RF Control Challenges for the X-FEL: Cavity Model and Controller Design

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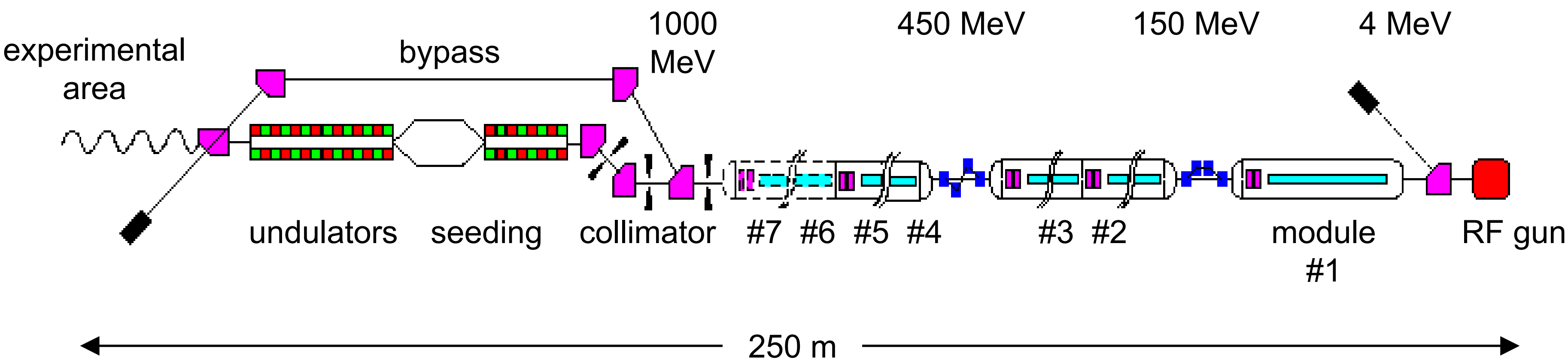
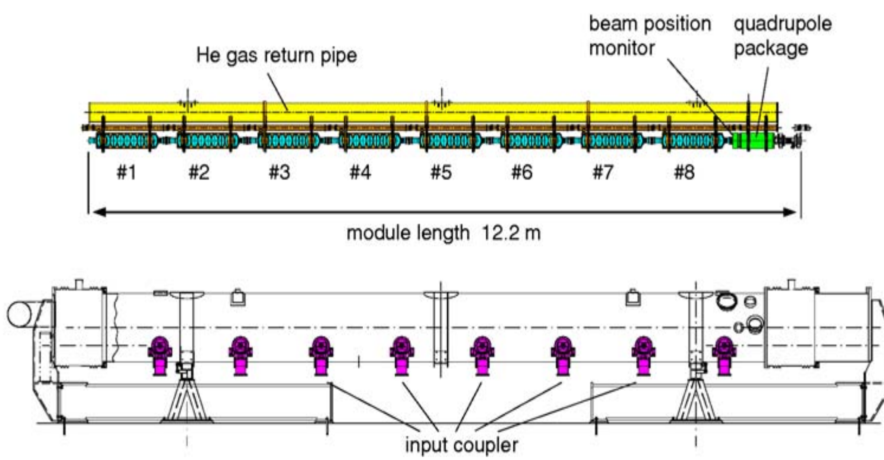
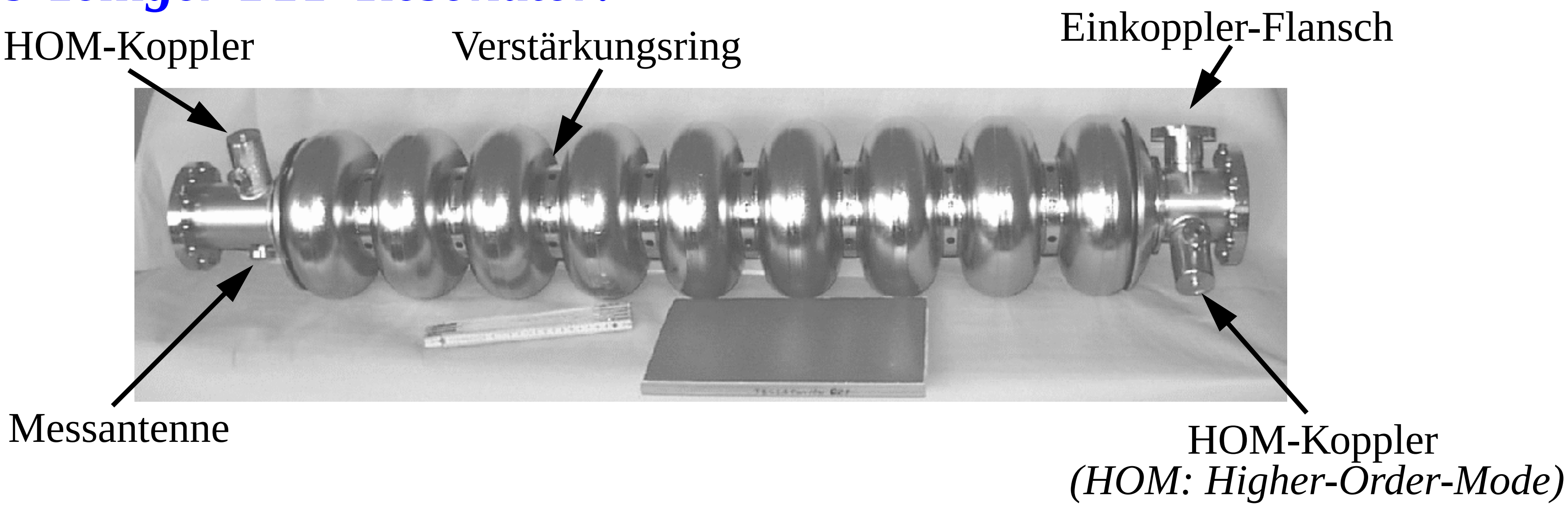
VUV-FEL


experimental area
bypass
1000 MeV
450 MeV
150 MeV
4 MeV
undulators
seeding
collimator
#7 #6 #5 #4
#3 #2
module #1
RF gun
250 m


He gas return pipe
beam position monitor
quadrupole package
#1
#2
#3
#4
#5
#6
#7
#8
module length 12.2 m
input coupler

- ***9-zelliger TTF Resonator:***

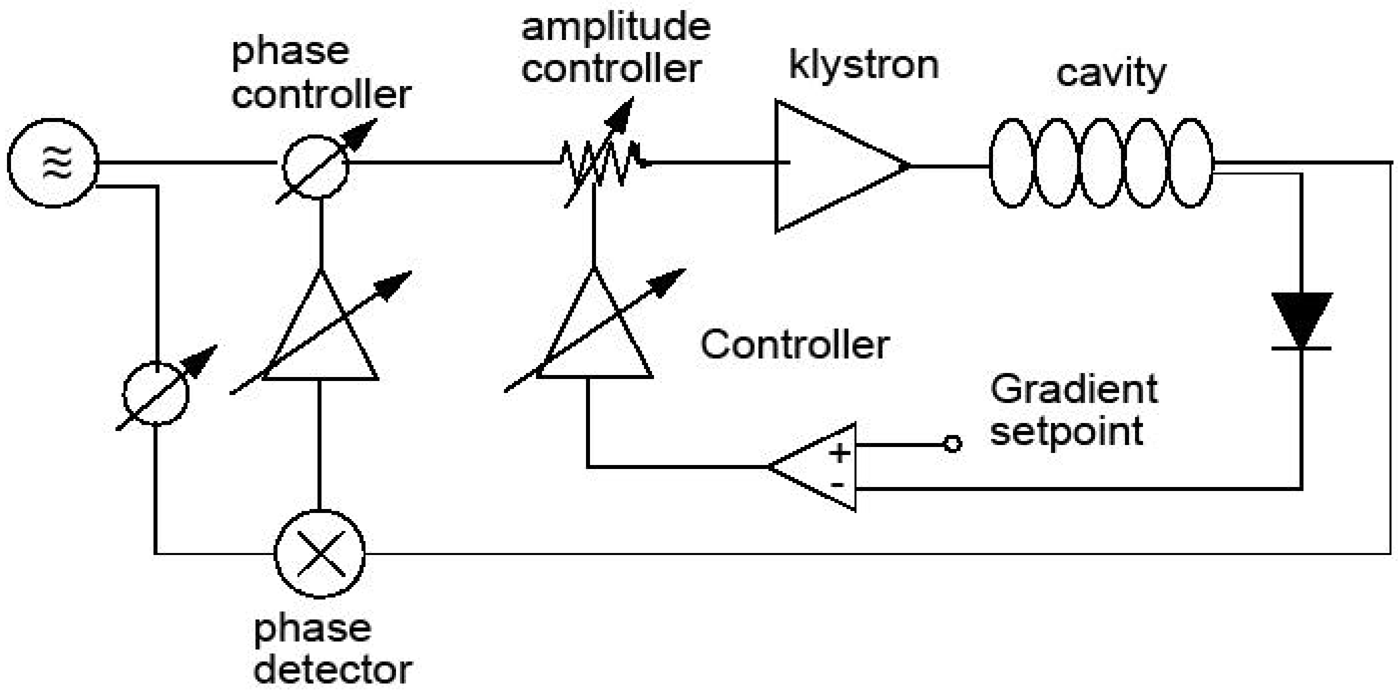


Parameter	Wert
Resonatortyp	Stehwelle, 9 Zellen
Beschleunigungsmode	TM_{010}
Frequenz der Beschl.-mode	1300 MHz
aktive Länge	1.038 m
Δf / ΔL	315 Hz / µm
unbelastete Güte	$>10^{10}$
belastete Güte, Bandbreite	$2.5\ 10^6$, 260 Hz

RF Control Model

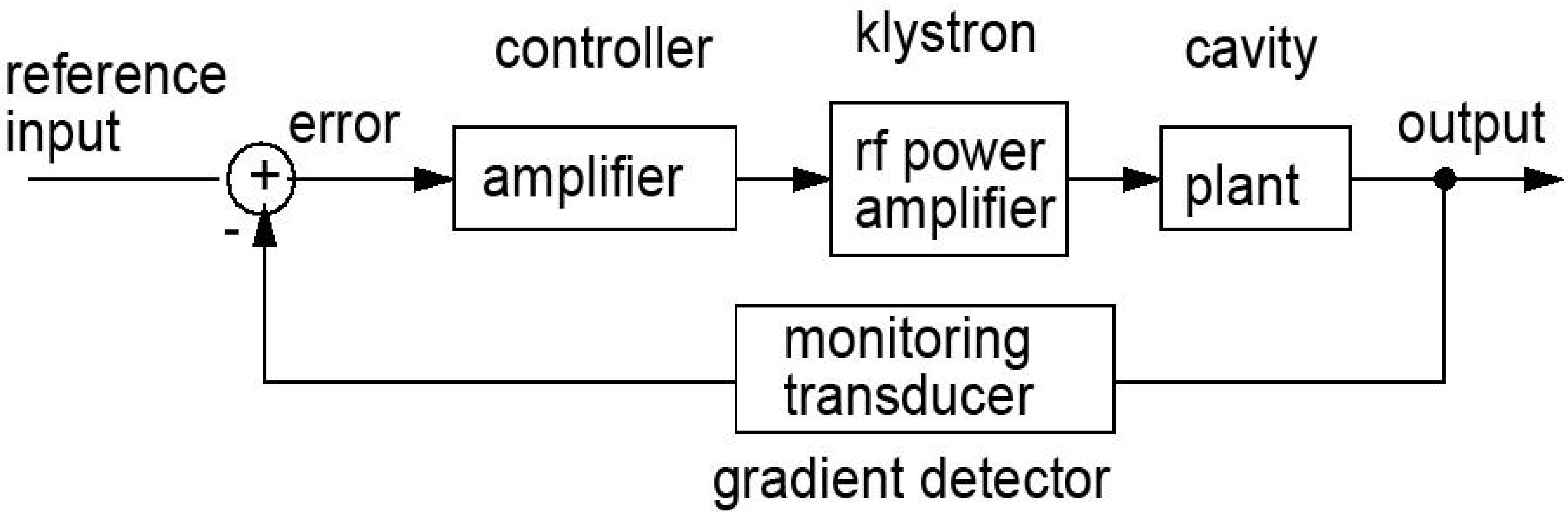
Goal:
Maintain stable gradient and phase

Solution:
Feedback for gradient amplitude and phase:

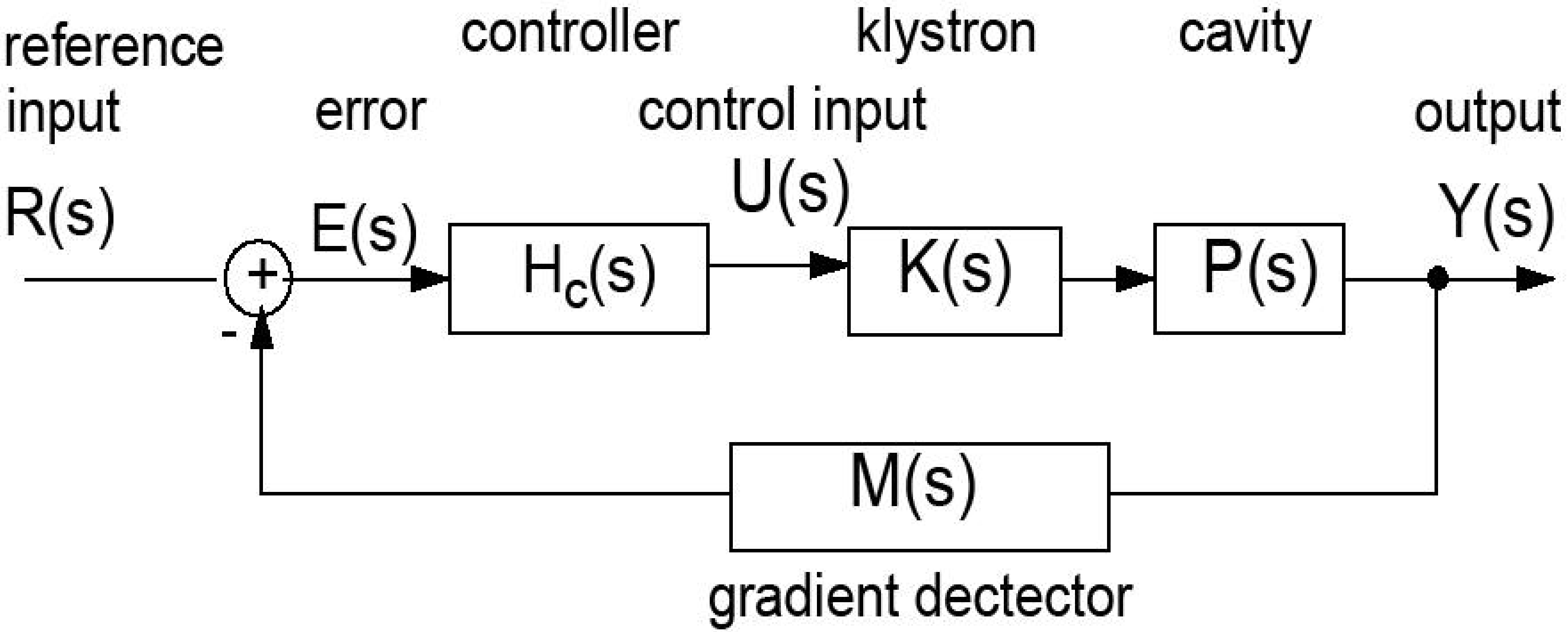


Model:
Mathematical description of input-output relation of components combined with block diagram:

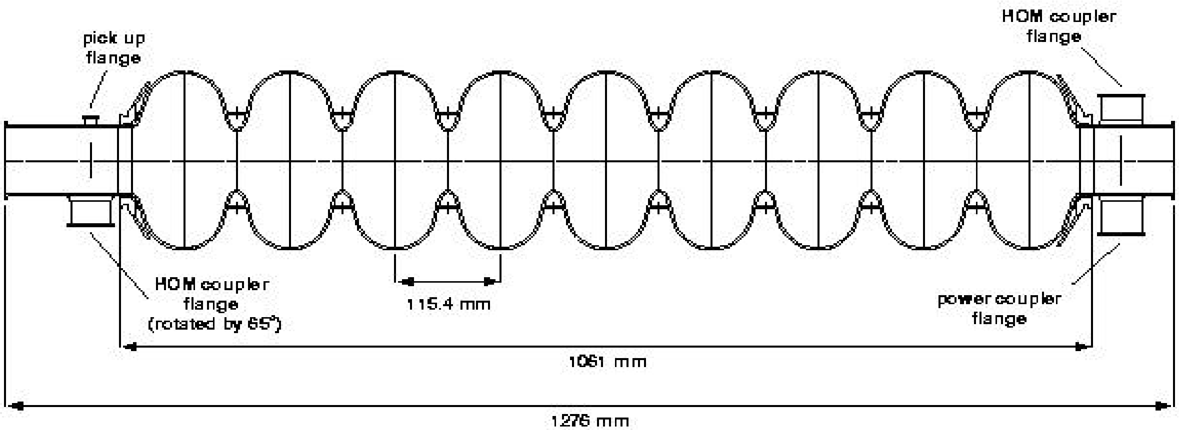
Amplitude Loop (general form):



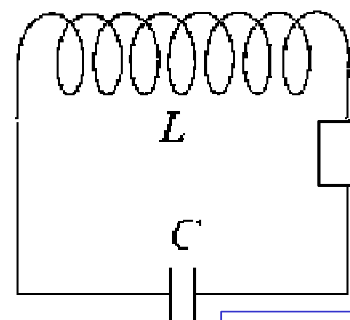
RF Control model using “transfer functions”



Kavitäten für TESLA

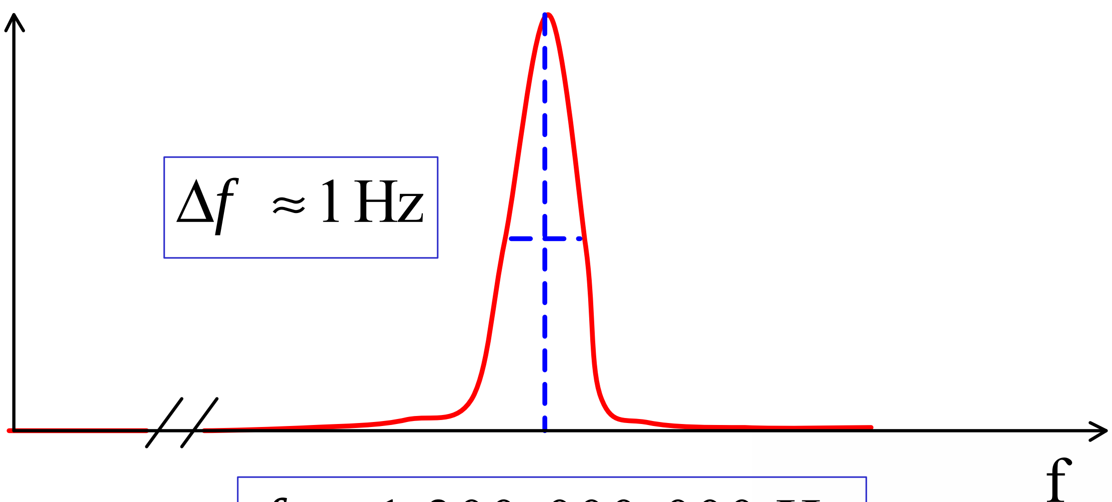


Schwingkreis:



Frequenz: $$f_o = \frac{1}{2p\sqrt{LC}}$$

Gütefaktor: $$Q_o = \frac{f}{\Delta f} = \frac{G}{R_s}$$

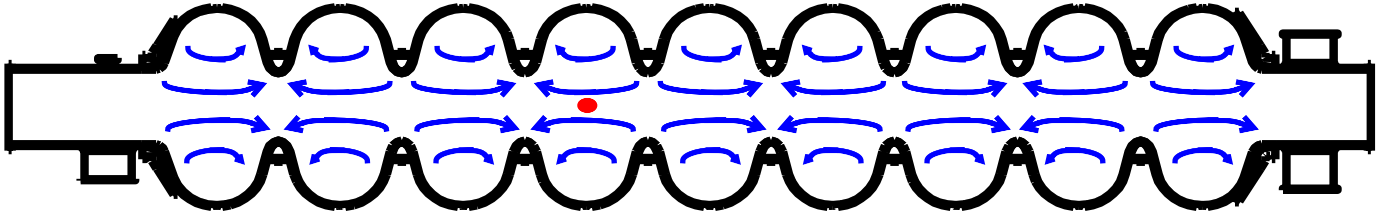


$$\Delta f \approx 1\,\text{Hz}$$

$$f_o = 1.300.000.000\ \text{Hz}$$

Þ $Q0$ » 10^{9} -10^{10}

- ***Beschleunigung:***



Beschleunigungsspannung:

$$V_{acc} = \frac{\text{maximaler Energiegewinn}}{\text{Ladung}} = \left| \int_{-L/2}^{L/2} E_z e^{i\omega(z/c)} dz \right|$$

Shunt-Impedanz:

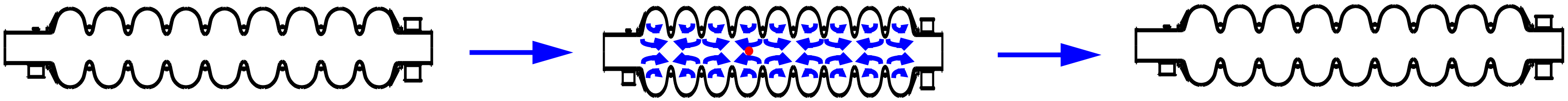
$$R_{\text{sh}} = \frac{(V_{acc})^2}{2P_{\text{Wand}}}$$

Unbelastete Güte:

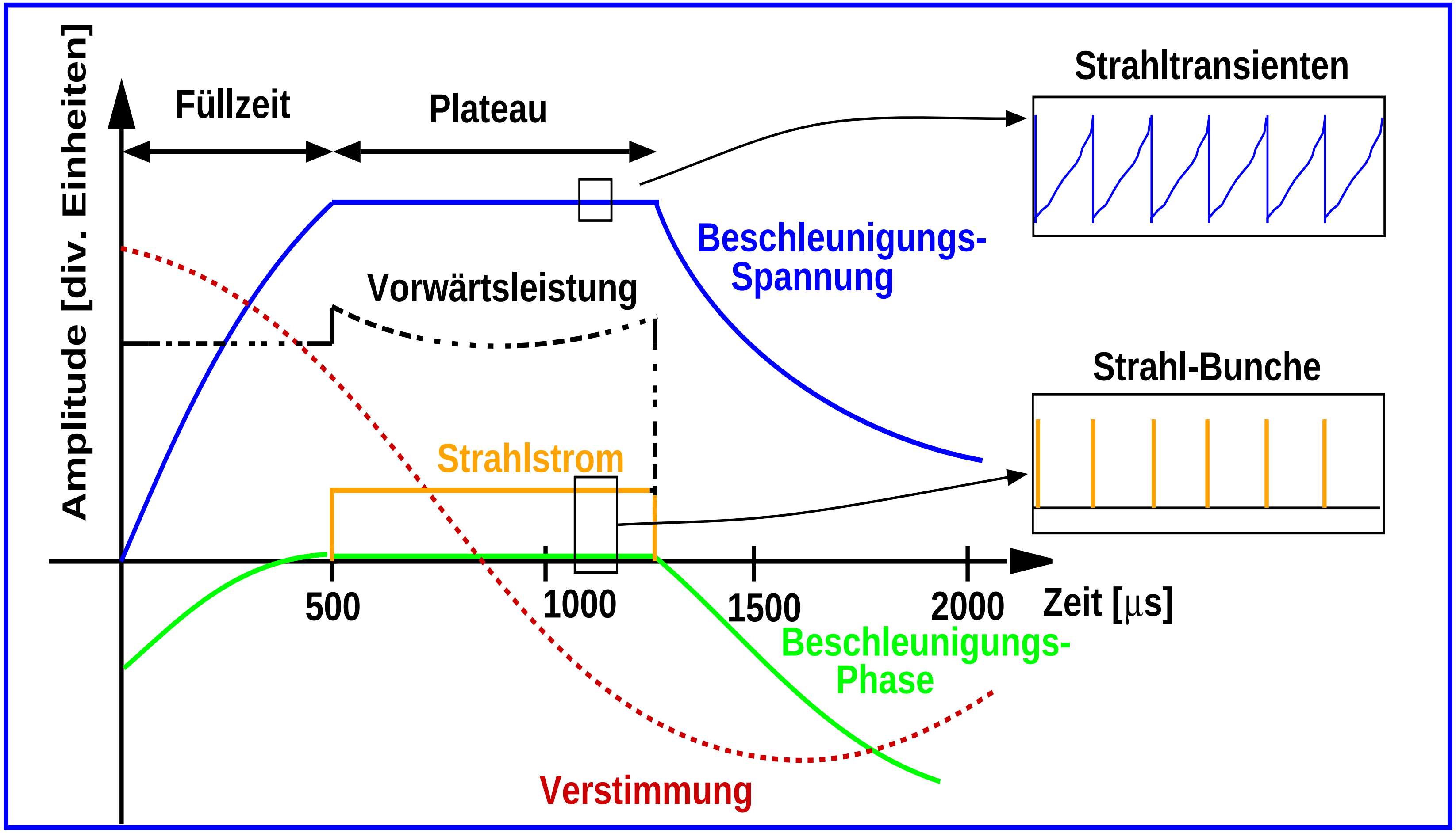
$$Q_0 = \frac{\omega W}{P_{\text{Wand}}}$$

$$\left(\frac{R_{\text{sh}}}{Q_0}\right) = \frac{(V_{acc})^2}{2\omega W} = 518\ \Omega$$

• ***Gepulster Beschleunigerbetrieb:***

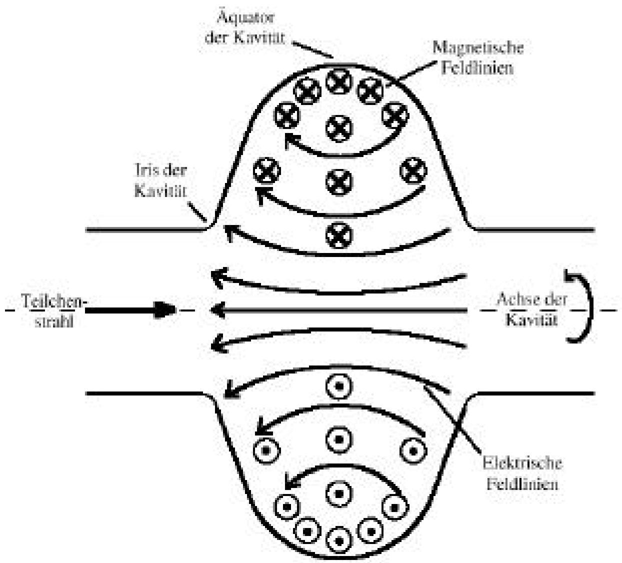


5 bis 10 Hz Wiederholrate, 950 µs konstantes Beschleunigungsfeld



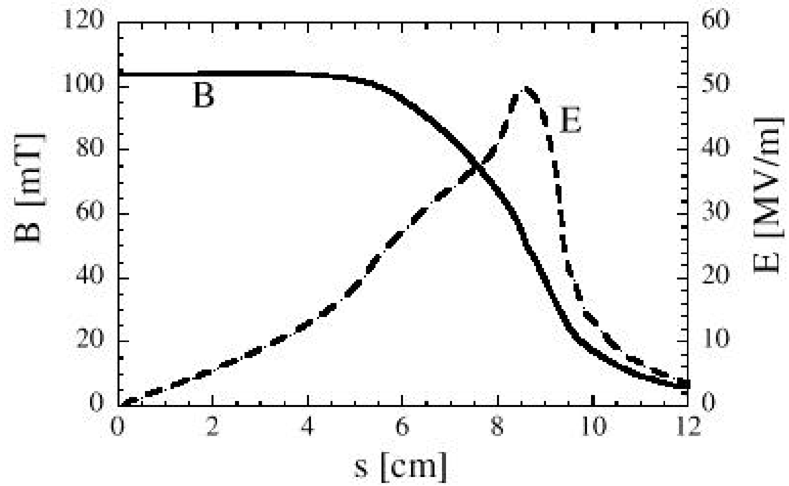
Field distributions in cavities

Elliptical cavity:



Numerical solution for surface fields:

