Implementation of Higher Order Mode Wakefields in MERLIN

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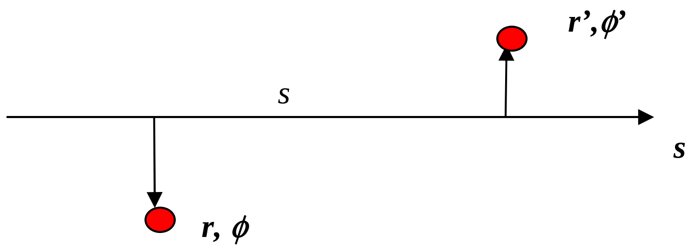
Introduction

- Extensive literature for wakefield effects and many computer codes for their calculations
  - concentrates on wake effects in RF cavities (axial symmetry)
  - only lower order modes are important
  - only long-range wakefields are considered
- For collimators:
  - particle bunches distorted from their Gaussian shape
  - short-range wakefields are important
  - higher order modes must be considered (particle close to the collimator edges)

Wake Effects from a Single Charge

- Investigate the effect of a leading unit charge on a trailing unit charge separated by distance s



- the change in momentum of the trailing particle is a vector *w* called 'wake potential'
- w is the gradient of the 'scalar wake potential': $w = \nabla W$
- W is a solution of the 2-D Laplace Equation where the coordinates refer to the trailing particle; W can be expanded as a Fourier series:

$$W(r, \theta, r', s) = \sum W_m(s)\, r'^m\, r^m \cos(m\theta) \quad (W_m \text{ is the 'wake function'})$$

- the transverse and longitudinal wake potentials w_L and w_T can be obtained from this equation

The Effect of a Slice

- the effect on a trailing particle of a bunch slice of N particles all ahead by the same distance s is given by simple summation over all particles in the slice

- if we write: $C^m = \Sigma\, r'^m \cos(m\theta')$ and $S^m = \Sigma\, r'^m \sin(m\theta')$ the combined kick is:

$$w_z = \Sigma\; W'_m(s)\, r^m\, [\, C^m \cos(m\theta) - S^m \sin(m\theta)]$$

$$w_x = \Sigma\, m\, W_m(s)\, r^{m-1}\, \{C^m \cos[(m-1)\theta] + S^m \sin[(m-1)\theta]\}$$

$$w_y = \Sigma\, m\, W_m(s)\, r^{m-1}\, \{S^m \cos[(m-1)\theta] - C^m \sin[(m-1)\theta]\}$$

- for a particle in slice *i*, a wakefield effect is received for all slices $j \geq i$:

$$\Sigma_j\, w_x = \Sigma_m\, m\, r^{m-1}\, \{ \cos[\,(m-1)\theta\,]\; \Sigma_j W_m(s_j)\, C_{mj} + \sin[\,(m-1)\theta\,]\; \Sigma_j W_m(s_j)\, S_{mj} \}$$

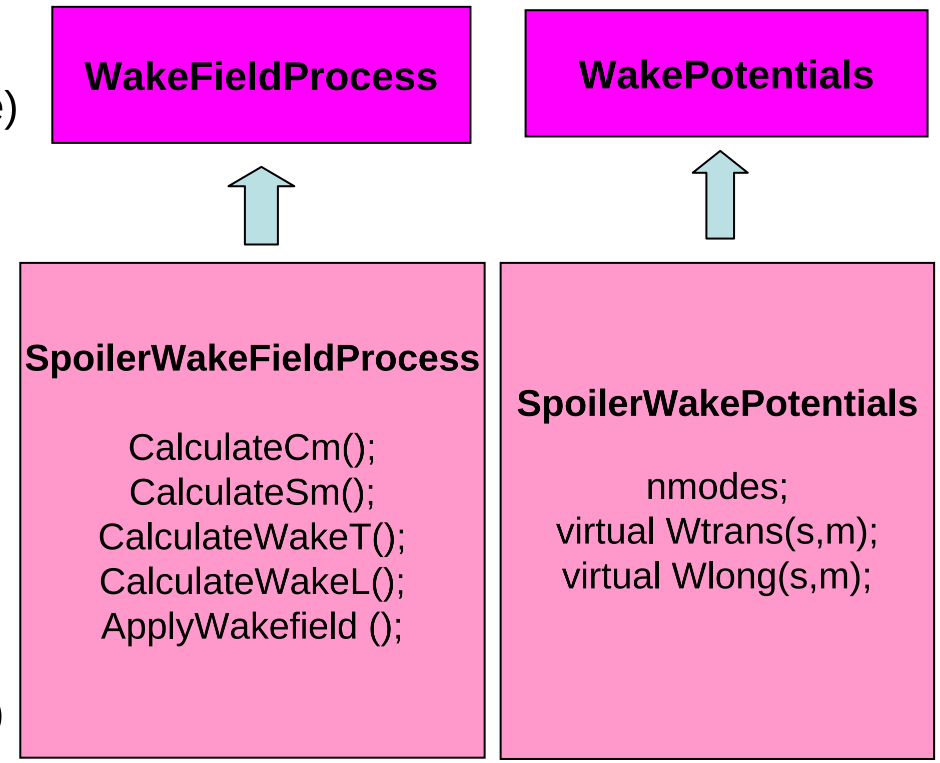
Changes to MERLIN

Previously in Merlin:

- Two base classes: WakeFieldProcess and WakePotentials
  - transverse wakefields (only dipole mode)
  - longitudinal wakefields

