Positron Capture Simulations:

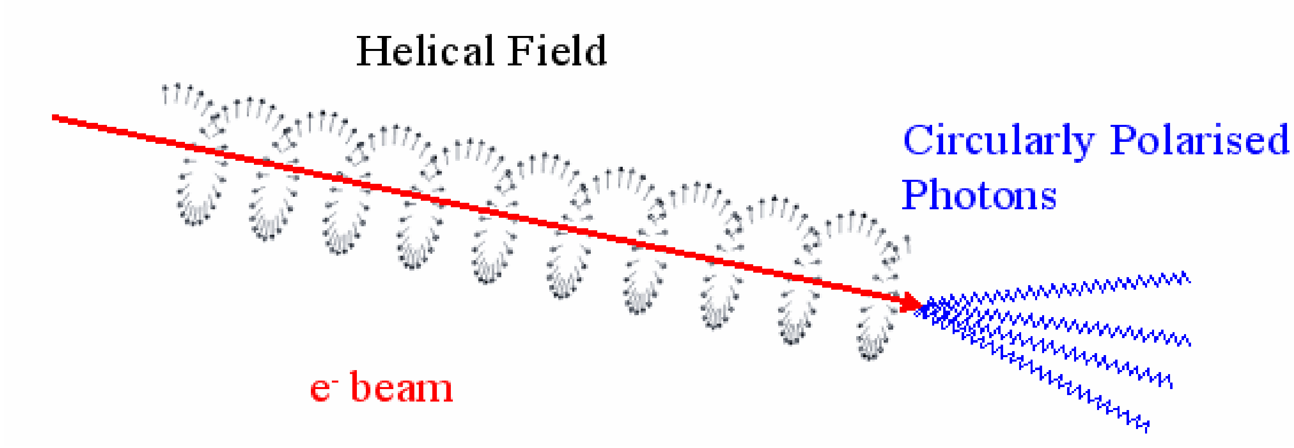
Runge-Kutta vs Boris

Leo Jenner, Daresbury Labs

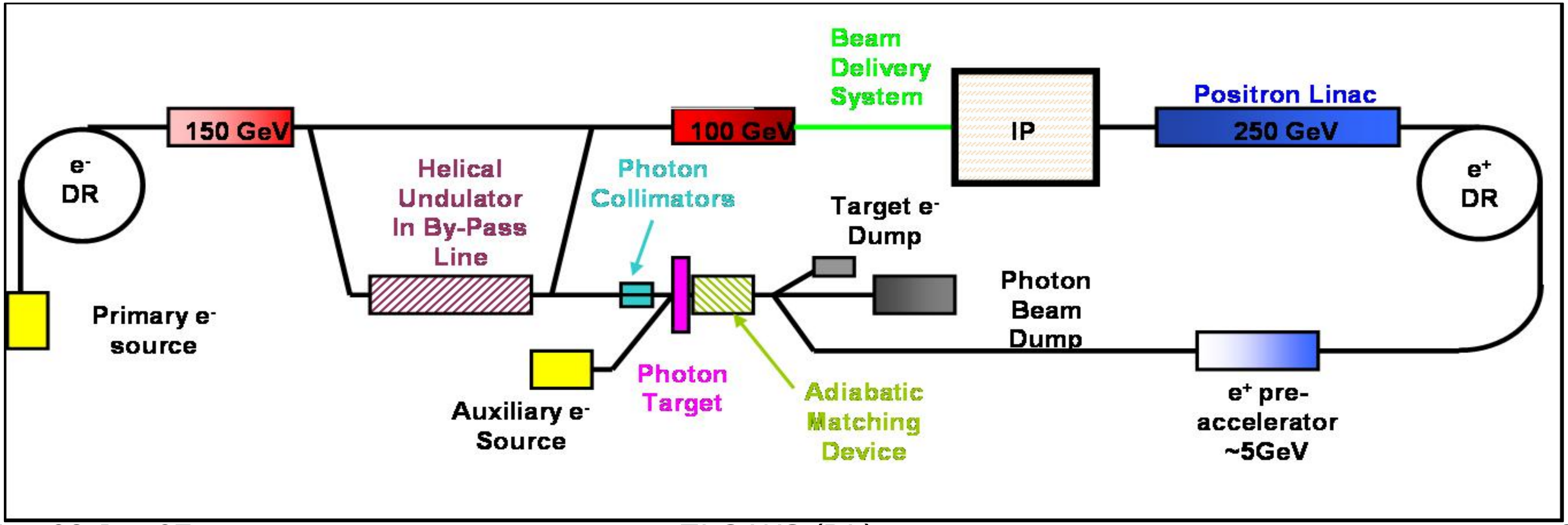
Outline…

- Positron capture optics
- What is Astra?
- Spin tracking…
- T-BMT equation
- Runge-Kutta integration method
- Boris' method