Þ

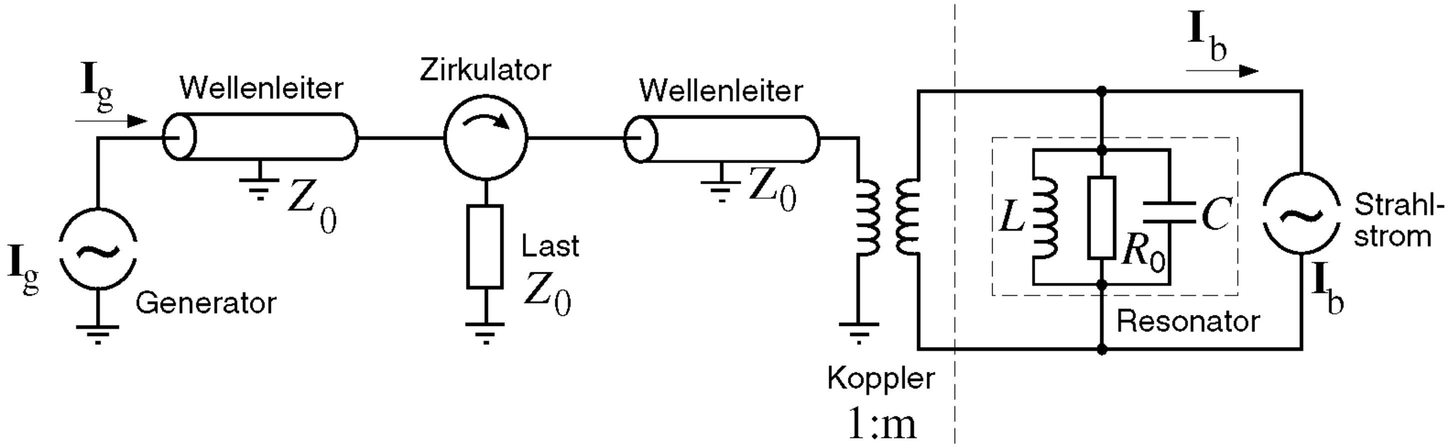


Relations for the surface fields to acclerating gradient:

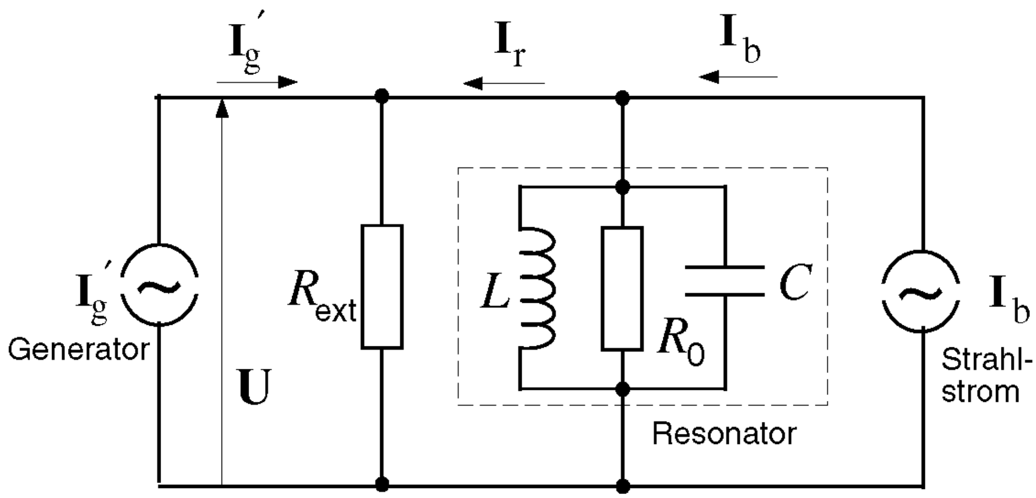
E_{peak}/E_{acc} = 1,98 **minimize this to reduce field emission**

Þ **B_{peak}/E_{acc} = 4,17 [mT]/[MV/m]** **minimize because of maximum critical field of the superconductor**

Cavity Model



Equivalent circuits



$$C \cdot \ddot{U} + \frac{1}{R_L} \cdot \dot{U} + \frac{1}{L} \cdot U = \dot{I}_g' + \dot{I}_b$$ L.O.D.E.

with $$\omega_{1/2} := \frac{1}{2R_L C} = \frac{\omega_0}{2Q_L}$$

$$\ddot{U} + 2\omega_{1/2} \cdot \dot{U} + \omega_0^2 \cdot U = 2R_L\omega_{1/2} \cdot (\frac{2}{m}\dot{I}_g + \dot{I}_b)$$

Only envelope of rf (real and imaginary part) is of interest:

$$\begin{aligned}\mathbf{U}(t) &= (U_r(t) + iU_i(t)) \cdot \exp(i\omega_{HF}t) \\ \mathbf{I}_g(t) &= (I_{gr}(t) + iI_{gi}(t)) \cdot \exp(i\omega_{HF}t) \\ \mathbf{I}_b(t) &= (I_{b\omega r}(t) + iI_{b\omega i}(t)) \cdot \exp(i\omega_{HF}t) = 2(I_{b0r}(t) + iI_{b0i}(t)) \cdot \exp(i\omega_{HF}t)\end{aligned}$$

Neglect small terms in derivatives for U and I

$$\begin{aligned}\ddot{U}_r + i\ddot{U}_i(t) &\ll \omega_{HF}^2(U_r(t) + iU_i(t)) \\ 2\omega_{1/2}(\dot{U}_r + i\dot{U}_i(t)) &\ll \omega_{HF}^2(U_r(t) + iU_i(t))\end{aligned}$$

$$\int_{t_1}^{t_2}(\dot{I}_r(t) + i\dot{I}_i(t))dt \ll \int_{t_1}^{t_2}\omega_{HF}(I_r(t) + iI_i(t))dt$$

Envelope equations for real and imaginary component

$$\begin{aligned}\dot{U}_r(t) + \omega_{1/2} \cdot U_r + \Delta\omega \cdot U_i &= \omega_{HF}\left(\frac{r}{Q}\right) \cdot \left(\frac{1}{m}I_{gr} + I_{b0r}\right) \\ \dot{U}_i(t) + \omega_{1/2} \cdot U_i - \Delta\omega \cdot U_r &= \omega_{HF}\left(\frac{r}{Q}\right) \cdot \left(\frac{1}{m}I_{gi} + I_{b0i}\right)\end{aligned}$$

Matrix equations:

$$\begin{bmatrix} \dot{U}_r(t) \\ \dot{U}_i(t) \end{bmatrix} = \begin{bmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{bmatrix} \cdot \begin{bmatrix} U_r(t) \\ U_i(t) \end{bmatrix}$$

$$+ \omega_{HF}\Big(\frac{r}{Q}\Big) \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{m} I_{gr}(t) + I_{b0r}(t) \\ \frac{1}{m} I_{gi}(t) + I_{b0i}(t) \end{bmatrix}$$

With System Matrices

$$\mathbf{A} = \begin{bmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{bmatrix} \qquad \mathbf{B} = \omega_{HF}\Big(\frac{r}{Q}\Big) \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\vec{x}(t) = \begin{bmatrix} U_r(t) \\ U_i(t) \end{bmatrix} \qquad \vec{u}(t) = \begin{bmatrix} \frac{1}{m} I_{gr}(t) + I_{b0r}(t) \\ \frac{1}{m} I_{gi}(t) + I_{b0i}(t) \end{bmatrix}$$

General Form

$$\dot{\vec{x}}(t) = \mathbf{A} \cdot \vec{x}(t) + \mathbf{B} \cdot \vec{u}(t)$$

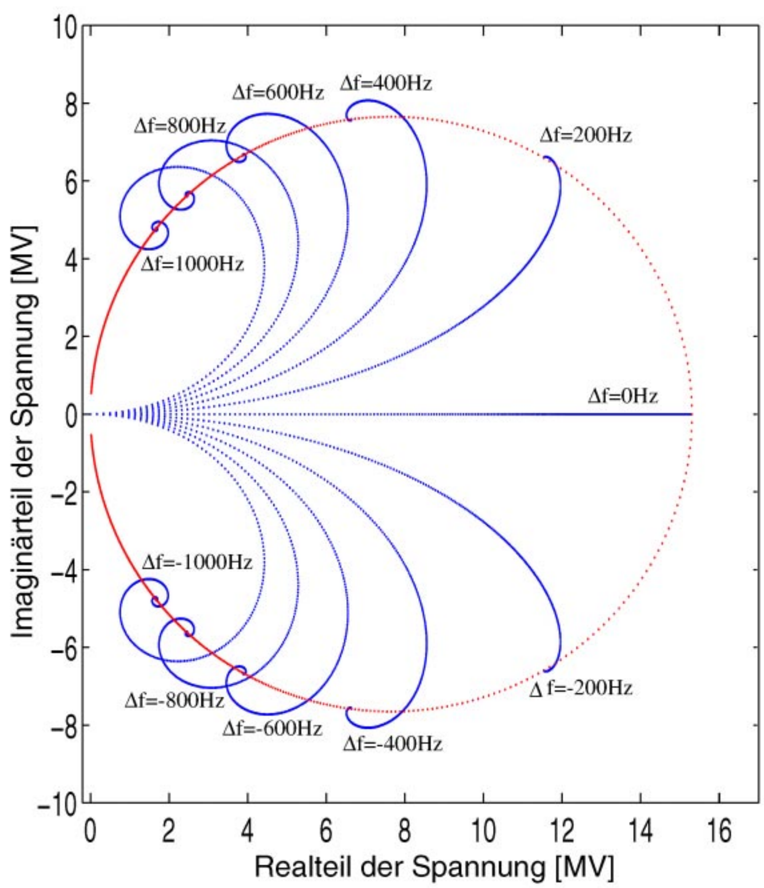
Solution

$$\vec{x}(t) = \mathbf{\Phi}(t) \cdot \vec{x}(0) + \int_0^t \mathbf{\Phi}(t-t') \cdot \mathbf{B} \cdot \vec{u}(t') dt'$$

$$\mathbf{\Phi}(t) \;=\; e^{-\omega_{1/2} t} \begin{bmatrix} \cos(\Delta\omega t) & -\sin(\Delta\omega t) \\ \sin(\Delta\omega t) & \cos(\Delta\omega t) \end{bmatrix}$$

special case $u(t) = \begin{bmatrix} \frac{1}{m} I_{gr}(t) + I_{b0r}(t) \\ \frac{1}{m} I_{gi}(t) + I_{b0i}(t) \end{bmatrix} =: \begin{bmatrix} I_r \\ I_i \end{bmatrix}$

$$\begin{bmatrix} U_r(t) \\ U_i(t) \end{bmatrix} = \frac{\omega_{HF}\left(\frac{r}{Q}\right)}{\omega_{1/2}^2 + \Delta\omega^2} \cdot \begin{bmatrix} \omega_{1/2} & -\Delta\omega \\ \Delta\omega & \omega_{1/2} \end{bmatrix}$$
$$\cdot \left\{ \mathbf{1} - \begin{bmatrix} \cos(\Delta\omega t) & -\sin(\Delta\omega t) \\ \sin(\Delta\omega t) & \cos(\Delta\omega t) \end{bmatrix} e^{-\omega_{1/2} t} \right\} \cdot \begin{bmatrix} I_r \\ I_i \end{bmatrix}$$



- Continuous Model

$$\begin{bmatrix} \dot{v}_r \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} -\omega_{1/2} & -\Delta\omega(t) \\ \Delta\omega(t) & -\omega_{1/2} \end{bmatrix} \cdot \begin{bmatrix} v_r \\ v_i \end{bmatrix} + \begin{bmatrix} R \cdot \omega_{1/2} & 0 \\ 0 & R \cdot \omega_{1/2} \end{bmatrix} \cdot \begin{bmatrix} I_r \\ I_i \end{bmatrix}$$

where $\omega_{1/2} = \frac{\omega_{rf}}{2Q}$ and $\Delta\omega(t) = \omega_0(t) - \omega_{rf}$

State Space Form

$$\dot{x} = A \cdot x + B \cdot u$$

$$y = C \cdot x + D \cdot u$$

with solution

$$x(t) = e^{A \cdot t} \cdot x(0) + \int_0^t e^{A \cdot \tau} \cdot B \cdot u(t - \tau) \cdot d\tau$$

• Discrete Model

State Space Form
$$x_{k+1} = A_d \cdot x_k + B_d u_k$$
$$y_k = C_d \cdot x_k + D_d u_k$$

where $A_d = e^{AT_s}$ $\quad B_d = \int_0^{T_s} e^{A\tau} B d\tau$ $\quad C_d = C$ $\quad D_d = D$

$$A_d = e^{-\omega_{1/2} \cdot T_s} \cdot \begin{bmatrix} \cos(\Delta\omega T_s) & -\sin(\Delta\omega T_s) \\ \sin(\Delta\omega T_s) & \cos(\Delta\omega T_s) \end{bmatrix} \approx \begin{bmatrix} 1 - \omega_{1/2} T_s & -\Delta\omega T_s \\ \Delta\omega T_s & 1 - \omega_{1/2} T_s \end{bmatrix}$$

$$B_d = \quad \ldots\ldots \quad \approx \begin{bmatrix} \omega_{1/2} T_s & \Delta\omega\omega_{1/2} T_s^2/2 \\ \Delta\omega\omega_{1/2} T_s^2/2 & \omega_{1/2} T_s \end{bmatrix}$$

with solution
$$x(k) = A^k \cdot x(0) + \sum_{i=1}^{k} A^{i-1} \cdot B \cdot u(k-i)$$

Discrete Cavity Model

Converting the transferfunction from the continuous cavity model to the discrete model:

$$H(s) = \frac{\omega_{12}}{\Delta\omega^2 + (s+\omega_{12})^2} \begin{bmatrix} s+\omega_{12} & -\Delta\omega \\ \Delta\omega & s+\omega_{12} \end{bmatrix}$$

The discretization of the model is represented by the z-transform:

$$H(z) = \left(1-\frac{1}{z}\right) Z\left(\frac{H(s)}{s}\right) = \frac{z-1}{z} \cdot Z\left\{L^{-1}\left\{\frac{H(s)}{s}\right\}\right\}\Bigg|_{t = kT_s}$$

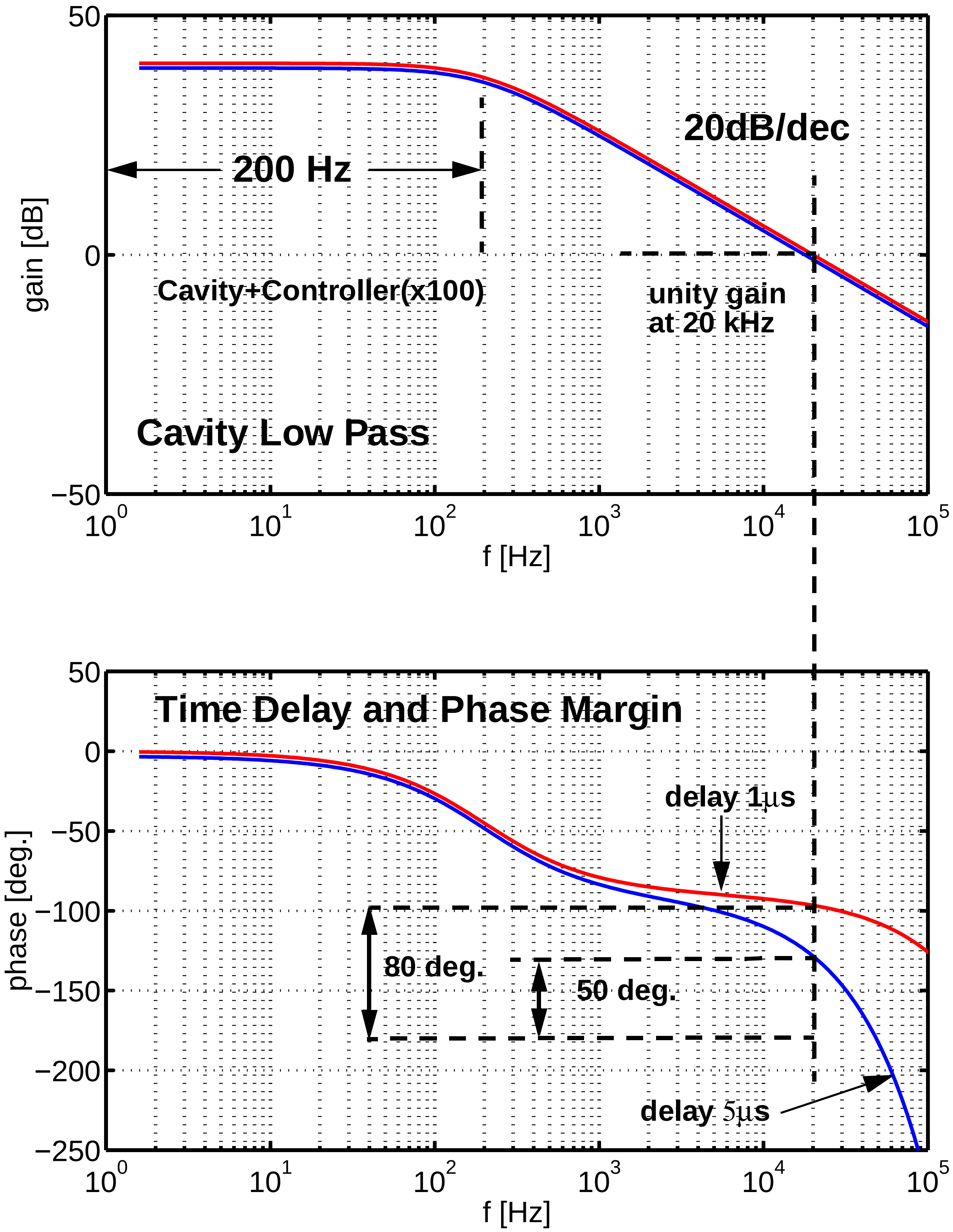
$$H(z) = \frac{\omega_{12}}{\Delta\omega^2 + \omega_{12}^2} \cdot \begin{bmatrix} \omega_{12} & -\Delta\omega \\ \Delta\omega & \omega_{12} \end{bmatrix}$$

$$-\left(\frac{\omega_{12}}{\Delta\omega^2 + \omega_{12}^2} \cdot \frac{z-1}{z^2 - 2ze^{\omega_{12}T_s} \cdot \cos(\Delta\omega T_s) + e^{2\omega_{12}T_s}}\right)$$

$$\cdot \left\{\left((z - e^{\omega_{12}T_s} \cdot \cos(\Delta\omega T_s)) \cdot \begin{bmatrix} \omega_{12} & -\Delta\omega \\ \Delta\omega & \omega_{12} \end{bmatrix}\right)\right.$$

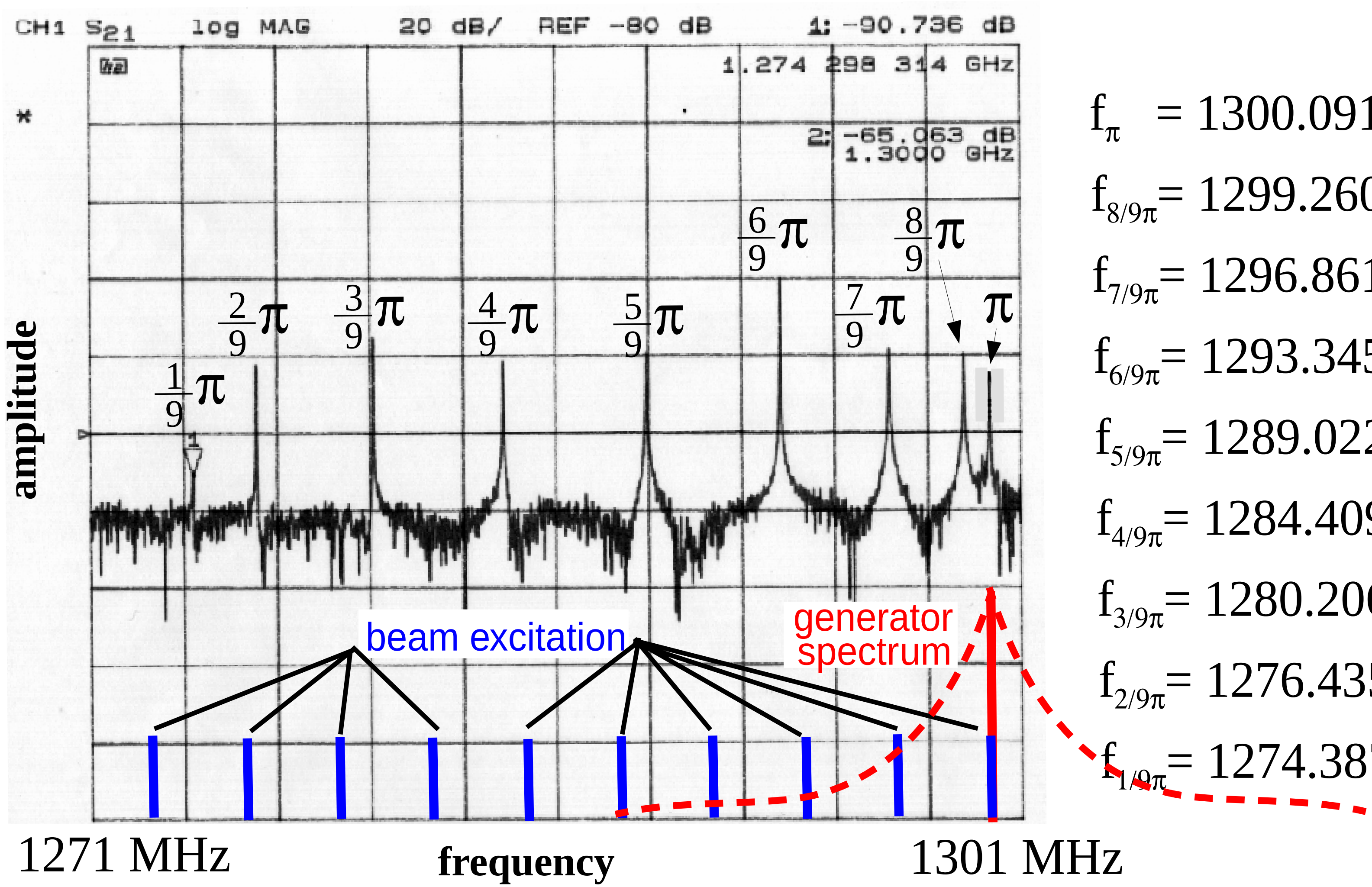
$$\left. - e^{\omega_{12}T_s} \cdot \sin(\Delta\omega T_s) \cdot \begin{bmatrix} \Delta\omega & \omega_{12} \\ -\omega_{12} & \Delta\omega \end{bmatrix} \right\}$$

Cavity Transfer Function



Excitation of other Passband Modes

Example: TESLA 9-cell cavity



f_{π} = 1300.091 MHz

$f_{8/9\pi}$ = 1299.260 MHz

$f_{7/9\pi}$ = 1296.861 MHz

$f_{6/9\pi}$ = 1293.345 MHz

$f_{5/9\pi}$ = 1289.022 MHz

$f_{4/9\pi}$ = 1284.409 MHz

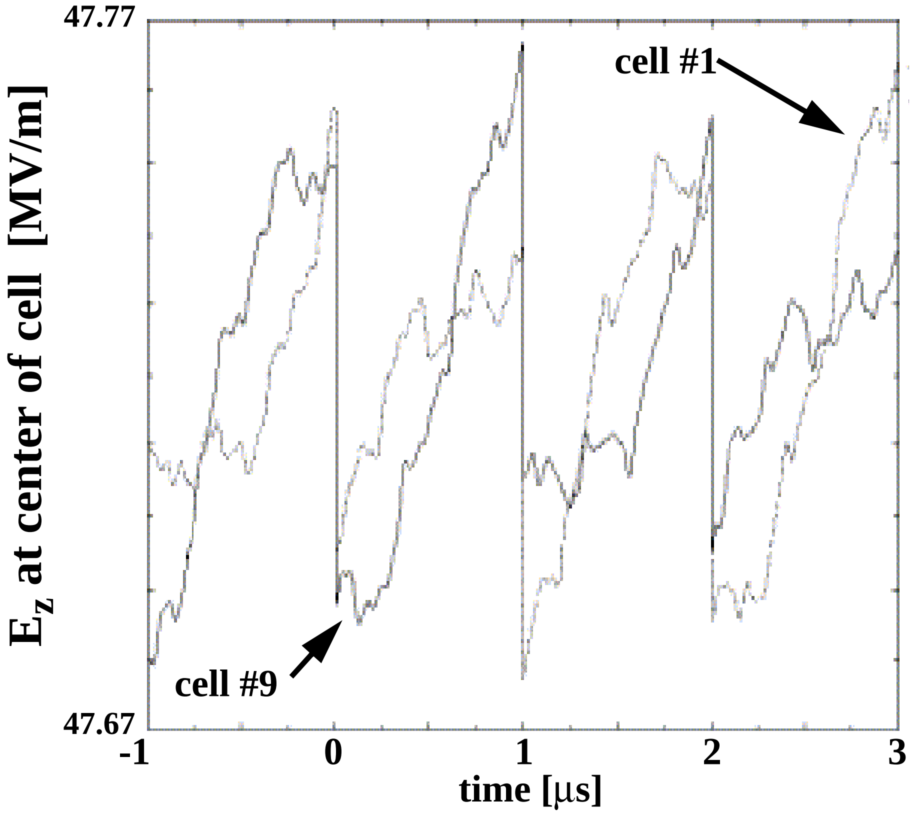
$f_{3/9\pi}$ = 1280.206 MHz

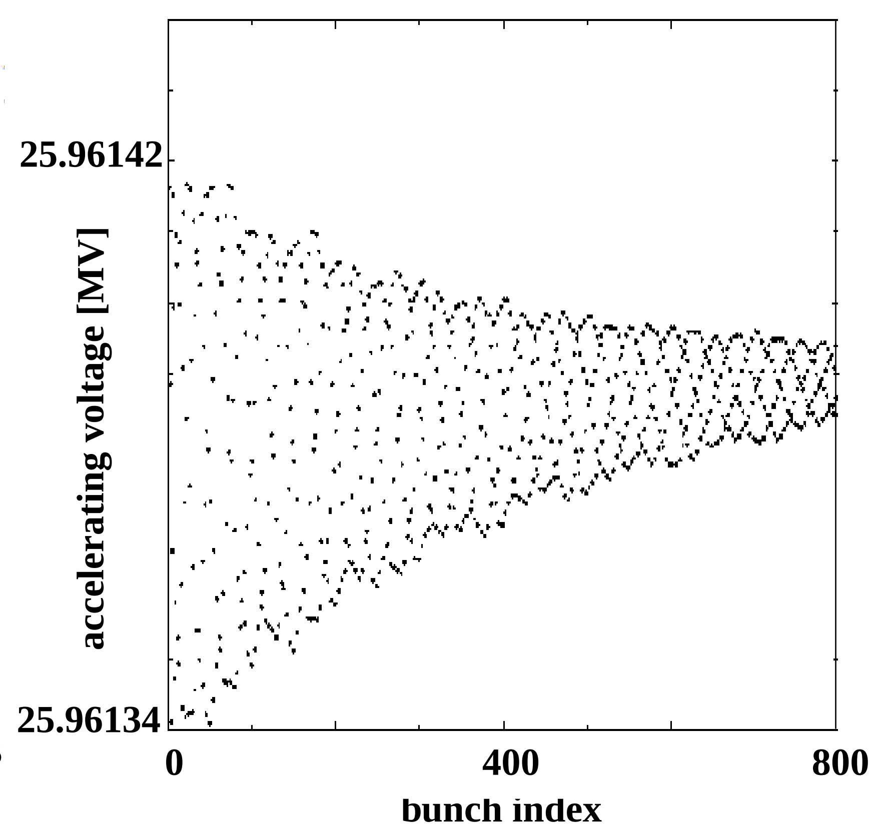
$f_{2/9\pi}$ = 1276.435 MHz

$f_{1/9\pi}$ = 1274.387 MHz

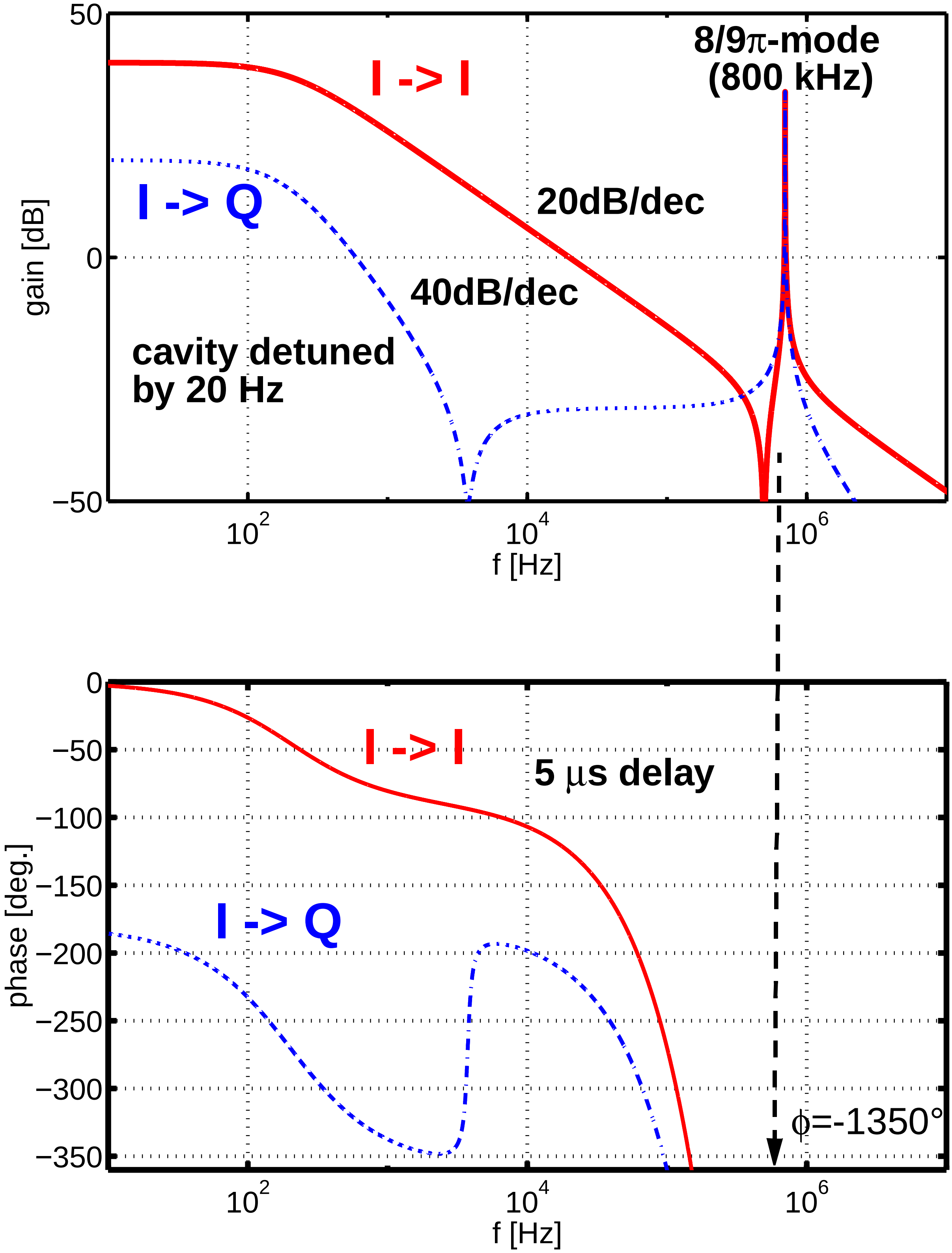
Excitation of other Passband Modes (2)

