CP violation in unpolarized

$$e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$$

Per Osland, LCWS, Hamburg 2007

Work done with Alexander Vereshagin

arXiv:0704.2165

Eigenstates of 2×2 chargino mass matrix

$$\mathcal{M}_\chi = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$$

Choose

$$M_2 \quad \text{real}$$

$$\mu = |\mu| e^{i\phi}$$

Notation: $$U^* \mathcal{M}_\chi V^\dagger = \begin{pmatrix} m_{\chi 1} & 0 \\ 0 & m_{\chi 2} \end{pmatrix}$$

Need two different matrices since $\mathcal{M}_\chi$ is non-Hermitian

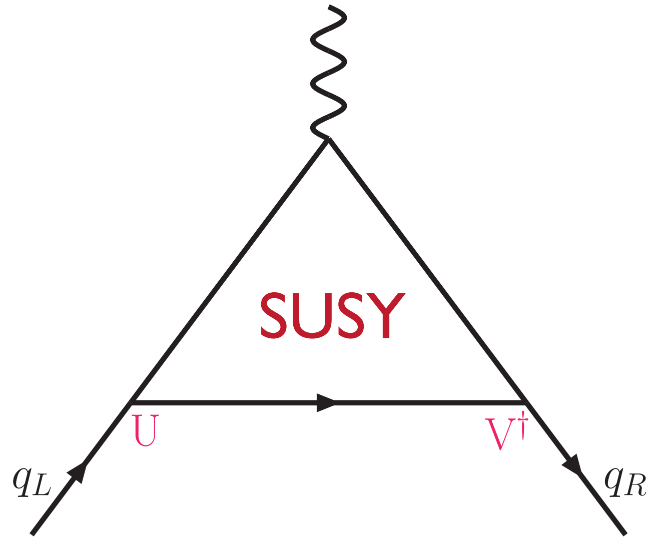
$$m_{\chi 1} < m_{\chi 2}$$

Phase leads to CP violation

Electron and neutron EDM's

Constraints on superparticle masses, order TeV

Ellis, Ferrara, Nanopoulos, 1982; Buchmuller, Wyler, 1983,...



Could be cancellations, allowing lighter superparticles

Kizukuri, Oshimo, 1992; Ibrahim, Nath, 1998,...

Chargino "pair" production:

$$e^+e^- \longrightarrow \begin{cases} \tilde{\chi}_2^+\tilde{\chi}_2^- & \text{CP eigenstate in c.m. frame} \\ \left.\begin{array}{l}\tilde{\chi}_2^+\tilde{\chi}_1^- \\ \tilde{\chi}_1^+\tilde{\chi}_2^-\end{array}\right\} & \text{not CP eigenstates} \\ \tilde{\chi}_1^+\tilde{\chi}_1^- & \text{CP eigenstate in c.m. frame} \end{cases}$$

Tree level results given by Bartl et al: hep-ph/0403265

Include polarization

Claim:

$$\sigma(e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_1^-) \neq \sigma(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-)$$

if μ is complex

Effect vanishes at tree level. Loop contribution is finite

Notation:

$$e^+(p_1, P_+) + e^-(p_2, P_-) \to \tilde{\chi}_i^+(k_1) + \tilde{\chi}_j^-(k_2)$$

$$i = 1, 2, \quad j = 1, 2$$

$p_1, p_2, \quad k_1, k_2$ momenta

positive negative

P_-, P_+ polarizations

Parity and charge transformation:

three-momentum

spin other QN's

c.m. frame

$$\mathrm{P} a^\dagger(\boldsymbol{p}, \sigma, n)\mathrm{P}^{-1} = \eta_n a^\dagger(-\boldsymbol{p}, \sigma, n)$$

$$\mathrm{C} a^\dagger(\boldsymbol{p}, \sigma, n)\mathrm{C}^{-1} = \xi_n a^\dagger(\boldsymbol{p}, \sigma, n^c)$$

S-matrix element:

$$\langle \tilde{\chi}_i^+(\boldsymbol{k}_1), \tilde{\chi}_j^-(\boldsymbol{k}_2)|S|e^+(\boldsymbol{p}_1, P_+), e^-(\boldsymbol{p}_2, P_-)\rangle$$

transforms under P, C, CP ...

$$\langle\tilde\chi_i^+(\boldsymbol{k}_1),\tilde\chi_j^-(\boldsymbol{k}_2)|S|e^+(\boldsymbol{p}_1,P_+),e^-(\boldsymbol{p}_2,P_-)\rangle$$

$$\xrightarrow{\text{P}}\ \langle\tilde\chi_i^+(-\boldsymbol{k}_1),\tilde\chi_j^-(-\boldsymbol{k}_2)|S|e^+(-\boldsymbol{p}_1,P_+),e^-(-\boldsymbol{p}_2,P_-)\rangle$$

$$\boldsymbol{p}_{1,2}\leftrightarrow-\boldsymbol{p}_{1,2},\quad \boldsymbol{k}_{1,2}\leftrightarrow-\boldsymbol{k}_{1,2}$$

$$\langle\tilde\chi_i^+(\boldsymbol{k}_1),\tilde\chi_j^-(\boldsymbol{k}_2)|S|e^+(\boldsymbol{p}_1,P_+),e^-(\boldsymbol{p}_2,P_-)\rangle$$

$$\xrightarrow{\text{C}}\ \langle\tilde\chi_i^-(\boldsymbol{k}_1),\tilde\chi_j^+(\boldsymbol{k}_2)|S|e^-(\boldsymbol{p}_1,P_+),e^+(\boldsymbol{p}_2,P_-)\rangle$$

$$\boldsymbol{p}_1\leftrightarrow\boldsymbol{p}_2,\quad \boldsymbol{k}_1\leftrightarrow\boldsymbol{k}_2,\quad m_i\leftrightarrow m_j,\quad P_+\leftrightarrow P_-$$