Changes to Merlin

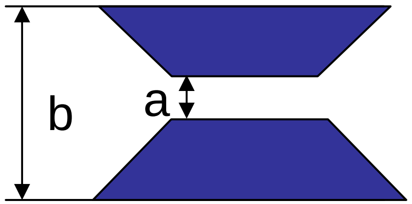
- Some functions made virtual in the base classes
- Two derived classes:
  - SpoilerWakeFieldProcess - does the summations
  - SpoilerWakePotentials - provides prototypes for W(m,s) functions (virtual)
- The actual form of W(m,s) for a collimator type is provided in a class derived from SpoilerWakePotentials



Example

Tapered collimator in

the diffractive regime:



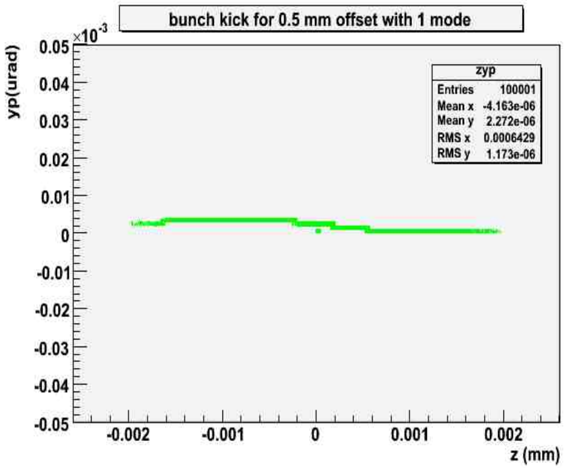
$$W_m(z) = 2\ (1/a^{2m} - 1/b^{2m})\ \exp\ (-mz/a)\ \Theta(z)$$

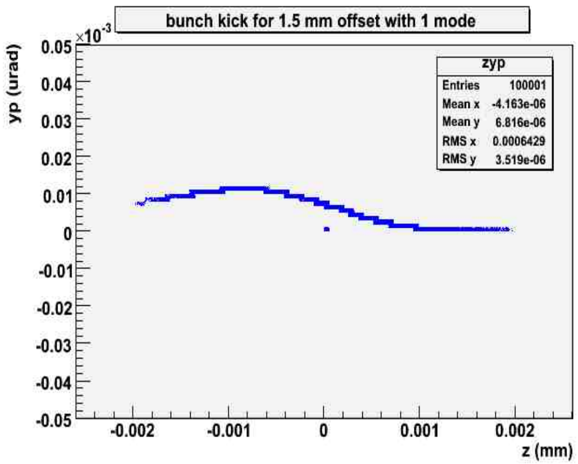
```
Class  TaperedCollimatorPotentials:  public   SpoilerWakePotentials
{   public:
    double a, b;
    double* coeff;
    TaperedCollimatorPotentials (int m, double rada, double radb) : SpoilerWakePotentials (m, 0. , 0. )
{         a = rada;
          b = radb;
          coeff = new double [m];
           for (int i=0; i<m; i++)
           {coeff [i] = 2*(1./pow(a, 2*i) - 1./pow(b, 2*i));}  }
     ~TaperedCollimatorPotentials(){delete [ ] coeff;}
      double Wlong (double z, int m) const {return z>0 ? -(m/a)*coeff [m]/exp (m*z/a) :  0 ;} ;
      double Wtrans (double z, int m) const { return z>0 ? coeff[m] / exp(m*z/a) : 0 ; } ;       };
```

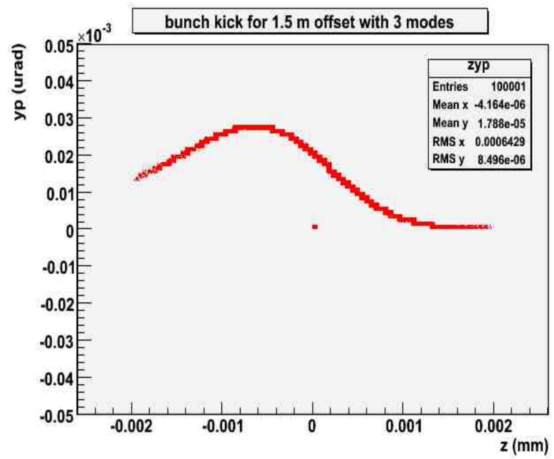
Simulations

SLAC beam tests simulated: energy - 1.19 GeV, bunch charge - $2*10^{10}$ e^-

Collimator half -width: 1.9 mm







- small displacement - 0.5 mm
- one mode considered
- effect is small
- adding m=2,3 etc does not change much the result

- large displacement - 1.5 mm
- one mode considered
- the bunch tail gets a bigger kick

- large displacement - 1.5 mm
- higher order modes considered (ie. m=3)
- the effect on the bunch tail is significant

Application to the ILC - BDS collimators

- beam is sent through the BDS off-axis (beam offset applied at the end of the linac)

- parameters at the end of linac:

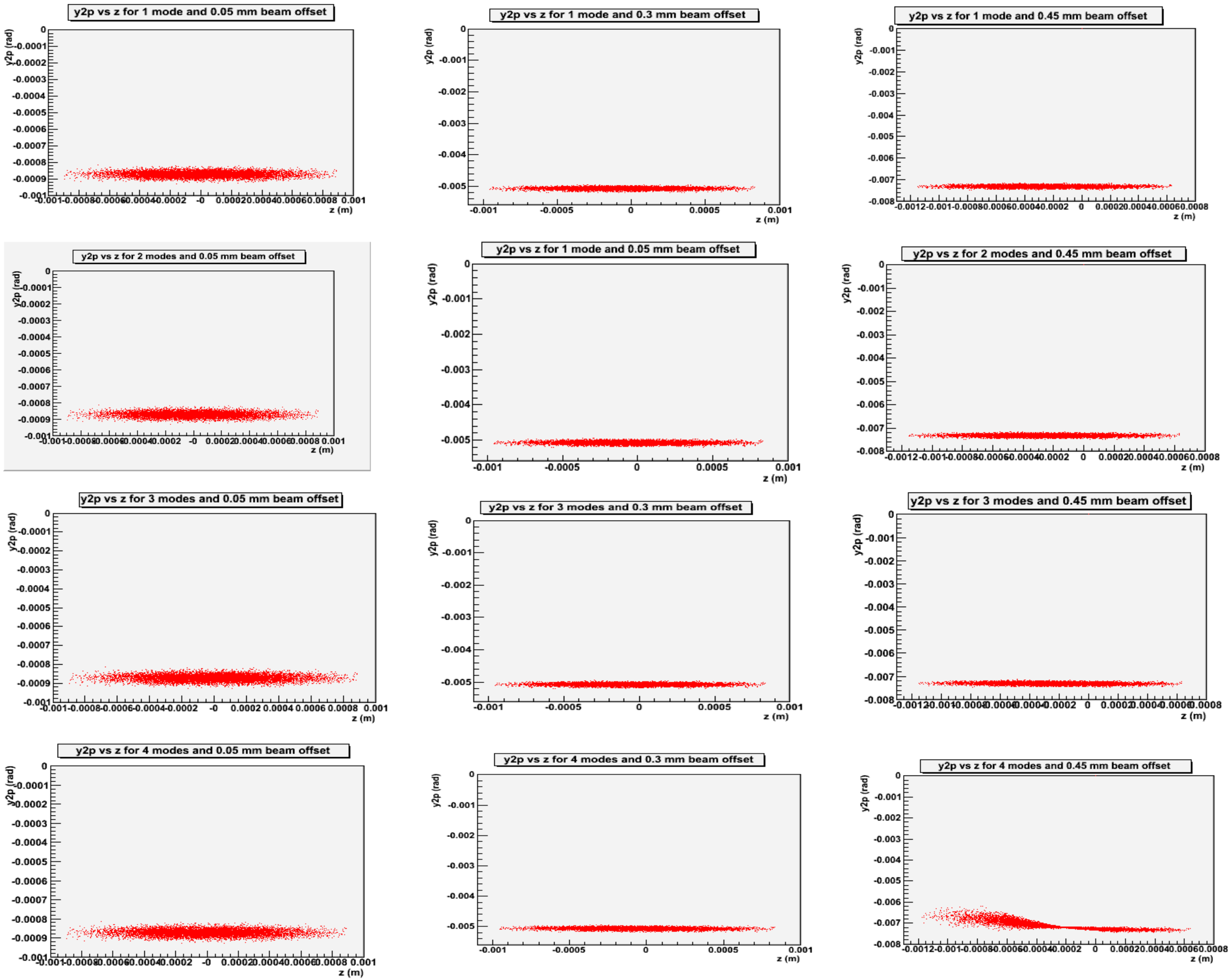
β_x=45.89 m	ε_x=2 10^{-11}	σ_x = 30.4 10^{-6} m
β_y =10.71 m	ε_y =8.18 10^{-14}	σ_y = 0.9 10^{-6} m

-interested in variation in beam sizes at the IP and in bunch shape due to wakefields

