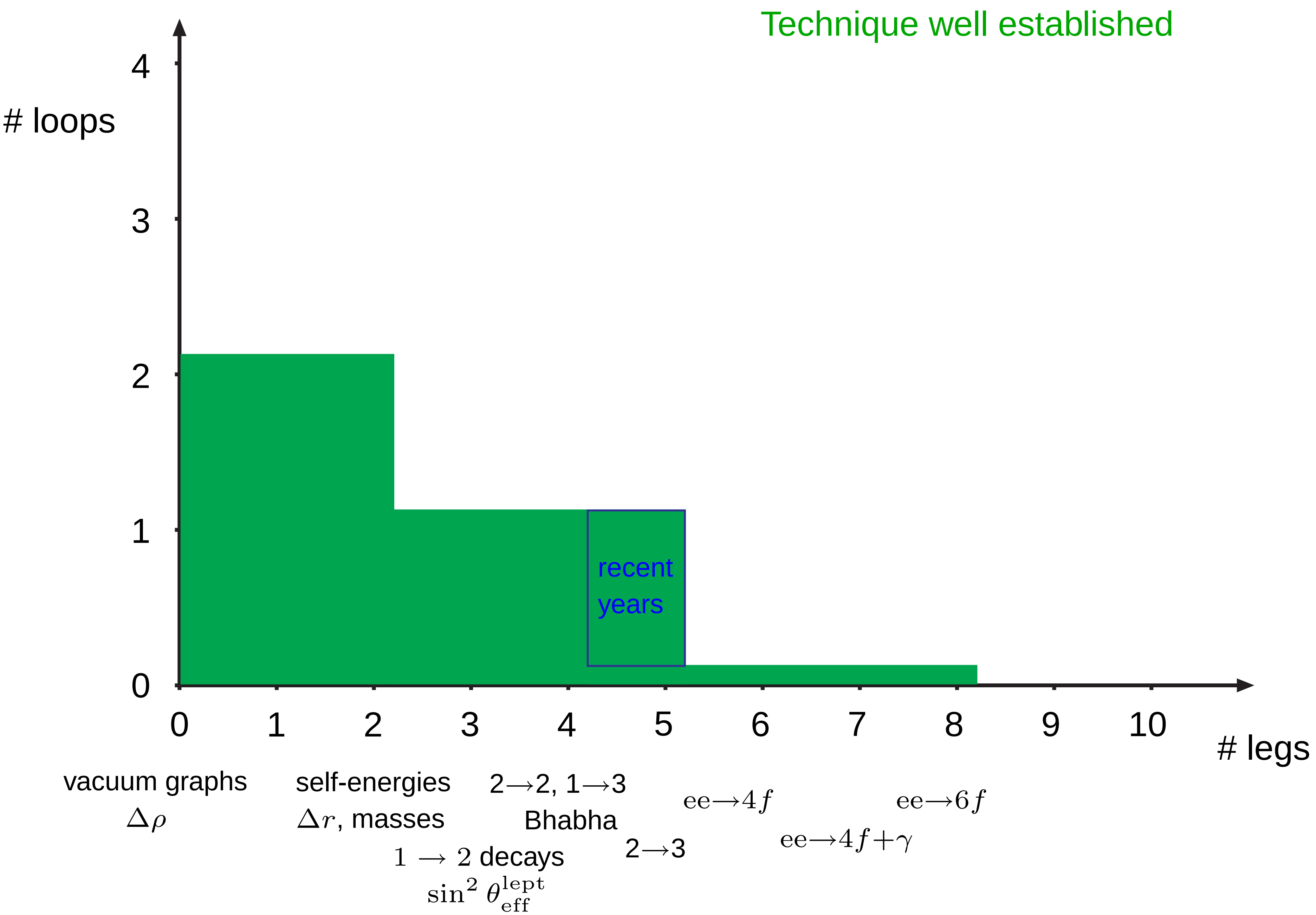
# Loopverein Summary Report

Stefan Dittmaier

