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Damping Rings

Solutions to Homework Problems

Useful physical constants:

$c = 2.998\times10^{8}$ m/s
$C_\gamma = 8.846\times10^{-5}$ m/GeV^3
$C_q = 3.832\times10^{-13}$ m

Some of the parameter specifications for the damping rings are as follows:

Circumference	6.6 km
Energy	5 GeV
Injected emittance (x and y)	1 μm
Extracted horizontal emittance	0.8 nm
Extracted vertical emittance	2 pm
Equilibrium vertical emittance	1.4 pm
Maximum extracted energy spread	0.13%
Beam store time	200 ms
Lattice type	TME
Number of dipoles	120
Dipole length	6 m

In these questions, you will work towards specifications for the parameters for the damping wiggler (peak field, total length, and period), given the above parameters for the damping rings.

1. Calculate the transverse damping times required to achieve the extracted emittances starting with the specified injected emittances, in the given store time.
2. Estimate (i) the damping times, and (ii) the natural emittance that would be achieved in the lattice without any damping wiggler (i.e. with the only synchrotron radiation energy loss provided by the dipoles). Assume that the lattice is properly tuned for the minimum possible natural emittance.
3. Estimate the maximum wiggler peak field allowed by the specified extracted energy spread.
4. Assuming the wiggler peak field is the maximum allowed by the energy spread, estimate the length of damping wiggler needed to achieve the required damping times.
5. Assuming an average horizontal beta function in the wiggler of 20 m, estimate the maximum wiggler period in order to achieve the specified extracted horizontal emittance.

Solution:

1. The emittance evolves as:

$$\varepsilon(t) = \varepsilon(0)\exp\left(-\frac{2t}{\tau}\right) + \varepsilon(\infty)\left[1 - \exp\left(-\frac{2t}{\tau}\right)\right]$$

$$= \varepsilon(\infty) + [\varepsilon(0) - \varepsilon(\infty)]\exp\left(-\frac{2t}{\tau}\right)$$

where τ is the damping time. Therefore:

$$\frac{t}{\tau} = \frac{1}{2}\ln\left(\frac{\varepsilon(0) - \varepsilon(\infty)}{\varepsilon(t) - \varepsilon(\infty)}\right)$$

Substituting in values for the injected, equilibrium and extracted vertical emittance, we get:

$$\frac{t}{\tau} \approx 7.16$$

So, with the store time t = 200 ms, we get:

$$\tau \approx \frac{t}{7.16} \approx 27.9\ \text{ms}$$

The horizontal damping time will be approximately the same; but since a very much smaller extracted emittance is required in the vertical, the damping time requirements are set by the vertical emittance.

2. (i) The dipoles would form a ring of circumference 720 m, so the bending radius must be 720 m / 2π = 114.6 m. Therefore, the dipoles make a contribution to the second synchrotron radiation integral:

$$I_{2,dip} = \frac{2\pi}{\rho} \approx 0.0548\ \text{m}^{-1}$$

If the dipoles were the only source of synchrotron radiation energy loss, the energy loss per turn would be:

$$U_0 = \frac{C_\gamma}{2\pi} E_0^4 I_{2,dip} \approx 482\ \text{keV}$$

The transverse damping times (assuming the damping partition numbers are equal to 1) would then be:

$$\tau_x = \tau_y = 2\frac{E_0}{U_0} T_0 \approx 457\ \text{ms}$$

(ii) For a TME lattice, the natural emittance is given by:

$$\varepsilon_0 = \frac{1}{12\sqrt{15}} C_q \gamma^2 \theta^3$$

So with a beam energy of 5 GeV, and 120 dipoles, we find:

$$\varepsilon_0 \approx 0.11\,\text{nm}$$

3. If the synchrotron radiation energy loss is dominated by the wiggler (which will need to be the case to achieve the specified damping times), the equilibrium energy spread is related to the wiggler peak field by:

$$\sigma_\delta^2 \approx \frac{4}{3\pi} C_q \frac{\gamma^2}{\rho_w} = \frac{4}{3\pi} \frac{e}{mc} C_q \gamma B_w$$

With the maximum equilibrium energy spread 0.15%, the maximum wiggler peak field is:

$$B_w \approx 1.81\,\text{T}$$

4. Let us start by calculating the required energy loss per turn. This is related to the transverse damping time by:

$$\tau_x = \tau_y = 2\frac{E_0}{U_0} T_0 \approx 27.9\,\text{ms}$$

With the given beam energy and circumference, we find:

$$U_0 \approx 7.89\,\text{MeV}$$

Now we calculate the required value for the second synchrotron radiation integral, I_2. This is related to the energy loss per turn by:

$$U_0 = \frac{C_\gamma}{2\pi} E_0^4 I_2 \approx 7.89\,\text{MeV}$$

Hence, we find that:

$$I_2 \approx 0.897\,\text{m}^{-1}$$

From part (b), we know that the dipoles make a contribution to the second synchrotron radiation integral:

$$I_{2,dip} = \frac{2\pi}{\rho} \approx 0.0548\,\text{m}^{-1}$$

which is much smaller than the total value of I_2 required. The rest must be contributed by the wiggler:

$$I_{2,wig} \approx 0.842\,\text{m}^{-1}$$

The wiggler contribution is related to the peak field, wiggler length, and beam rigidity by:

$$I_{2,wig} = \frac{1}{(B\rho)^2} \frac{B_w^2 L_w}{2} \approx 0.842\,\text{m}^{-1}$$

With a beam energy of 5 GeV, the rigidity is 16.68 Tm; so with a peak field of 1.81 T, the total length of wiggler required is:

$$L_w \approx 143\,\mathrm{m}$$

5. The synchrotron radiation energy loss is dominated by the wiggler (94%), so we assume that we can neglect the dipole contribution to the natural emittance (this isn't completely true, but a good approximation).

   In this case, the natural emittance is given by:

$$\varepsilon_0 \approx \frac{8}{15\pi} C_q \gamma^2 \frac{\langle \beta_x \rangle}{\rho_w^3 k_w^2}$$

   where ρ_w is the bending radius corresponding to the peak field of the wiggler, and:

$$k_w = \frac{2\pi}{\lambda_w}$$

   where λ_w is the wiggler period.

   Assuming an average beta function of 20 m, and a natural emittance close to the specified extracted emittance, we find:

$$\lambda_w \approx 0.445\,\mathrm{m}$$

   In practice, the wiggler period will probably need to be somewhat smaller than this, to allow some margin between the natural emittance and the specified extracted emittance, and (in the likely case that the lattice isn't perfectly tuned for the lowest possible emittance) to allow for the quantum excitation from the main dipole magnets in the arcs.