TESLA 9-cell cavity with 1 MHz beam (M. Ferrario)

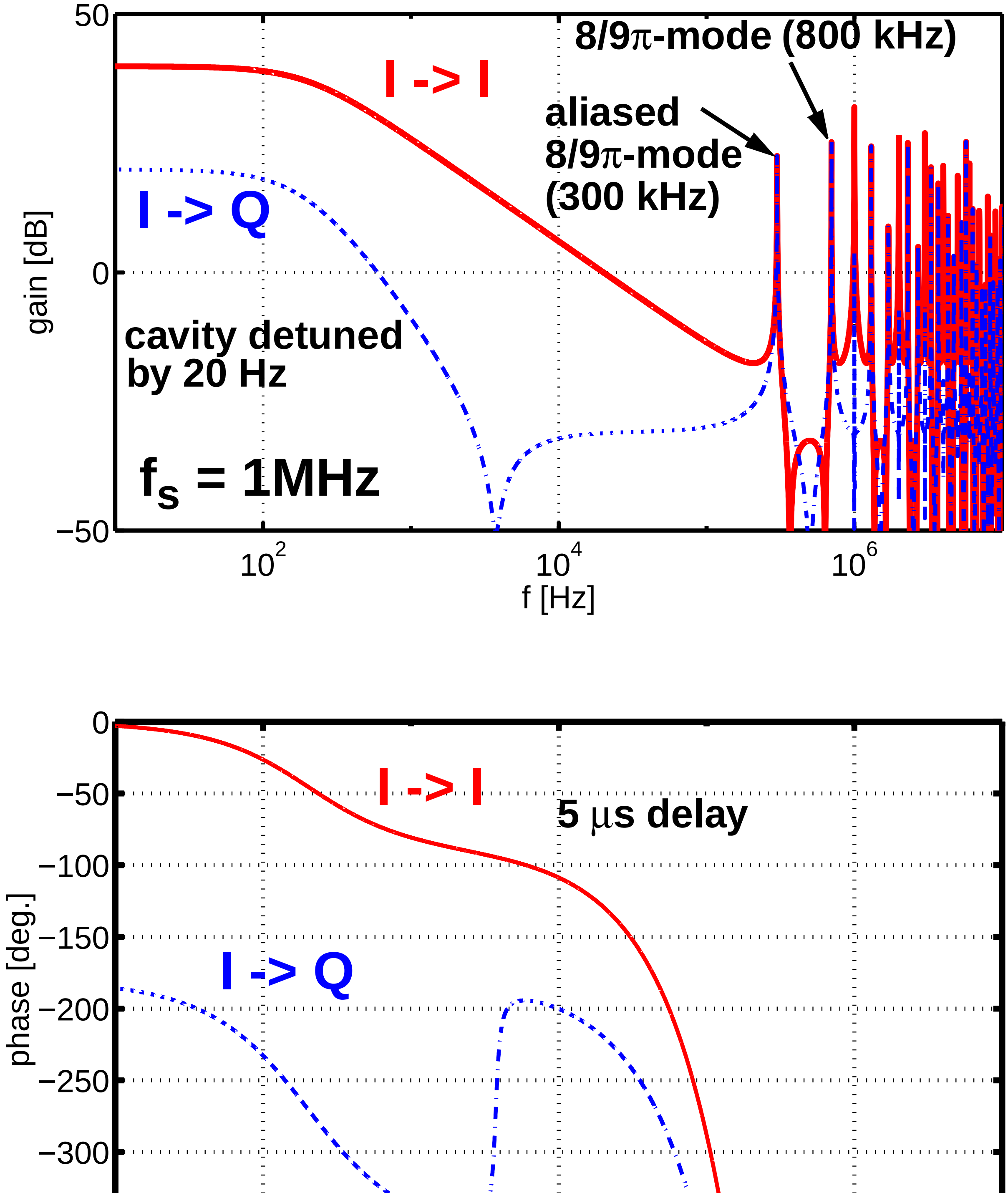




Cavity Transfer Function



Discrete Cavity TF



Cavity Model

Cavity Field

$$\begin{bmatrix} \dot{v}_r \\ v_i \end{bmatrix} = \begin{bmatrix} -\omega_{12} & -\Delta\omega \\ \Delta\omega & -\omega_{12} \end{bmatrix} \cdot \begin{bmatrix} v_r \\ v_i \end{bmatrix} + R \cdot \omega_{12} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_r \\ I_i \end{bmatrix}$$

Mechanical Properties

$$\begin{bmatrix} \dot{\Delta\omega} \end{bmatrix} = \begin{bmatrix} -1/\tau_m \end{bmatrix} \cdot \begin{bmatrix} \Delta\omega \end{bmatrix} + \begin{bmatrix} -2\pi/\tau_m K_m \end{bmatrix} \cdot \begin{bmatrix} (v_r^2 + v_i^2) \end{bmatrix}$$

or

$$\begin{bmatrix} \dot{\Delta\omega} \\ \dot{\Delta\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_m^2 & -1/\tau_m \end{bmatrix} \cdot \begin{bmatrix} \Delta\omega \\ \dot{\Delta\omega} \end{bmatrix} + 2\pi\omega_m^2 K_m \cdot \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ (v_r^2 + v_i^2) \end{bmatrix}$$

Typical Parameters

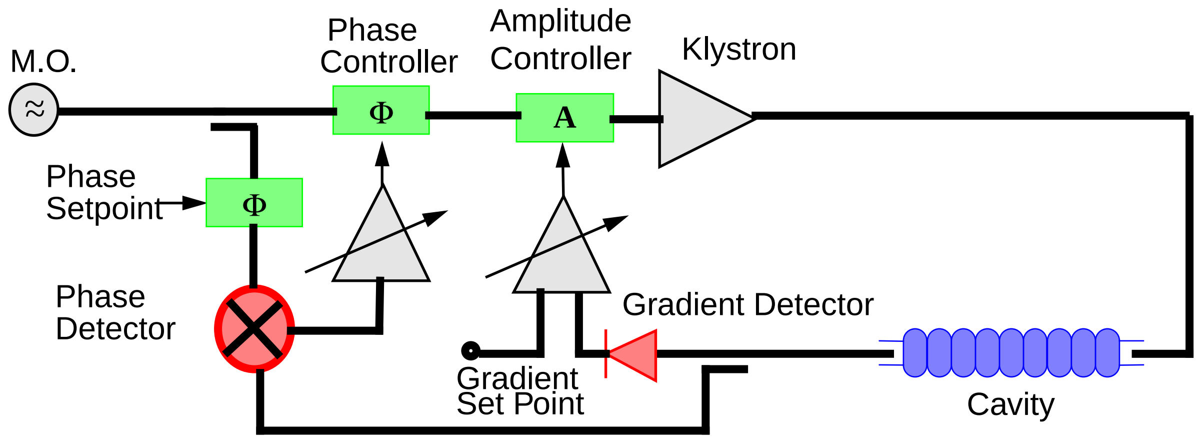
$\Delta\omega = \omega_0 - \omega_{rf}$, $\omega_{12} = \dfrac{\omega_0}{2 \cdot Q_L}$, $R = \left(\dfrac{r}{Q}\right) \cdot Q_L$,

$\omega_0 = 2\pi \cdot 1.3 \cdot 10^9$, $Q_L = 3 \cdot 10^6$, $\left(\dfrac{r}{Q}\right) = 1030\ \dfrac{\Omega}{m}$, $K_m = -1\ \mathrm{Hz/(MV/m)}^2$

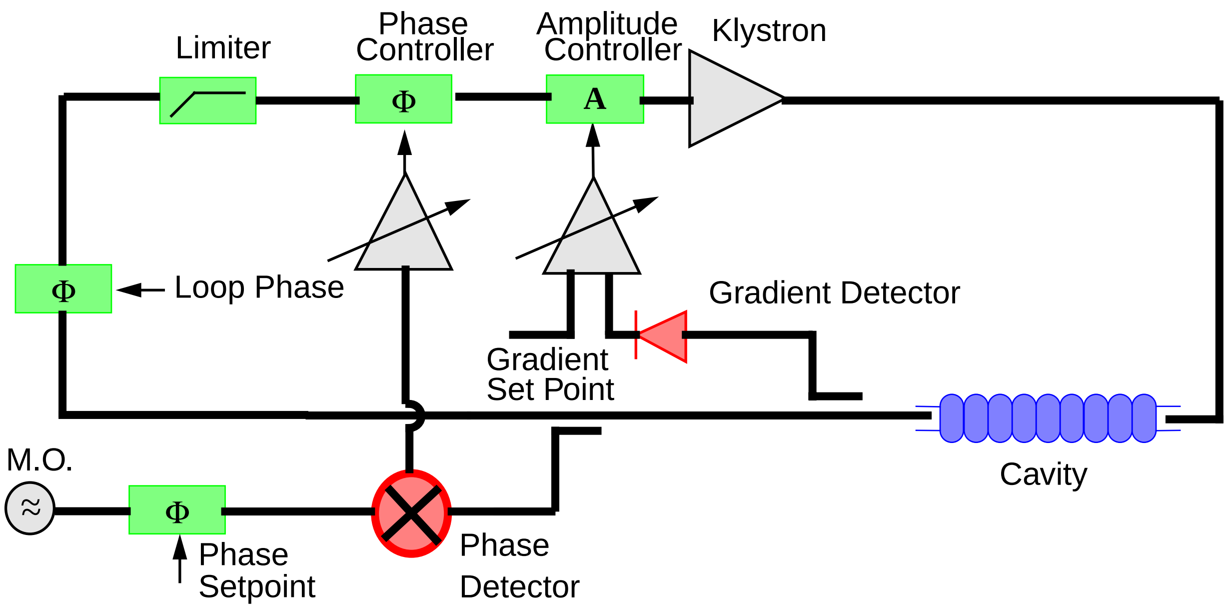
Control Choices (1)

- Self-excited Loop (SEL) vs
  Generator Driven System (GDR)
- Vector-sum (VS) vs individual cavity control
- Analog vs Digital Control Design
- Amplitude and Phase (A&P) vs
  In-phase and Quadrature (I/Q) detector and controller

Control Choices (2)

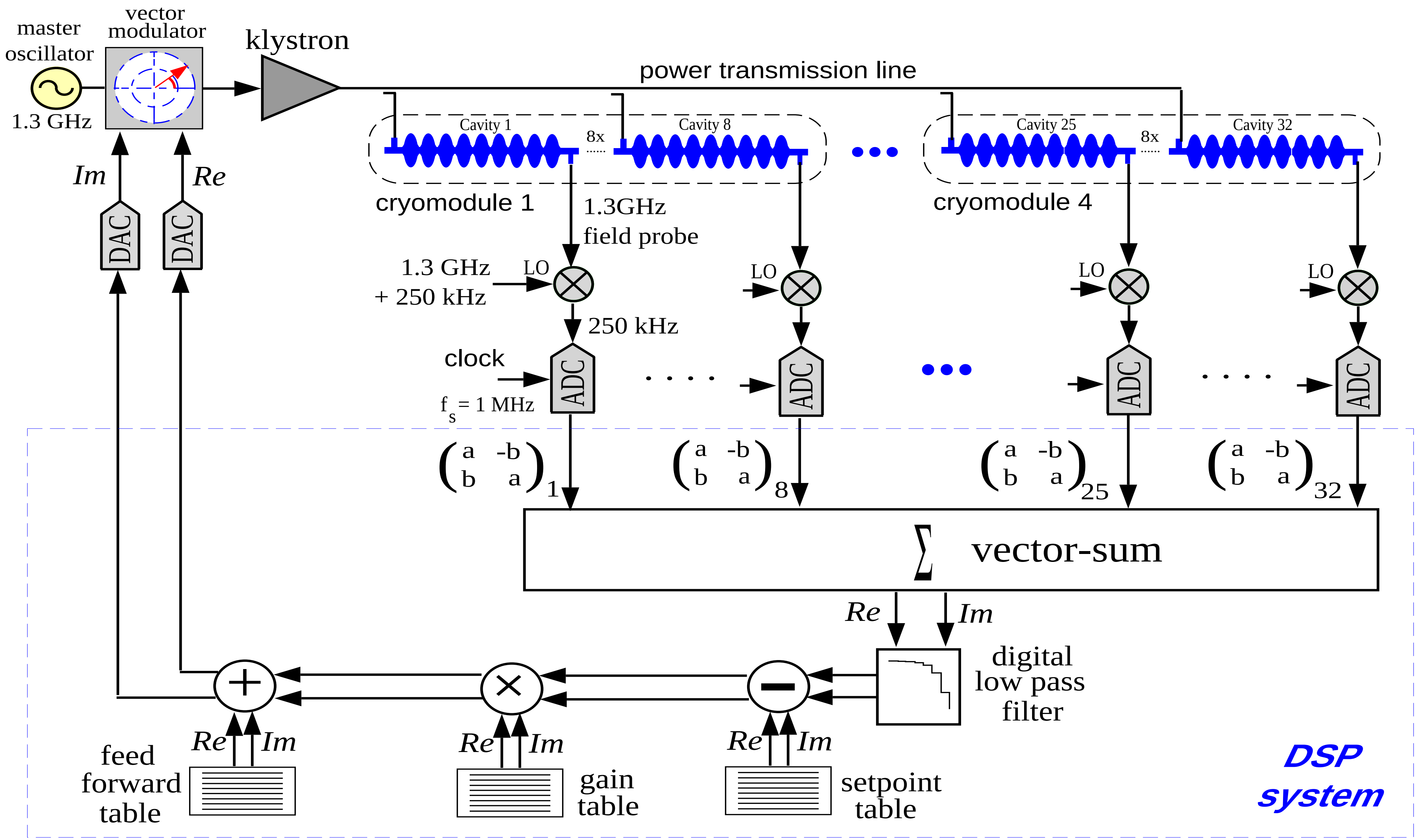


Generator Driven Resonator

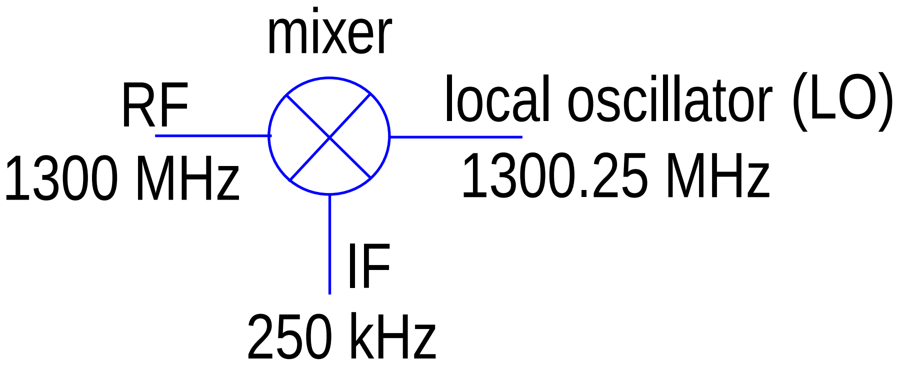


Self Excited Loop

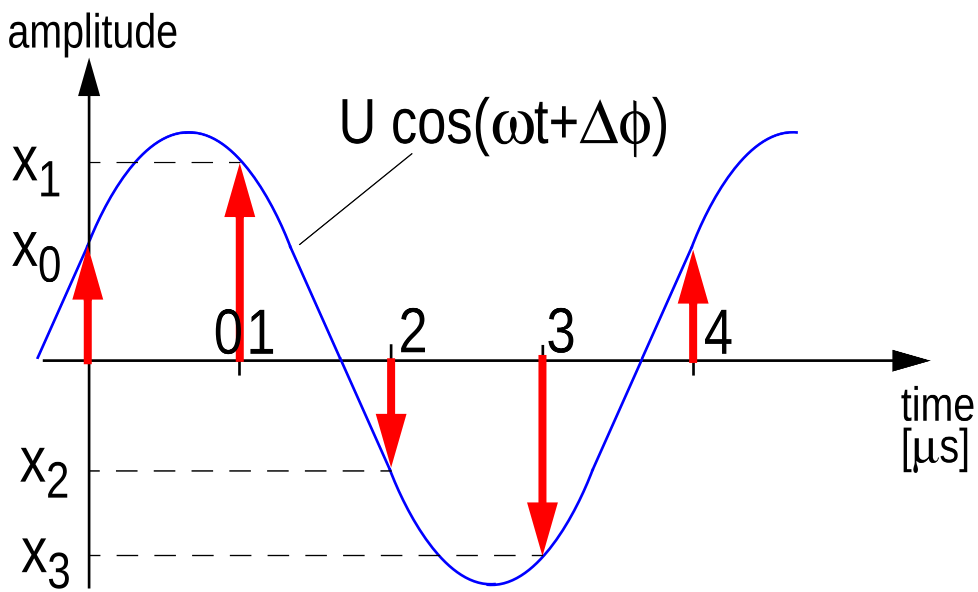
Digital Control at the TTF



Digital I/Q Detection



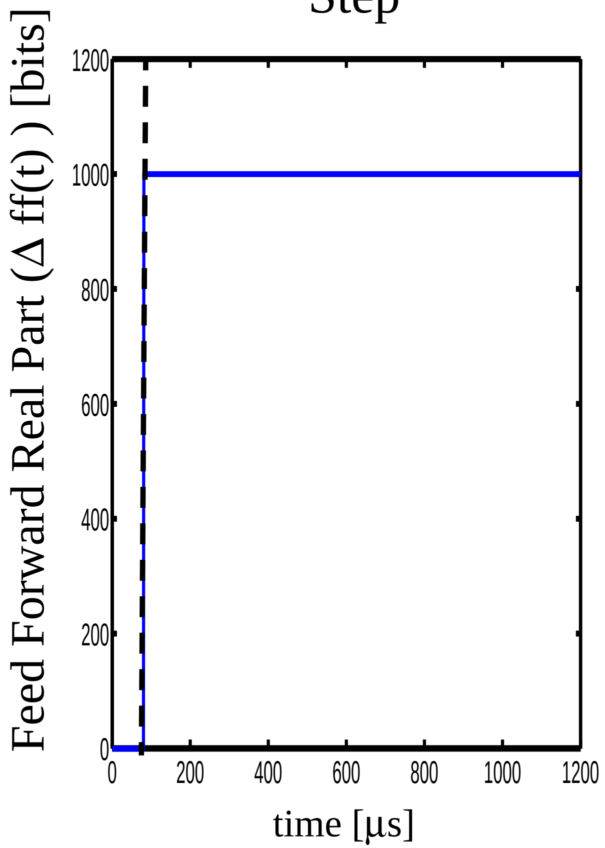
- downconversion of cavity field to IF frequency at 250 kHz
- complete phase and amplitude information of the accelerating field is preserved.



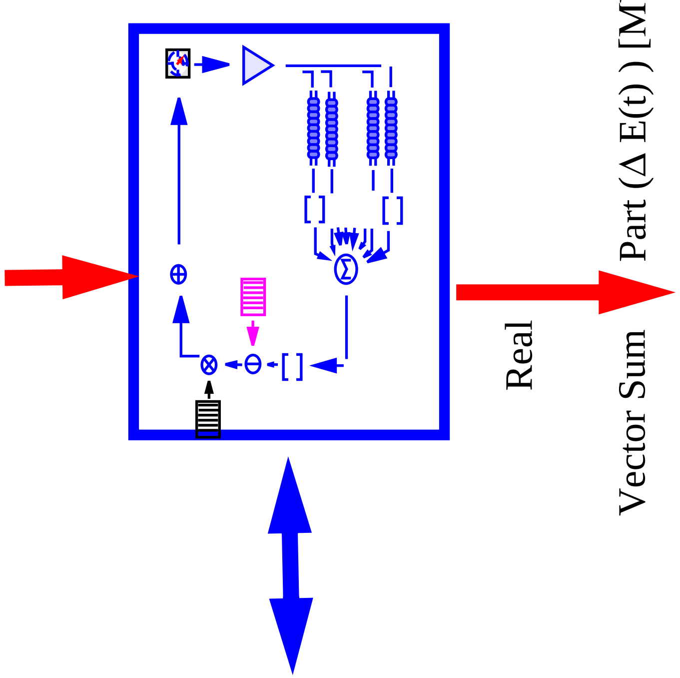
- sample IF signal at 1MHz rate
- subsequent samples describe real and imaginary component of the cavity field.

Adaptive Feedforward

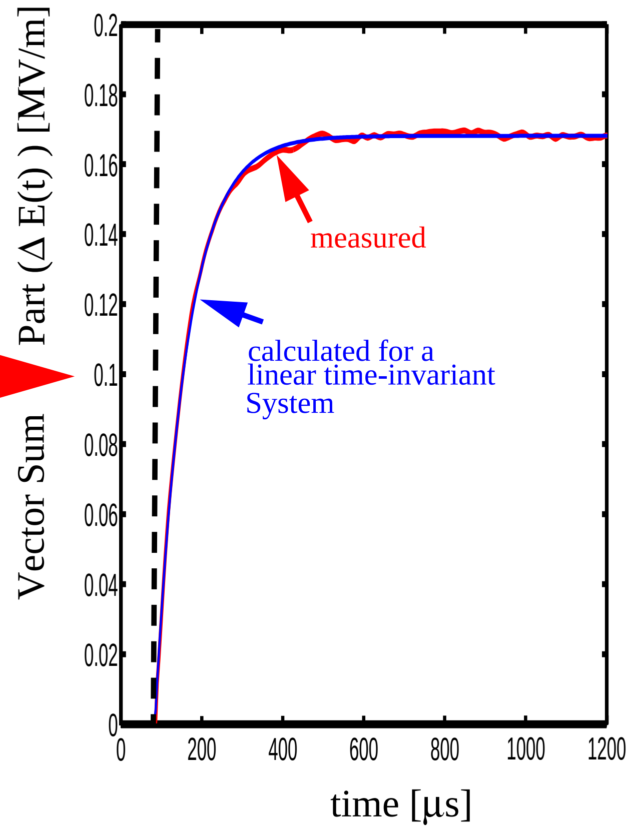
Step



Closed Loop System



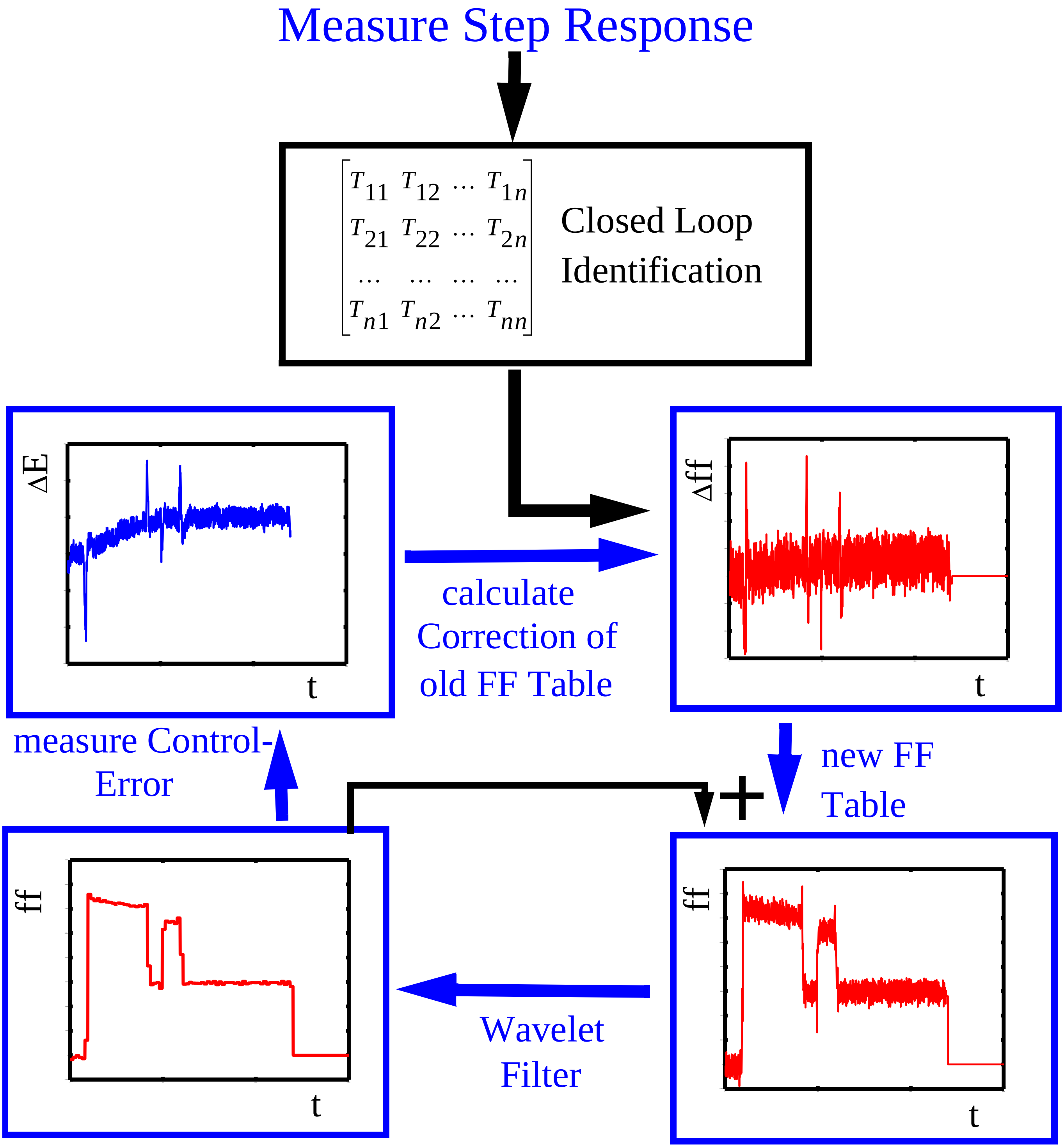
Step Response



$$\begin{bmatrix} \Delta E(\tau_1) \\ \Delta E(\tau_2) \\ \dots \\ \Delta E(\tau_n) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & \dots & T_{1n} \\ T_{21} & T_{22} & \dots & T_{2n} \\ \dots & \dots & \dots & \dots \\ T_{n1} & T_{n2} & \dots & T_{nn} \end{bmatrix} \begin{bmatrix} \Delta ff_1 \\ \Delta ff_n \\ \dots \\ \Delta ff_n \end{bmatrix}$$

$$\Delta ff(t) = \sum_j \Delta ff_j \Theta(t - t_j).$$

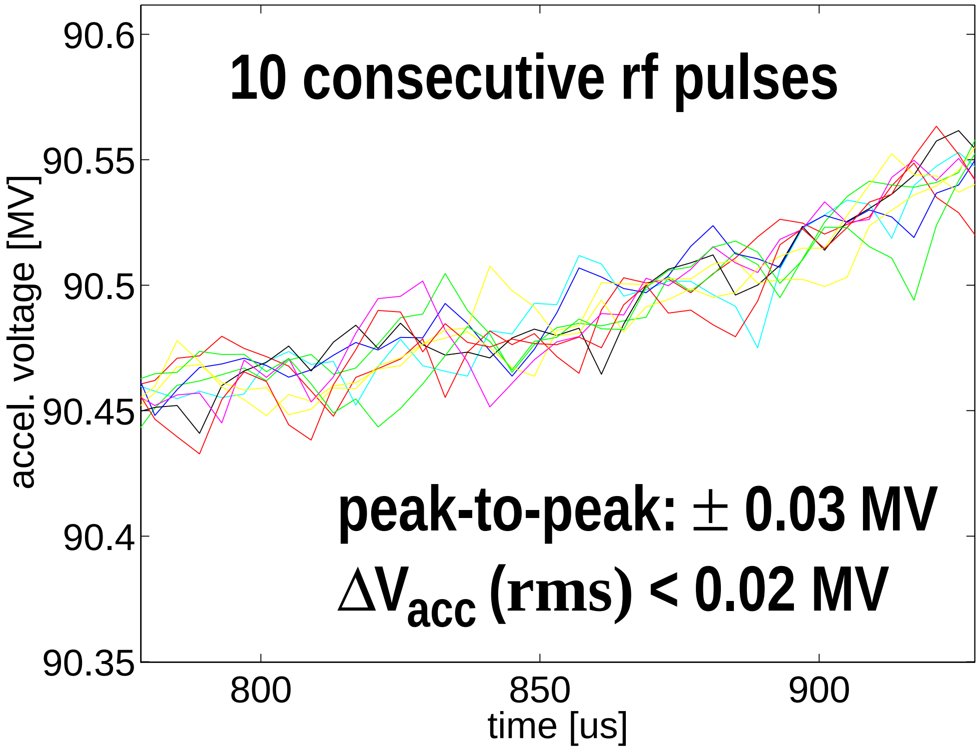
Adaptive Feed Forward



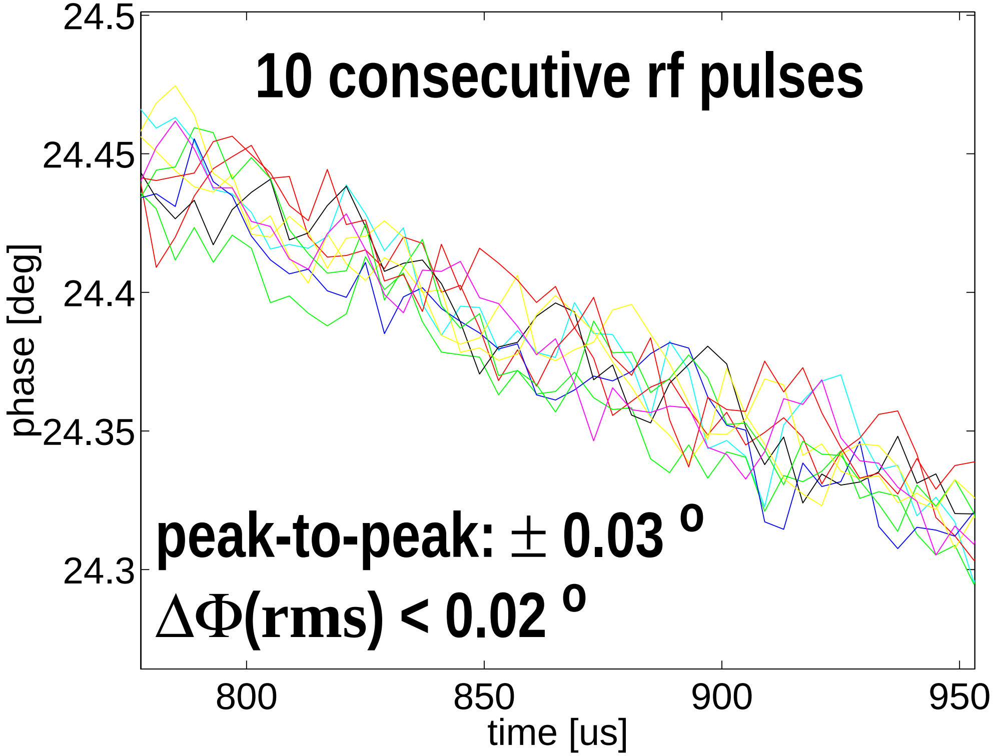
Adaptive Feed Forward can handle nonlinear systems through linarisation around the operating point.