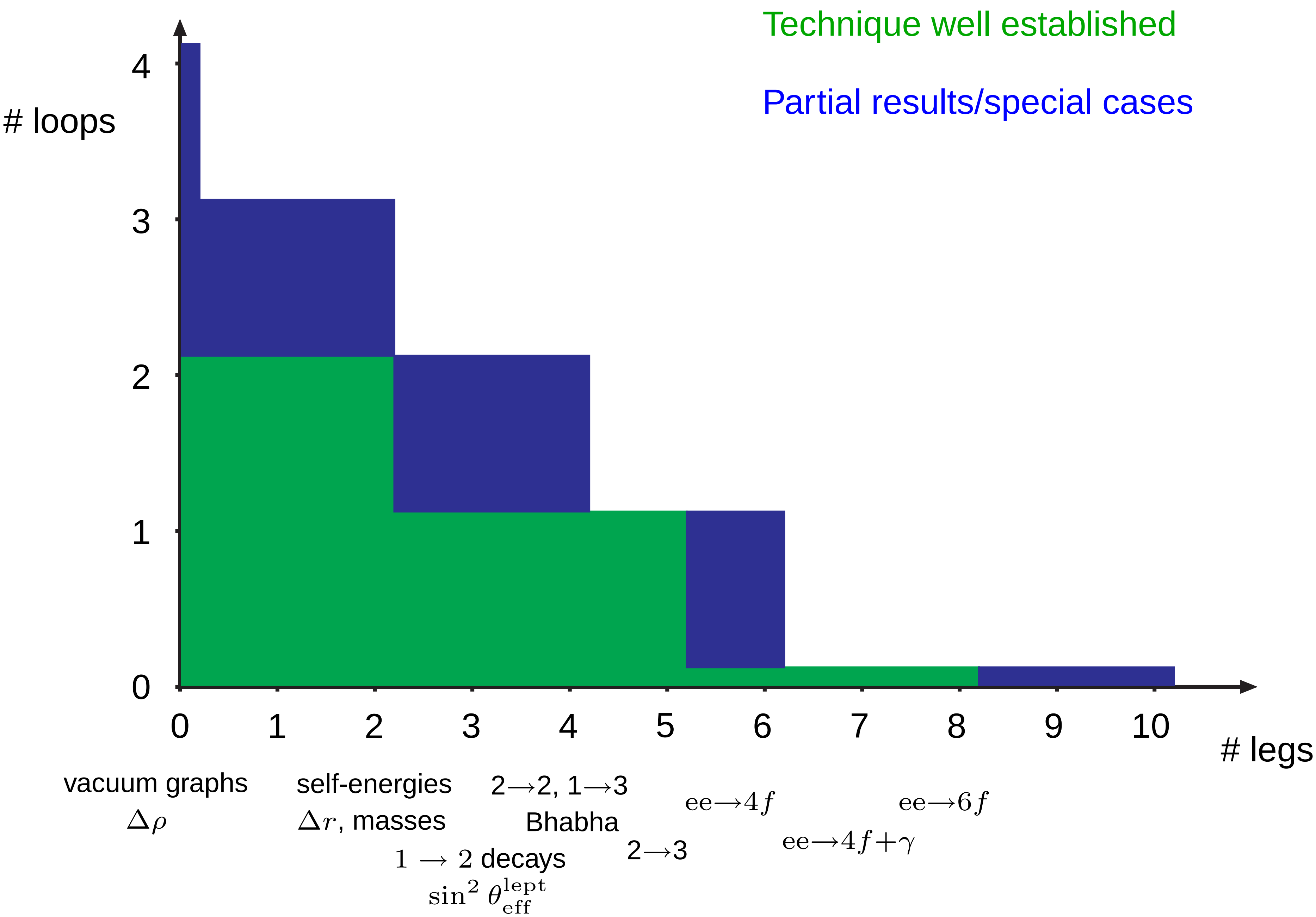
MPI Munich

[Loopverein conveners: S.Dittmaier and W.Hollik (MPI Munich)]