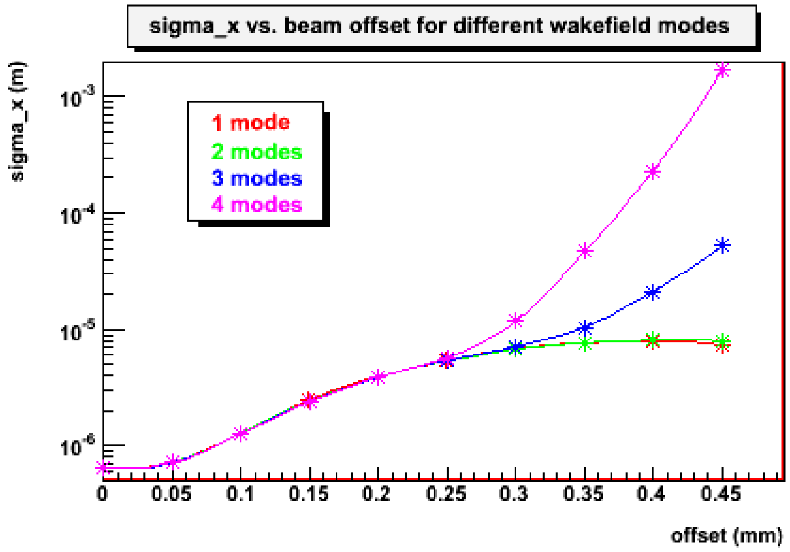
ILC-BDS colimators

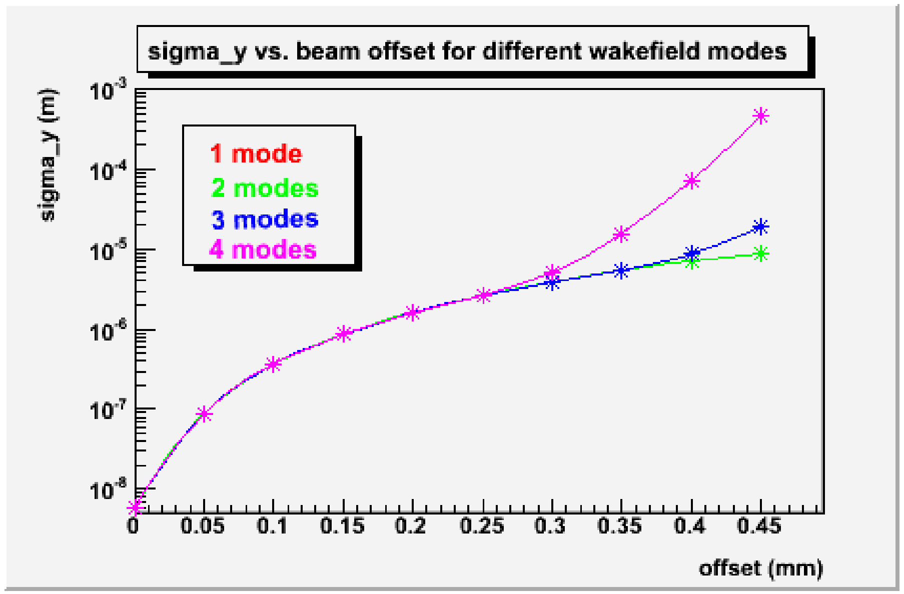
No	Name	Type	Z (m)	Aperture
1	CEBSY1	Ecollimator	37.26	~
2	CEBSY2	Ecollimator	56.06	~
3	CEBSY3	Ecollimator	75.86	~
4	CEBSYE	Rcollimator	431.41	~
5	SP1	Rcollimator	1066.61	x99y99
6	AB2	Rcollimator	1165.65	x4y4
7	SP2	Rcollimator	1165.66	x1.8y1.0
8	PC1	Ecollimator	1229.52	x6y6
9	AB3	Rcollimator	1264.28	x4y4
10	SP3	Rcollimator	1264.29	x99y99
11	PC2	Ecollimator	1295.61	x6y6
12	PC3	Ecollimator	1351.73	x6y6
13	AB4	Rcollimator	1362.90	x4y4
14	SP4	Rcollimator	1362.91	x1.4y1.0
15	PC4	Ecollimator	1370.64	x6y6
16	PC5	Ecollimator	1407.90	x6y6
17	AB5	Rcollimator	1449.83	x4y4

No	Name	Type	Z (m)	Aperture
18	SP5	Rcollimator	1449.84	x99y99
19	PC6	Ecollimator	1491.52	x6y6
20	PDUMP	Ecollimator	1530.72	x4y4
21	PC7	Ecollimator	1641.42	x120y10
22	SPEX	Rcollimator	1658.54	x2.0y1.6
23	PC8	Ecollimator	1673.22	x6y6
24	PC9	Ecollimator	1724.92	x6y6
25	PC10	Ecollimator	1774.12	x6y6
26	ABE	Ecollimator	1823.21	x4y4
27	PC11	Ecollimator	1862.52	x6y6
28	AB10	Rcollimator	2105.21	x14y14
29	AB9	Rcollimator	2125.91	x20y9
30	AB7	Rcollimator	2199.91	x8.8y3.2
31	MSK1	Rcollimator	2599.22	x15.6y8.0
32	MSKCRAB	Ecollimator	2633.52	x21y21
33	MSK2	Rcollimator	2637.76	x14.8y9


y2p vs z for 1 mode and 0.05 mm beam offset
y2p vs z for 1 mode and 0.3 mm beam offset
y2p vs z for 1 mode and 0.45 mm beam offset
y2p vs z for 2 modes and 0.05 mm beam offset
y2p vs z for 1 mode and 0.05 mm beam offset
y2p vs z for 2 modes and 0.45 mm beam offset
y2p vs z for 3 modes and 0.05 mm beam offset
y2p vs z for 3 modes and 0.3 mm beam offset
y2p vs z for 3 modes and 0.45 mm beam offset
y2p vs z for 4 modes and 0.05 mm beam offset
y2p vs z for 4 modes and 0.3 mm beam offset
y2p vs z for 4 modes and 0.45 mm beam offset
y2p (rad)
z (m)

Emittance dilution due to wakefield





- beam size at the IP in absence of wakefields:

  $\sigma_x = 6.51*10^{-7}$ m

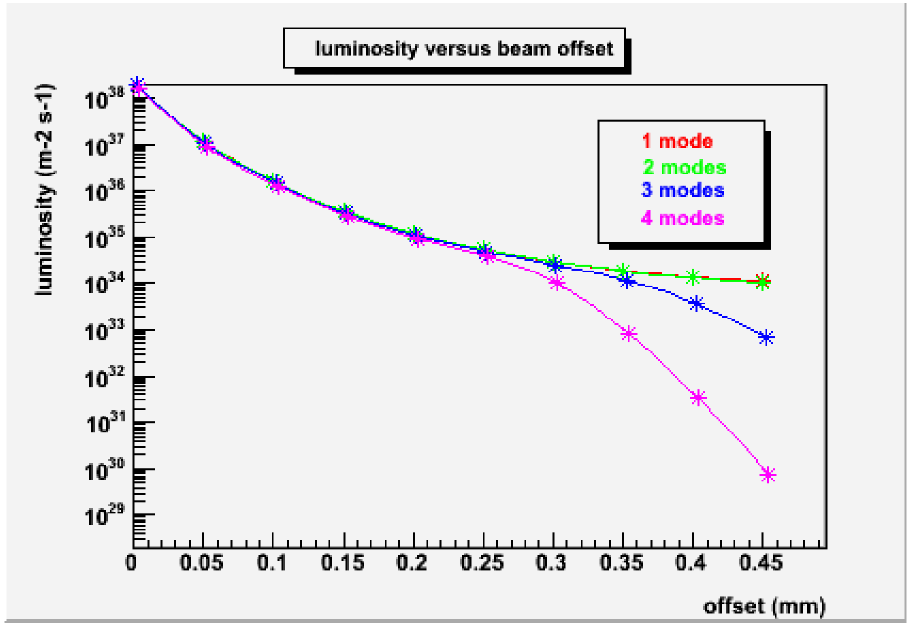
  $\sigma_y = 5.69*10^{-9}$ m

- wakefields switched on -> an increase in the beamsize
- higher order modes are not an issue when the beam offset in increased up to 0.25 mm
- from 0.3 mm beam offset, higher order modes become important
- beam size for an offset of 0.45 mm:

  $\sigma_x = 1.70*10^{-3}$ m

  $\sigma_y = 4.77*10^{-4}$ m

Luminosity loss due to wakefields



- luminosity in absence of wakefields:

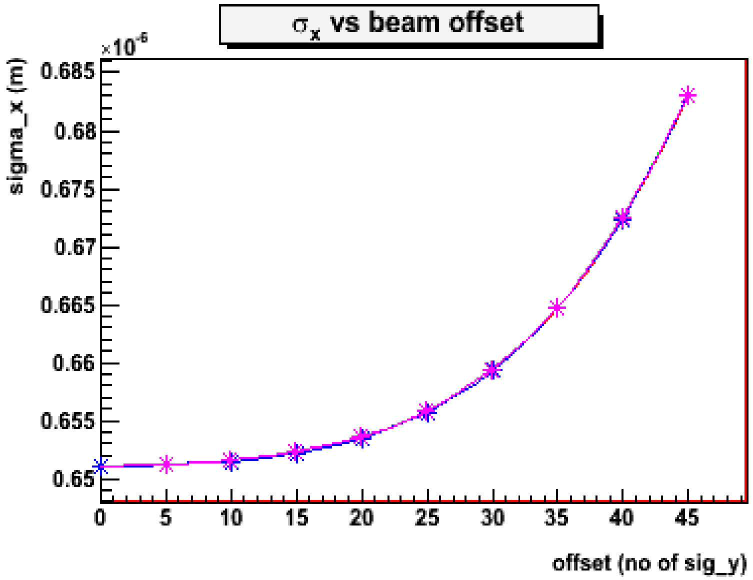
  $L = 2.03 \cdot 10^{38}\ m^{-2}\ s^{-1}$

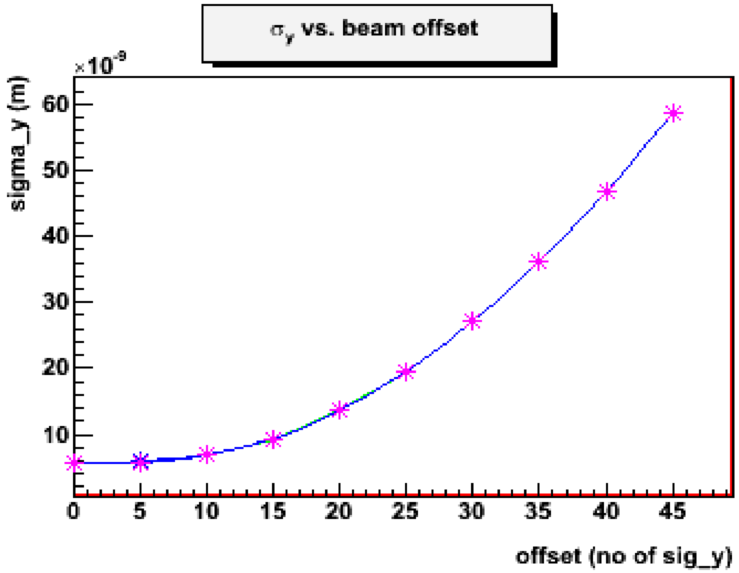
- at 0.25 mm offset: $L \sim 10^{34}$
- at 0.45 mm offset: $L \sim 10^{29}$

-> Catastrophic!

How far from the axis can be the beam to avoid a drop in the luminosity from $L \sim 10^{38}$ to $L \sim 10^{37}\ m^{-2}\ s^{-1}$?

Emittance dilution for very small offsets



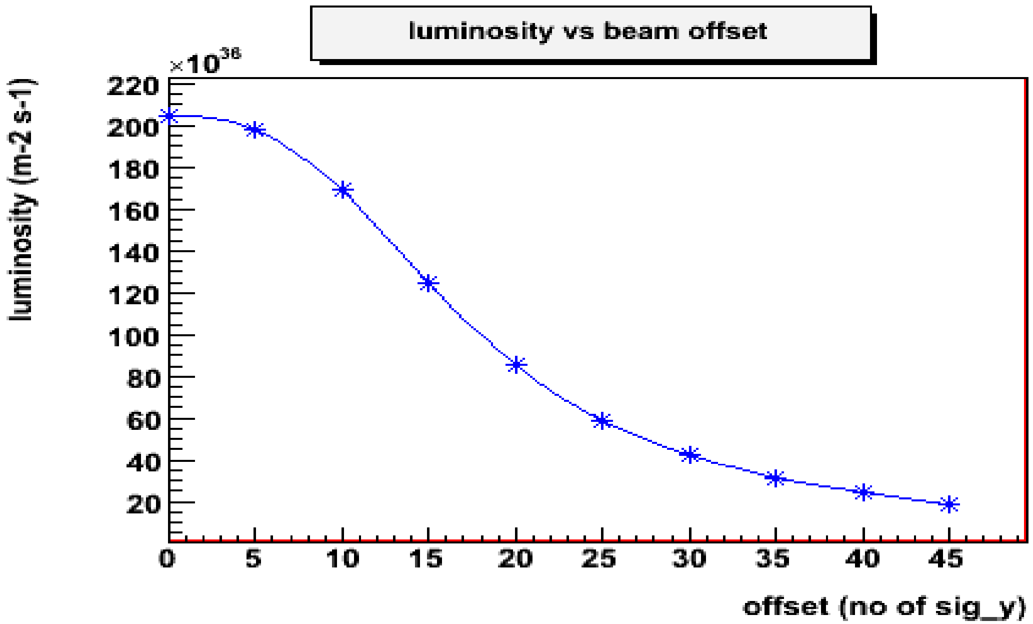


Luminosity

- Luminosity is stable ($L \sim 10^{38}$) for beam offsets up to 16 sigmas

- At beam offsets of 45 sigmas (approx. 40 um) luminosity drops from $L \sim 10^{38}$ to $L \sim 10^{37}$

