

Precise Charm and Bottom Quark Masses

Matthias Steinhauser

TTP, University of Karlsruhe

in collaboration with Hans Kühn and Christian Sturm

SFB/TR 9



Universität Karlsruhe (TH)
Forschungsuniversität • gegründet 1825



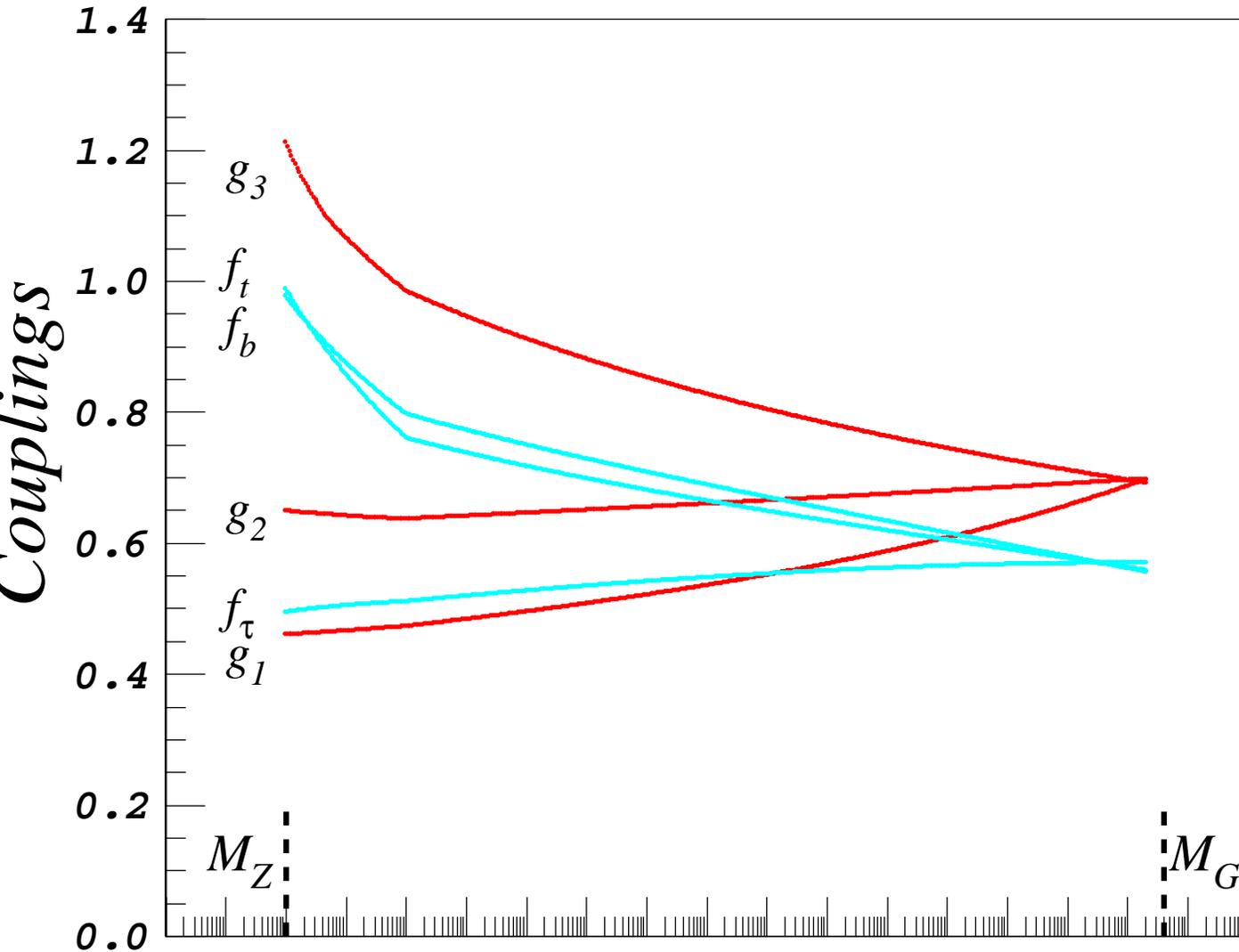
Quark masses

- Fundamental parameters of the Standard Model
- B decays: $\Gamma \sim m_b^5 \dots$
- Spectroscopy
- Higgs decay \Leftrightarrow ILC

$$\Gamma(H \rightarrow b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2 (1 + \mathcal{O}(\alpha_s) + \dots)$$

Yukawa unification

[Auto,Baer,Balázs,Beyaev,Ferrandis,Tata'03]



⇒ needed:

$$\frac{\delta m_t}{m_t} \approx \frac{\delta m_b}{m_b}$$

$$\delta m_t = 1 \text{ GeV}$$

⇒ $\delta m_b = 25 \text{ MeV}$
necessary

Quark mass definitions

- pole mass
- $\overline{\text{MS}}$ mass
- kinetic mass
- 1S mass
- PS mass
- ...