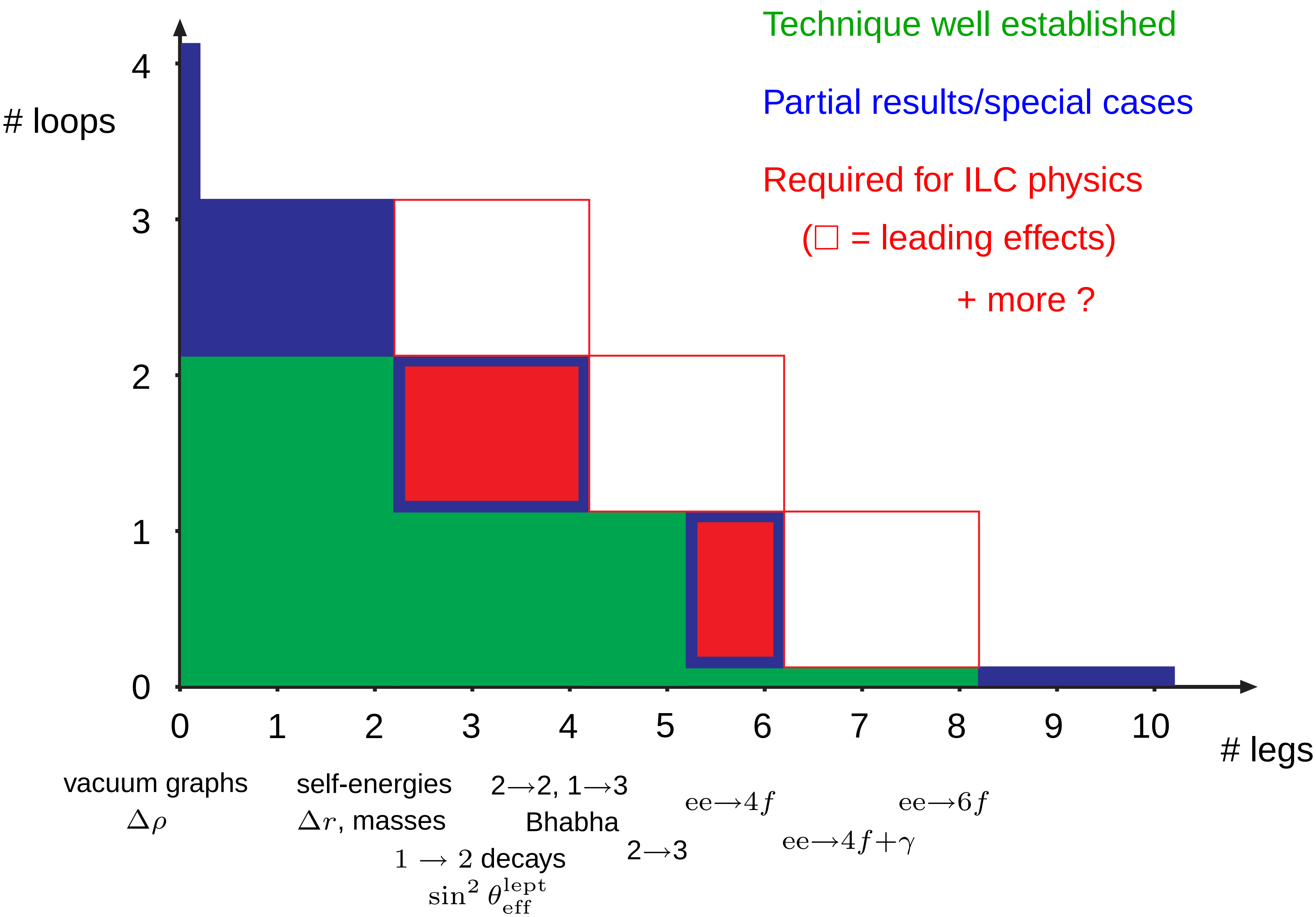
## State-of-the-art in precision calculations



## State-of-the-art in precision calculations



# State-of-the-art in precision calculations



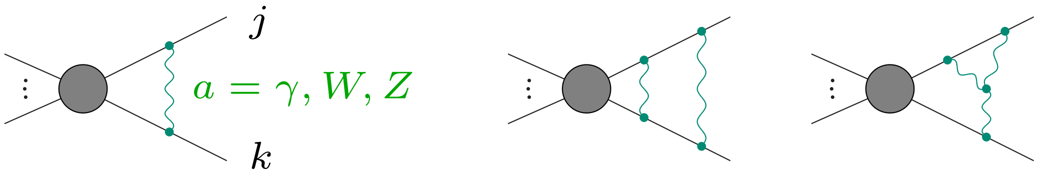
## Issues discussed in the Loopverein sessions:

- Electroweak (EW) corrections at high energies
  - ◇ Jantzen: EW high-energy log's (NLL) to $f\bar{f} \to f\bar{f}$
  - ◇ Penin: EW high-energy log's for $\mathrm{ee} \to \mathrm{WW}$ at 2 loops
- Explicit 2-loop calculations for specific processes / observables
  - ◇ Actis: Bhabha scattering (closed muon loops)
  - ◇ Schöfbeck: $m_{\tilde{\chi}_0^i}$ in NNLO SQCD
  - ◇ Uccirati: $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ (full EW at 2 loops)
  - ◇ Weinzierl: $A_{\mathrm{FB}}$ (QCD at 2 loops)
- Multi-loop calculations
  - ◇ Boughezal: $\Delta\rho$ in $\mathcal{O}(G_\mu m_\mathrm{t}^2 \alpha_\mathrm{s}^3)$
  - ◇ Steinhauser: $\mathrm{b} \to \mathrm{s}\gamma$ in NNLO QCD
  - ◇ Sturm: 4-loop results for $m_\mathrm{c}$, $m_\mathrm{b}$ determinations
- Technical progress for loop calculations
  - ◇ Mihaila: dimensional reduction in QCD at 3/4 loops
  - ◇ Rodrigo: proposal for numerical evaluation of 1-loop integrals

# Electroweak corrections at high energies

# Electroweak radiative corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange




etc.

+ sub-leading logarithms from collinear singularities

Typical impact on $2 \to 2$ reactions at $\sqrt{s} \sim 1\,\mathrm{TeV}$:

$$\delta_{\mathrm{LL}}^{1-\mathrm{loop}} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -26\%, \qquad \delta_{\mathrm{NLL}}^{1-\mathrm{loop}} \sim +\frac{3\alpha}{\pi s_{\mathrm{W}}^2} \ln\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 16\%$$

$$\delta_{\mathrm{LL}}^{2-\mathrm{loop}} \sim +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 3.5\%, \qquad \delta_{\mathrm{NLL}}^{2-\mathrm{loop}} \sim -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -4.2\%$$

$\Rightarrow$ Corrections still relevant at 2-loop level

Note: differences to QED / QCD where Sudakov log's cancel

- massive gauge bosons $\mathrm{W}$, $\mathrm{Z}$ can be reconstructed
  $\hookrightarrow$ no need to add “real $\mathrm{W}$, $\mathrm{Z}$ radiation”
- non-Abelian charges of $\mathrm{W}$, $\mathrm{Z}$ “open” $\rightarrow$ Bloch–Nordsieck theorem not applicable

# Two-loop electroweak NLL corrections

## Massless fermionic processes $f_1 f_2 \longrightarrow f_3 \cdots f_n$

with different $|(p_i + p_j)^2| \gg M_\mathrm{W}^2$ and different masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{top}^2 \sim M_\mathrm{Higgs}^2$:

- complete electroweak next-to-leading logarithmic (NLL) corrections in $D = 4 - 2\epsilon$ dimensions $\longrightarrow$ Denner, B.J., Pozzorini, **hep-ph/0608326**
- factorizable contributions calculated with 2 independent methods: 1.) sector decomposition, 2.) expansion by regions & Mellin–Barnes
- non-factorizable contributions shown to vanish due to collinear Ward identities
- result expressed by exponentiated 1-loop terms and $\beta$-function coefficients
- agreement with existing results

## Towards electroweak NLL corrections for arbitrary processes

- advantage: large parts of method are general
- extension to massive fermionic & arbitrary processes

  $\hookrightarrow$ work in progress . . .

## Talk of A.Penin

- 2-loop electroweak corrections to $\mathrm{e}^+\mathrm{e}^-\to\mathrm{W}^+\mathrm{W}^-$

  at high energies calculated to NNLL level: $\alpha^2 \ln^n\left(\frac{s}{M_\mathrm{W}^2}\right)$ with $n = 4, 3, 2$

- calculated corrections to $\sigma(\mathrm{e}^+\mathrm{e}^-\to\mathrm{W}^+\mathrm{W}^-)$
  needed at accuracy of $1-2\%$ for $\sqrt{s} > 500\,\mathrm{GeV}$

- work in progress / for the future:
  - ◇ Yukawa enhanced terms of order $\alpha^2\left(\frac{m_\mathrm{t}^2}{M_\mathrm{W}^2}\right)^n$ with $n = 2, 1$
  - ◇ log's of order $\alpha^2 \ln^1\left(\frac{s}{M_\mathrm{W}^2}\right)$
  - ◇ leading 2-loop corrections at small W scattering angles (Regge regime)