ILC Positron Source Layout

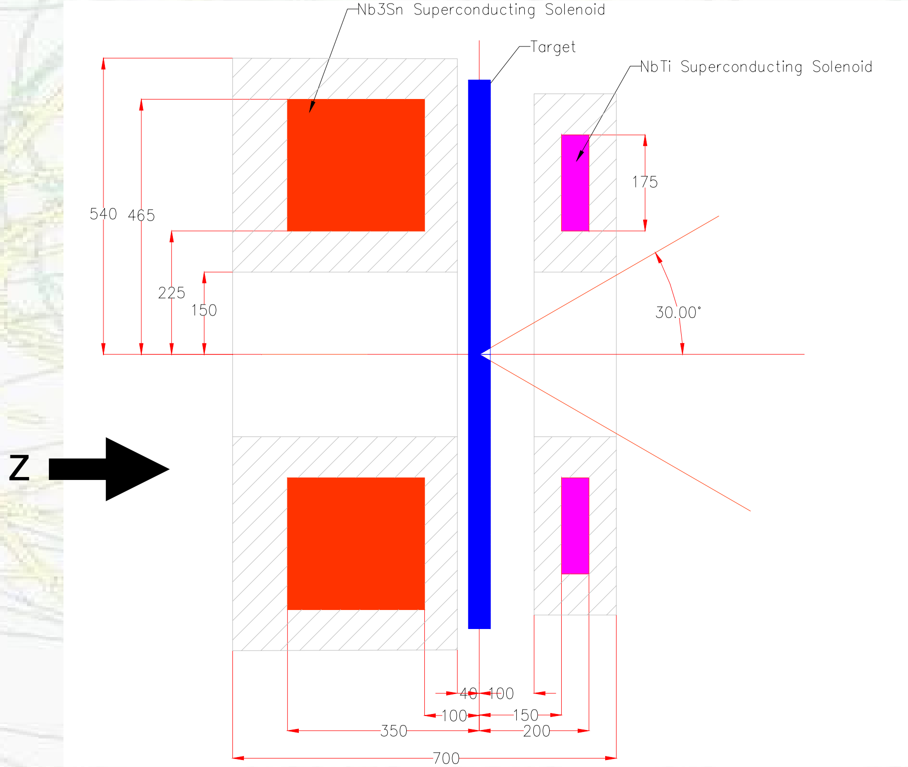


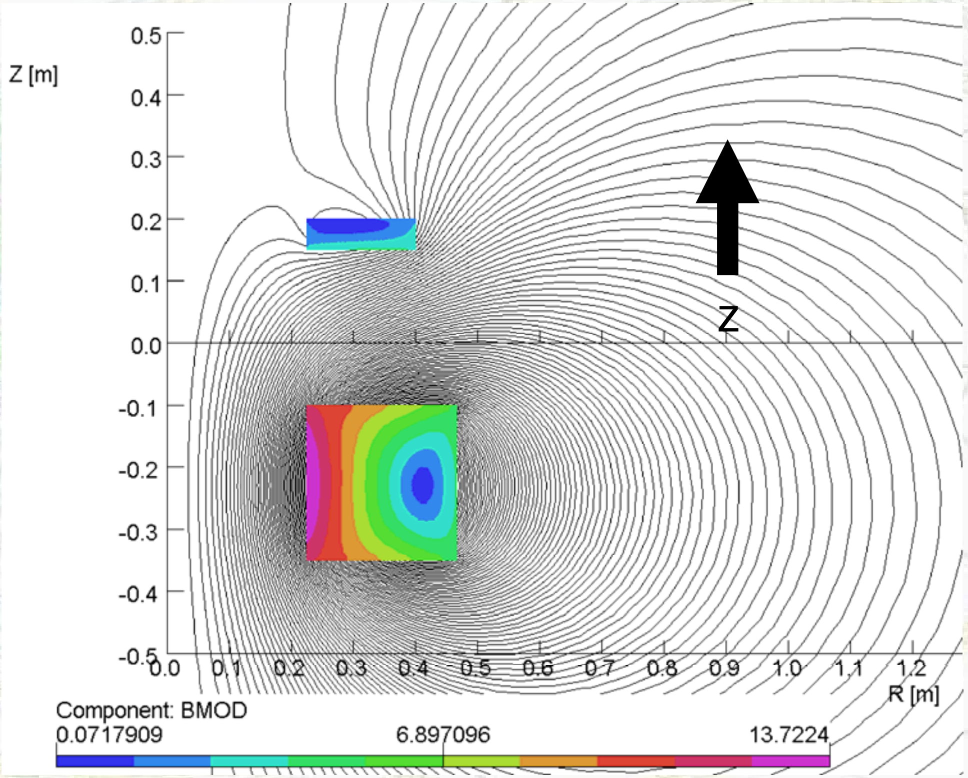
Original baseline layout of ILC with undulator at 150GeV position in main linac.