The calculation of a new feed forward table needs only a few seconds.

Reproducibility of Subsequent Pulses of Vector-Sum



Gradient



Phase

Goals for System Identification

- Measure the correct detuning $\Delta\omega(t)$:
  For minimizing the rf-power it is desired to know the curve $\Delta\omega(t)$, because then one can find an optimal value for the predetuning.
- Find the correct model for the Lorentz-Force-Detuning by performing System Identification on the $\Delta\omega(t)$-curve.
- Detect the current beam-phase relative to $\vec{E}_{acc}$ of cavity one of each cryo-module,
  adjust it without need of transients,
  detect drifts in the reference signal with respect to the beam.
- Calibrate all signals delivered by low-level-rf based on the calibration of cavity-fields, especially deliver the correct phases.
- Provide the signals for automation of the low-level-rf-system, gradient, beam-phase, optimal predetuning, loop-stability

Fitting Models to Data

One of the most common ways of fitting functions to data is least squares method. It delivers the best approximation if one assumes a normal distribution for the measurement errors. One can show this with the maximum likelihood method.

$$\rho(f(x_i) - y_i) \equiv \sum_{i=1}^{N} (f(x_i) - y_i)^2 \rightarrow \text{Min!}$$

$y_1 \ldots y_N$ shall be expressed as function $f(x_i)$.

The x_i's are assumed to be error-free. (sampling time)

let f be a linear combination of n functions f_i:

$$f(x;(a_1, \ldots, a_n)) = a_1 f_1(x) + \ldots + a_n f_n(x)$$

then the condition for minimum error is

$$\begin{pmatrix} [f_1(x)f_1(x)] & \ldots & [f_1(x)f_n(x)] \\ \vdots & \ddots & \vdots \\ [f_n(x)f_1(x)] & \ldots & [f_n(x)f_n(x)] \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} [yf_1(x)] \\ \vdots \\ [yf_n(x)] \end{pmatrix}$$

$$[f(x)] \equiv \sum_{i=1}^{N} f(x_i)$$

Now apply it to the difference equation:

$$y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) = b_1 u(t-1) + \ldots + b_m u(t-m)$$

$$\theta \equiv \left[a_1 \ldots a_n, b_1 \ldots b_m\right]^{\mathrm{T}}$$

$$\varphi(t) \equiv \left[-y(t-1) \ldots -y(t-n), u(t-1) \ldots u(t-m)\right]^{\mathrm{T}}$$

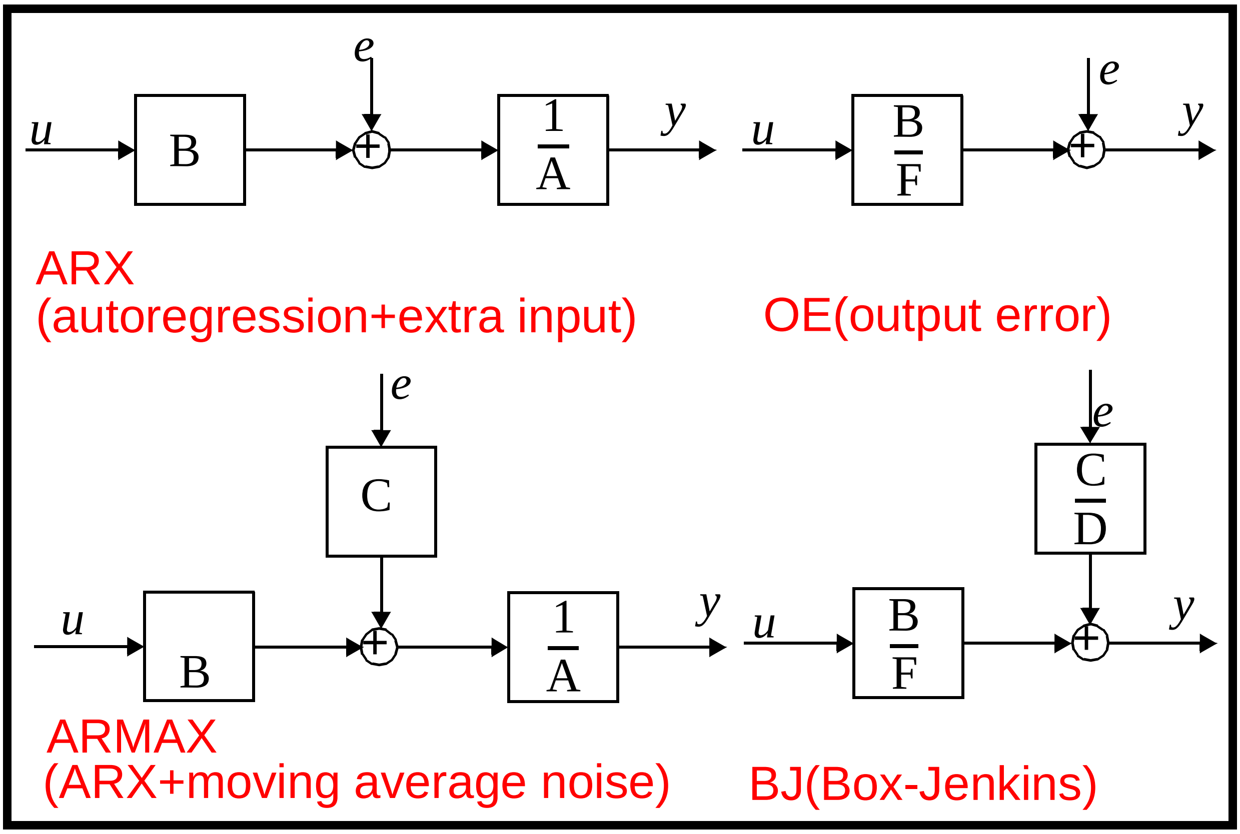
With the above formulas one finds:

$$\theta = \left(\sum_{t=1}^{N} \varphi(t)\varphi^{\mathrm{T}}(t)\right)^{-1} \sum_{t=1}^{N} \varphi(t) y(t)$$

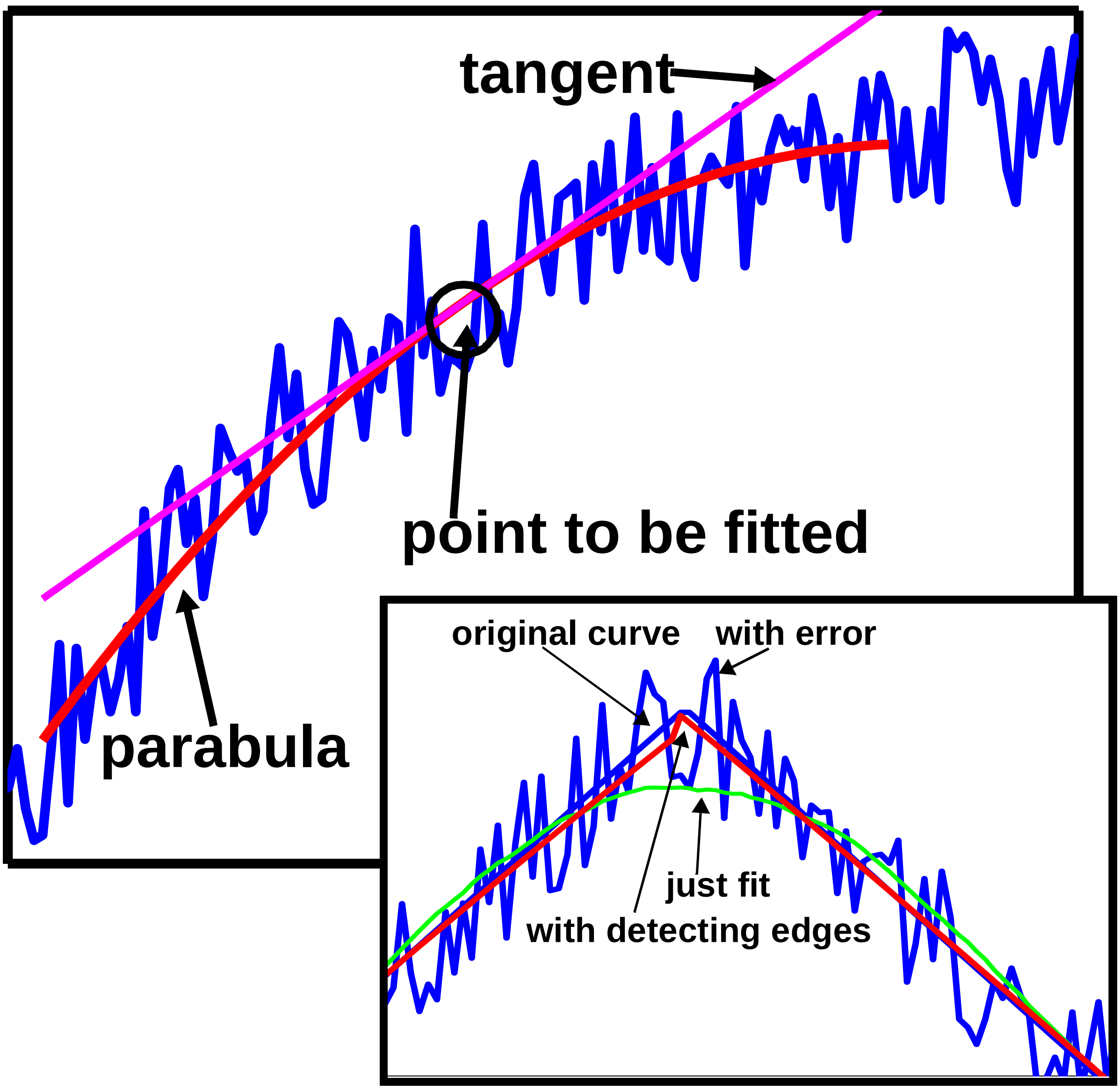
all coefficients have to be constant.
It’s discrete transfer function is the following:

$$\mathrm{H}(z) = \frac{b_1 z^{-1} + \ldots + b_m z^{-m}}{1 + a_1 z^{-1} + \ldots + a_n z^{-n}} \equiv \frac{\mathrm{B}}{\mathrm{A}} \qquad (\mathrm{ARX})$$

In picture together with some other ready-made models:

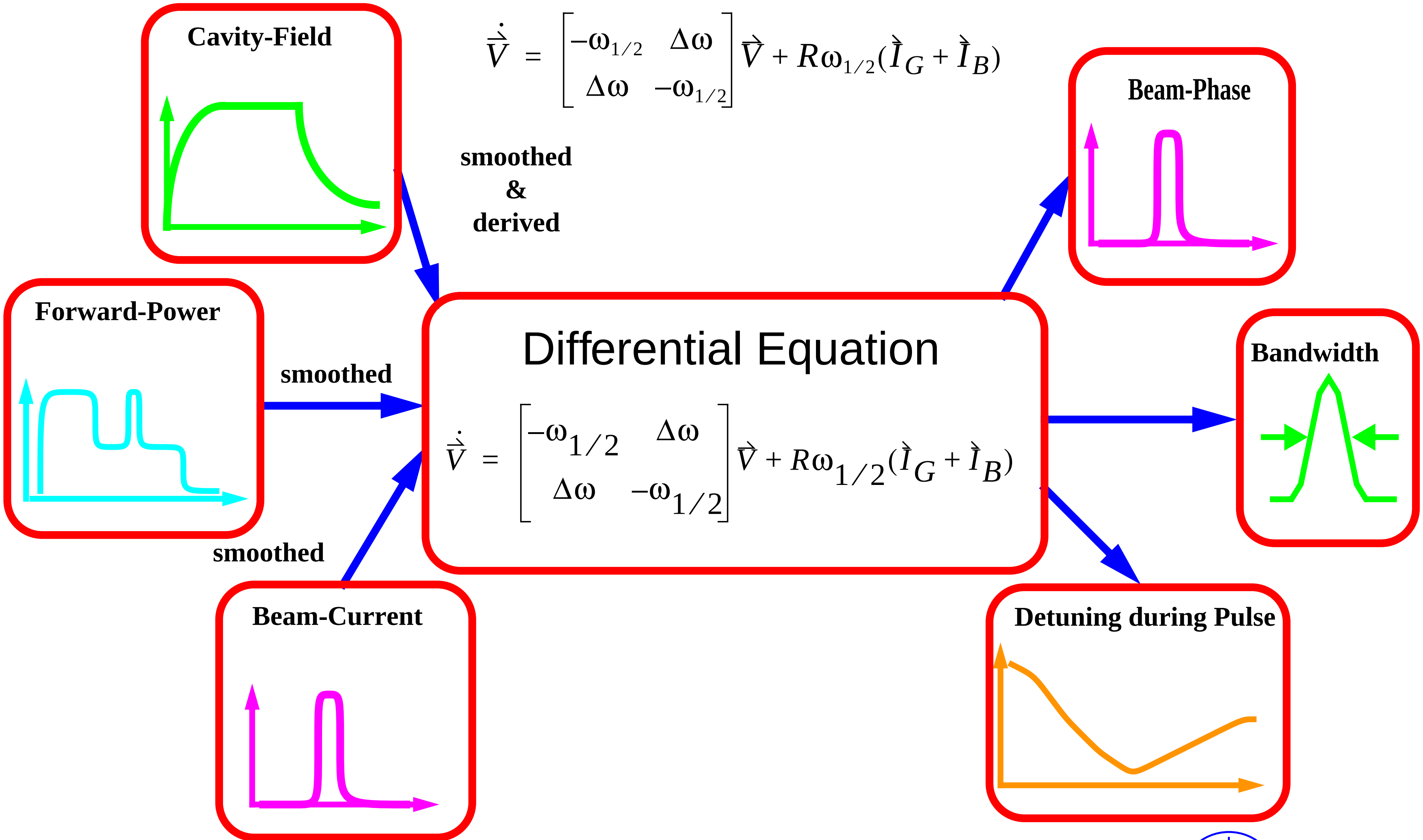


Another Way of fitting the system's differential equation to the data is to calculate the derivation from the measured data. I did this by laying a parabolic curve through every point of the measured curve and then calculated it's derivation. Uncontinuities are recognized by comparing the prediction of neighbouring parabulas for the same point.

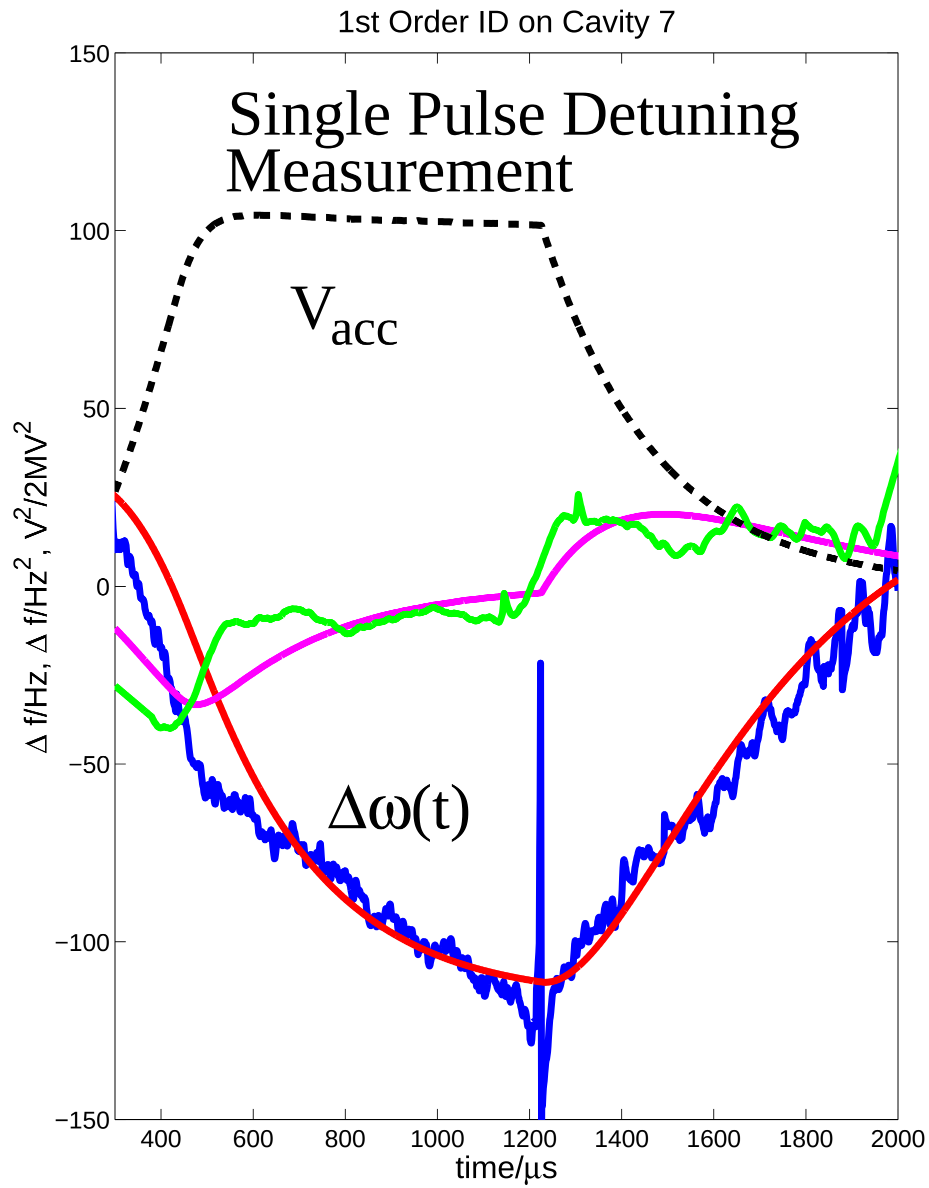


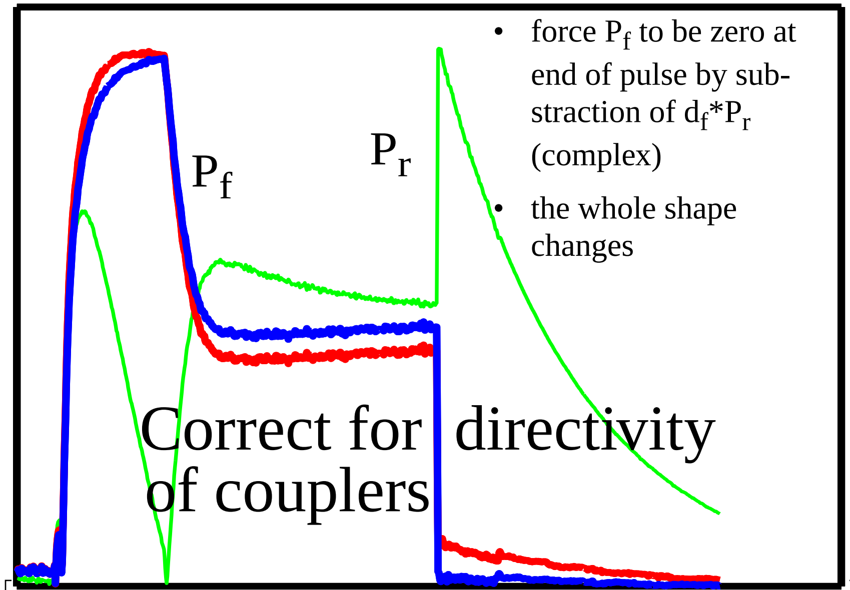
System Identification (1)

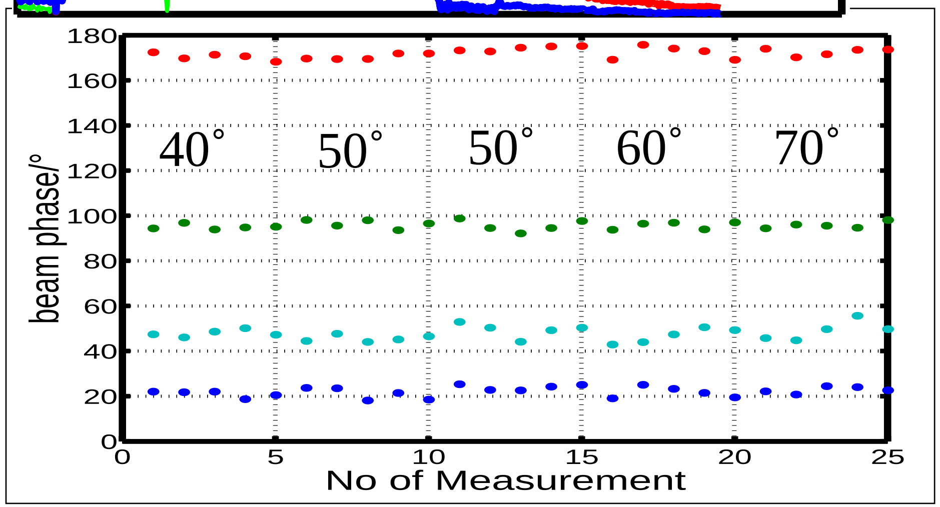
$$\dot{\vec{V}} = \begin{bmatrix} -\omega_{1/2} & \Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{bmatrix} \vec{V} + R\omega_{1/2}(\vec{I}_G + \vec{I}_B)$$



System Identification (2)



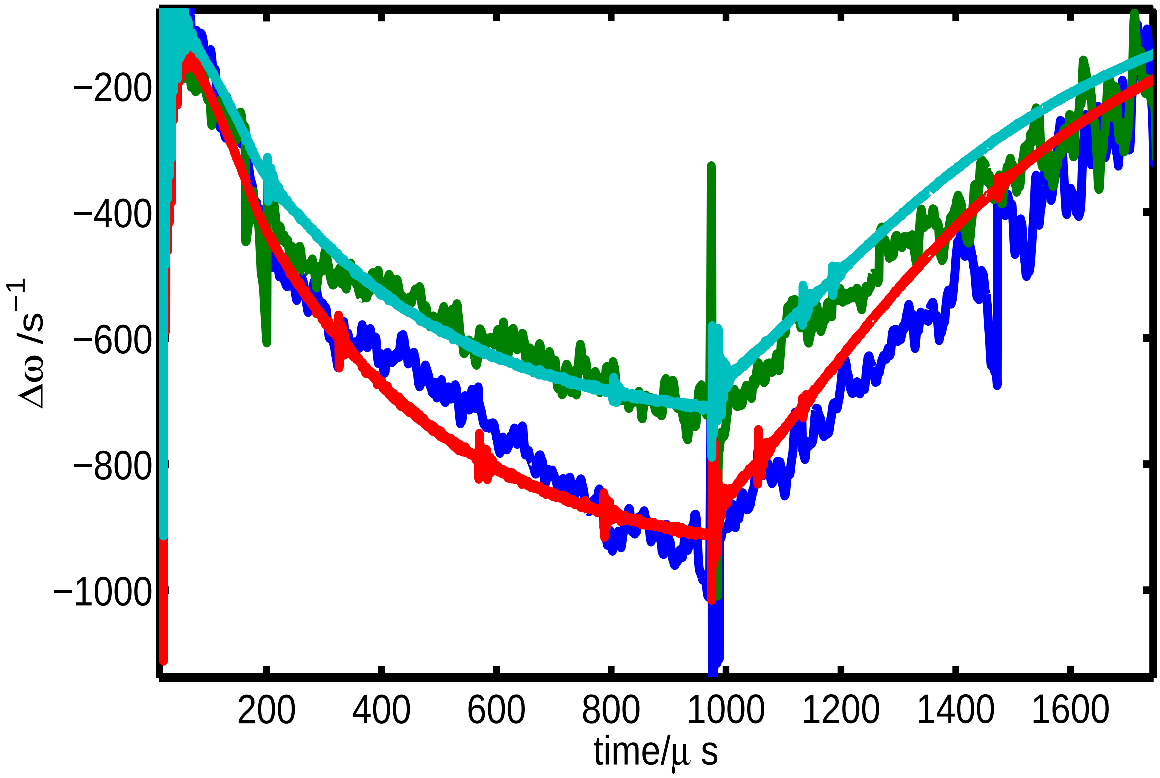




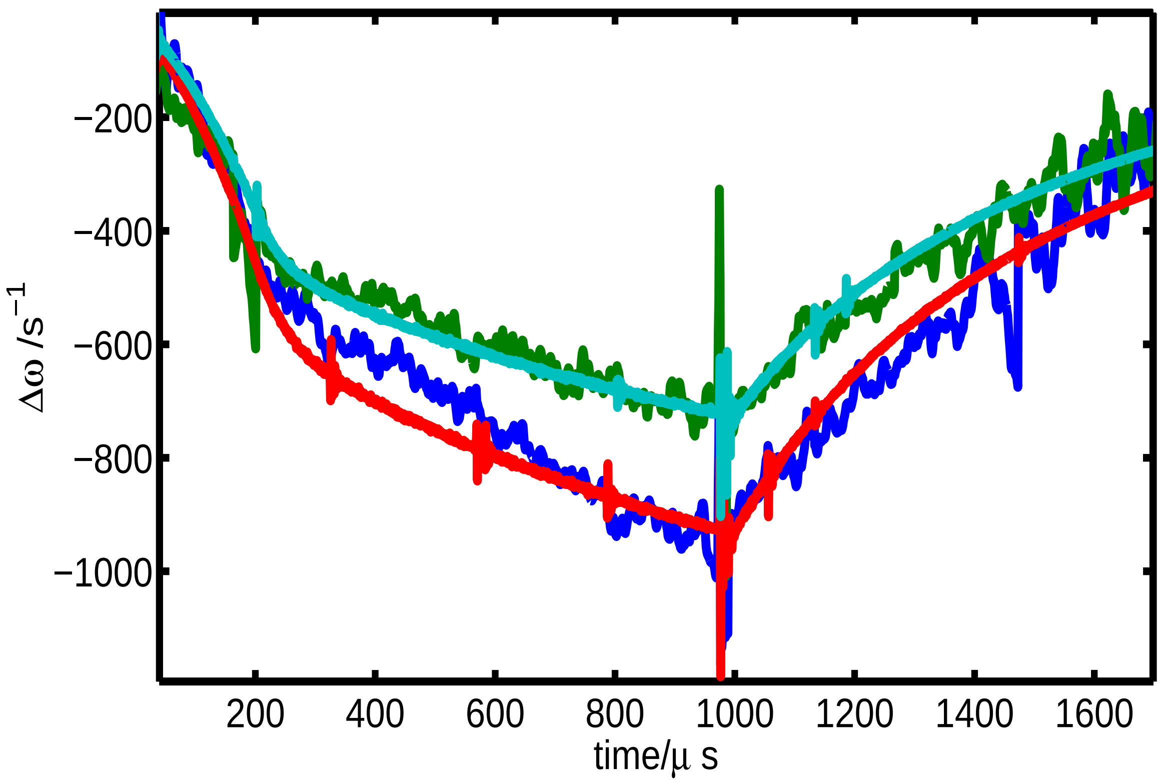
Beam phase of 4 cavities for different phase of V_{acc}