$$\langle\tilde\chi_i^+(\boldsymbol{k}_1),\tilde\chi_j^-(\boldsymbol{k}_2)|S|e^+(\boldsymbol{p}_1,P_+),e^-(\boldsymbol{p}_2,P_-)\rangle$$

$$\xrightarrow{\text{CP}}\ \langle\tilde\chi_j^+(-\boldsymbol{k}_2),\tilde\chi_i^-(-\boldsymbol{k}_1)|S|e^+(-\boldsymbol{p}_2,P_-),e^-(-\boldsymbol{p}_1,P_+)\rangle$$

$$\boldsymbol{p}_1\leftrightarrow-\boldsymbol{p}_2,\quad \boldsymbol{k}_1\leftrightarrow-\boldsymbol{k}_2,\quad m_i\leftrightarrow m_j,\quad P_+\leftrightarrow P_-$$

General structure of cross section:

$$d\sigma = d\sigma_0 + (\text{terms linear in } |\boldsymbol{P}_\pm|) + (\ldots)|\boldsymbol{P}_-||\boldsymbol{P}_+|$$

Unpolarized part can only depend on Mandelstam invariants

$$s \equiv (p_1 + p_2)^2 \qquad t \equiv (p_1 - k_1)^2$$

and masses m_i m_j

$$s \quad \text{and} \quad t = (p_1 - k_1)^2 = (p_2 - k_2)^2$$

are invariant under CP

However, some terms change under $m_i \longleftrightarrow m_j$

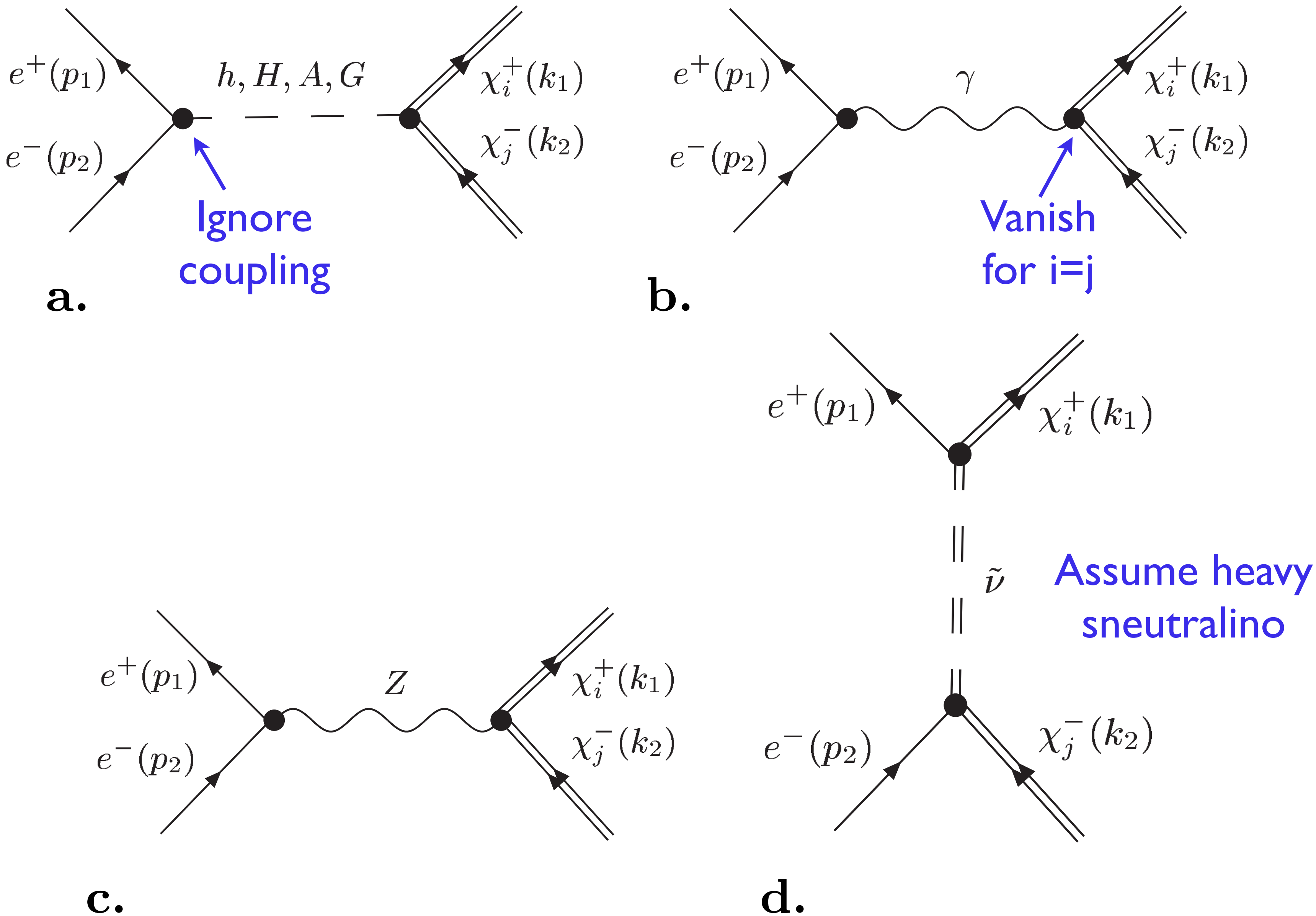
Will consider ratio $$\frac{d\sigma_0^{\rm odd}}{d\sigma_0}$$

where

$$d\sigma_0^{\rm odd} = \frac{1}{2}\Big[d\sigma_0 - d\sigma_0^{\rm CP}\Big] \qquad d\sigma_0^{\rm CP} \equiv d\sigma_0\Big|_{m_i \leftrightarrow m_j}$$

