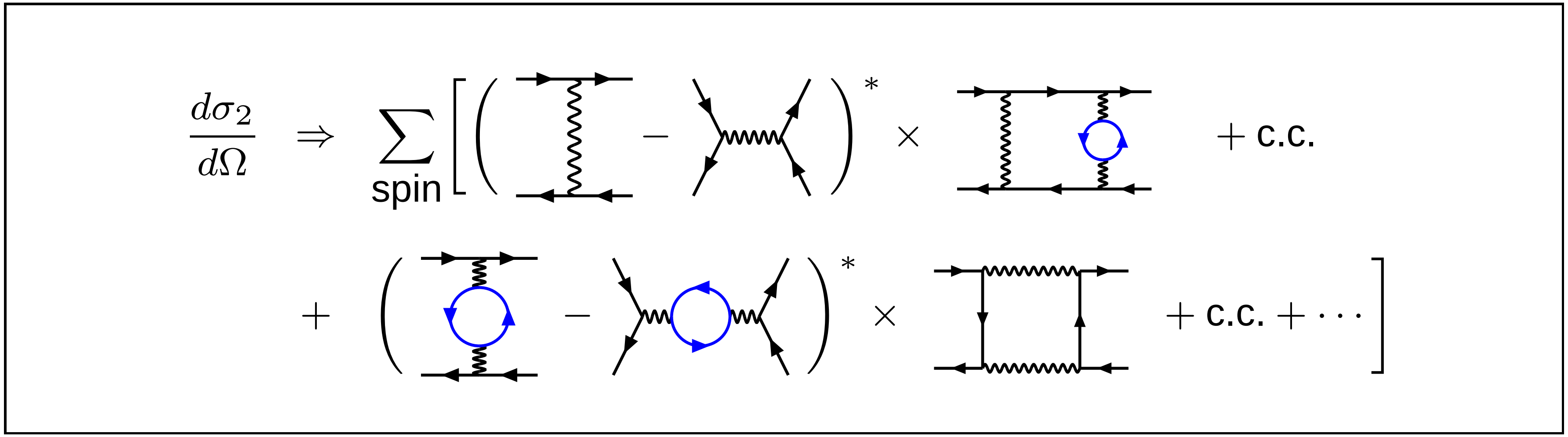
$$B_0[(p_1+k_1)^2;\ldots] - B_0(s;\ldots) = \frac{1}{\pi}\int_{4M^2}^{\infty} d\sigma \, \mathrm{Im} B_0(\sigma;M^2,M^2)\left[\frac{1}{\sigma-(p_1+k_1)^2} - \frac{1}{\sigma-s}\right]$$

$$\Rightarrow \frac{1}{\pi}\int_{4M^2}^{\infty} d\sigma \, \mathrm{Im} B_0(\sigma;M^2,M^2)\left[C_0(m^2,s,m^2;\underline{m}^2,0,\sigma) + \frac{B_0(m^2;\underline{m}^2,0)}{\sigma-s}\right]$$

numerical integration

Results for fermion-loop corrections

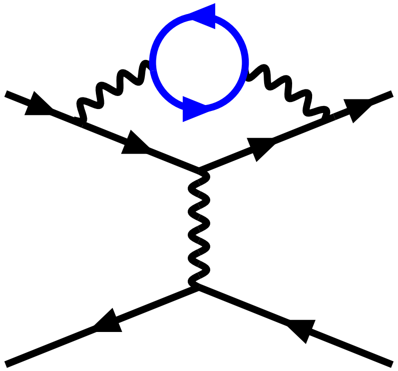
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} + \left(\frac{\alpha}{\pi}\right)\frac{d\sigma_1}{d\Omega} + \left(\frac{\alpha}{\pi}\right)^2\frac{d\sigma_2}{d\Omega}$$



- electron loops $\longrightarrow$ complete agreement with the exact result of **[Bonciani-Ferroglia-Mastrolia-Remiddi-van der Bij]** (2004)

- heavy-lepton loops $\longrightarrow$ approx. analytical result neglecting m_e^2/m_f^2, m_e^2/s, m_f^2/s $+$ numerical cross check (dispersion integrals)

Example: vertex form factor



$$L_e(t) = \ln(-m_e^2/t)$$

$$L(R_f) = \ln(m_e^2/m_f^2)$$

$$F_{\mathrm{V},e}^{(2)}(t) = \frac{1}{4}\left(\frac{383}{27} - \zeta_2\right) + \frac{1}{6}\left(\frac{265}{36} + \zeta_2\right) L_e(t) + \frac{19}{72} L_e^2(t) + \frac{1}{36} L_e^3(t)$$

$$\begin{aligned} F_{\mathrm{V},f}^{(2)}(t) = \frac{1}{6}\Big\{ & \frac{1}{6}\left(\frac{3355}{36} + 19\,\zeta_2 - 12\,\zeta_3\right) - \left(\frac{265}{36} + \zeta_2\right) L(R_f) + \frac{25}{12} L^2(R_f) \\ & - \frac{1}{6} L^3(R_f) + \left[\frac{265}{36} + \zeta_2 - \frac{19}{6} L(R_f) + L^2(R_f)\right] L_e(t) \\ & + \frac{1}{2}\left[\frac{19}{6} - L(R_f)\right] L_e^2(t) + \frac{1}{6} L_e^3(t) \Big\} \end{aligned}$$

Summary

- Small-angle Bhabha scattering $\Rightarrow$ luminosity monitor at ILC
- Two-loop available corrections:
  - Bonciani, Ferroglia *et al* $\Rightarrow$ non-approximated electron-loop
  - Glover-Tausk-van der Bij $+$ Penin $\Rightarrow$ approximated photonic ($\mathcal{O}(m_e^2/s)$)
- Goal: independent cross-check $+$ improvement

  Actis, Czakon, Gluza, Riemann

  - fermionic corrections $\Rightarrow$ agreement with Bonciani, Ferroglia *et al*

    $+$ heavy-lepton (muons, taus) loops allowed
  - photonic corrections $\Rightarrow$ exact result ?

    Mellin-Barnes method $+$ summation techniques for non-planar double-box diagrams