OE Model (3,4,0)

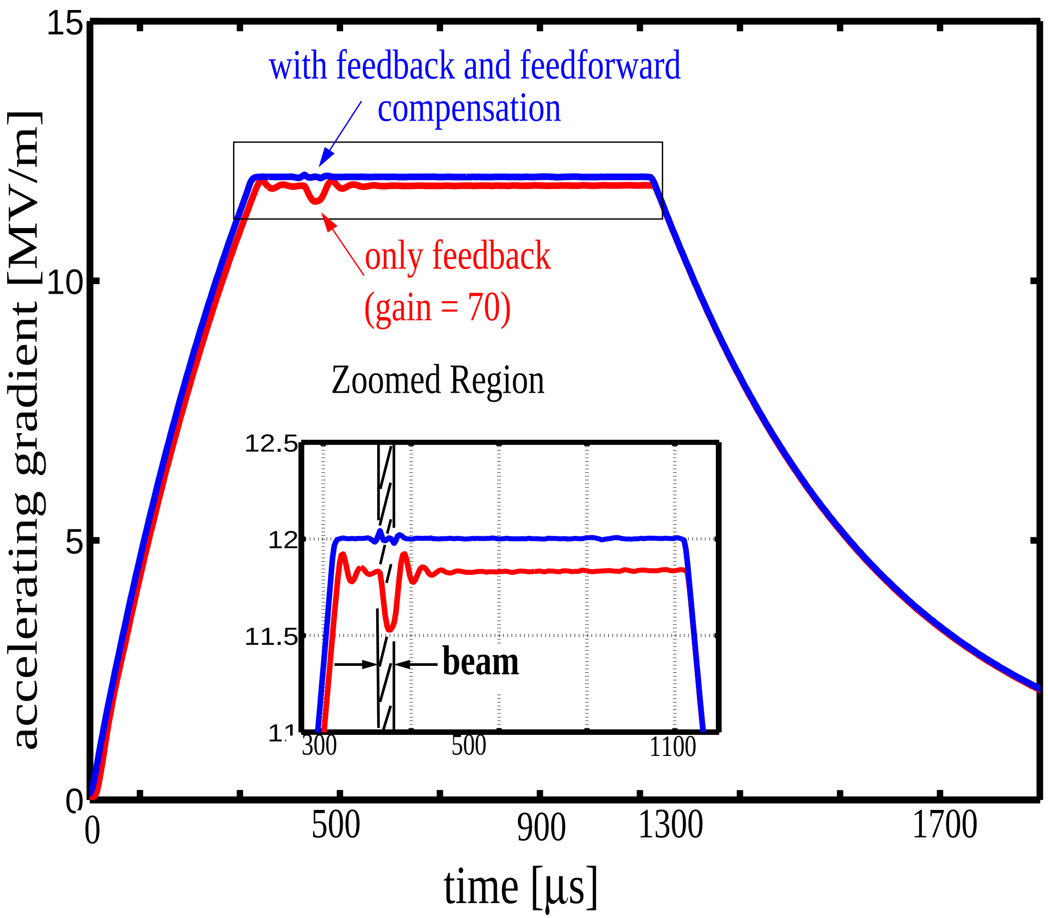
Taking only Decay for Identification



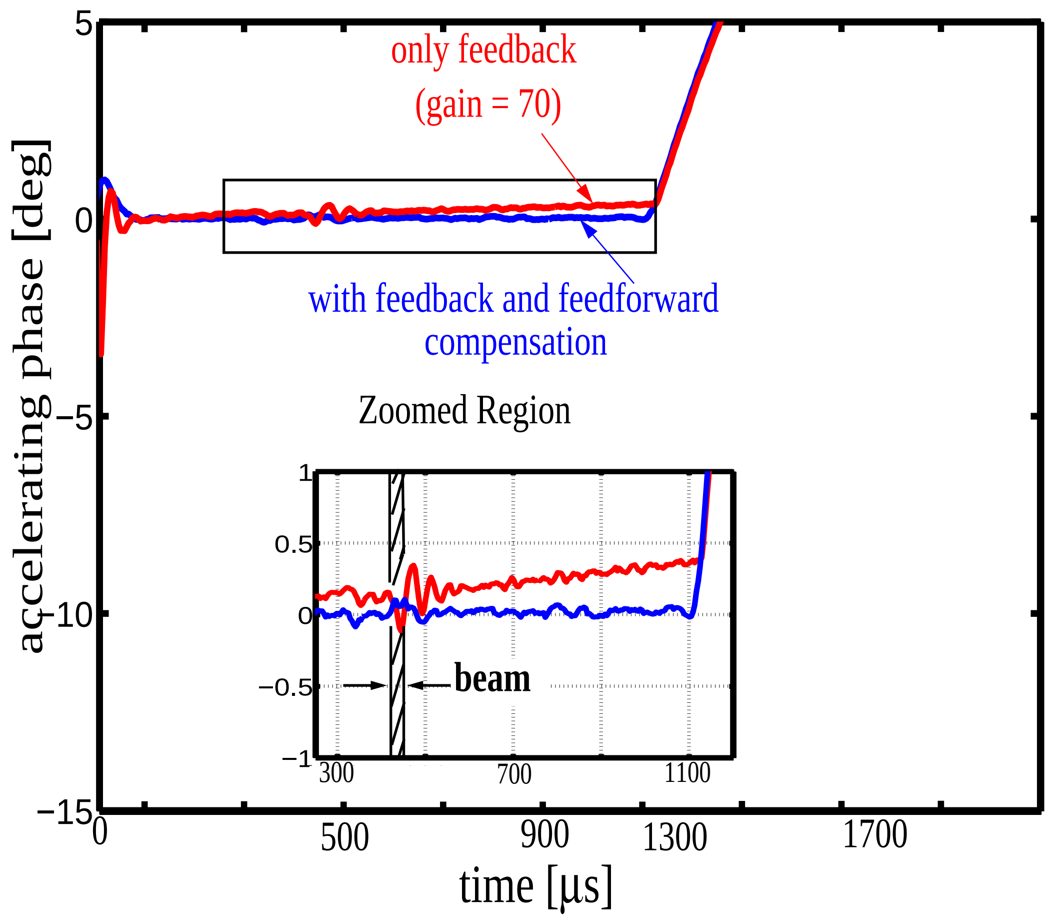
Taking whole Pulse for Identification



Performance at TTF (1)

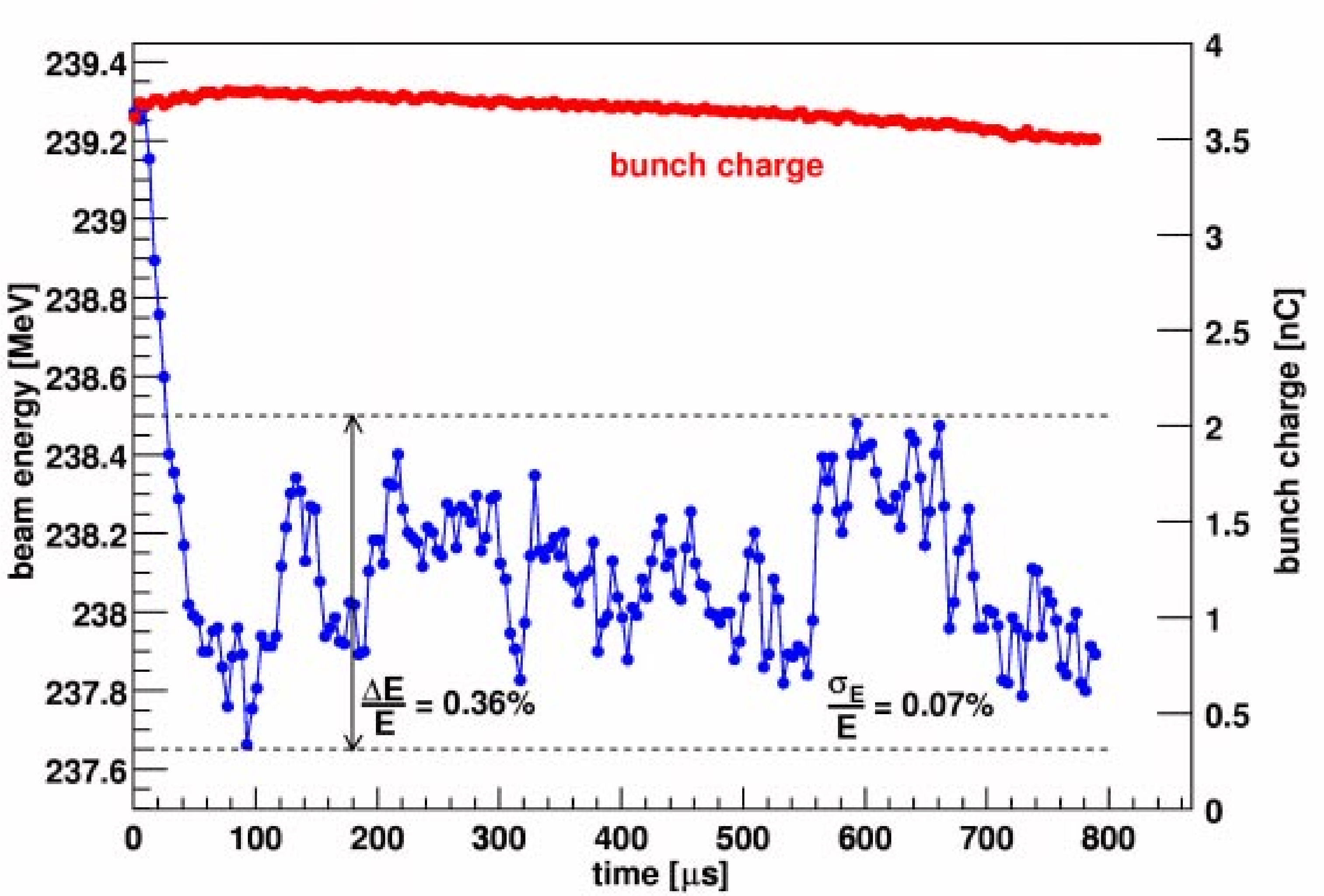


Amplitude



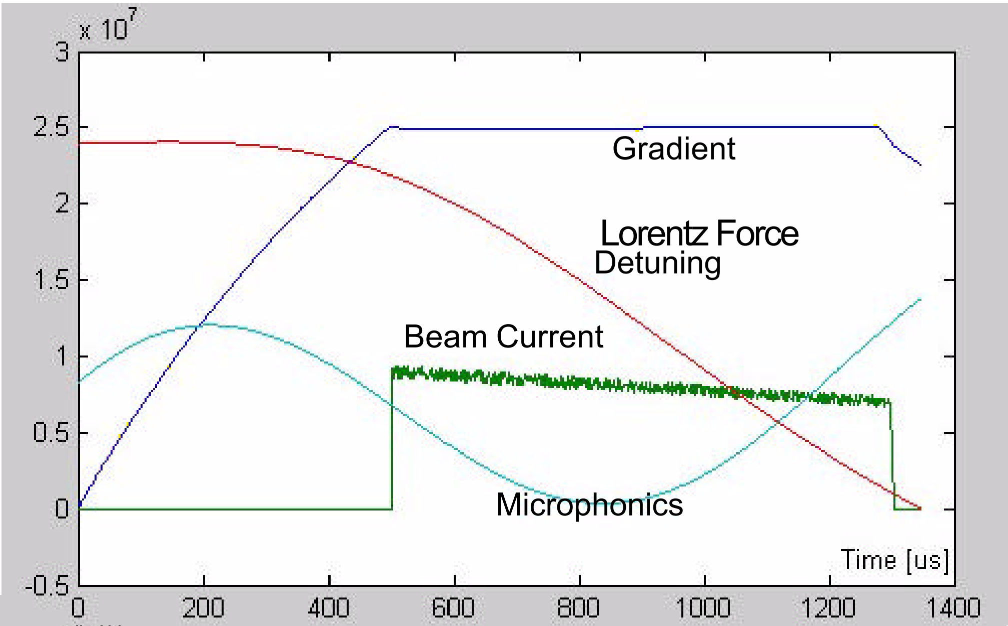
Phase

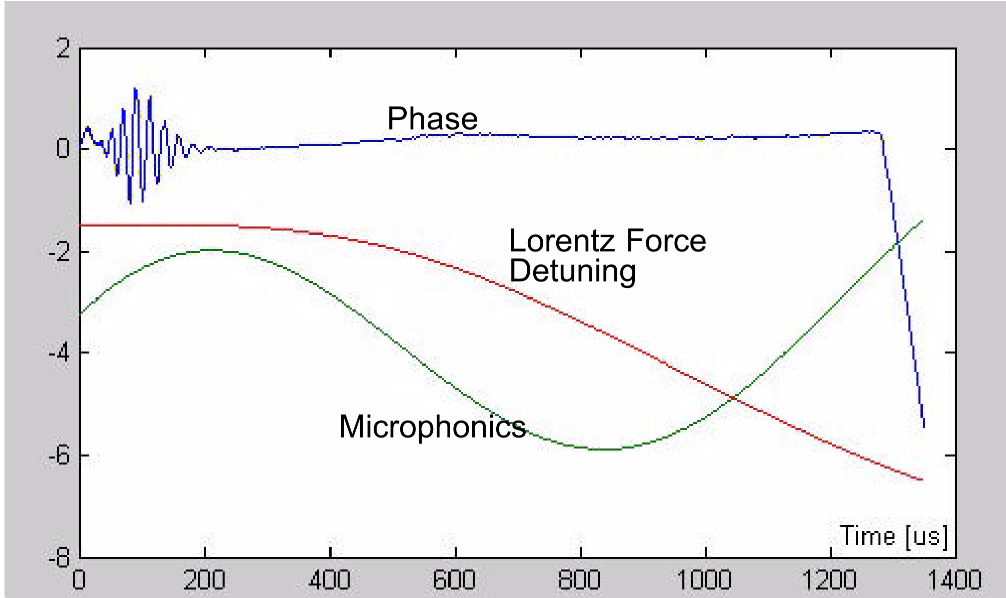
Performance at TTF (2)

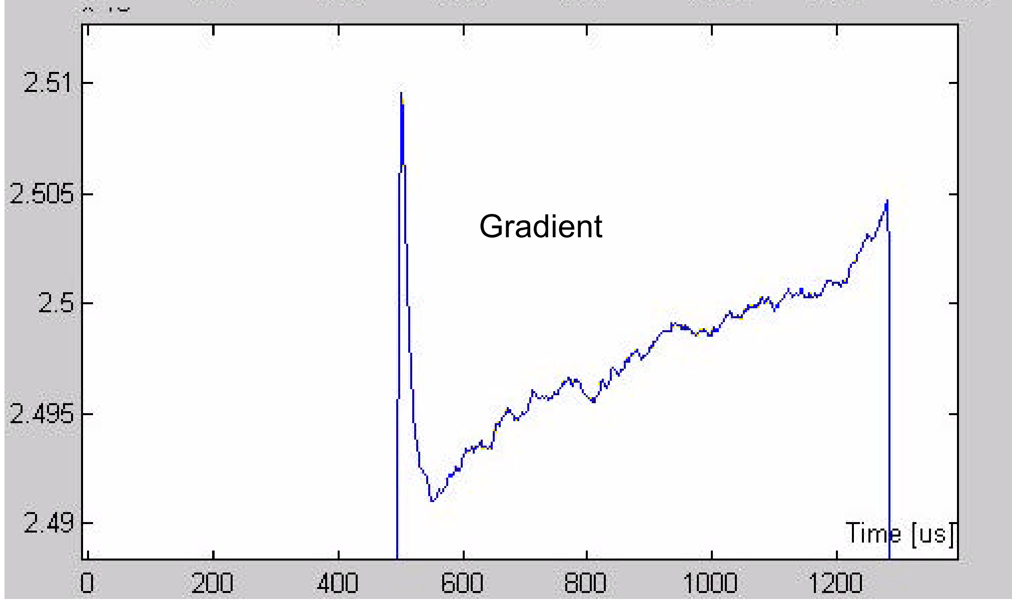


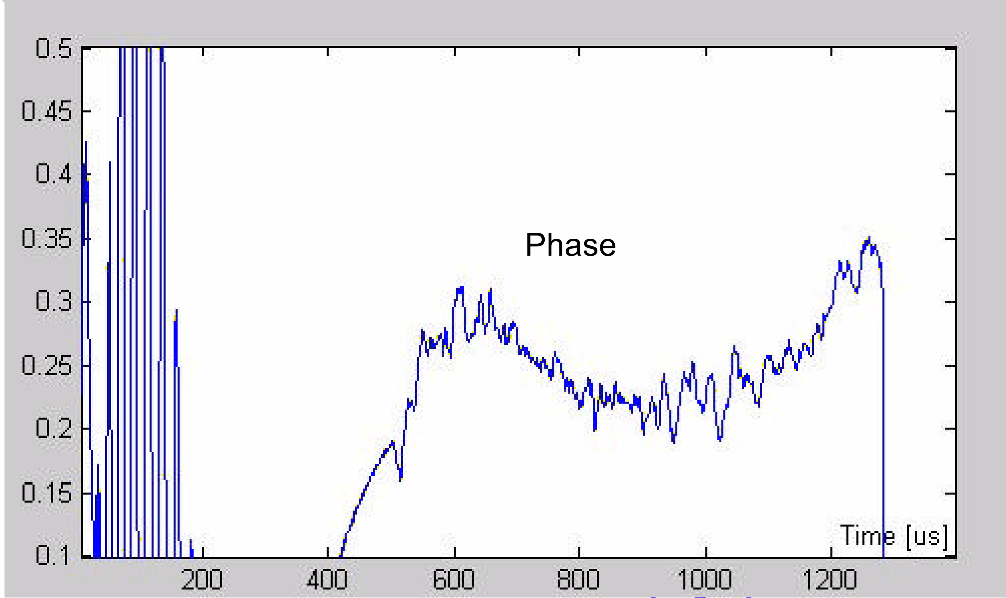
Operation with long beam pulses

RF Regulation TESLA Cavity (Simulation)

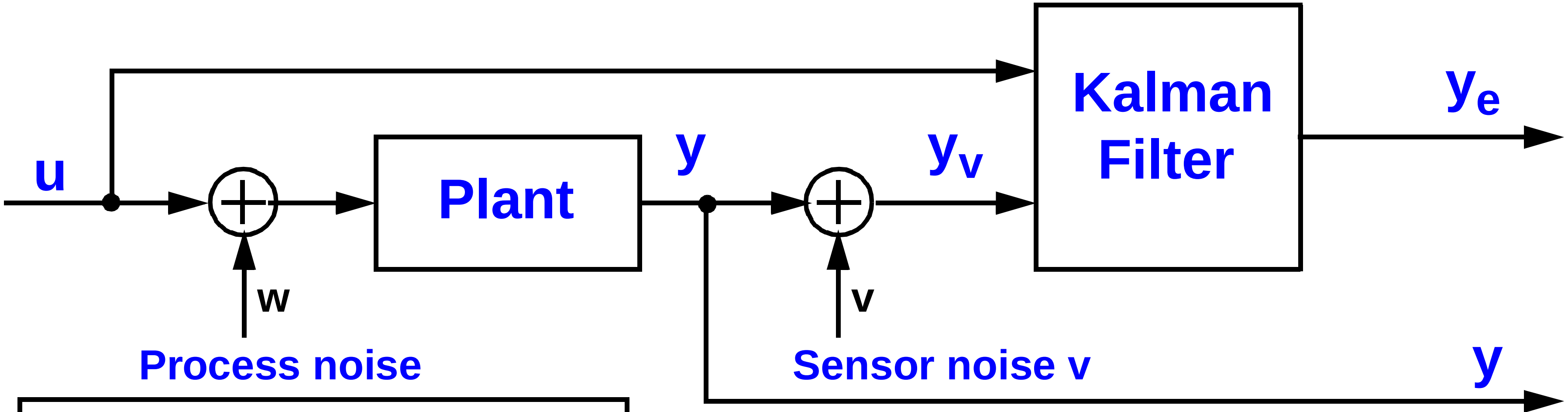








Principle Kalman Filter (steady state)



Discrete Plant:

$x[n+1]=Ax[n]+B(u[n]+w[n])$

$y[n]=Cx[n]$

Noisy output measurement: $y_v[n]=Cx[n]+v[n]$

Measurement update:

$$\hat{x}[n|n]=\hat{x}[n|n-1]+M(y_v[n]-C\hat{x}[n|n-1])$$

Time update: $\hat{x}[n+1|n]=A\hat{x}[n|n]+Bu[n]$

The correction term is a function of the innovation, i.e. the discrepancy

$$y_v[n+1]-C\hat{x}[n+1|n]=C(x[n+1]-\hat{x}[n+1|n])$$

The innovation gain matrix M is chosen to minimize steady-state covariance of the estimation error given the noise covariances $E(w[n]w[n]^T)=Q$ and $E(v[n]v[n]^T)=R$

Kalman Filter (Cnt'd)

where M is the solution of the Riccati Equation:

$M=Q+AMA^T-AMC^T(R+CMC^T)^{-1}CMA^T$

Combining time and measurement update into state space model (the kalman filter):

$$\hat{x}[n+1|n]] = A(I-MC)\cdot\hat{x}[n|n-1]+[B\ AM]\begin{bmatrix}u[n]\\ y_v[n]]\end{bmatrix}$$

$$\hat{y}[n|n] = C(I-MC)\cdot\hat{x}[n|n-1]+CMy_v[n]$$

This filter generates and optimal estimate $\hat{y}[n|n]$ of y[n]. Note that filter state is $\hat{x}[n|n-1]$

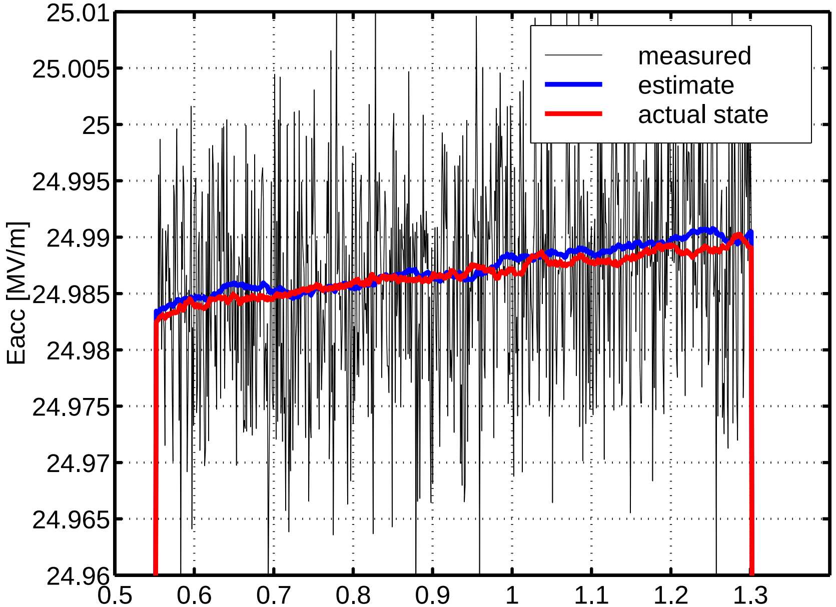
Example: TTF Cavity $Q_L = 3*10^6$

$\omega_0 = 1.3*10^9$ Hz

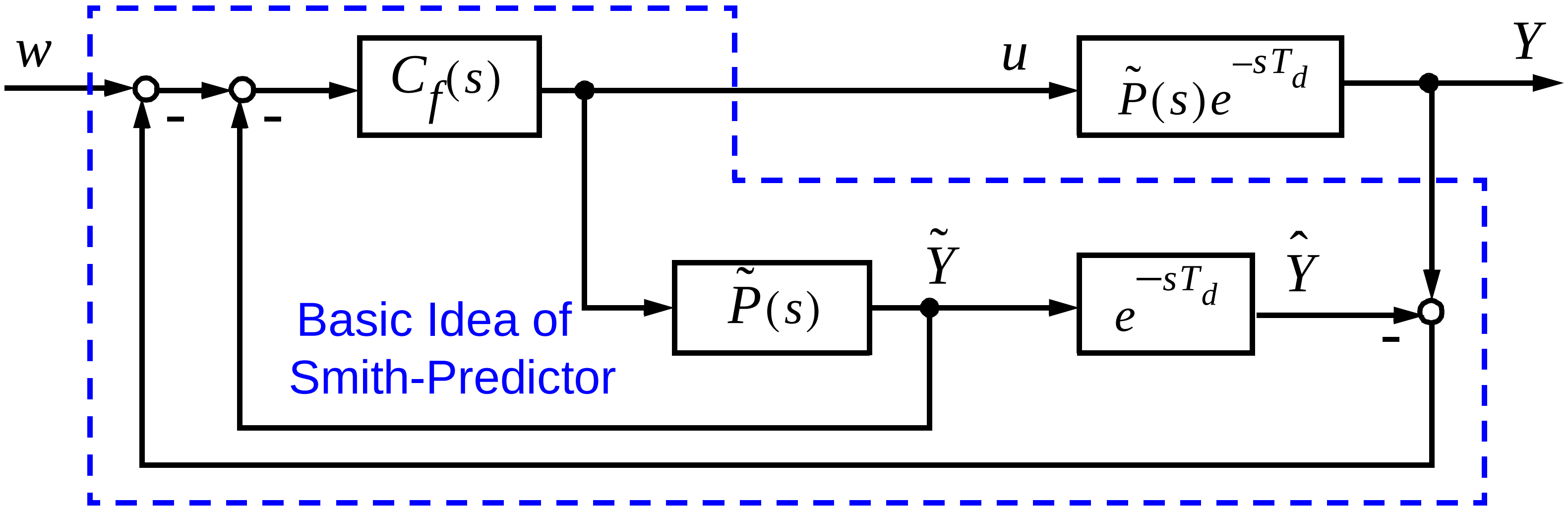
Beam noise : $\sigma(I_b)/I_b = 0.1$

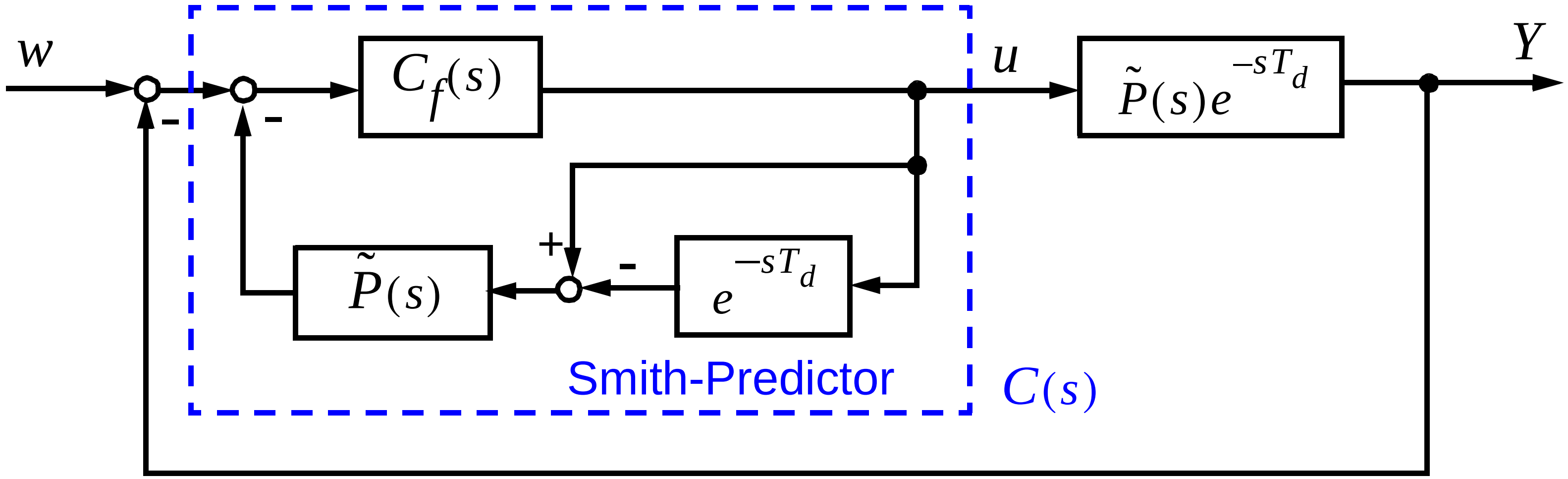
Sensor noise : $\sigma(V_d)/V_d = 0.01$

$\Longrightarrow$ $\sigma_y / y = 0.0009$

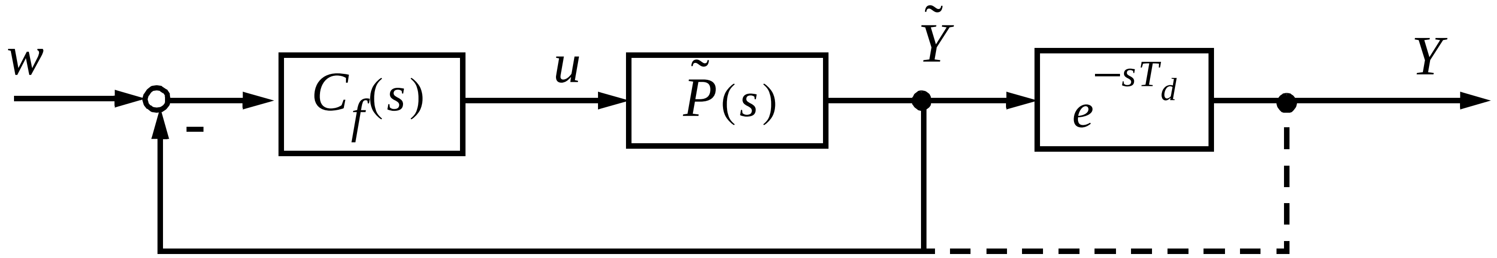


Principle of Smith Predictor





Smith Predictor (Cnt'd)