Light quark masses, top quark mass

PDG:

$$m_u = 1.5 \dots 3.0 \text{ MeV}$$

$$m_d = 3 \dots 7 \text{ MeV}$$

$$\bar{m} = \frac{m_u + m_d}{2} = 2.5 \dots 5.5 \text{ MeV}$$

$$m_s = 95 \pm 25 \text{ MeV}$$

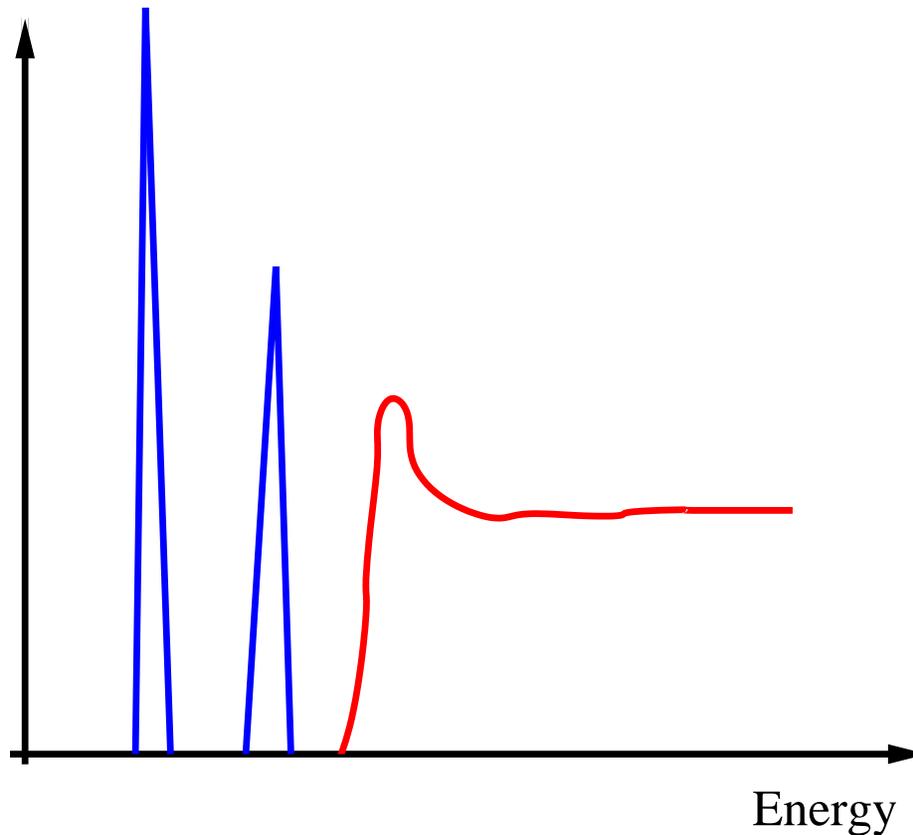
⇒ less accurately known than heavy quark masses

$$m_t = 170.9 \pm 1.8 \text{ GeV}$$

Charm/Bottom

- Consider $\sigma(e^+e^- \rightarrow \text{hadrons})$

Cross section



- \Rightarrow sum rules, (“SVZ” sum rules)

[Novikov et al.'78]

Sum rules

- Idea: consider moments

$$\mathcal{M}_n \equiv \int \frac{ds}{s^{n+1}} R_Q(s)$$

$$R_Q = \frac{\sigma(e^+e^- \rightarrow Q\bar{Q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

- Dispersion relation \Leftrightarrow

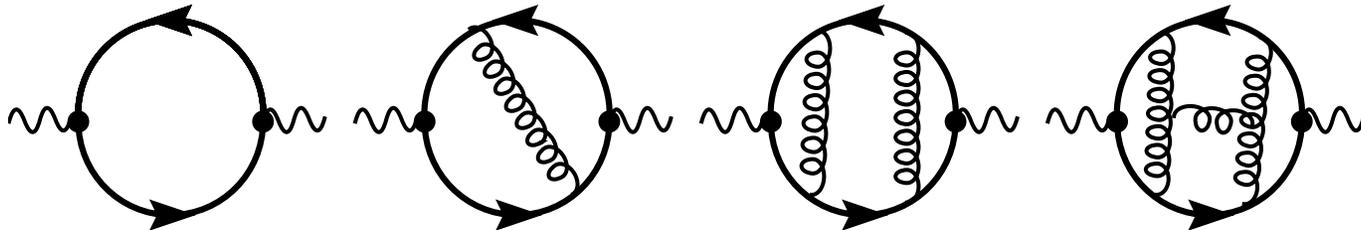
$$\mathcal{M}_n = \frac{12\pi^2}{n!} \left(\frac{d}{dq^2} \right)^n \Pi_Q(q^2) \Big|_{q^2=0}$$

$$R_Q = 12\pi \text{Im} [\Pi_Q(q^2 = s + i\varepsilon)]$$

$$\mathcal{M}_n^{\text{th}}$$

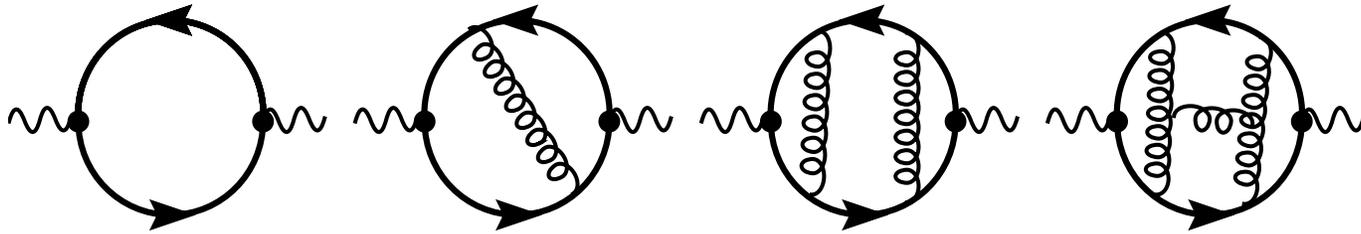
$$\mathcal{M}_n = \frac{12\pi^2}{n!} \left(\frac{d}{dq^2} \right)^n \Pi_Q(q^2) \Big|_{q^2=0}$$

$$\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n z^n \quad z = \frac{q^2}{4m_Q^2}$$



$$\mathcal{M}_n^{\text{th}} = \frac{9}{4} Q_c^2 \left(\frac{1}{4m_c^2} \right)^n \bar{C}_n$$

C_n to 4 loops



- 1, 2 and 3 loops: MATAD

[MS'96-'00]

- 4 loops:

- method: 1. reduce to master integrals

[Laporta,Remiddi'96; Laporta'01]

2. compute masters

- several Million equations; several GB tables

- all steps cross-checked

1. [Chetyrkin,Kühn,Sturm'06; Boughezal,Czakon,Schutzmeier'06]

2. [Schröder,Vuorinen'05; Chetyrkin,Faisst,Sturm,Tentyukov'06],...