Approximate as $$\frac{d\sigma_0^{\rm odd}\big|_{1\ \rm loop}}{d\sigma_0\big|_{\rm tree}}$$

Tree level



$e^+(p_1)$
$e^-(p_2)$
h, H, A, G
$\chi_i^+(k_1)$
$\chi_j^-(k_2)$
Ignore coupling
a.
$e^+(p_1)$
$e^-(p_2)$
γ
$\chi_i^+(k_1)$
$\chi_j^-(k_2)$
Vanish for i=j
b.
$e^+(p_1)$
$e^-(p_2)$
Z
$\chi_i^+(k_1)$
$\chi_j^-(k_2)$
c.
$e^+(p_1)$
$\chi_i^+(k_1)$
$\tilde{\nu}$
Assume heavy sneutralino
$e^-(p_2)$
$\chi_j^-(k_2)$
d.

Tree level

$$\frac{d\sigma}{d\Omega} = \frac{\beta}{64\pi^2 s}|\mathcal{M}|^2 \ , \quad \beta \equiv \frac{|\boldsymbol{p}_{\rm out}|}{|\boldsymbol{p}_{\rm in}|}$$

$$\begin{aligned} &|\mathcal{M}_{Z,\ \rm tree}|^2 \\ &= \chi^2\Big((g_V^2 + g_A^2)\{|G_V|^2[\mathcal{A} - 2(m_i - m_j)^2/s] \\ &\quad +|G_A|^2[\mathcal{A} - 2(m_i + m_j)^2/s]\} \\ &\quad -4g_V g_A(G_V^* G_A + G_V G_A^*)\beta\cos\theta\Big) \end{aligned}$$

CP even

$$\chi = \left(\frac{g}{4\cos\theta_W}\right)^2 \frac{s}{s - M_Z^2}, \quad \mathcal{A} = 2 - \beta^2\sin^2\theta$$

$$g_V = 1 - 4\sin^2\theta_W \qquad g_A = -1$$

plus sneutrino contributions

$Z\chi\chi$: $\quad G_V \equiv G_{V\,j,i} \qquad G_A \equiv G_{A\,j,i}$

$$L = \frac{g}{4\cos\theta_W}\bar{\Psi}_{\chi_j}\gamma^\rho\Big\{\Big[2\delta_{kj}\cos 2\theta_W + U_{k1}U^\dagger_{1j} + V_{j1}V^\dagger_{1k}\Big] + \gamma^5\Big[U_{k1}U^\dagger_{1j} - V_{j1}V^\dagger_{1k}\Big]\Big\}\Psi_{\chi_k}Z_\rho$$

$$\equiv \frac{g}{4\cos\theta_W}\bar{\Psi}_{\chi_j}\gamma^\rho\Big\{G_{V\,k,j} + \gamma^5 G_{A\,k,j}\Big\}\Psi_{\chi_k}Z_\rho$$

First index corresponds to annihilated positive particle

$$\mathrm{CP}: \quad G_{V(A)\,j,i} \leftrightarrow G_{V(A)\,i,j}, \quad m_i \leftrightarrow m_j$$

Hermiticity of Lagrangian:

$$G_{V(A)\,j,i} = G^*_{V(A)\,i,j}$$

Recall origin of $U \quad V$:

$$\mathcal{M}_\chi = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$$

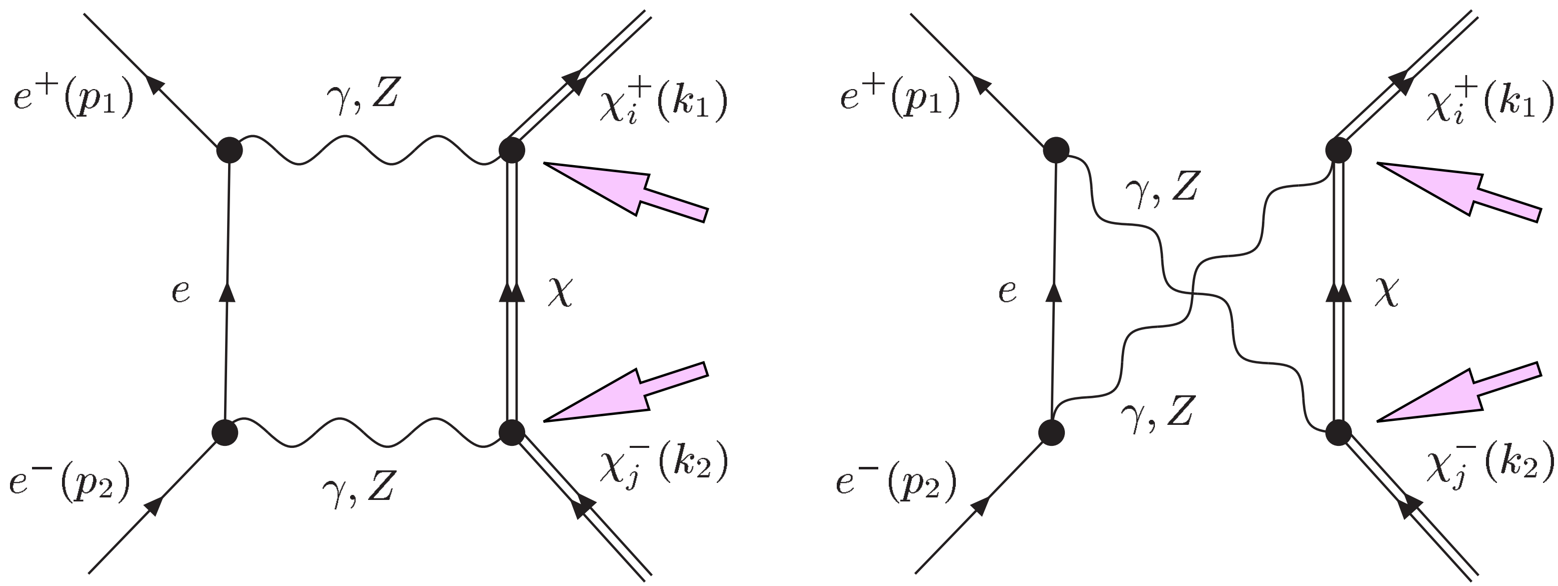
Choose

$$M_2 \quad \text{real}$$

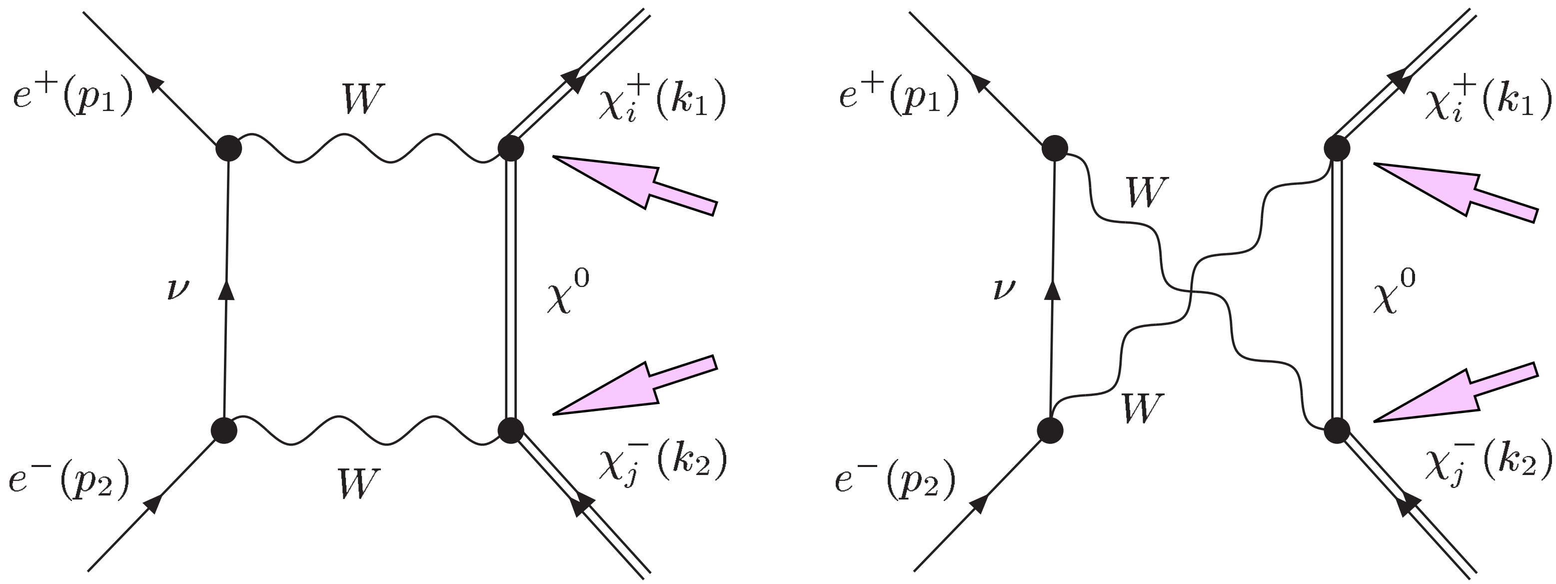
$$\mu = |\mu| e^{i\phi}$$

Notation: $$U^* \mathcal{M}_\chi V^\dagger = \begin{pmatrix} m_{\chi_1} & 0 \\ 0 & m_{\chi_2} \end{pmatrix}$$

Box diagrams



One $Z\chi\chi$ vertex is diagonal, the other is not



Similar to ZZ boxes, but now intermediate neutralino

Other box diagrams vanish in heavy sneutrino limit

$W\chi\chi_0$:

$$L = \frac{g}{2}\Big\{\bar{\Psi}_{\chi_j}\gamma^\rho(\mathcal{V} + \gamma^5\mathcal{A})\Psi_{\chi_a^0}W_\rho + \bar{\Psi}_{\chi_a^0}\gamma^\rho(\mathcal{V}^* + \gamma^5\mathcal{A}^*)\Psi_{\chi_j}W_\rho^\dagger\Big\}$$

$$\mathcal{V} = -Z_{a2}U_{1j}^\dagger - Z_{2a}^\dagger V_{j1} - \frac{Z_{a3}U_{2j}^\dagger}{\sqrt{2}} + \frac{Z_{4a}^\dagger V_{j2}}{\sqrt{2}},$$

$$\mathcal{A} = -Z_{a2}U_{1j}^\dagger + Z_{2a}^\dagger V_{j1} - \frac{Z_{a3}U_{2j}^\dagger}{\sqrt{2}} - \frac{Z_{4a}^\dagger V_{j2}}{\sqrt{2}}.$$

diagonalize neutralino mass matrix

$$Z^* M_{\chi^0} Z^\dagger = \mathrm{diag}\{m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_2^0}, m_{\chi_3^0}\}$$

could also be complex

$$M_{\chi^0} = \begin{pmatrix} M_1 & 0 & -m_Z \cos\beta \sin\theta_W & m_Z \sin\beta \sin\theta_W \\ 0 & M_2 & m_Z \cos\beta \cos\theta_W & -m_Z \sin\beta \cos\theta_W \\ -m_Z \cos\beta \sin\theta_W & m_Z \cos\beta \cos\theta_W & 0 & -\mu \\ m_Z \sin\beta \sin\theta_W & -m_Z \sin\beta \cos\theta_W & -\mu & 0 \end{pmatrix}$$