Capture Optics Design

V.S.Kashikhin

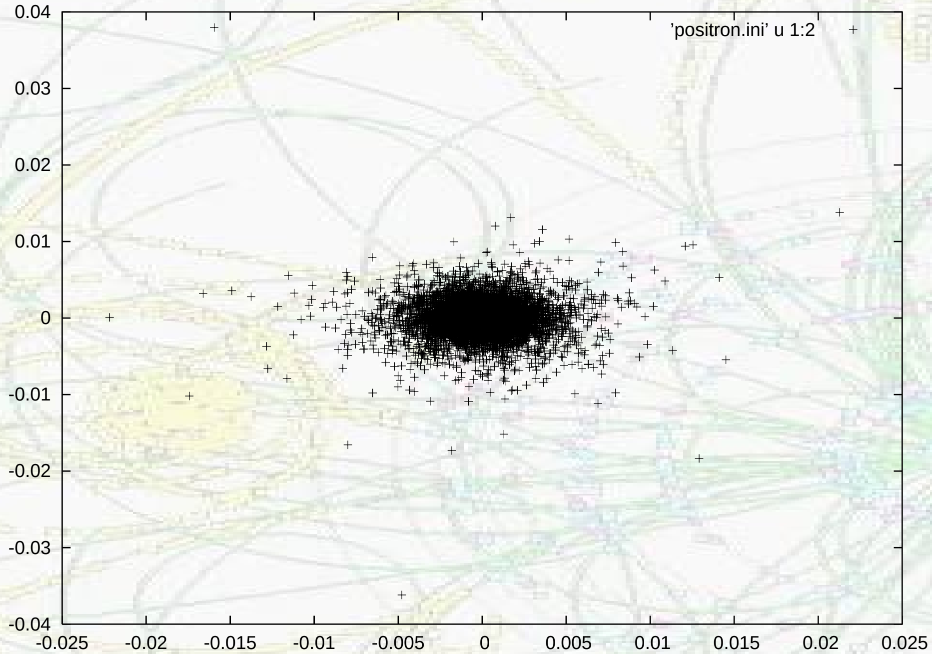


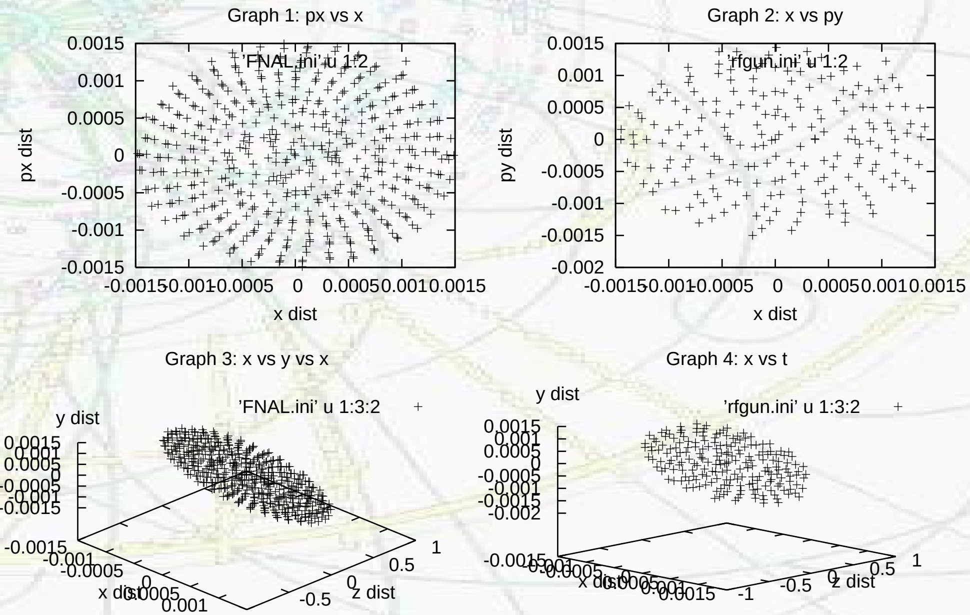


Two coil magnet gives focussing solenoid field

ASTRA

Klaus Floettmann





No spin tracking…

T-BMT Equation

$$\frac{d\vec{S}}{ds} = \vec{\Omega}(\vec{E}, \vec{B}, \gamma, \vec{v}) \times \vec{S}$$