C_n to 4 loops

● result (charm):

n	$\bar{C}_n^{(0)}$	$\bar{C}_n^{(10)}$	$\bar{C}_n^{(11)}$	$\bar{C}_n^{(20)}$	$\bar{C}_n^{(21)}$	$\bar{C}_n^{(22)}$	$\bar{C}_n^{(30)}$	$\bar{C}_n^{(31)}$	$\bar{C}_n^{(32)}$	$\bar{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	2.4967	3.3130	-0.0889	-5.6404	4.0669	0.9590	0.0642
2	0.4571	1.1096	1.8286	2.7770	5.1489	1.7524	—	6.7216	6.4916	-0.0974
3	0.2709	0.5194	1.6254	1.6388	4.7207	3.1831	—	7.5736	13.1654	1.9452
4	0.1847	0.2031	1.4776	0.7956	3.6440	4.3713	—	4.9487	17.4612	5.5856

$$\begin{aligned} \bar{C}_n = & \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi} \left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c} \right) + \\ & \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2 \right) + \\ & \left(\frac{\alpha_s(\mu)}{\pi} \right)^3 \left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{m_c} + \bar{C}_n^{(32)} l_{m_c}^2 + \bar{C}_n^{(33)} l_{m_c}^3 \right) \\ & l_{m_c} = \ln(m_c^2/\mu^2) \end{aligned}$$

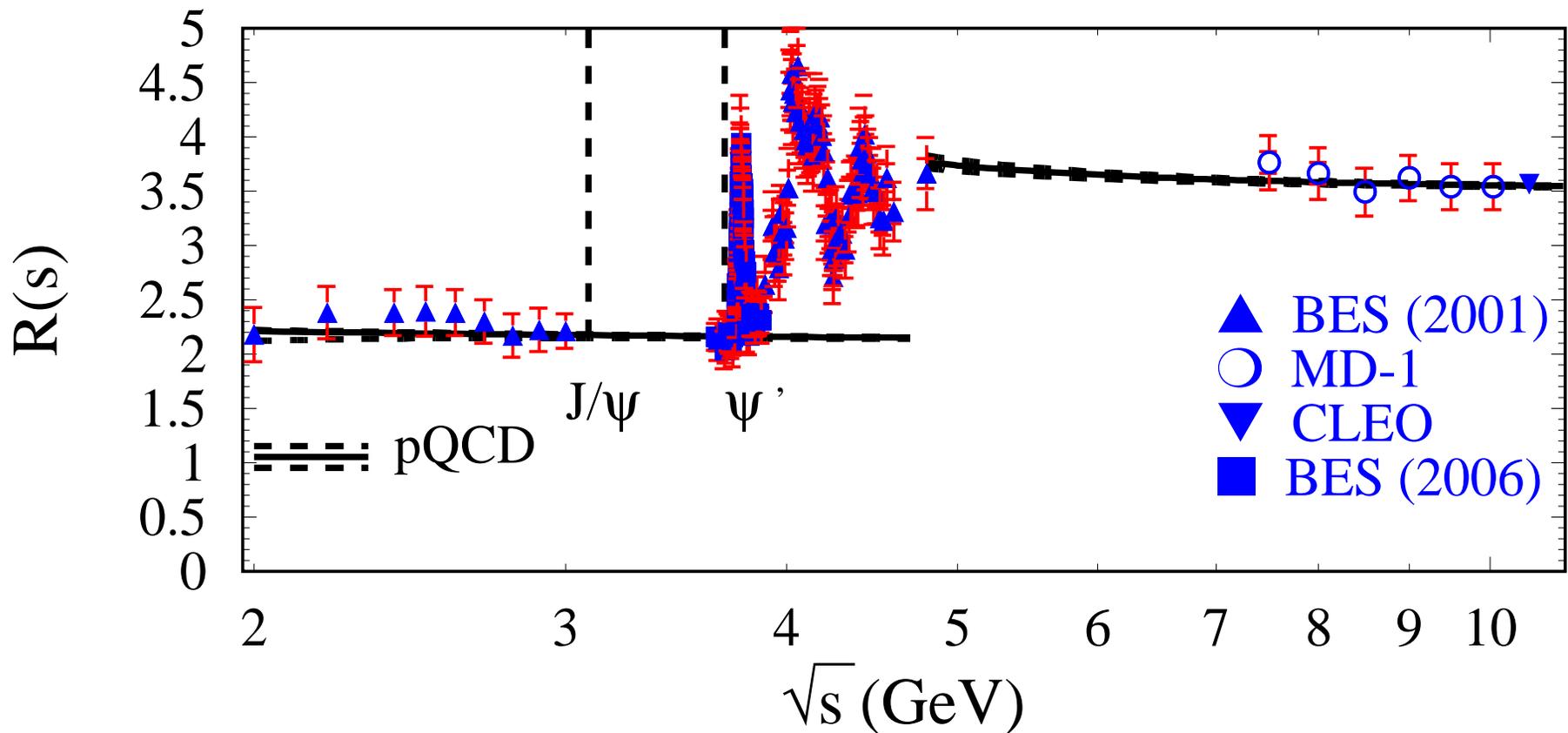
\mathcal{M}^{exp}

$$\mathcal{M}^{\text{exp}} = \mathcal{M}^{\text{res}} + \mathcal{M}^{\text{thresh}} + \mathcal{M}^{\text{cont}}$$