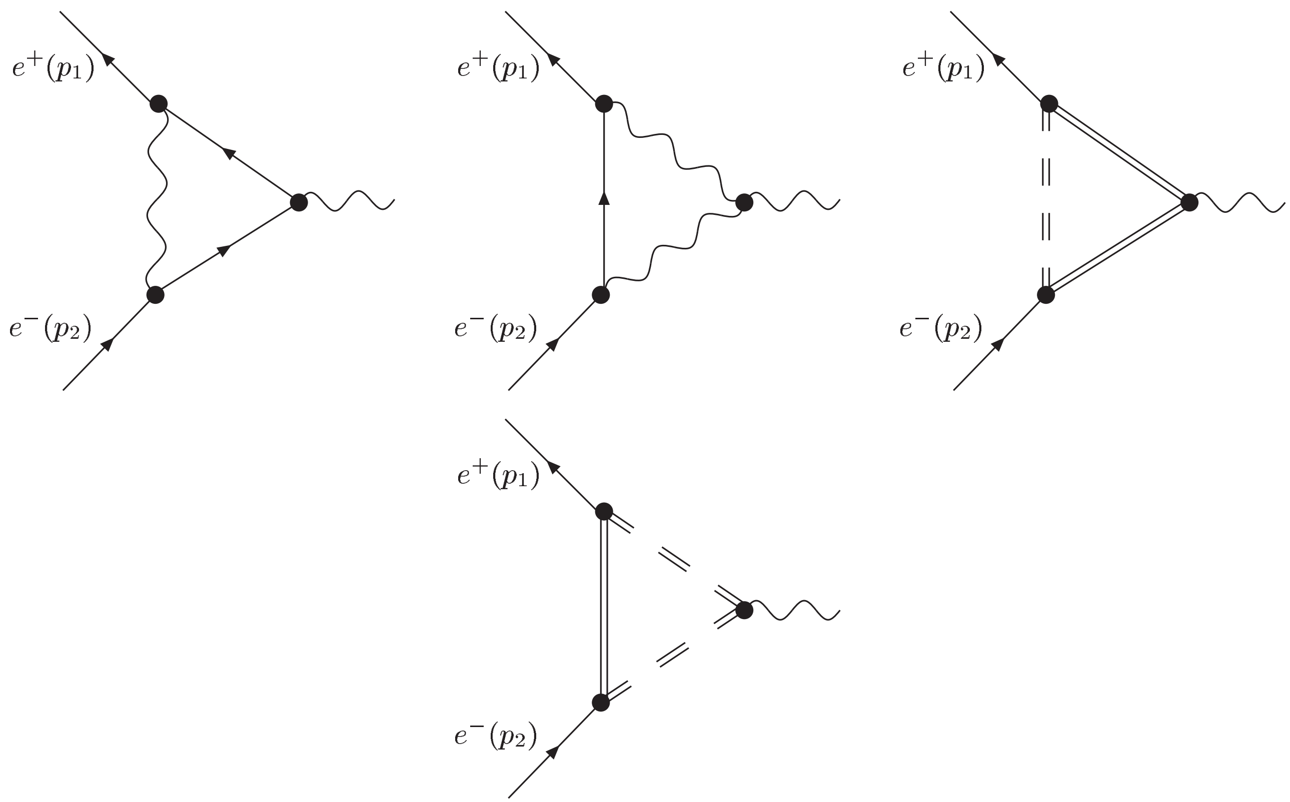
taken complex

$W^\pm \chi^\mp \chi_0$ coupling depends on two phases,

phase of μ and phase of M_1

take M_1 real

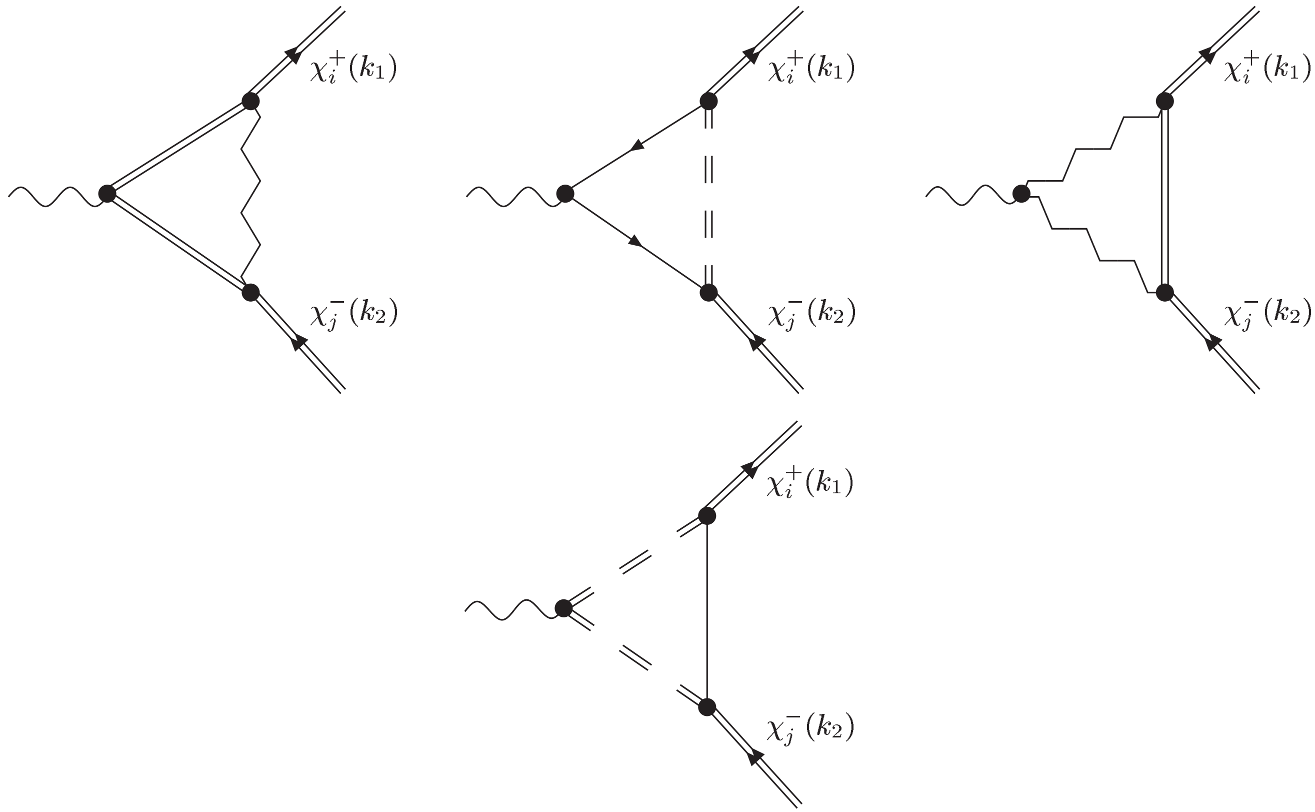
Triangle diagrams (not calculated)