Desired Feedback produced by Smith predictor

Command Step Response of Smith Predictor:

$$G_w(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{\tilde{P}(s)e^{-sT_d}C(s)}{1 + \tilde{P}(s)e^{-sT_d}C(s)}$$

Command Step Response of desired feedback:

$$G_w(s) = \frac{\tilde{G}(s)C_f(s)}{1 + \tilde{G}(s)C_f(s)}e^{-sT_d}$$

$\Longrightarrow$

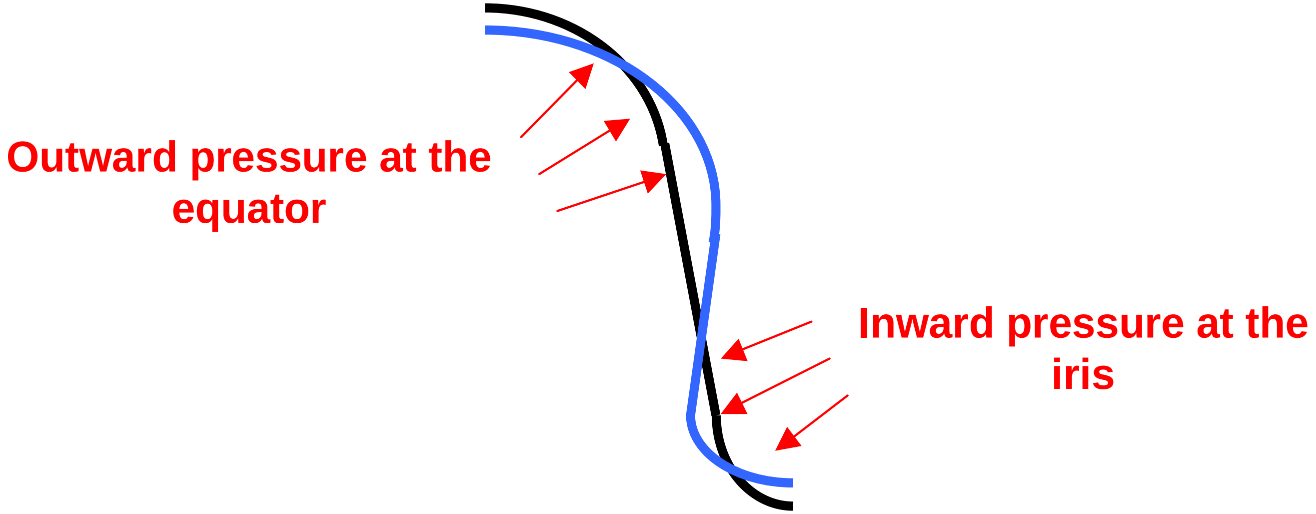
Smith Predictor :

$$C(s) = \frac{C_f(s)}{1 + C_f(s)\tilde{P}(s)(1 - e^{-sT_d})}$$

Lorentz Force Detuning (1)

- Radiation pressure : $P = (\mu_0 H^2 - \varepsilon_0 E^2)/4$

- Deformation of the cavity shape:

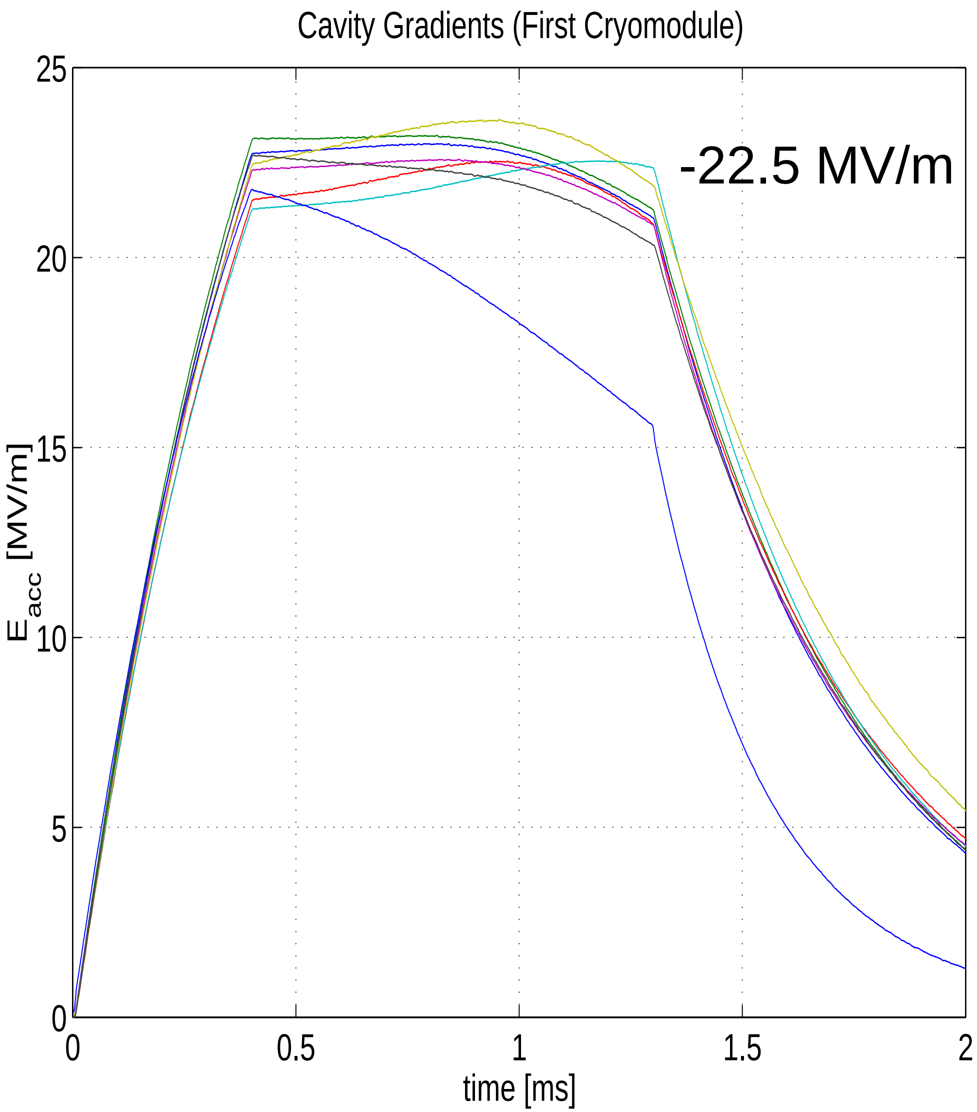


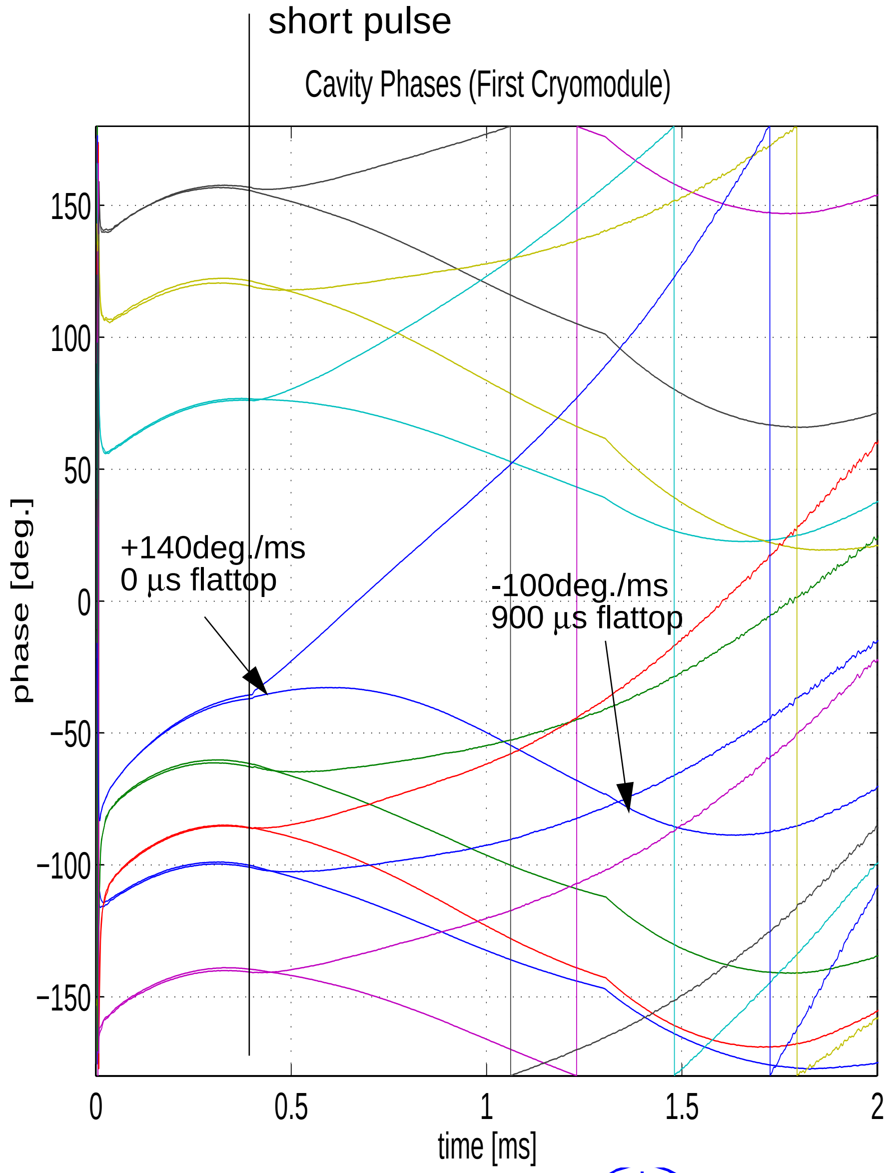
- Frequency shift : $\Delta f = KL{*}E^2_{acc}$

Lorentz Force Detuning (2)

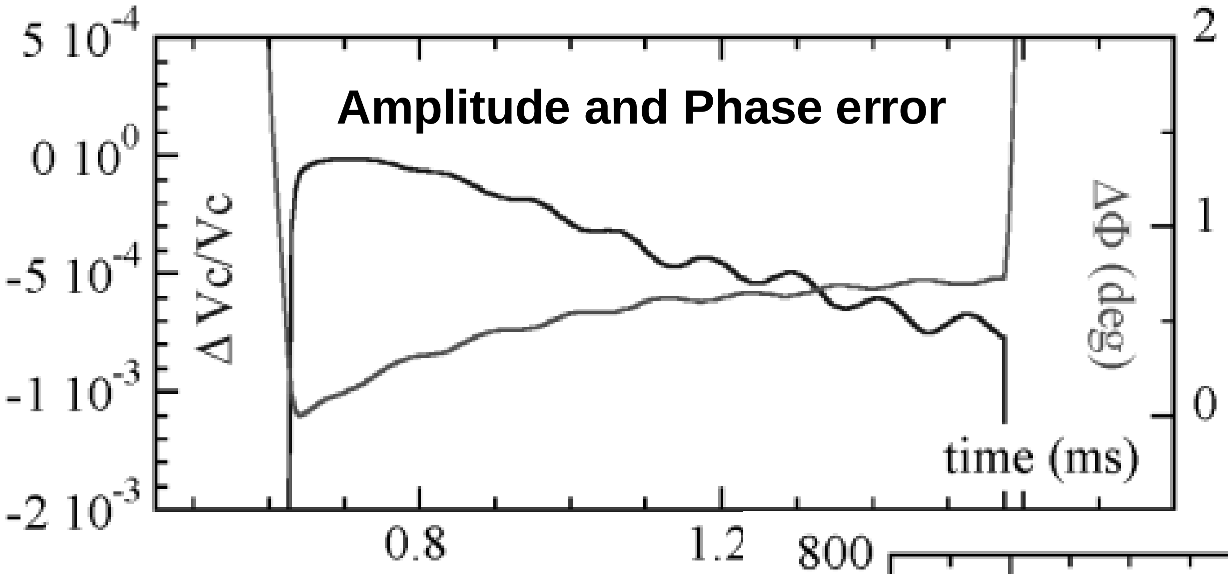
- Time varying detuning induced by Lorentz Force results in
  - Amplitude and Phase errors
  - Increase in peak power requirements
  - Increased stress on main power coupler
  - Reduced accelerator availability

Impact of Lorentz Force Detuning

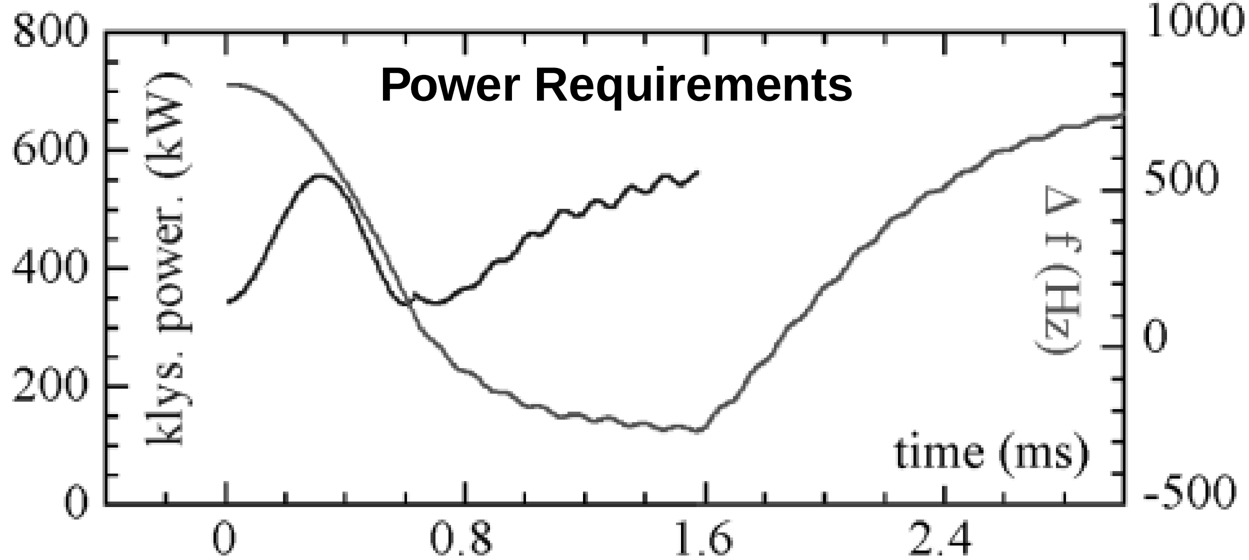




Impact of Lorentz Force Detuning



TESLA cavity at 35 MV/m with realistic feedback performance



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Modelling Lorentz Force Detuning

$$
\begin{bmatrix} \Delta\dot{\omega}_1 \\ \Delta\ddot{\omega}_1 \\ \vdots \\ \Delta\dot{\omega}_N \\ \Delta\ddot{\omega}_N \end{bmatrix} = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ -\omega_1^2 & -\frac{1}{\tau_1} & \dots & 0 & 0 \\ & & \vdots & & \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & -\omega_N^2 & -\frac{1}{\tau_N} \end{bmatrix} \cdot \begin{bmatrix} \Delta\omega_1 \\ \Delta\dot{\omega}_1 \\ \vdots \\ \Delta\omega_N \\ \Delta\dot{\omega}_N \end{bmatrix} + 2\pi \begin{bmatrix} 0 \\ -K_1\omega_1^2 \\ \vdots \\ 0 \\ -K_N\omega_N^2 \end{bmatrix} \cdot \begin{bmatrix} \mathrm{V}_{acc}^2 \end{bmatrix}
$$

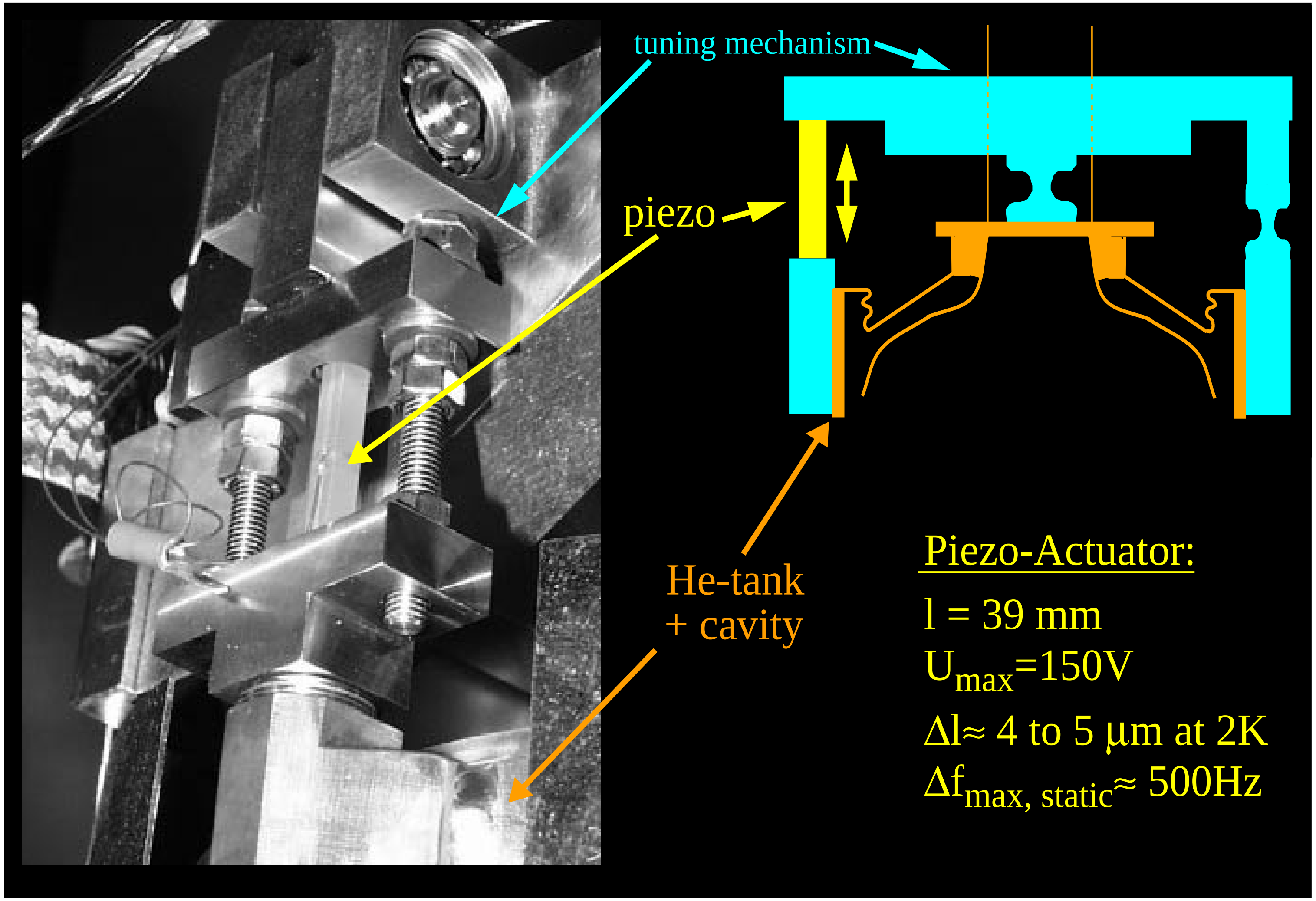
where $\Delta\omega_m$: detuning of mode *m*, V_{acc} : accelerating voltage, τ_m : mechanical time constant of mode *m* and K_m : Lorentz force detuning constant of mode *m*.

Active Lorentz Force Compensation

- Introduce counteracting force such that the resulting cavity detuning is constant.
  - Use fast piezo frequency tuner
  - Similar coupling to mechanical modes for Lorentz force and fast piezo tuner

Integration with Frequency Tuner

- ***Proof of Principle Setup of a fast Piezo-Tuner:***

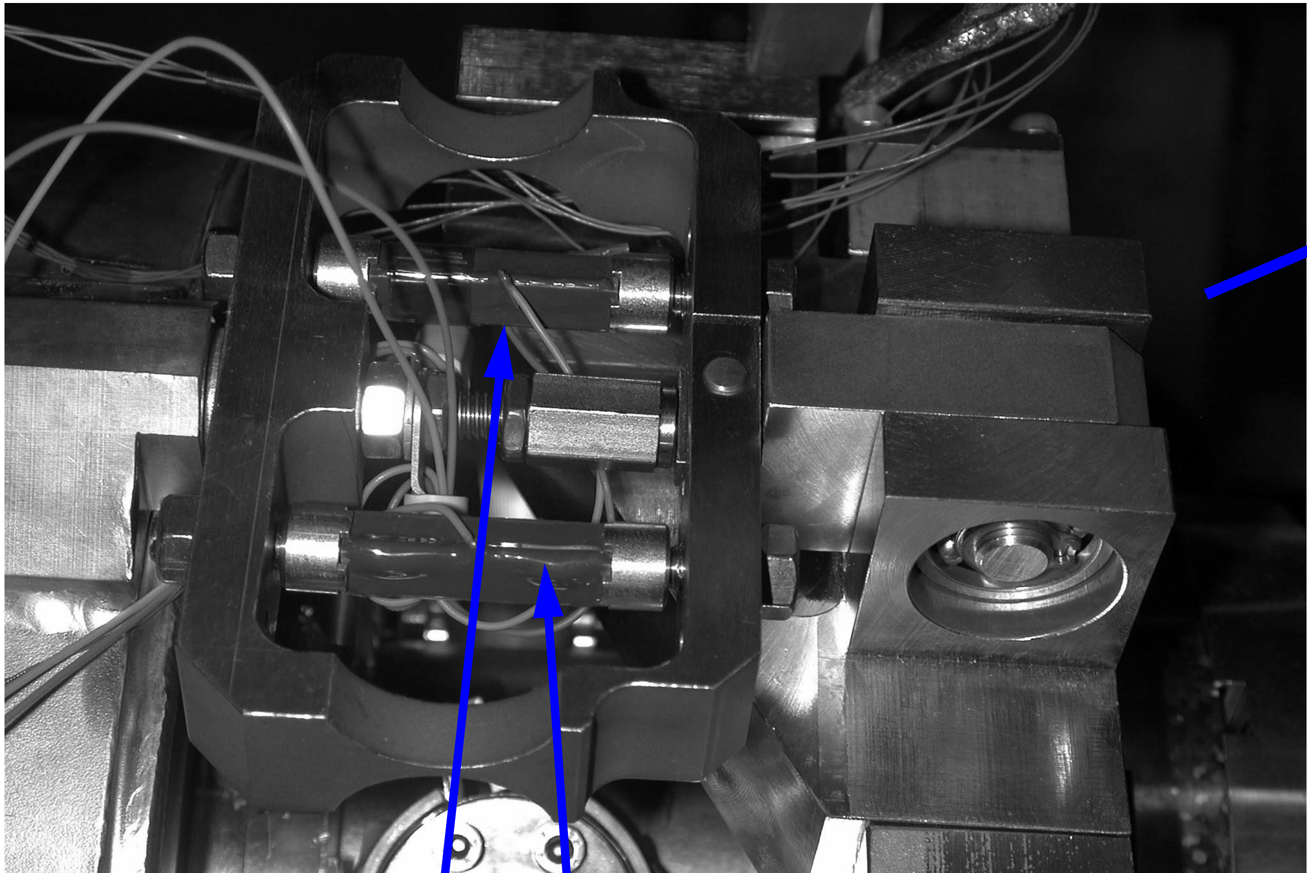


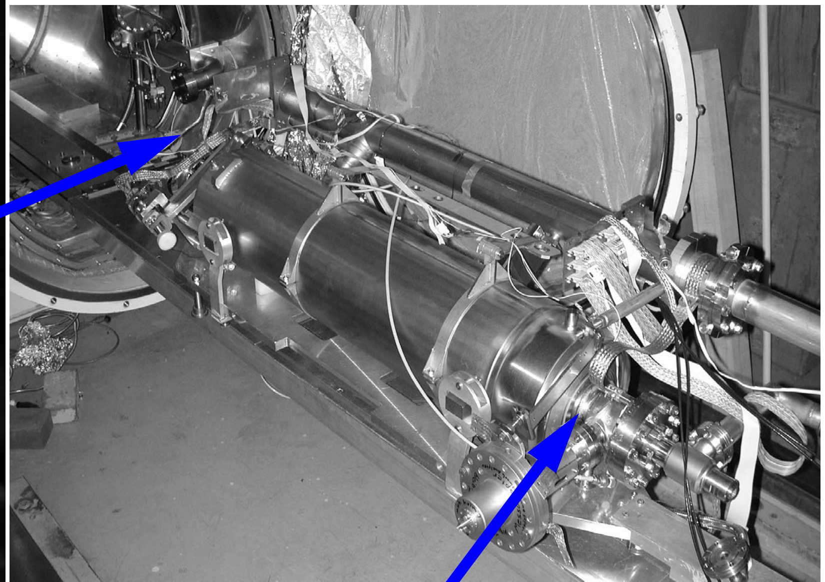
Feedforward vs Feedback

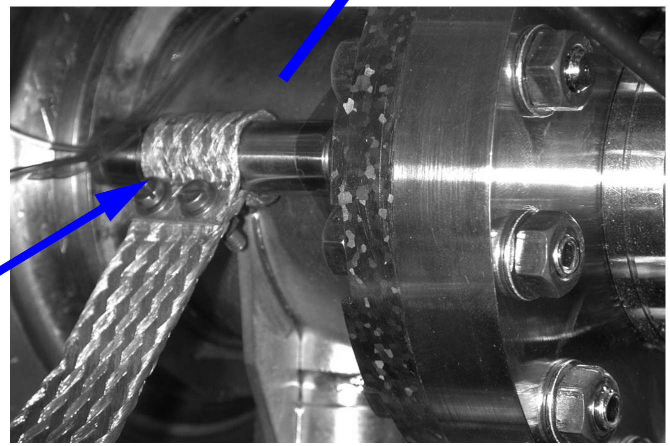
- Feedback
  - stability requirement must be fullfilled (i.e. sufficient phase margin must be guaranteed)
- Feedforward
  - requires repetitive perturbation
  - slow drifts can be corrected by adaptive Feedforward

Note: Control in both cases is only possible if the coupling factor of piezo to detuning is similar to that of Lorentz force to detuning.