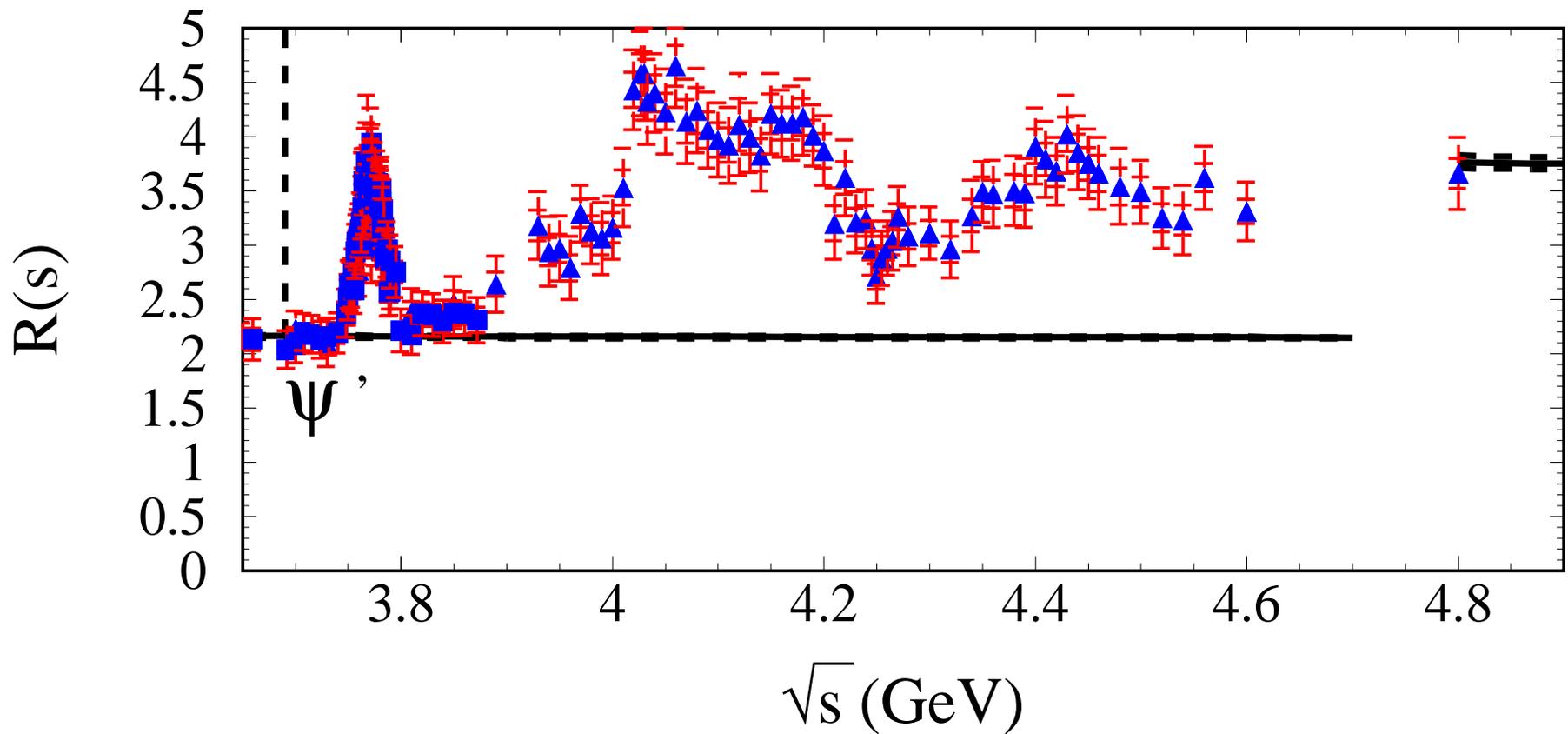
$$R^{\text{res}}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2} \left(\frac{\alpha}{\alpha(s)} \right)^2 \delta(s - M_R^2)$$

	J/Ψ	$\Psi(2S)$
M_Ψ (GeV)	3.096916(11)	3.686093(34)
Γ_{ee} (keV)	5.55(14)	2.48(6)
$(\alpha/\alpha(M_\Psi))^2$	0.957785	0.95554