> 8 diagrams

\times choice of chargino, neutralino, selectron

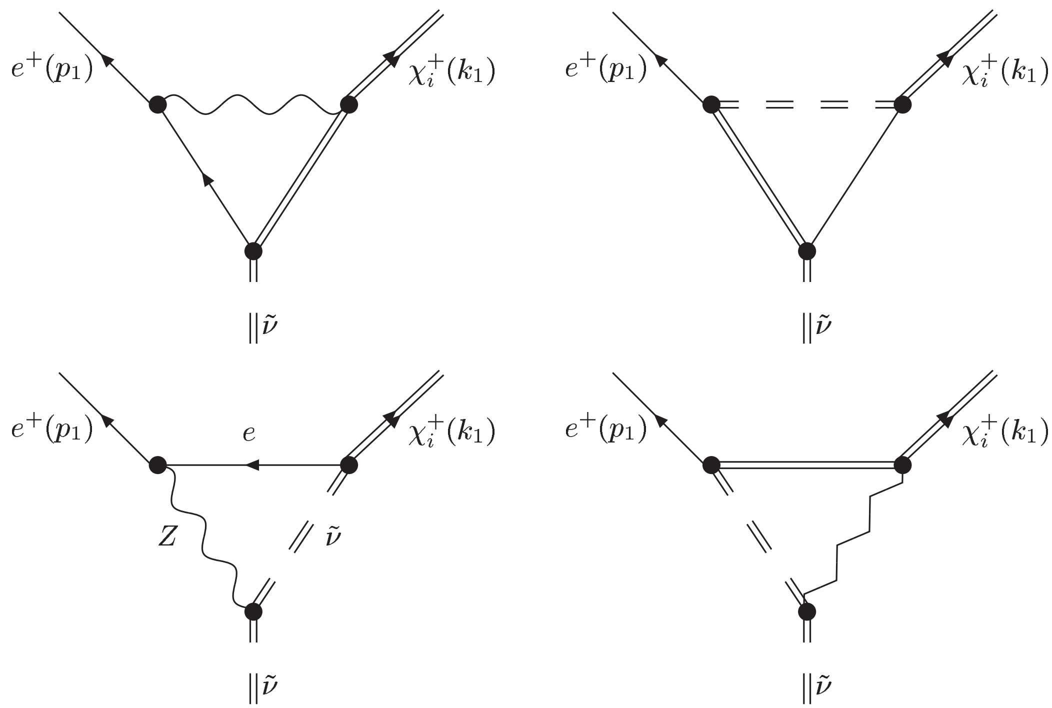
Triangle diagrams (not calculated)



18 diagrams

\times choice of chargino, neutralino, selectron

Triangle diagrams (not calculated)



9 diagrams

\times choice of chargino, neutralino, selectron, Higgs

Heavy sneutrino limit gives no simplification

Box diagrams:

- $\Upsilon\Upsilon$ give no contribution
- ΥZ cancel (D-integral part)
- ZZ “simple”
- WW complicated

Box diagram results:

$Z\chi\chi$

$$\begin{aligned}
&\left|\mathcal{M}^2\right|_{\substack{\text{CP}-\text{odd,}\\ \text{Z}-\text{box, D}}} \\
&= \frac{1}{(2\pi)^4} 2\text{Re}\Big[\frac{ig^6 m_i m_j}{(4\cos\theta_W)^6}(G_{A\,ij}G_{V\,ji} - G_{A\,ji}G_{V\,ij}) \\
&\times \big\{g_A(g_A^2 + 3g_V^2)m_Z^2(G_{V\,ii}I_{i;ji} + G_{V\,jj}I_{j;ji}) \\
&\;+ g_V(3g_A^2 + g_V^2)\big[(2m_i^2 - m_Z^2 - 2t)G_{A\,ii}I_{i;ji} \\
&\qquad\qquad\qquad + (2m_j^2 - m_Z^2 - 2t)G_{A\,jj}I_{j;ji}\big] \\
&\;+ g_A(g_A^2 + 3g_V^2)m_Z^2(G_{V\,ii}I^{\text{cr}}_{i;ji} + G_{V\,jj}I^{\text{cr}}_{j;ji}) \\
&\;- g_V(3g_A^2 + g_V^2)\big[(2m_i^2 - m_Z^2 - 2u)G_{A\,ii}I^{\text{cr}}_{i;ji} \\
&\qquad\qquad\qquad + (2m_j^2 - m_Z^2 - 2u)G_{A\,jj}I^{\text{cr}}_{j;ji}\big]\big\}\Big]
\end{aligned}$$