Piezo fixture with parallel PZTs at TTF



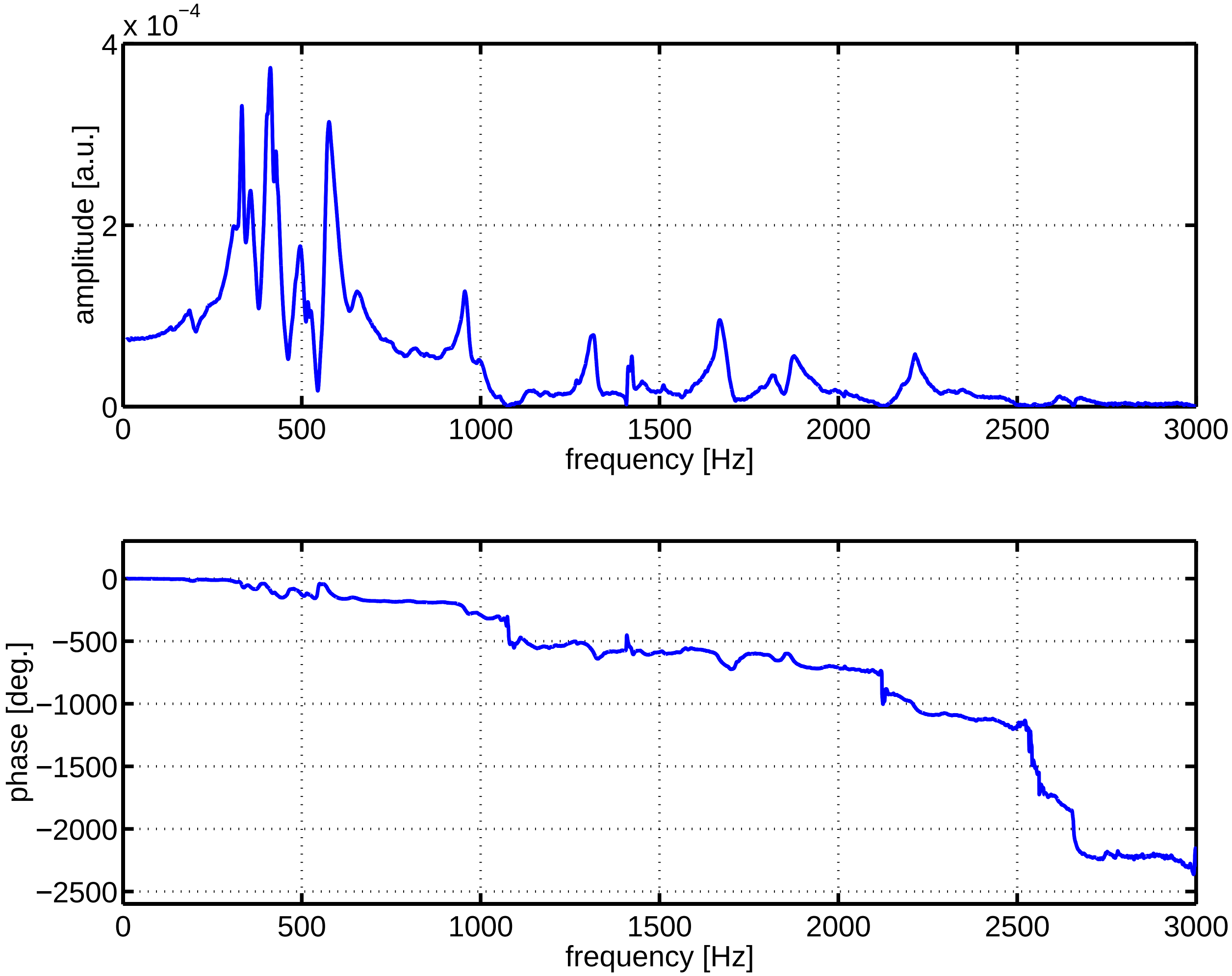




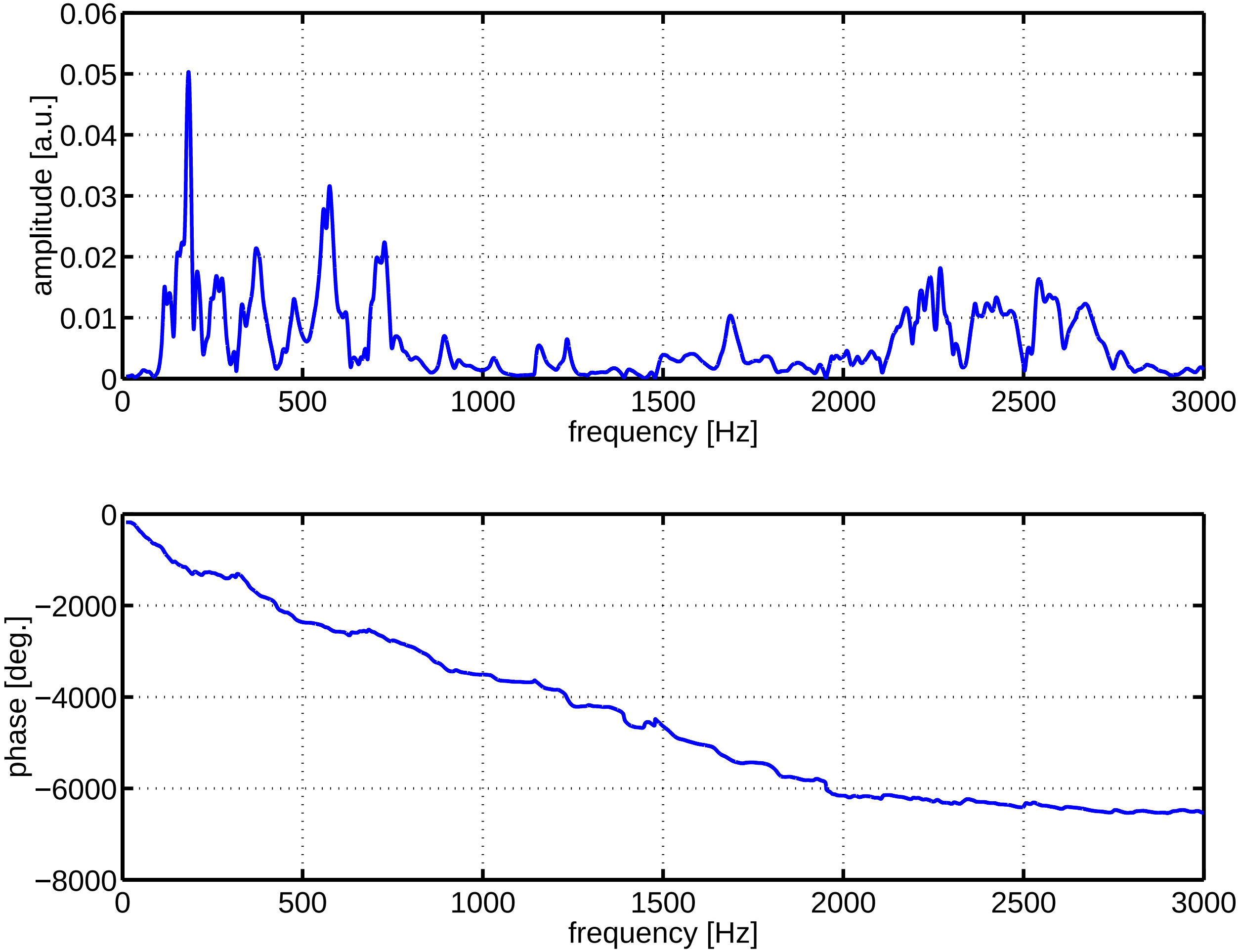
PZT 1 + 2

PZT 3

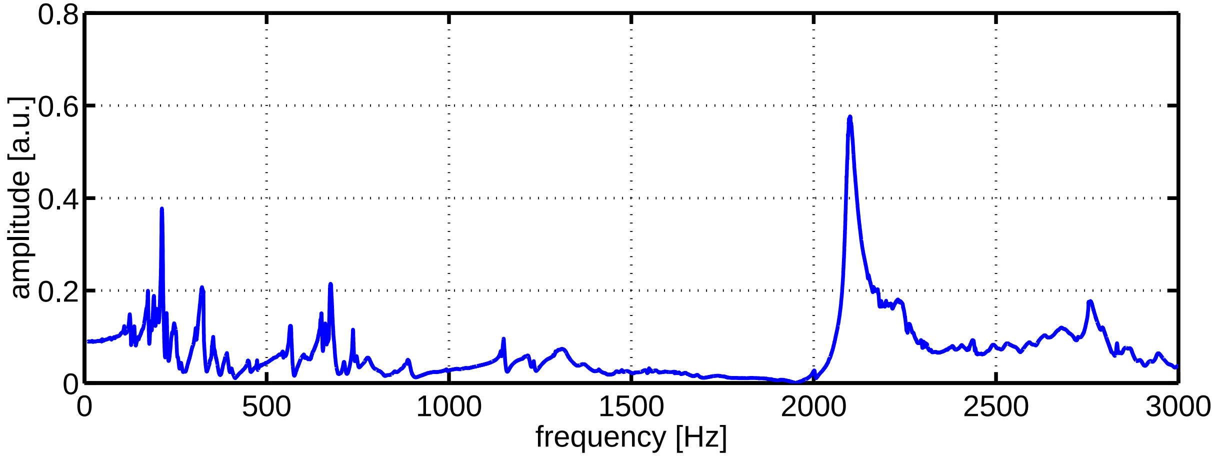
Transferfunction (PZT - RF)

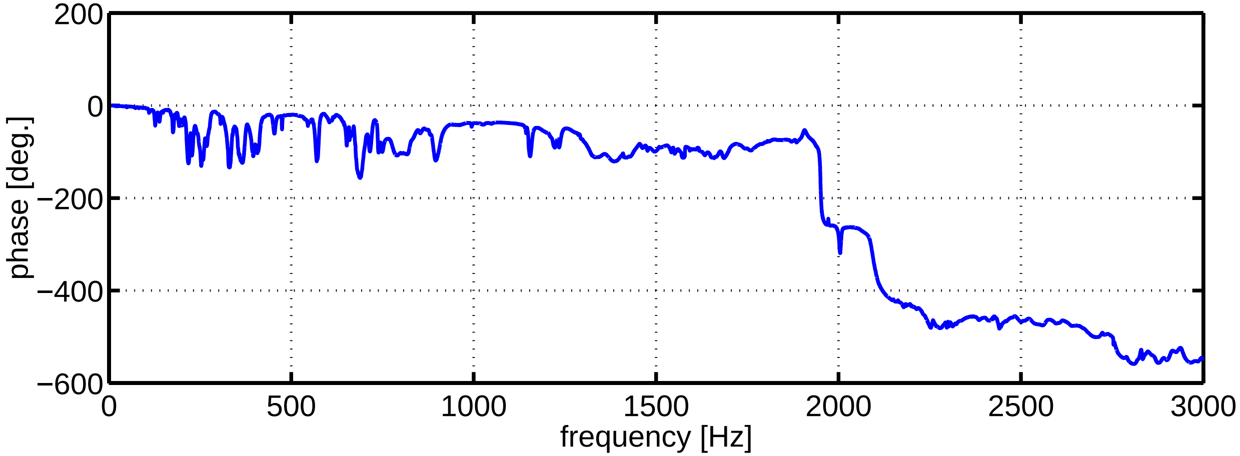


Transferfunction (PZT 1 - PZT 3)

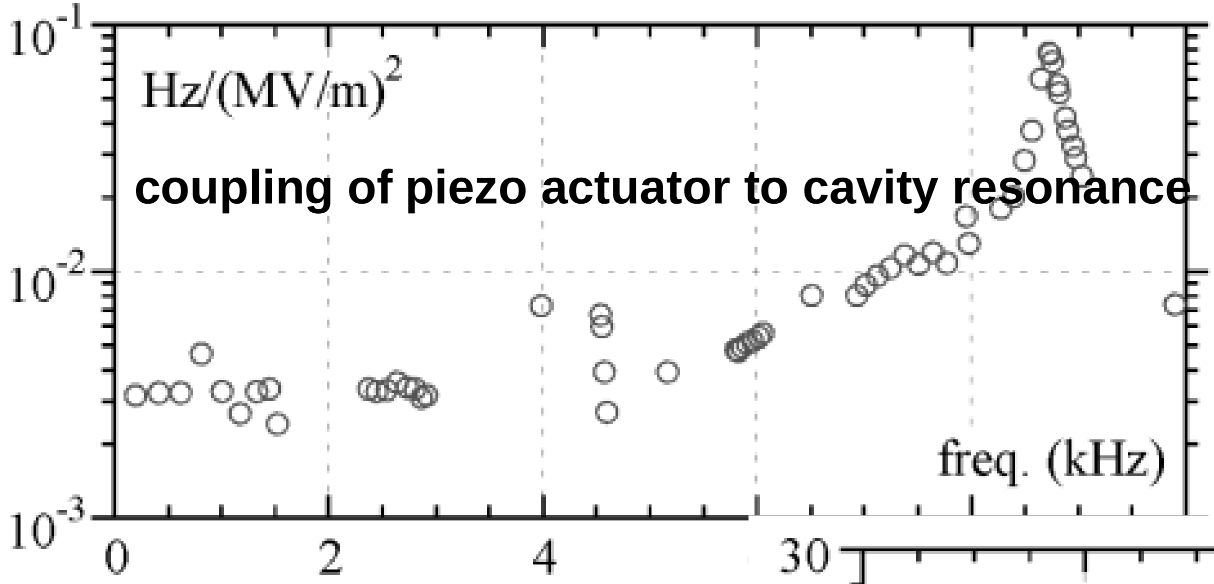


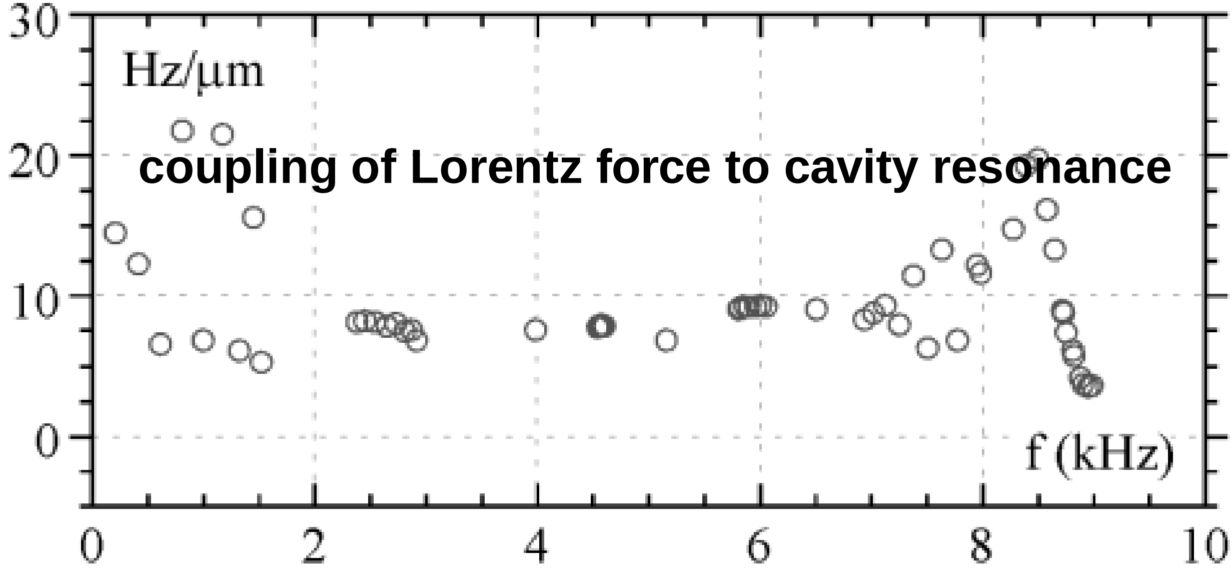
Transferfunction (PZT 1 - PZT 2)





Controllability





Similar coupling of Lorentz Force and Piezo actuator to cavity resonance frequencies are required.

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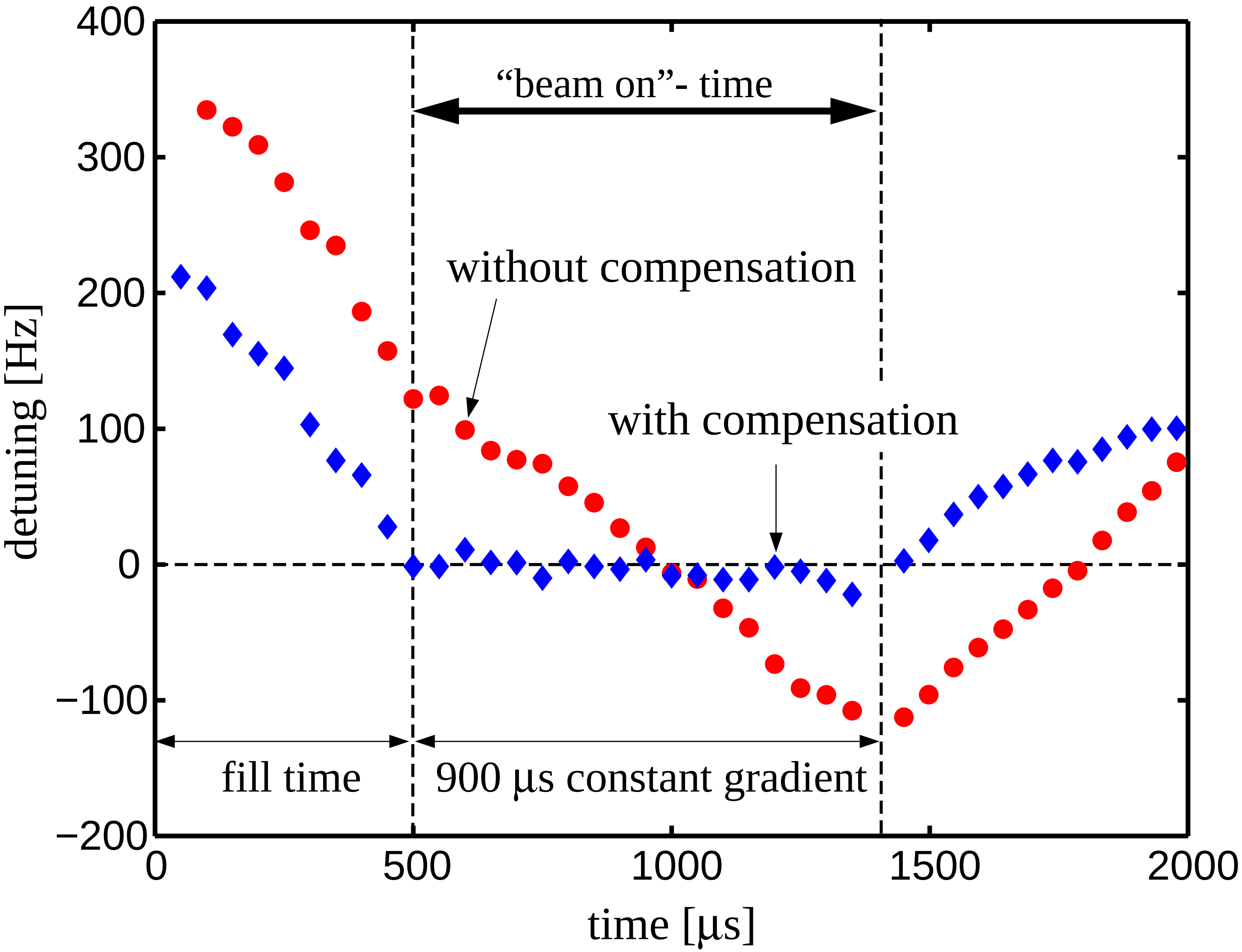
(simulation)

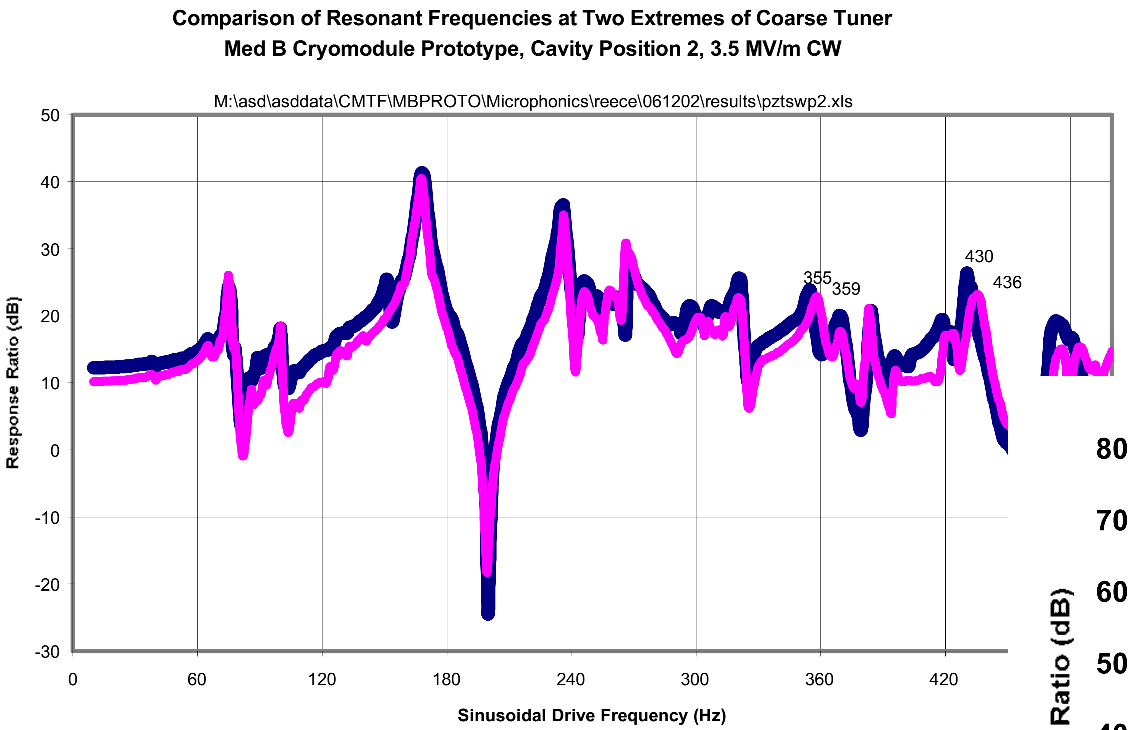
Active Compensation of Lorentz Force Detuning (2)



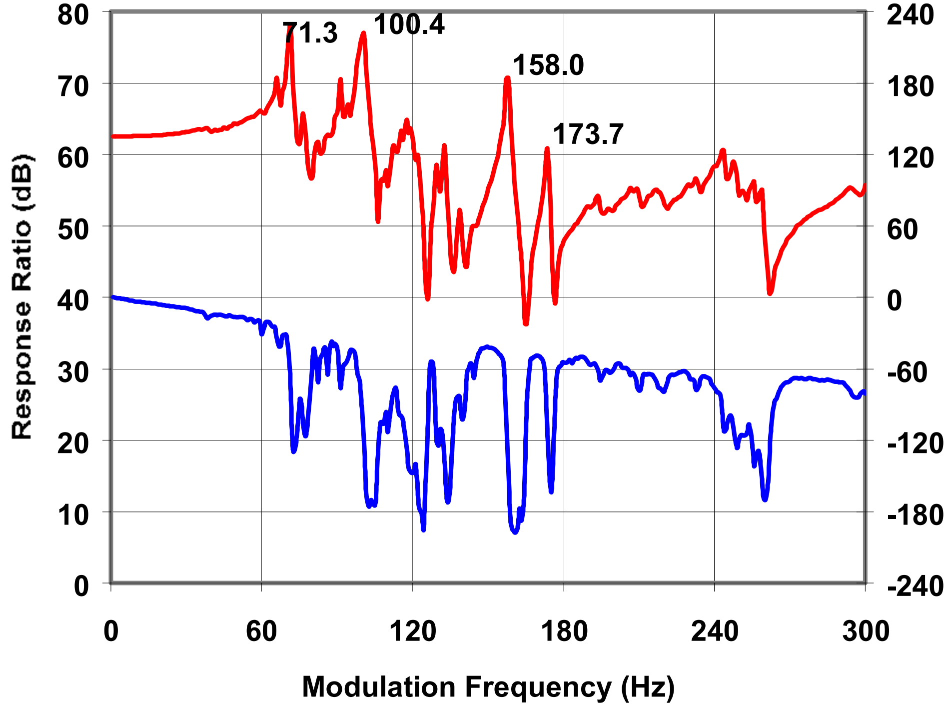
9-cell cavity operated at 23.5 MV/m

Lorentz force compensated with fast piezoelectric tuner

Experience with SNS cavity (β=0.61)



coupling Lorentz force to cavity detuning



Coupling piezo to cavity detuning

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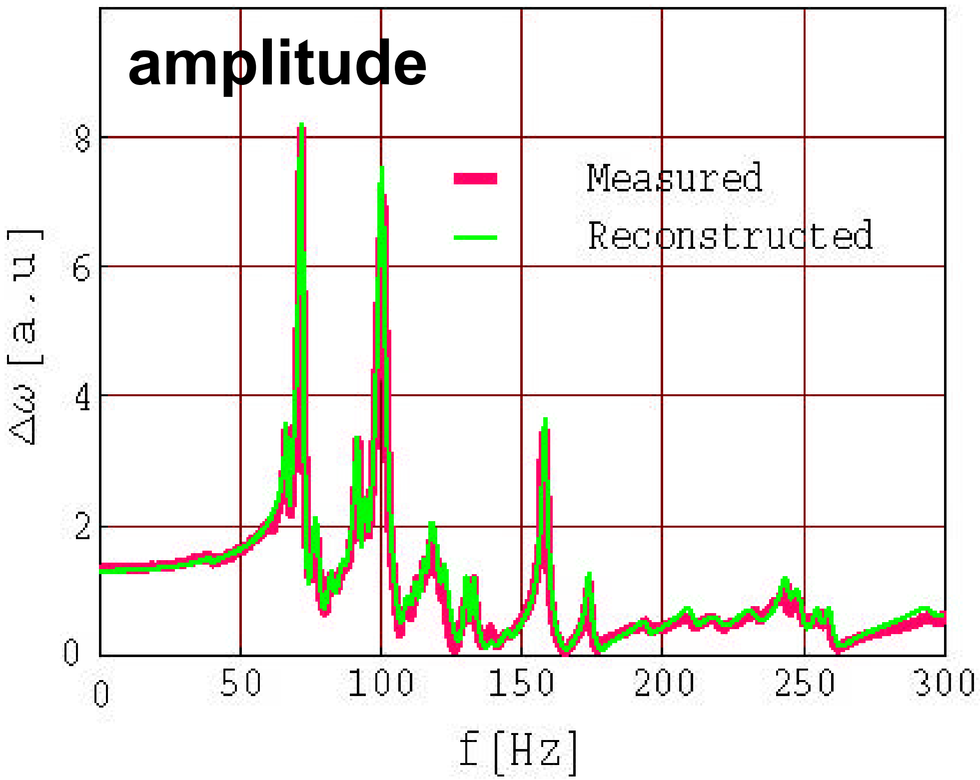
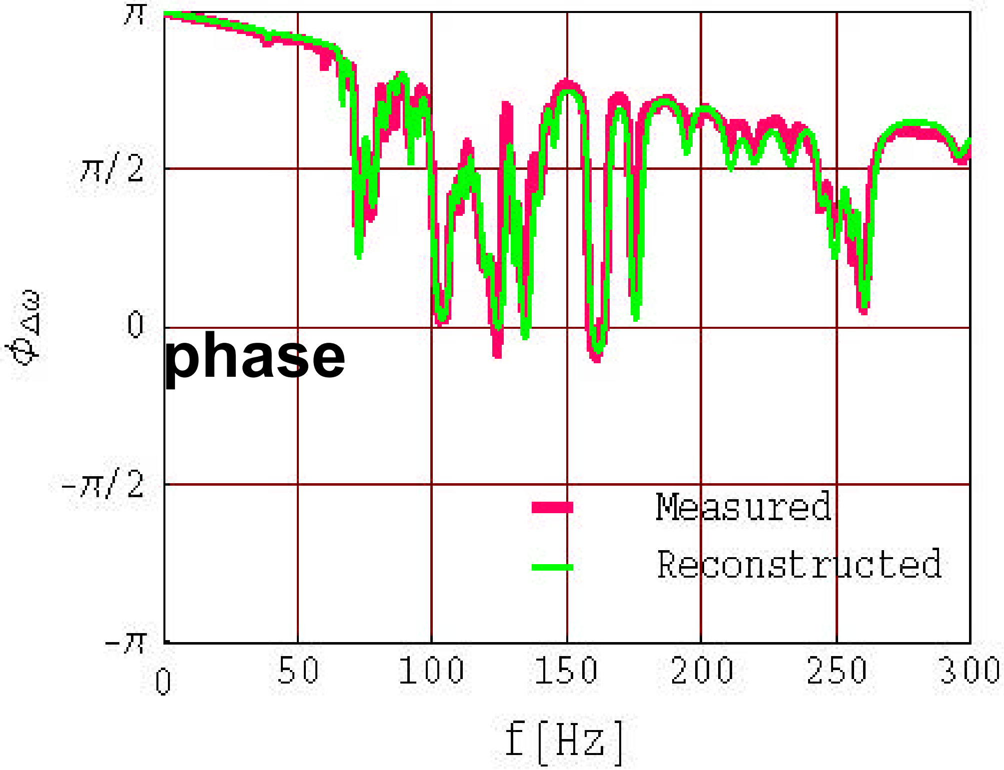
Transfer Function