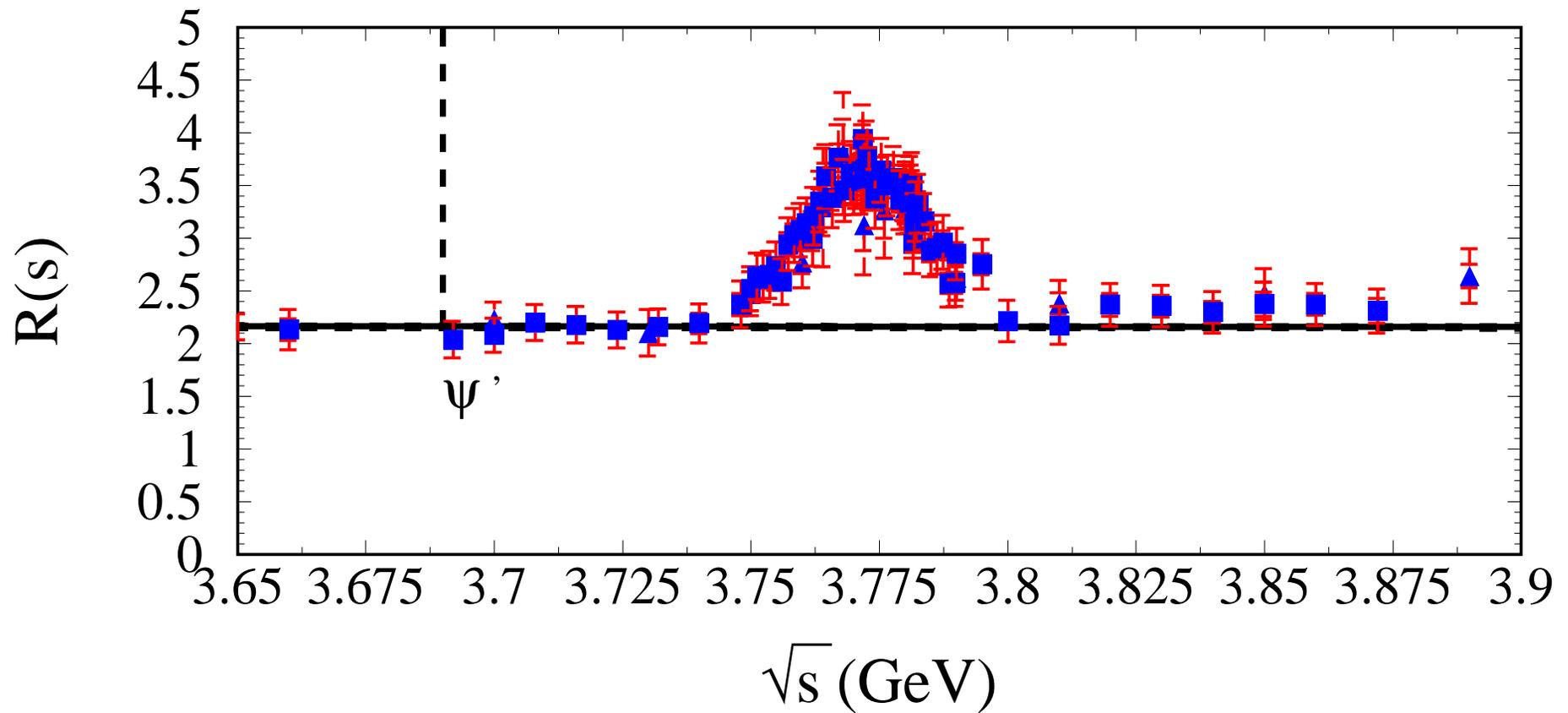
$R(s)$ in charm threshold region



$R(s)$ in charm threshold region

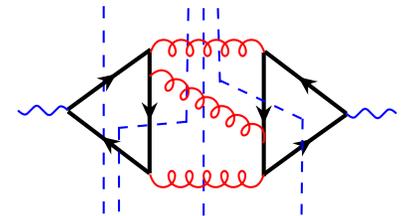
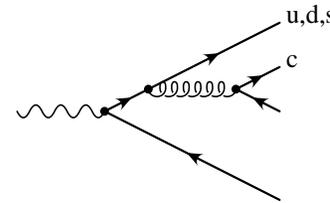


$R(s)$ in charm threshold region



M^{thresh}

- subtract R_{uds}
- \bar{R} from data below 3.73 GeV
- \sqrt{s} -dependence from theory



$\mathcal{M}^{\text{cont}}$ and \mathcal{M}^{exp}

- $\mathcal{M}^{\text{cont}}$
 - $\sqrt{s} \geq 4.8 \text{ GeV}$
 - no data
 - $R(s)$ with full mass dependence
- \mathcal{M}^{exp}

rhad: [Harlander,MS'02]

n	$\mathcal{M}_n^{\text{res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{np}}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	-0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

m_c

$$\mathcal{M}^{\text{th}} + \mathcal{M}^{\text{np}} \stackrel{!}{=} \mathcal{M}^{\text{exp}}$$

$$m_c(\mu) = \frac{1}{2} \left(\frac{\bar{C}_n}{\mathcal{M}_n^{\text{exp}} - \mathcal{M}^{\text{np}}} \right)^{1/(2n)}$$

$$\alpha_s(M_Z) = 0.1189 \pm 0.002, \quad \mu = (3 \pm 1) \text{ GeV}$$

n	$m_c(3 \text{ GeV})$	exp	α_s	μ	np	total	$\delta\bar{C}_n^{(30)}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.979	0.006	0.014	0.005	0.000	0.016	0.006	1.280
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$$-6.0 \leq \bar{C}_2^{(30)} \leq 7.0, \quad -6.0 \leq \bar{C}_3^{(30)} \leq 5.2, \quad -6.0 \leq \bar{C}_4^{(30)} \leq 3.1$$

m_c

$$\mathcal{M}^{\text{th}} + \mathcal{M}^{\text{np}} \stackrel{!}{=} \mathcal{M}^{\text{exp}}$$