# Explicit 2-loop calculations

# for specific processes / observables

Talk of S.Actis

# Two-loop QED corrections to Bhabha scattering

- Small-angle Bhabha scattering $\Rightarrow$ luminosity monitor at ILC
- Available corrections:
  - Bonciani, Ferroglia *et al* $\Rightarrow$ non-approximated fermionic ($N_f = 1$)
  - Glover-Tausk-van der Bij $+$ Penin $\Rightarrow$ approximated photonic ($\mathcal{O}(m_e^2/s)$)
- Goal: independent cross-check $+$ improvement
  Actis, Czakon, Gluza, Riemann
  - fermionic corrections $\Rightarrow$ agreement with Bonciani, Ferroglia *et al*
    $+$ heavy-lepton (muons, taus) loops allowed
  - bosonic corrections $\Rightarrow$ testing ground for
    Mellin-Barnes method $+$ summation techniques

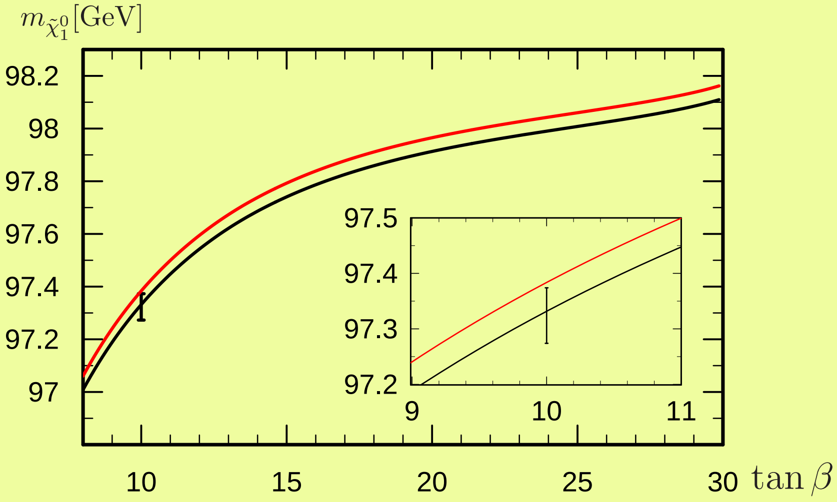
Talk of R.Schöfbeck

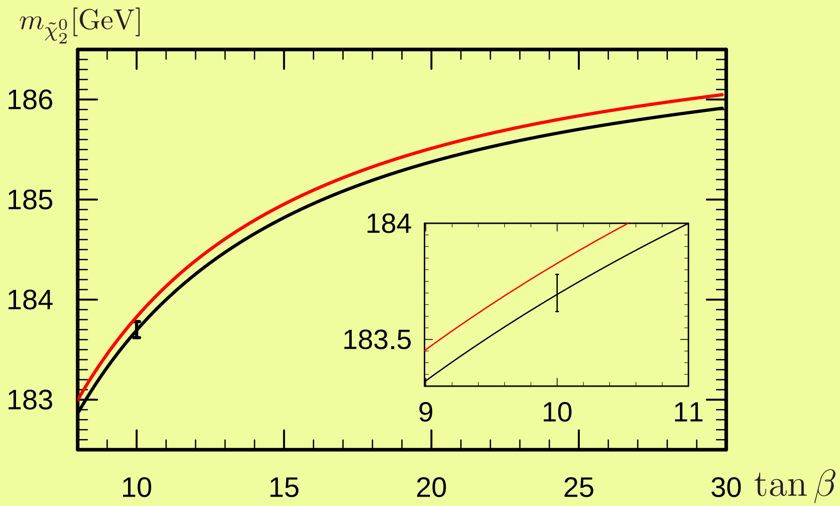
# NNLO SQCD corrections to the Neutralino pole masses in the MSSM

- motivated by the LHC/LC Study Group
  $\Delta m_{\tilde{\chi}_1^0} = 0.05$GeV (experimental)
- pole mass calculation in $\overline{DR}\,'$-scheme
- auto-generated code:
  FeynArts→FeynCalc→Tarcer
  Fortran, TSIL, LoopTools

Results:

- SPS1a': NNLO must be included
- ren. scale dep: greatly improved
- two-loop Yukawa corrections
  need to be included