W box results much more complicated

Loop (box) integrals:

$$I_{k;ij} \equiv D(p_1, p_2, -k_2, -k_1, m_Z, 0, m_Z, m_{\chi_k})$$
$$I^{\mathrm{cr}}_{k;ij} \equiv D(p_1, p_2, -k_1, -k_2, m_Z, 0, m_Z, m_{\chi_k})$$

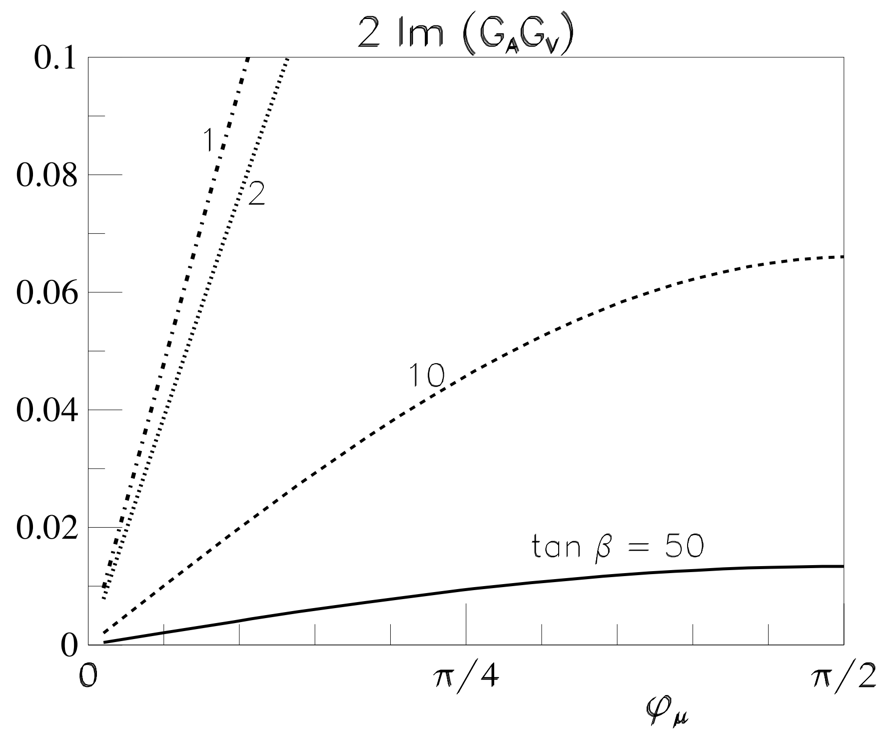
$$\begin{aligned} &D(l_1, l_2, l_3, l_4, m_1, m_2, m_3, m_4) \\ &\equiv \int \mathrm{d}^4 q \big\{ (q^2 - m_1^2)[(q + l_1)^2 - m_2^2] \\ &\qquad \times [(q + l_1 + l_2)^2 - m_3^2] \\ &\qquad \times [(q + l_1 + l_2 + l_3)^2 - m_4^2] \big\}^{-1} \end{aligned}$$

't Hooft, Veltman; Passarino, Veltman; LoopTools

Box diagram results, cont:

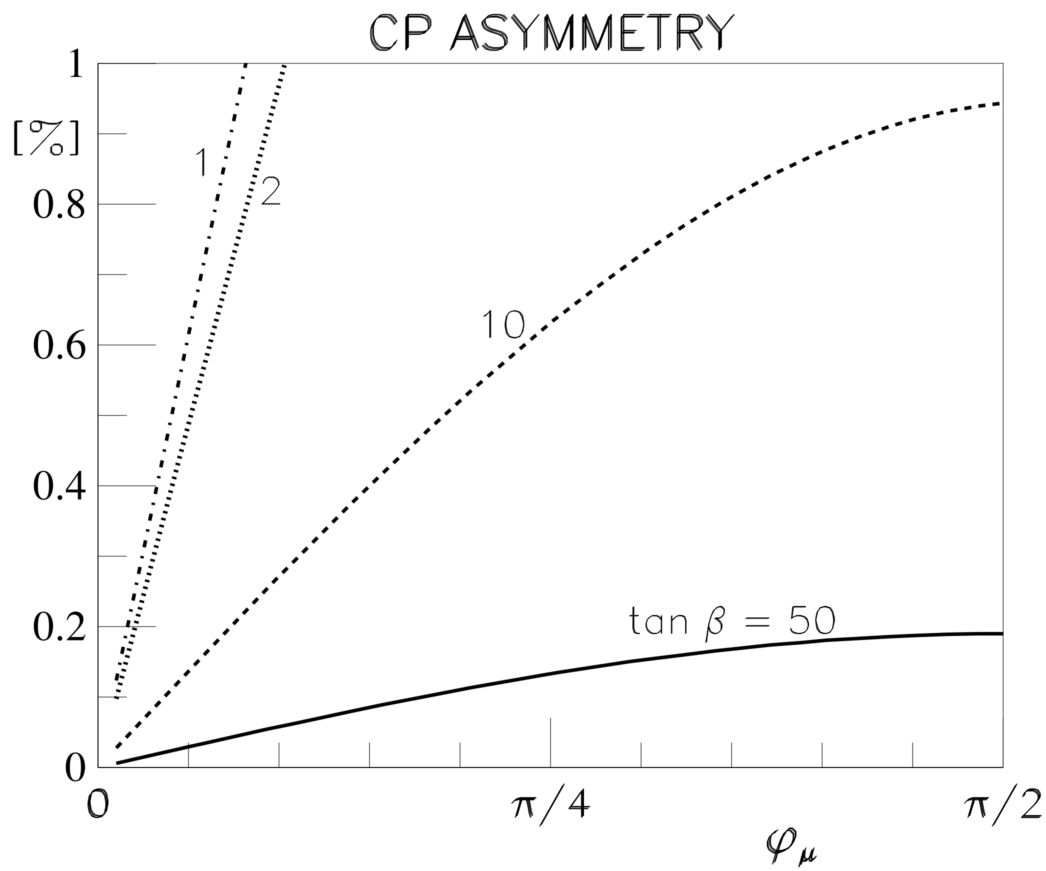
One nondiagonal $Z\chi\chi$ coupling at tree level, one at loop level