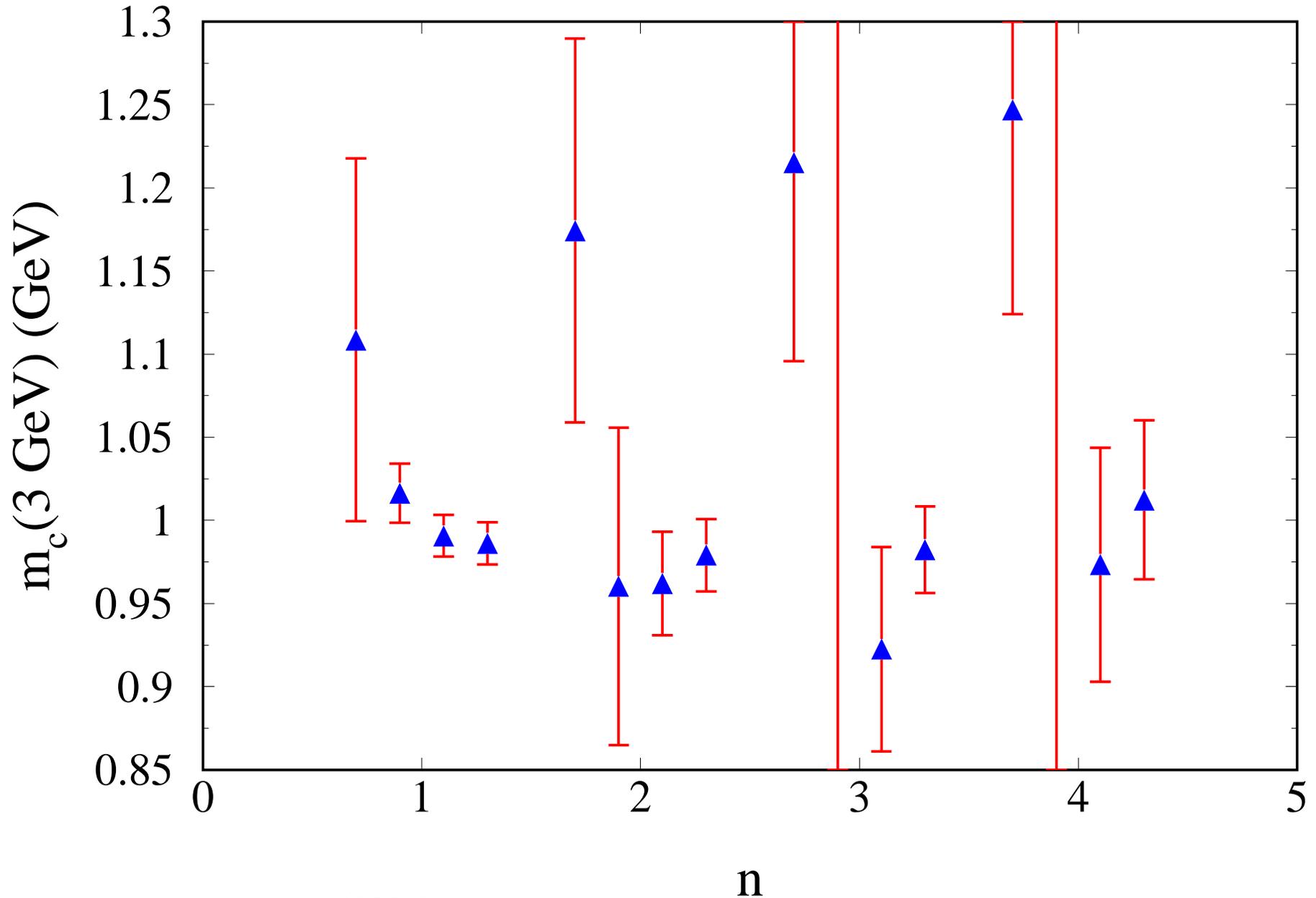
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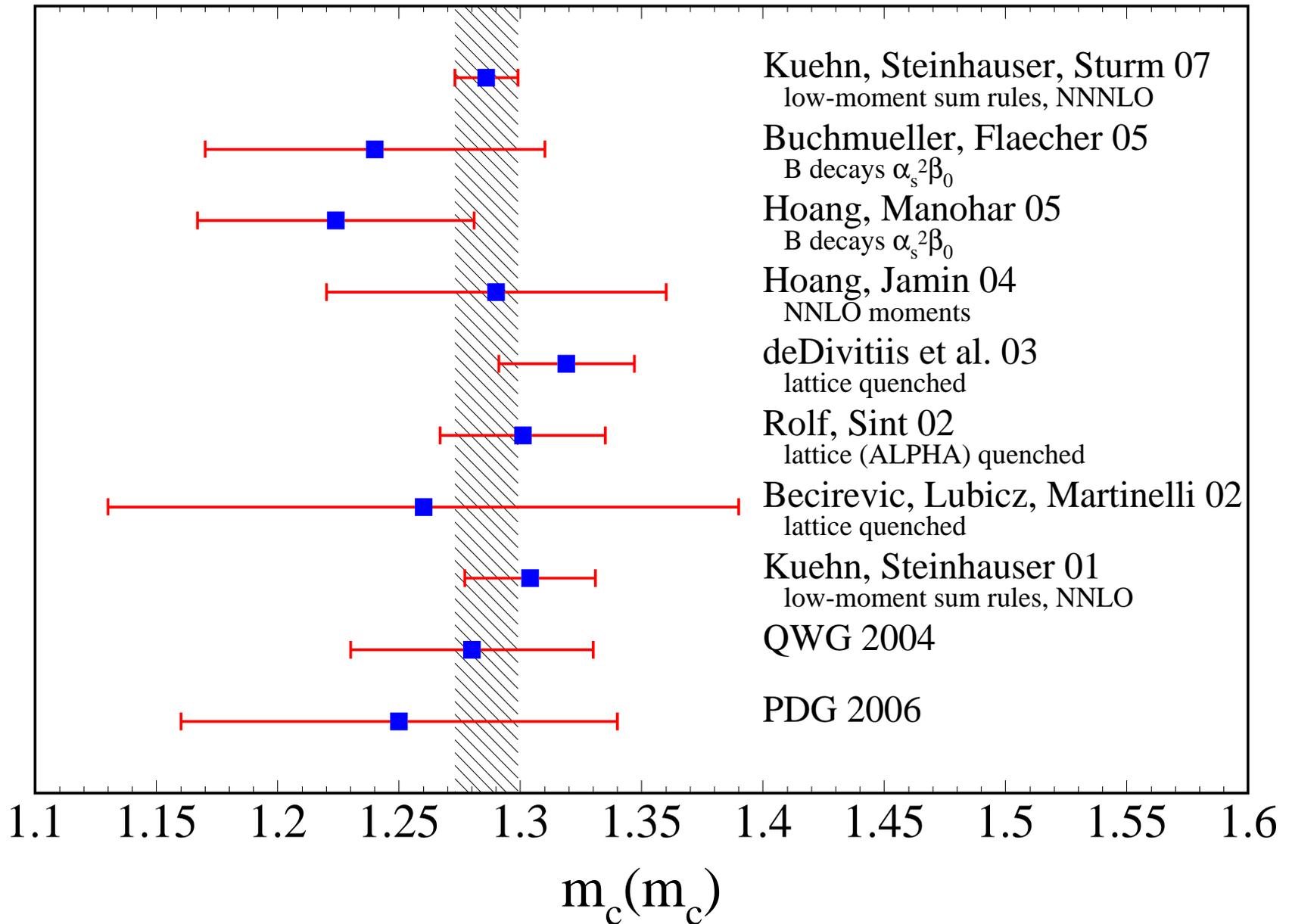
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$$m_c(3 \text{ GeV}) = 0.986(13) \text{ GeV}$$