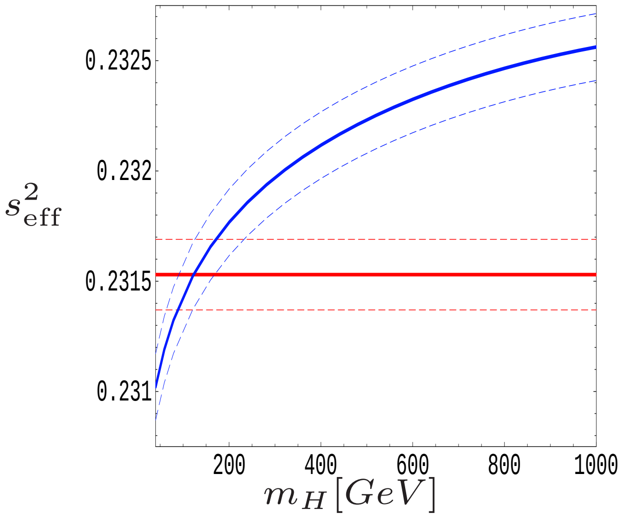


# Two-loop corrections to the effective electroweak mixing angle

$$\sin^2\theta^{\rm lept}_{\rm eff} = \frac{1}{4}\left[1 - {\rm Re}\left(\frac{g_v}{g_a}\right)\right] = \left(1 - \frac{m_W^2}{m_Z^2}\right)\kappa \qquad s = m_Z^2$$

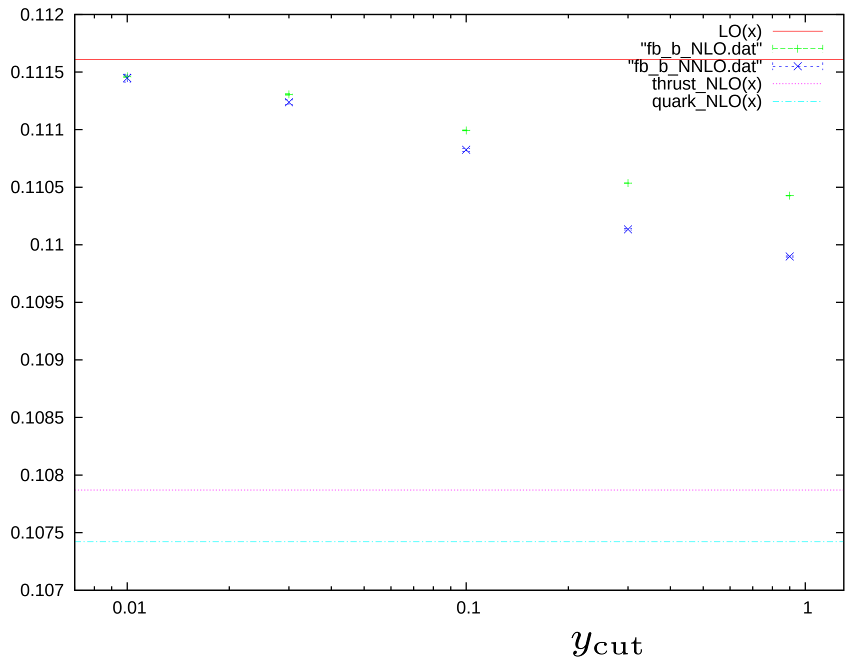
| $m_H[GeV]$ | $\Delta\kappa^{(1)}[\times 10^{-4}]$ | $\Delta\kappa^{(2)}_{fer}[\times 10^{-4}]$ | $\Delta\kappa^{(2)}_{bos}[\times 10^{-4}]$ |
|---|---|---|---|
| 100 | 413.325 | 1.07 | -0.74 |
| 200 | 394.023 | -0.33 | -0.47 |
| 600 | 354.060 | -2.89 | 0.18 |
| 1000 | 333.159 | -2.61 | 1.11 |




- Two-loop corrections → **completed**
- Fermionic-bosonic cancellation for $\Delta\kappa^{(2)}$
- First application of new general methods to compute Feynman diagrams

# The forward-backward asymmetry

- The forward-backward asymmetry shows the largest discrepancy in a fit of the Standard Model parameter.
- Experimental analysis based on an infrared-unsafe definition.
- Infrared-safe definition of the forward-backward asymmetry.
- Calculation of the NLO and NNLO QCD corrections.
- The corrections are small, useful observable also for a future linear collider.