They combine to: $G_{A\,ij}G_{V\,ji} - G_{A\,ji}G_{V\,ij} = 2i\,\mathrm{Im}\,G_{A\,ij}G_{V\,ji}$



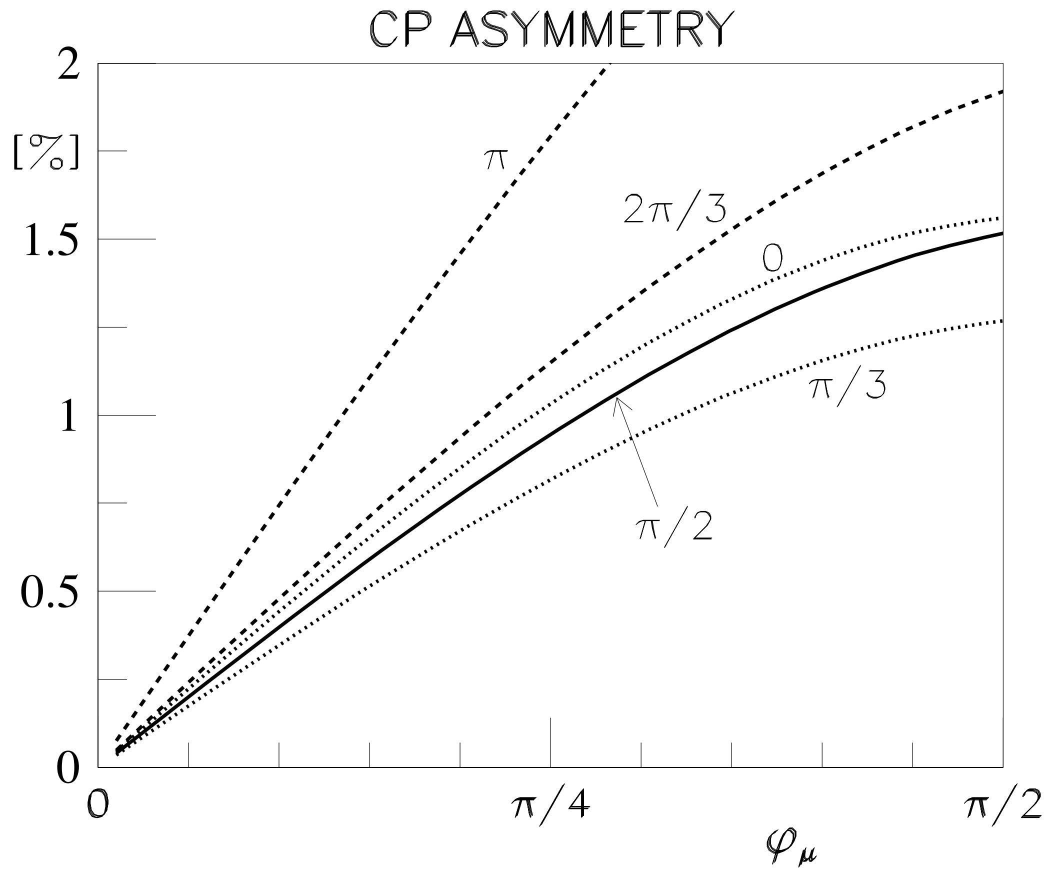
$$|\mu| = 300\ \mathrm{GeV}, \quad M_2 = 200\ \mathrm{GeV}$$

Box diagrams only (polar angle π/2):



Effect decreases with increasing energy

Smooth variation with polar angle:



Angles:
"0" means positive chargino along electron direction
(asymmetry between heavier and lighter positive chargino)

Related studies

Choi et al, hep-ph/9812236

Determine CP-violating phase ϕ_μ from asymmetries in chargino production

Choi et al, hep-ph/0001175, hep-ph/0002033

Determine $\cos\phi_\mu$, twofold asymmetry resolved by measuring chargino normal polarization

Choi et al, hep-ph/0108117, hep-ph/0202039

Unitarity quadrangles from neutralino mixing matrix give phase of M_1

Choi et al, hep-ph/021107

Review

Related studies, cont

Yang and Du, PRD 67, 055004 (2003)

$$A_{CP} = \frac{\Gamma(\tilde{\chi}_i^+ \to \tilde{\chi}_1^0 W^+) - \Gamma(\tilde{\chi}_i^- \to \tilde{\chi}_1^0 W^-)}{\Gamma(\tilde{\chi}_i^+ \to \tilde{\chi}_1^0 W^+) + \Gamma(\tilde{\chi}_i^- \to \tilde{\chi}_1^0 W^-)}$$

loop effect

Bartl et al, hep-ph/0403265

No CP violating effect involving beam polarization in triple product at tree level if at least one chargino decay is not observed

Bartl et al, hep-ph/0406309

CP violating effect involving chargino polarization measurable in chargino decay to lepton and sneutrino

Conclusions

- Non-zero CP-violating asymmetry in unpolarized cross sections
- Order of magnitude:
  - Possibly 1% (hard to measure)
- Only calculated in part
  - Triangle diagrams missing (lots of them)