Lorentz Factor

$$\Omega \propto \frac{(g-2)}{2}\gamma$$

Gyromagnetic anomaly

Differential Equation…

Runge-Kutta Integrator Scheme

Andriy Ushakov

Let an initial value problem be specified as follows.

$$y' = f(t, y), \quad y(t_0) = y_0$$

Then, the RK4 method for this problem is given by the following equation:

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = f\left(t_n, y_n\right)$$

$$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right)$$

$$k_4 = f\left(t_n + h, y_n + hk_3\right)$$

Thus, the next value (y_{n+1}) is determined by the present value (y_n) plus the product of the size of the interval (h) and an estimated slope. The slope is a weighted average of slopes:

- k_1 is the slope at the beginning of the interval;
- k_2 is the slope at the midpoint of the interval, using slope k_1 to determine the value of y at the point $t_n + h/2$ using Euler's method;
- k_3 is again the slope at the midpoint, but now using the slope k_2 to determine the y-value;
- k_4 is the slope at the end of the interval, with its y-value determined using k_3.

Boris' Numerical Integration Scheme

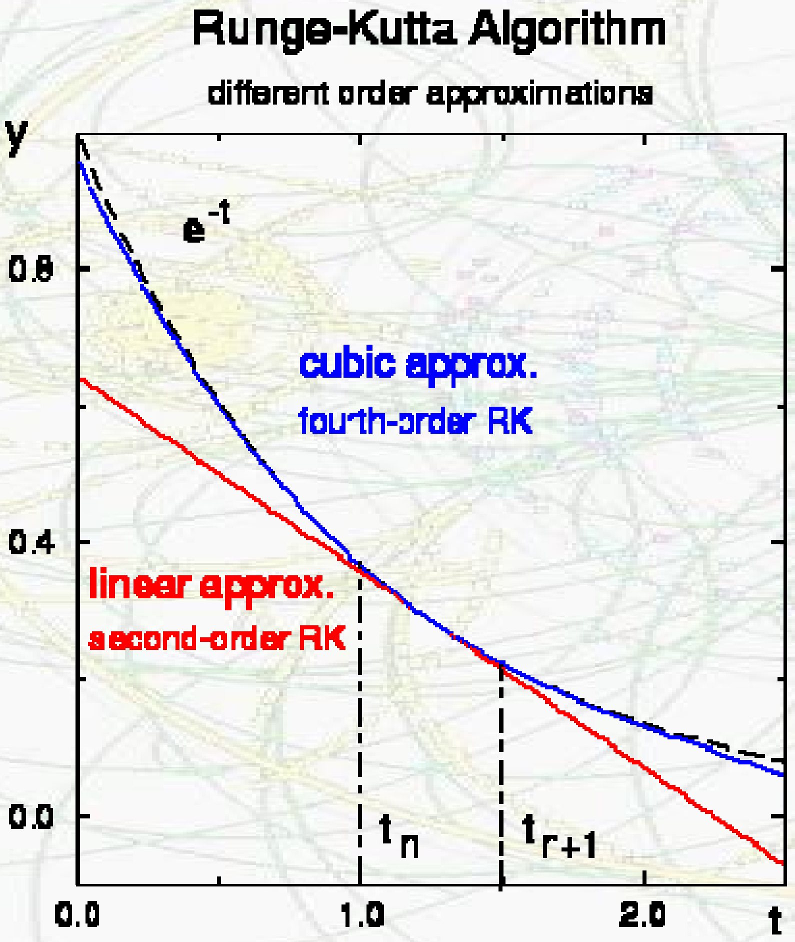
$$\frac{\mathrm{dw}}{\mathrm{dz}} = Mw + b$$

$$M = \frac{q}{p_z}\begin{pmatrix} 0 & B_z & E_x/c \\ -B_z & 0 & E_y/c \\ E_x/c & E_y/c & 0 \end{pmatrix} \qquad w = \begin{pmatrix} p_x \\ p_y \\ U/c \end{pmatrix} \qquad b = q\begin{pmatrix} -B_y \\ +B_x \\ E_z/c \end{pmatrix}$$

$$\frac{ds}{dz} = \frac{w}{p_z}$$

1. Advance w by vector term b a half-step
2. Advance w by M a full step
3. Advance w by b a half-step

R-K vs Boris



Boris: 2nd order, good at preserving conserving quantities, 1 calculation per step

R-K: 4th order, 4 calculations per step (speed!)

Will try both!

Conclusions...

- ASTRA can be used to simulate the motion of particles/bunches in the capture optics section
- …but doesn't have spin tracking included…
- Spin tracking routine using Boris integrator 1st draft ready
- Spin tracking subroutine using Runge-Kutta integrator underway – **cross-check**
- Merlin cross-check (Andy Wolski)