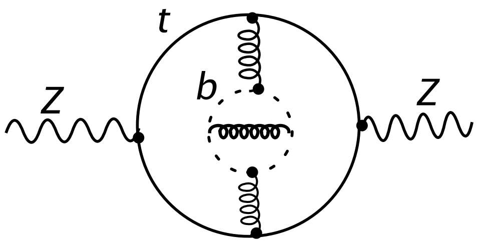


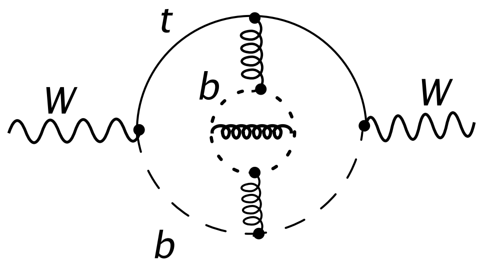

Talk of S.Weinzierl

# Multi-loop calculations

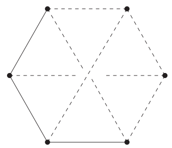
# Four-loop tadpoles and $\mathcal{O}(G_F m_t^2 \alpha_s^3)$ corrections to $\rho$

## Four-loop non-singlet contribution to $\Delta\rho$ using a large $M_t$




new set of high precision numerical values of $4$-loop single scale tadpoles



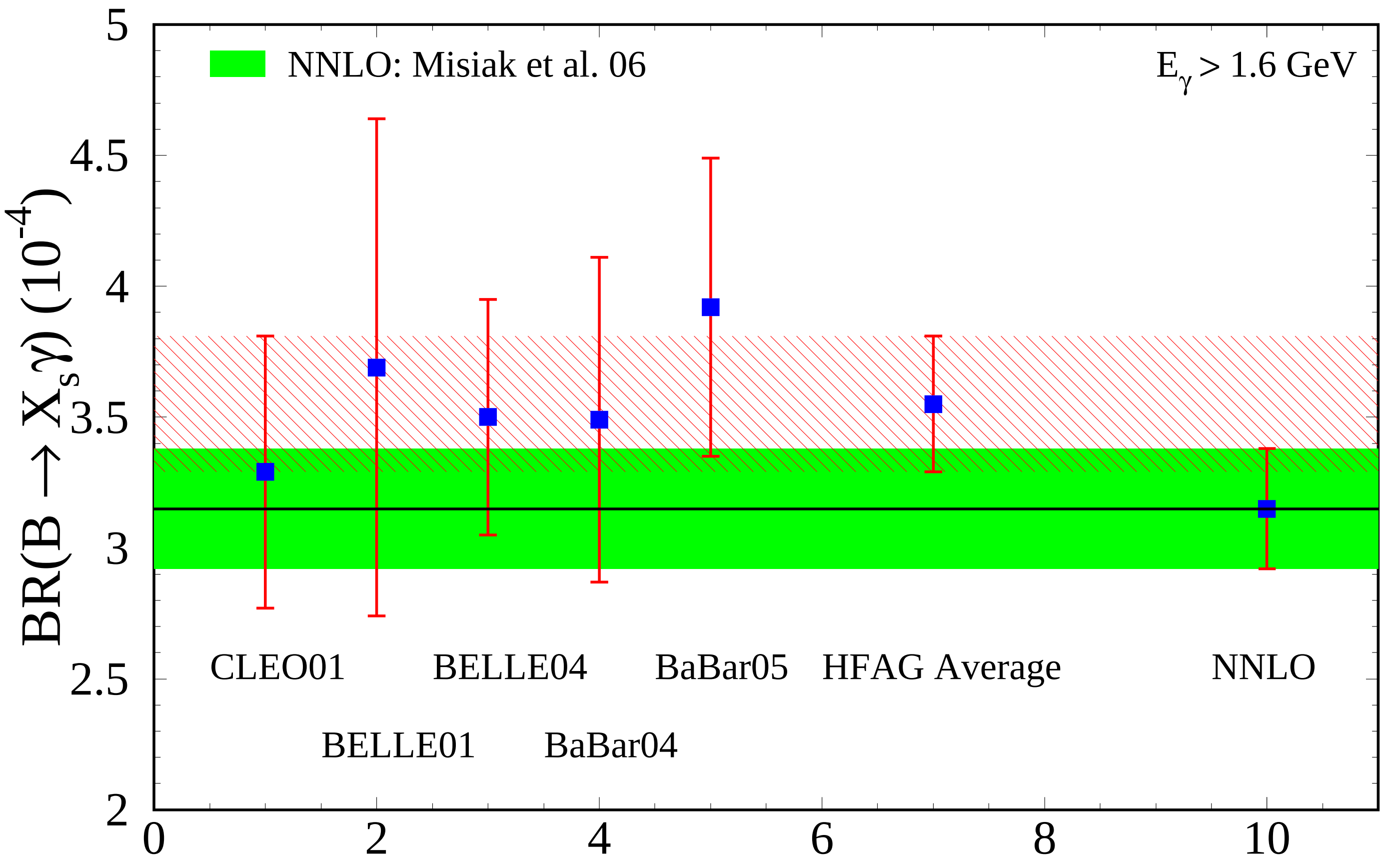
$$= -6.7284705600856810554718897752 1 - 26.087646599966615538965977071 7\,\varepsilon - 214.6477179124113620280527270 52\,\varepsilon^2 - 613.71520309662607565487490883 8\,\varepsilon^3$$