-> contribution from higher order modes is very small when beam is close to the axis

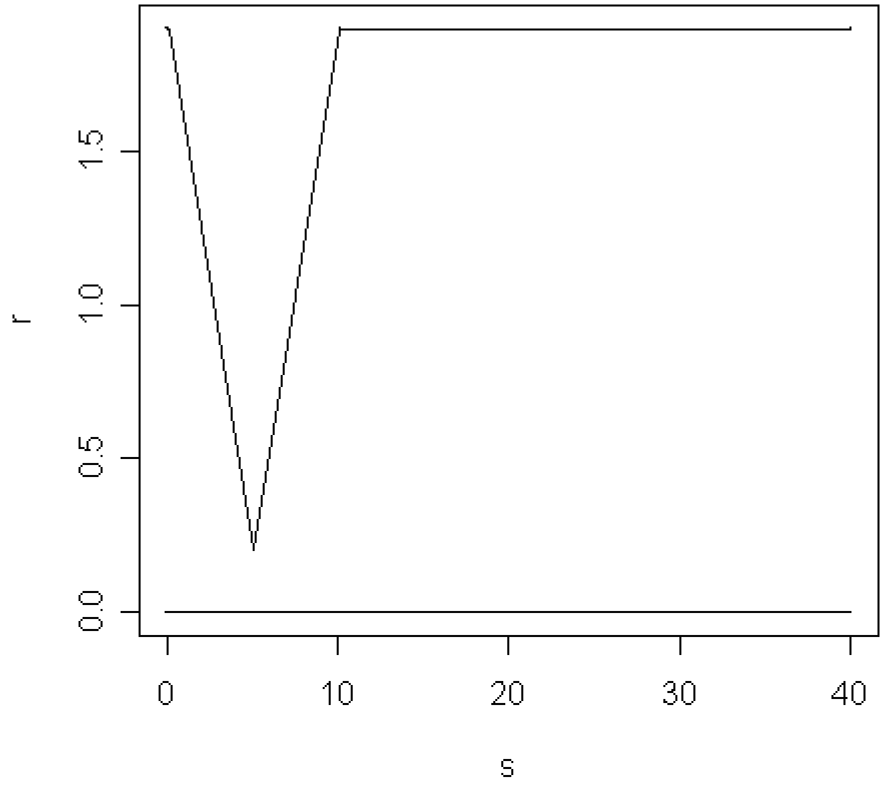


Extracting Delta Wakes from EM simulations

- Problem: how to extract delta wakes used by Merlin, Placet, etc. from bunch wakes available from EM simulations
- Wake functions $W_m(s)$ depend on component. Give variation with longitudinal co-ordinate $s=z_1-z_2$. (Variation with transverse coordinates specified by axial symmetry and Maxwell's equations)
- Analytic formulae available but only for some shapes and with arguable regions of validity
- EM simulators (ECHO, GDFIDL, HFSS etc) give wake functions due to bunches with some finite σ
- Taking limit of small σ needs small mesh size and computing time explodes