$m_c(3 \text{ GeV})$

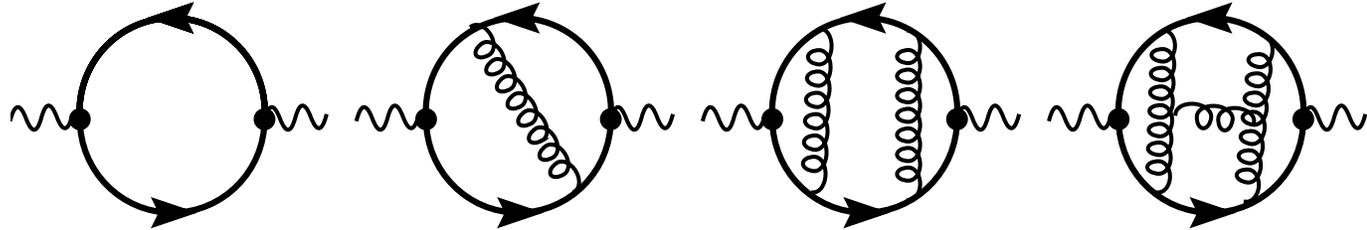


Charm — comparison



Bottom quark

- $\mathcal{M}_n^{\text{th}}$: see charm, $n_f = 5$



- $\mathcal{M}_n^{\text{np}}$: negligible
- \mathcal{M}^{res} : $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$
- $\mathcal{M}^{\text{thresh}}$: CLEO data up to 11.24 GeV
- $\mathcal{M}^{\text{cont}}$: pQCD above 11.24 GeV

m_b

$$\mathcal{M}^{\text{th}} \stackrel{!}{=} \mathcal{M}^{\text{exp}}$$

$$m_b(\mu) = \frac{1}{2} \left(\frac{1}{4} \frac{\bar{C}_n}{\mathcal{M}_n^{\text{exp}}} \right)^{1/(2n)}$$

n	$m_b(10 \text{ GeV})$	exp	α_s	μ	total	$\delta \bar{C}_n^{(30)}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.609	0.014	0.012	0.003	0.019	0.006	4.164
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