$$\Delta\rho^{\rm OS} = 3\left(\frac{G_F M_t^2}{8\sqrt{2}\pi^2}\right)\Bigg[1 - 2.8599\left(\frac{\alpha_s}{\pi}\right) + (\underbrace{-4.2072}_{\textbf{singlet}}\ \underbrace{-10.387}_{\textbf{non-singlet}})\left(\frac{\alpha_s}{\pi}\right)^2 + (7.9326 - 101.0827)\left(\frac{\alpha_s}{\pi}\right)^3\Bigg],$$

Talk of R.Boughezal

R.Boughezal Valencia November 2006 – p.1/1

# $\mathcal{B}(\bar{B} \to X_s\gamma)|_{E_\gamma > 1.6\text{GeV}}$ **to NNLO**




$$\mathcal{B}(\bar{B} \to X_s\gamma)|_{E_\gamma > 1.6\text{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$

[Misiak et al.'06], [Misiak,Steinhauser'06]

Universität Karlsruhe (TH)
Forschungsuniversität • gegründet 1825

Karlsruhe Institute of Technology

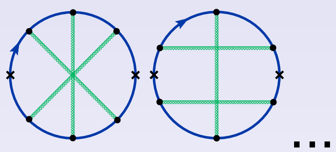
Talk of M.Steinhauser

Matthias Steinhauser, $\bar{B} \to X_s\gamma$ to NNLO – p.16

# 4-loop calculations for a precise charm- and bottom-quark mass determination

K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, C. Sturm

- Method: Determination of charm- and bottom-quark mass with the help of sum rules
- Experimental input: measurable R-ratio
- Theoretical input: Low energy expansion of vacuum polarization function $\Pi(q^2)$
- Expansion of $\Pi(q^2)$ leads to tadpole diagrams




- Solution of problem with Laporta's algorithm, mapping all integrals on a small set of master integrals
- First two expansion coefficients of $\Pi(q^2)$ have been calculated in 4-loop order in pQCD K.G. Chetyrkin, J. H. Kühn, C.Sturm, confirmed by R. Boughezal, M. Czakon,T. Schutzmeier
- Recent improved experimental data + new multi-loop result from pQCD lead to significantly reduced errors
- Preliminary result for $m_c(m_c) = 1290 \pm 13$ MeV ($N^3LO$ analysis)

Talk of C.Sturm

# Technical progress for loop calculations

## Talk of L.Mihaila: **DRED applied to QCD**

- 4-loop QCD $\beta$-function and mass anomalous dimension $\gamma_m$ computed within DRED
  - ◇ explicit calculation of $\beta$ and $\gamma_m$ at 3-loop order
  - ◇ 4-loop relation between DRED and DREG established
- Equivalence of DRED and DREG at 3-loop order
- SUSY-YM: supersymmetry preserved through 3-loop order
- Careful treatment of the evanescent couplings important in phenomenological applications

Talk of G.Rodrigo

The aim: numerical evaluation of any 1-loop diagram

The (not so new) idea: integration directly in momentum space

$$\int \mathrm{d}^4 q = \underbrace{\int \mathrm{d}^3 \mathbf{q}} \; \underbrace{\int \mathrm{d}q_0}$$

analytically via Cauchy's theorem $\Rightarrow$ loop cut into trees!

$\downarrow$

numerically via Monte Carlo methods
(similar to phase-space integrals)

Subtleties / bottlenecks::

- separation of IR (soft and collinear) divergences before MC integration
- numerical performance in “real life” not yet known

Current status / work in progress:

- done: scalar integrals for massless particles
- in progress: massive particles, tensor integrals
- future work: application to cross sections at NLO, automatization

# Conclusion

## Higher-order calculations

- are indispensible for almost all kinds of measurements
- are difficult, long(er and longer)-termed projects
- are being done

$\hookrightarrow$ please give young people needed credit + support !