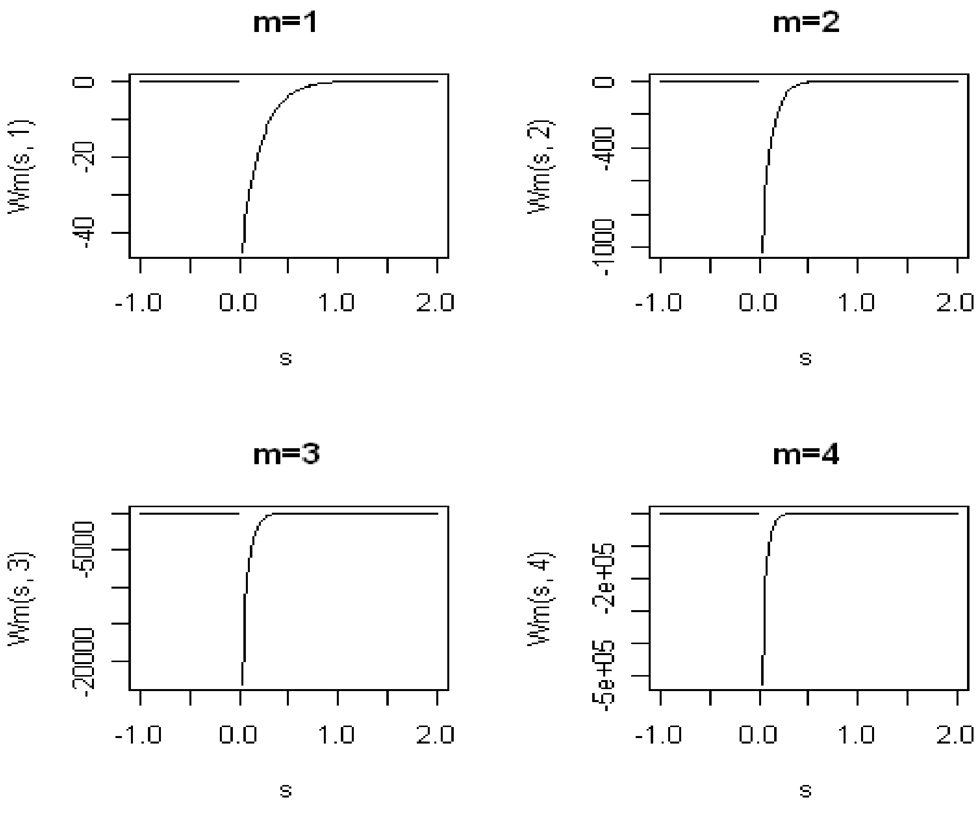
Tapered collimator

- Radius a=0.2 cm
- Beam pipe b=1.9 cm
- 10 cm long



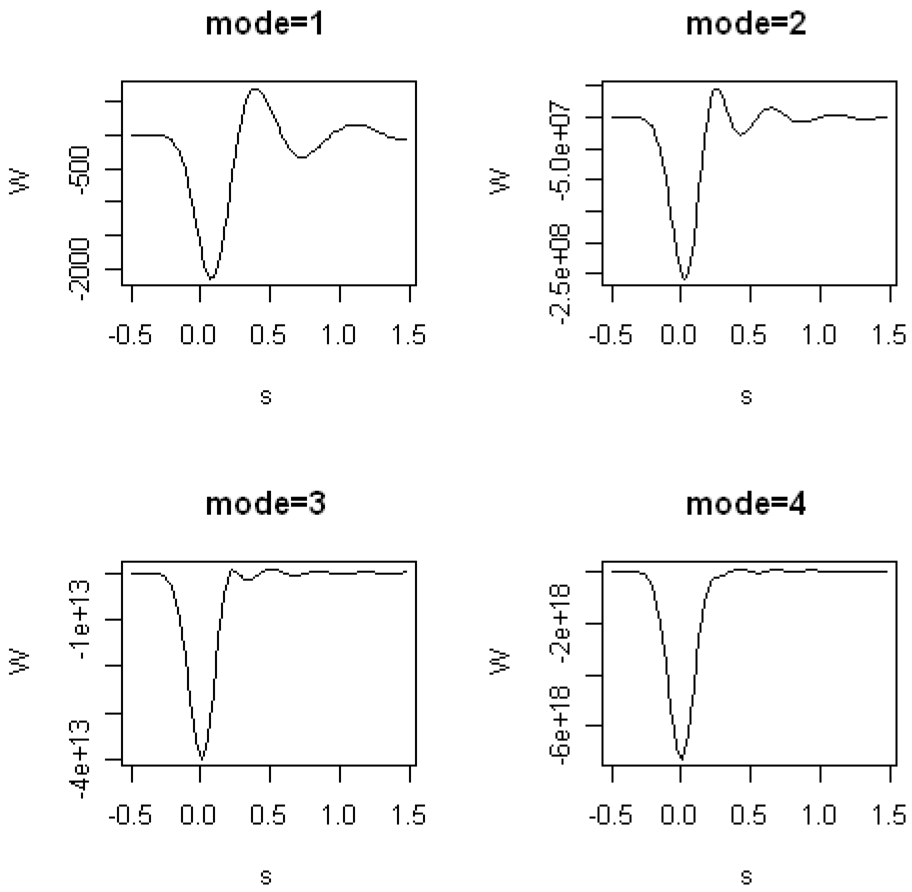
Analytic formulae

$W_m(s)=2(1/a^{2m}-1/b^{2m})\exp(-ms/a)$ (Zotter & Kheifets)



EM simulation

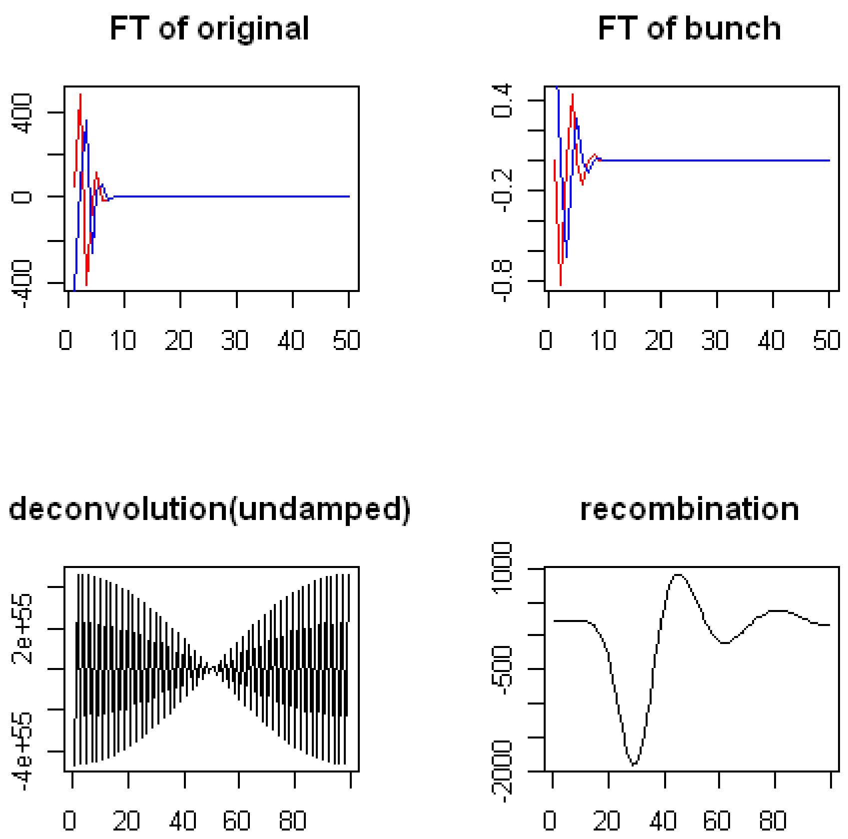
- Simulated using Echo-2D (Igor Zagorodnov)
- Gaussian beam, s=0.1 cm



Fourier Deconvolution

$$W_{bunch}(s,m)=W_{delta}(s,m) \otimes Gaussian$$

- Take FT of ECHO result and FT of Gaussian
- Divide to obtain FT of delta wake
- Back-transform.
- Horrible! But mathematically correct
- Due to noise in spectra. Well known problem