m_b

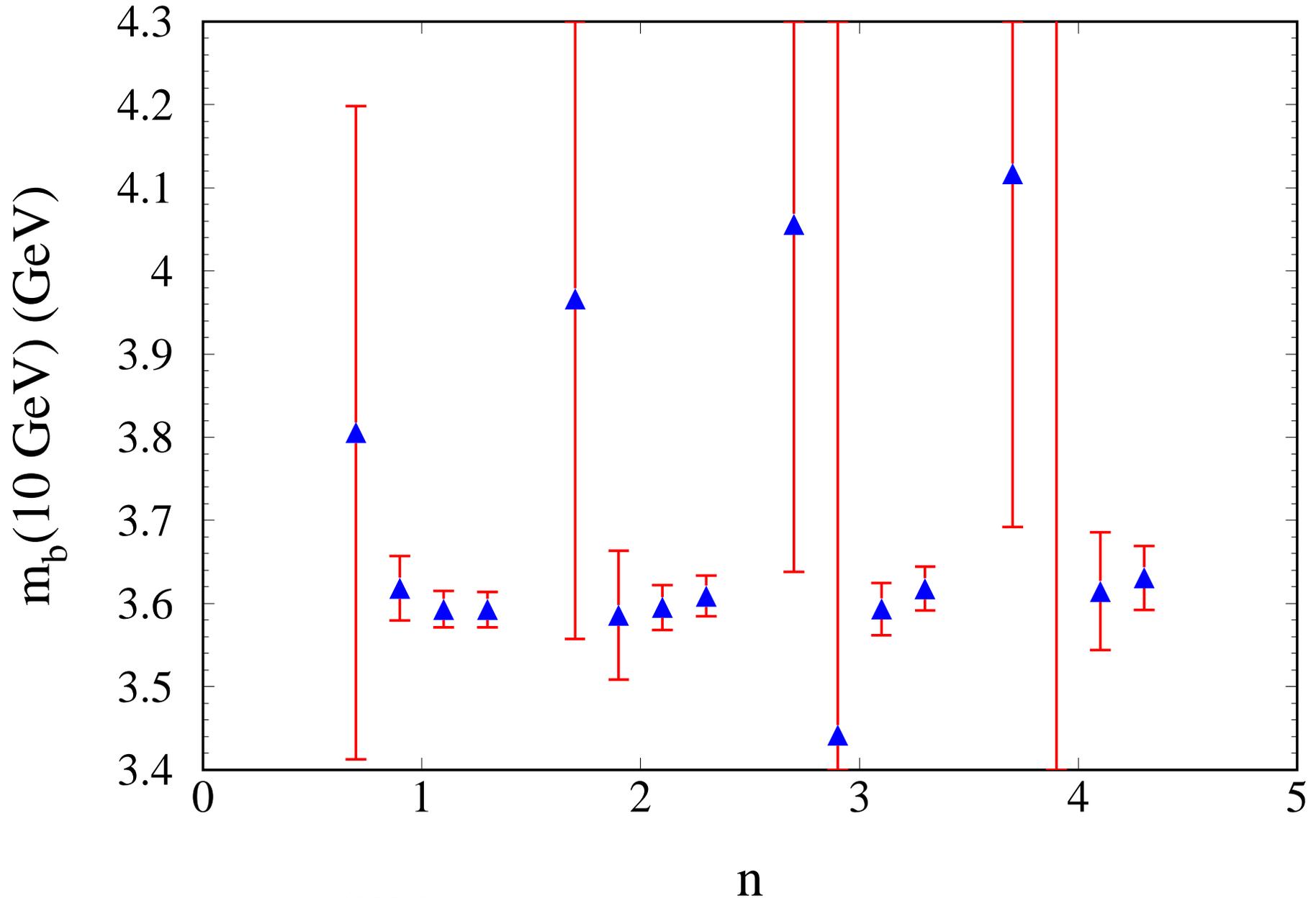
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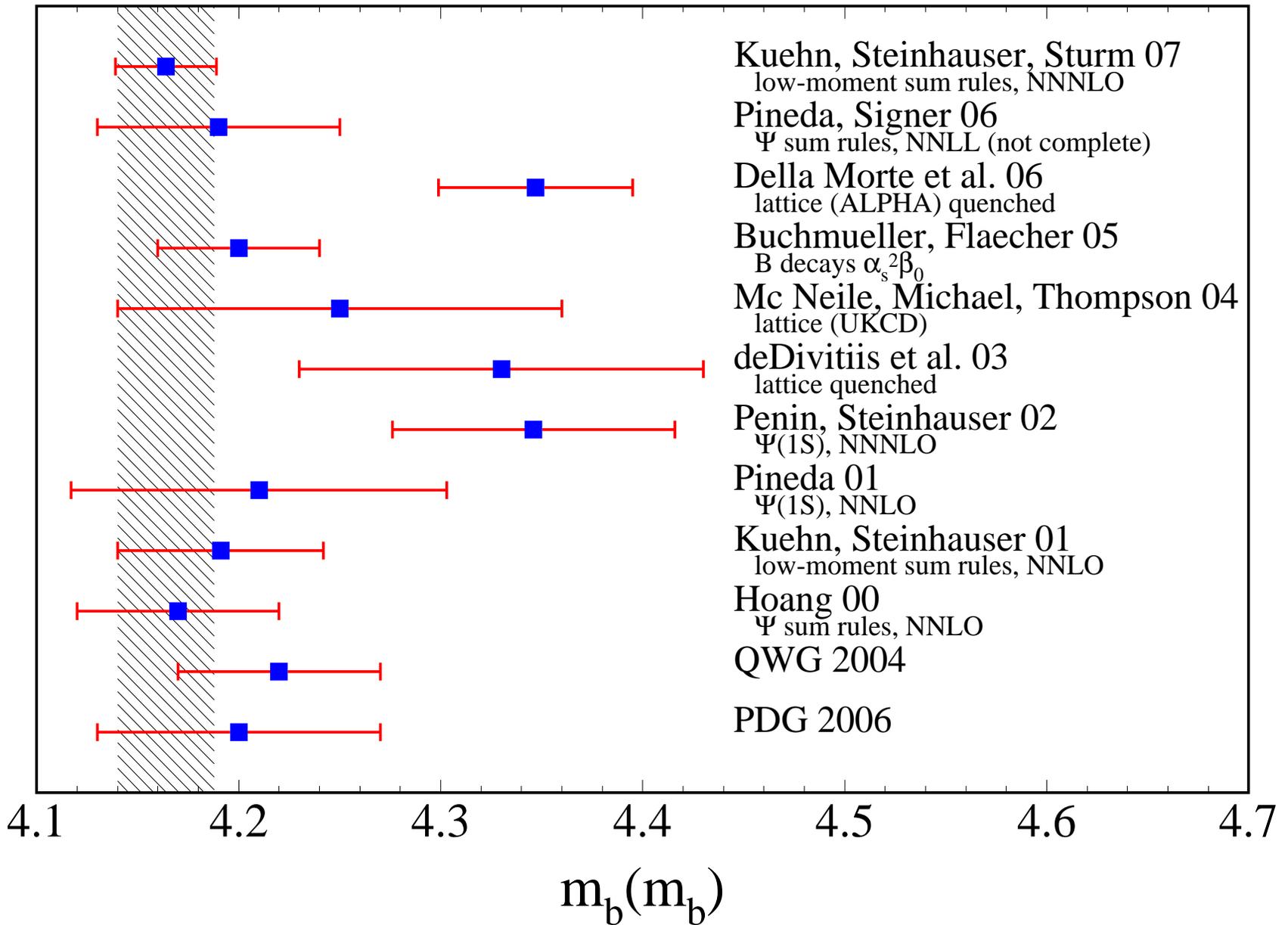
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$$m_b(10 \text{ GeV}) = 3.609(25) \text{ GeV}$$

$m_b(10 \text{ GeV})$



Bottom — comparison



Conclusions

- Most precise values for m_c and m_b
 $m_c(m_c) = 1.286(13) \text{ GeV}$ $m_b(m_b) = 4.164(25) \text{ GeV}$
- NNNLO analysis
- $\overline{\text{MS}}$ mass
- Possible improvements: experimental measurements:
 $R(s), \Gamma_{ee}$
- $\frac{\delta m_s}{m_s} \approx 10\%$
 $\frac{\delta m_c}{m_c} \approx 1\%$
 $\frac{\delta m_b}{m_b} \approx 0.6\%$
 $\frac{\delta m_t}{m_t} \approx 1\%$