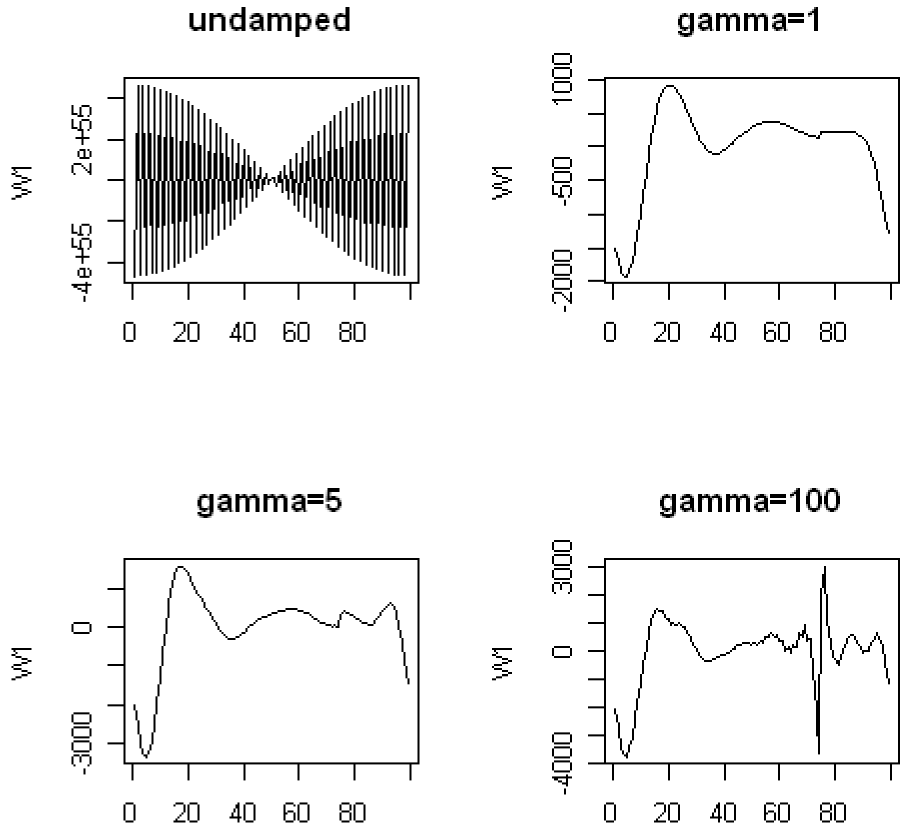
Try simple Inverse Filter

Cap factor

1./FTdenom(k)

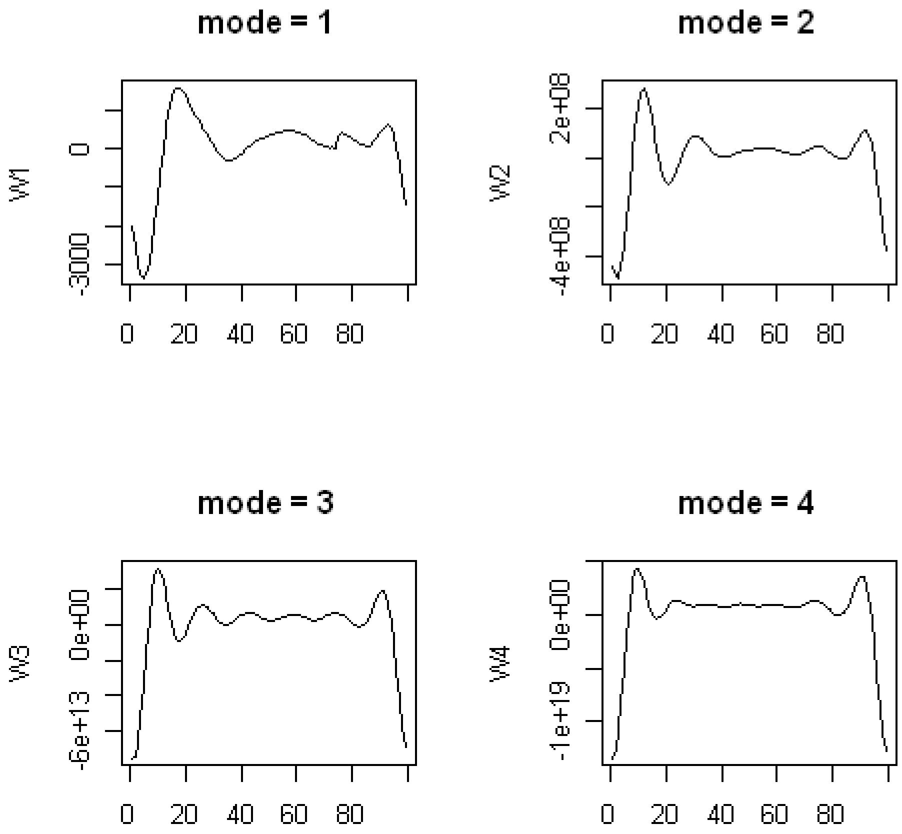
at value gamma

gamma=5 seems reasonable



Reconstructed delta wakes

- Compare with analytic formula
- Qualitative agreement on increase in size and decrease in width for higher modes
- Positive excursions not reproduced by formula
- Still problems with deconvolution: hard to synthesise necessary step function when higher modes damped



Next steps

- Use more sophisticated filter, incorporating causality ($W(s)=0$ for $s<0$)
- Compare simulations and formulae and establish conditions for validity
- Delta wakes extracted from simulations usable in Merlin (numerical tables) for collimators where analytical formulae not known
- Extend to non-axial collimators.