Transfer function Lorentz Force --> Detuning, SNS cavity

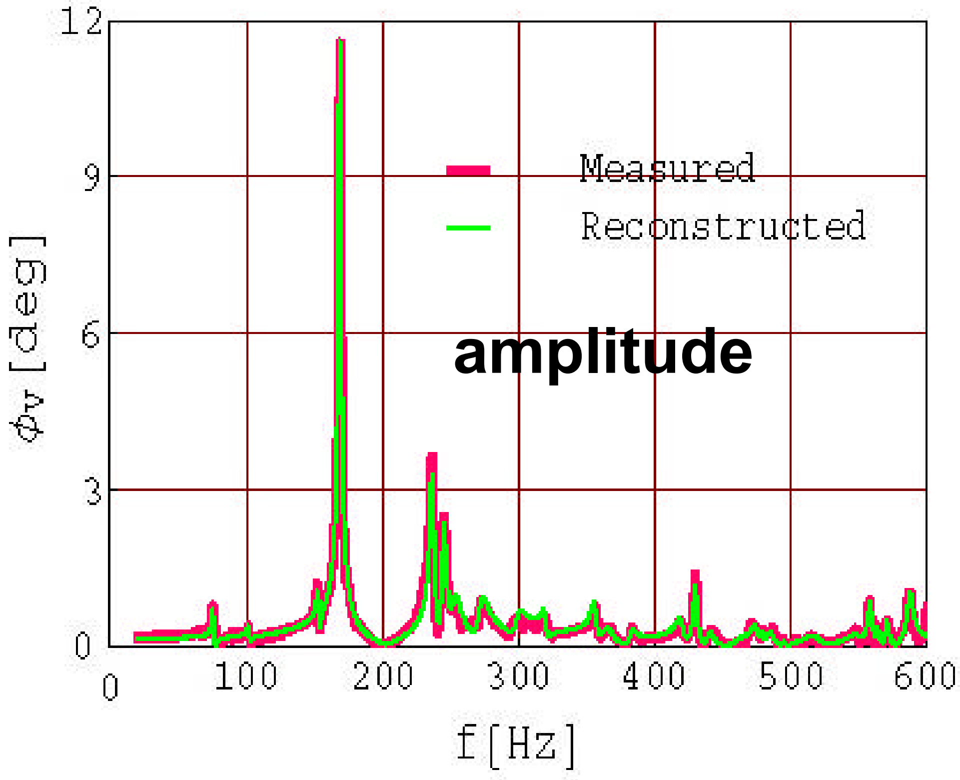


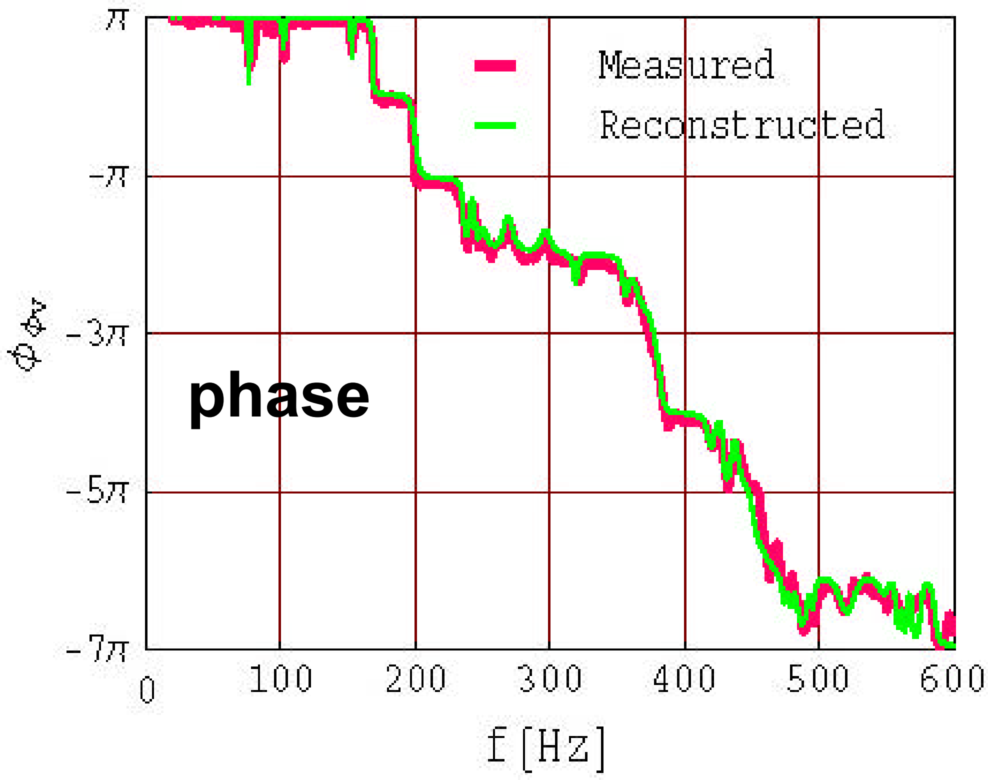


courtesy: J. Delayen, JLAB, M. Doleans, ORNL

Transfer Function

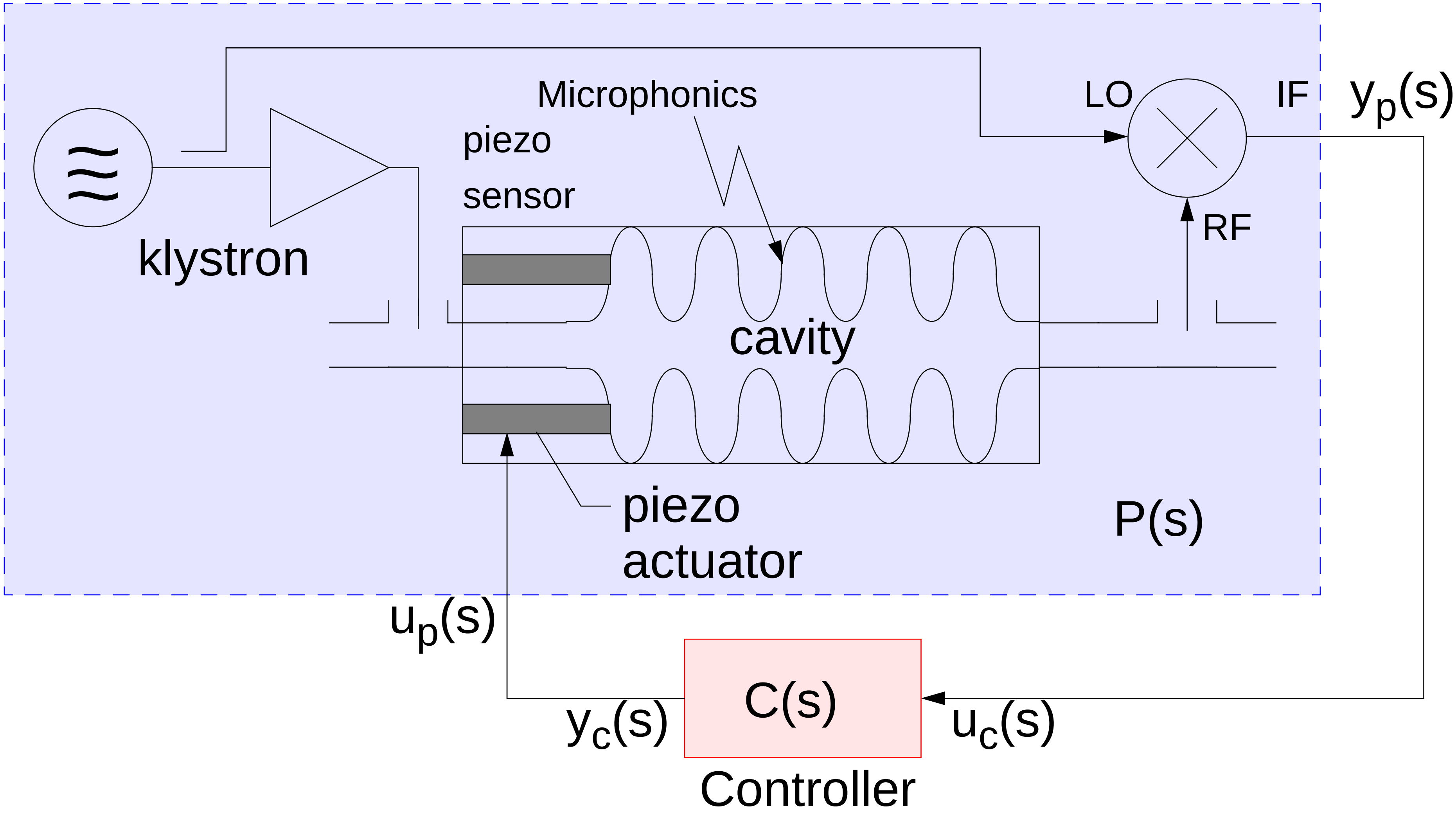
Transfer function Piezo Tuner --> Detuning, SNS cavity



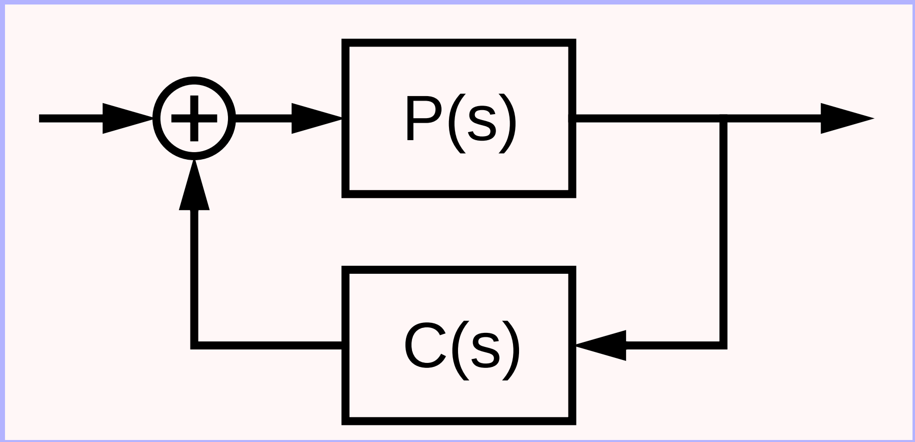


courtesy: J. Delayen, JLAB, M. Doleans, ORNL

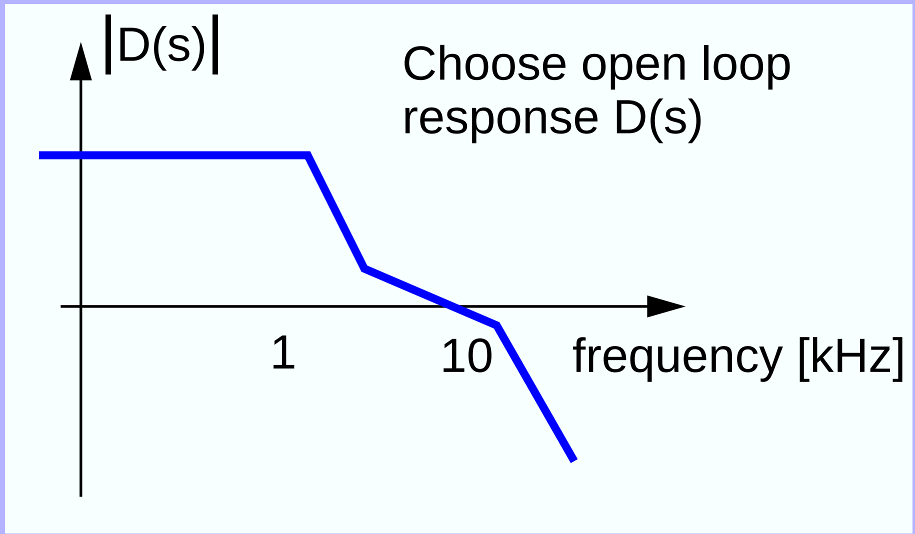
Microphonics Control



Controller Design



$$\Rightarrow \quad C(s) = \frac{D(s)}{P(s)}$$



D(s) : stability criteria fulfilled
high gain at low freq.
fast roll-off at high freq.

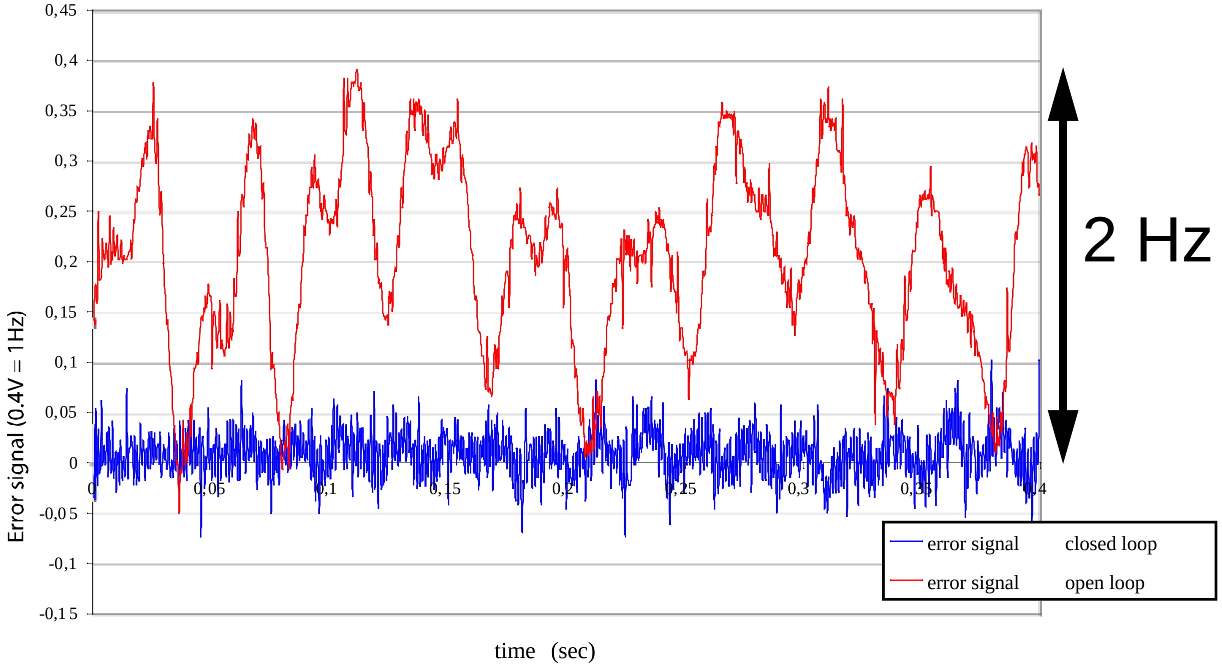
Digital Controller

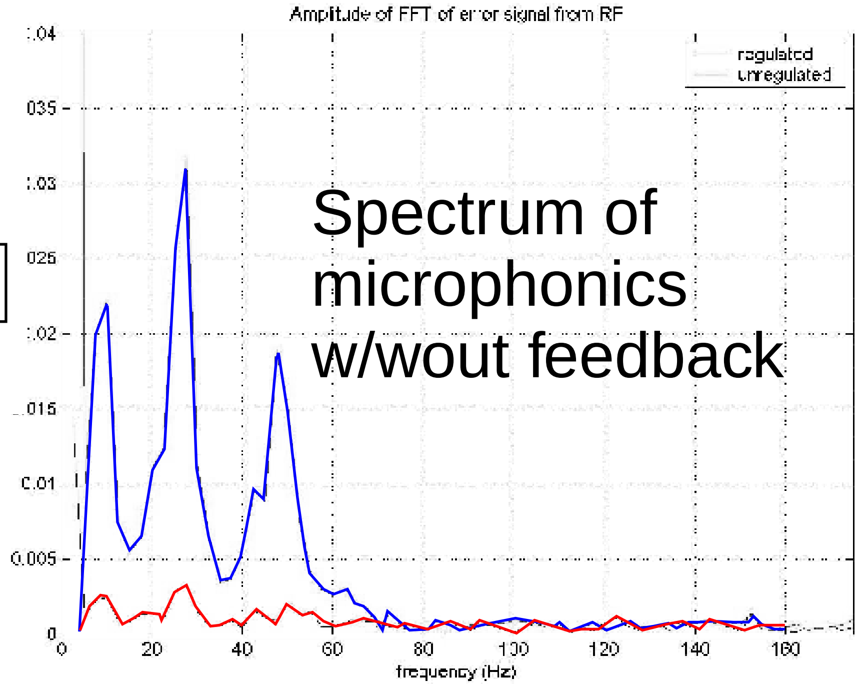
- C6701 processor from TI on PCI board (M67) with 4 ADCs and DACs (200kHz sampling rate)
- Programmed state space equation for 20th order system:

$$\vec{x}_{k+1} = A\vec{x}_k + B\vec{u}_k$$
$$\vec{y}_{k+1} = C\vec{x}_{k+1} + D\vec{u}_{k+1}$$

- Latency only 20 µs for 20x20 matrix multiplication (C++)
- Applied only notchfilter (672 Hz) and low pass (1kHz) to control microphonics in Q

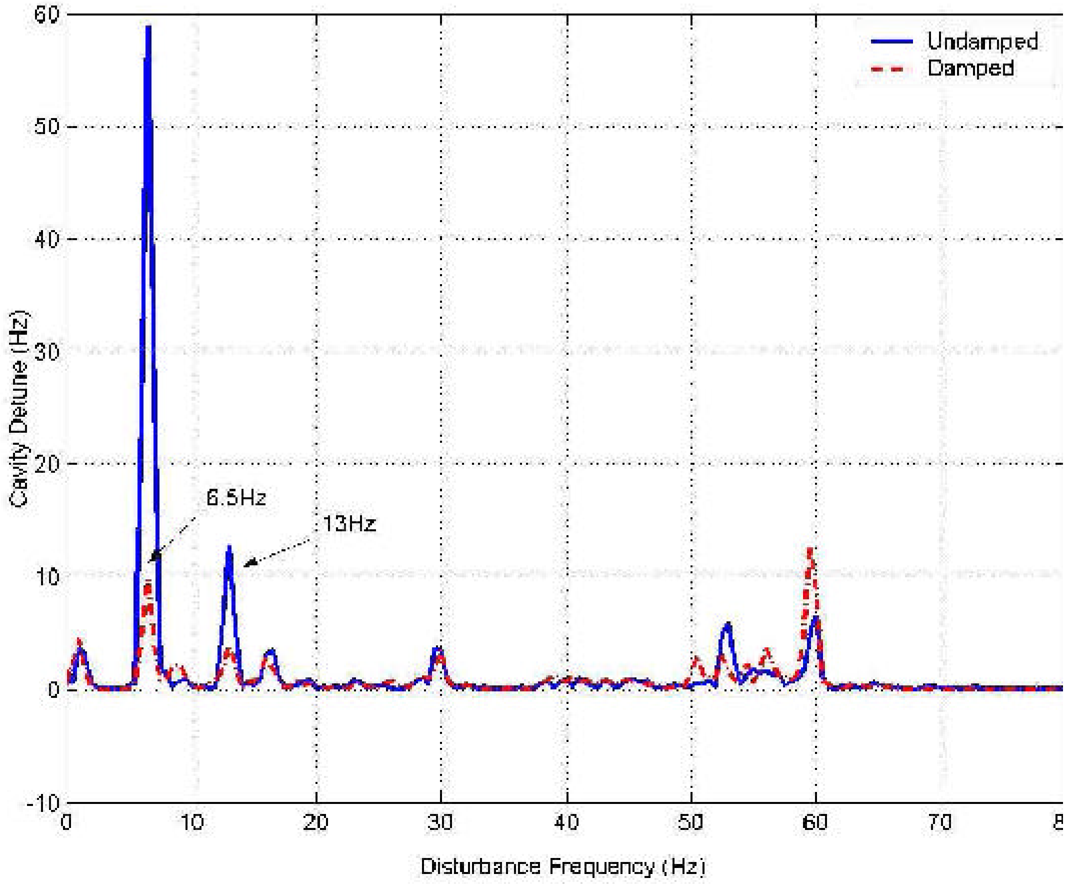
Control of Microphonics



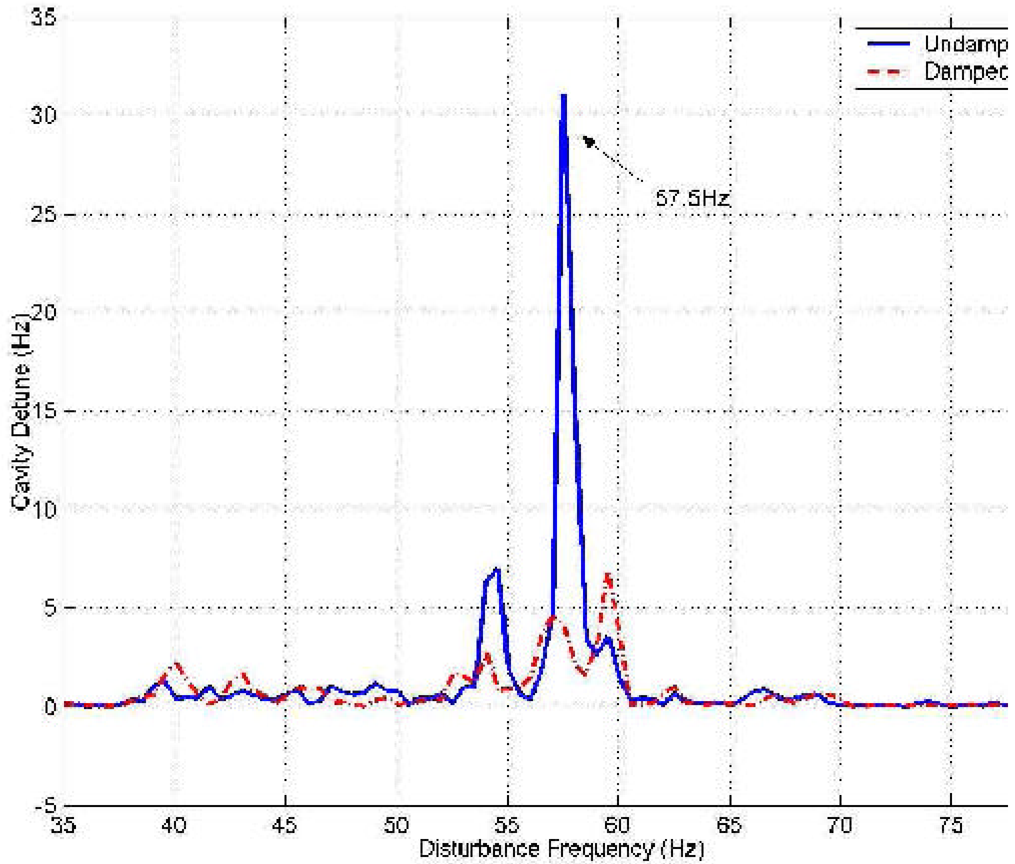


Microphonics suppressed by factor of 10 with feedback

Microphonics Suppression with Feedforward



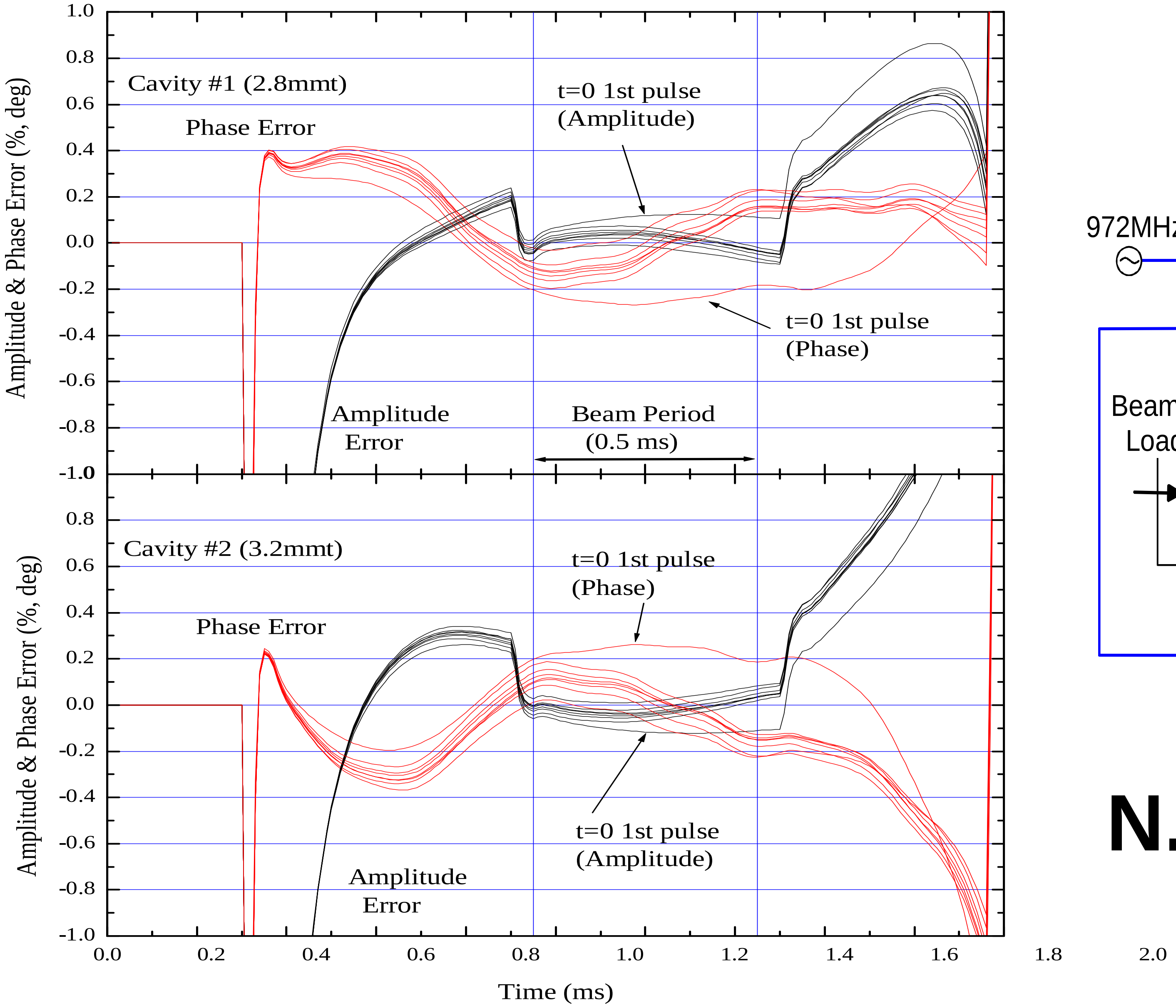
Active damping of helium oscillations at 2K.

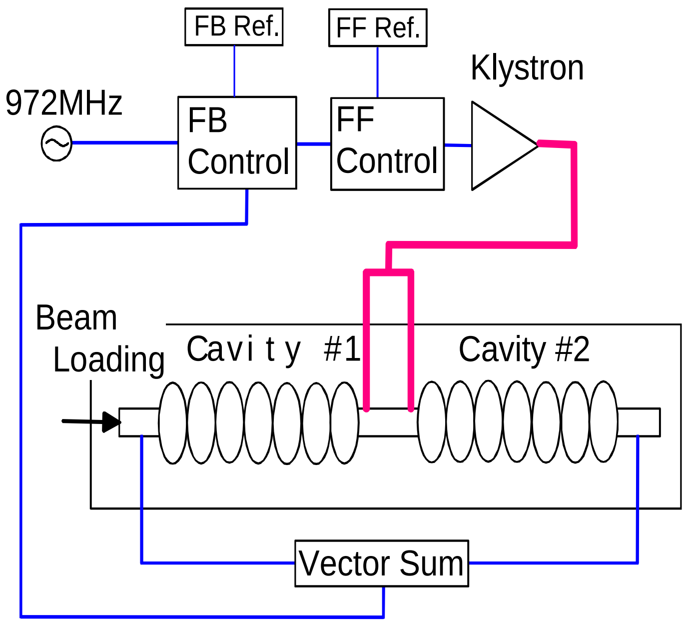


Active damping of external vibration at 2K.

T. Grimm

Vector-Sum Control





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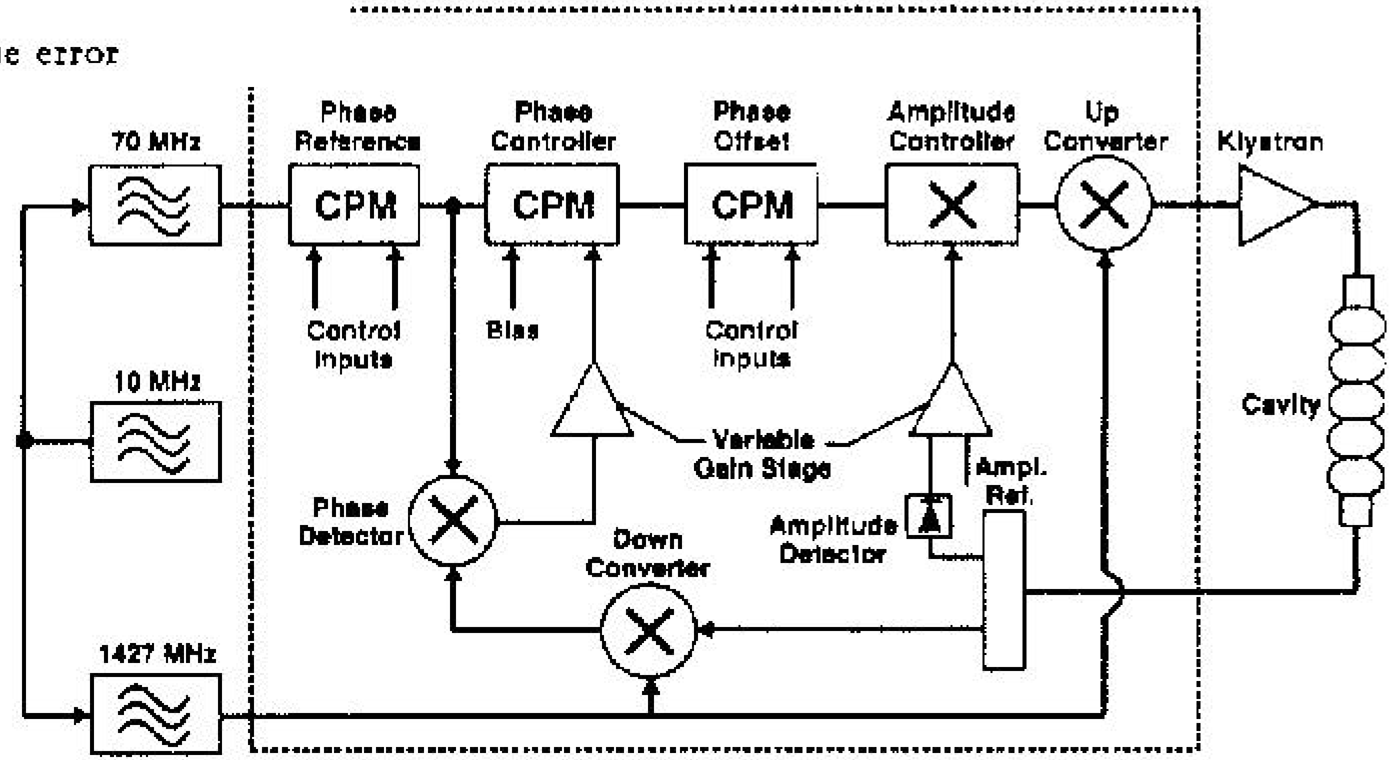
Requirements JLAB

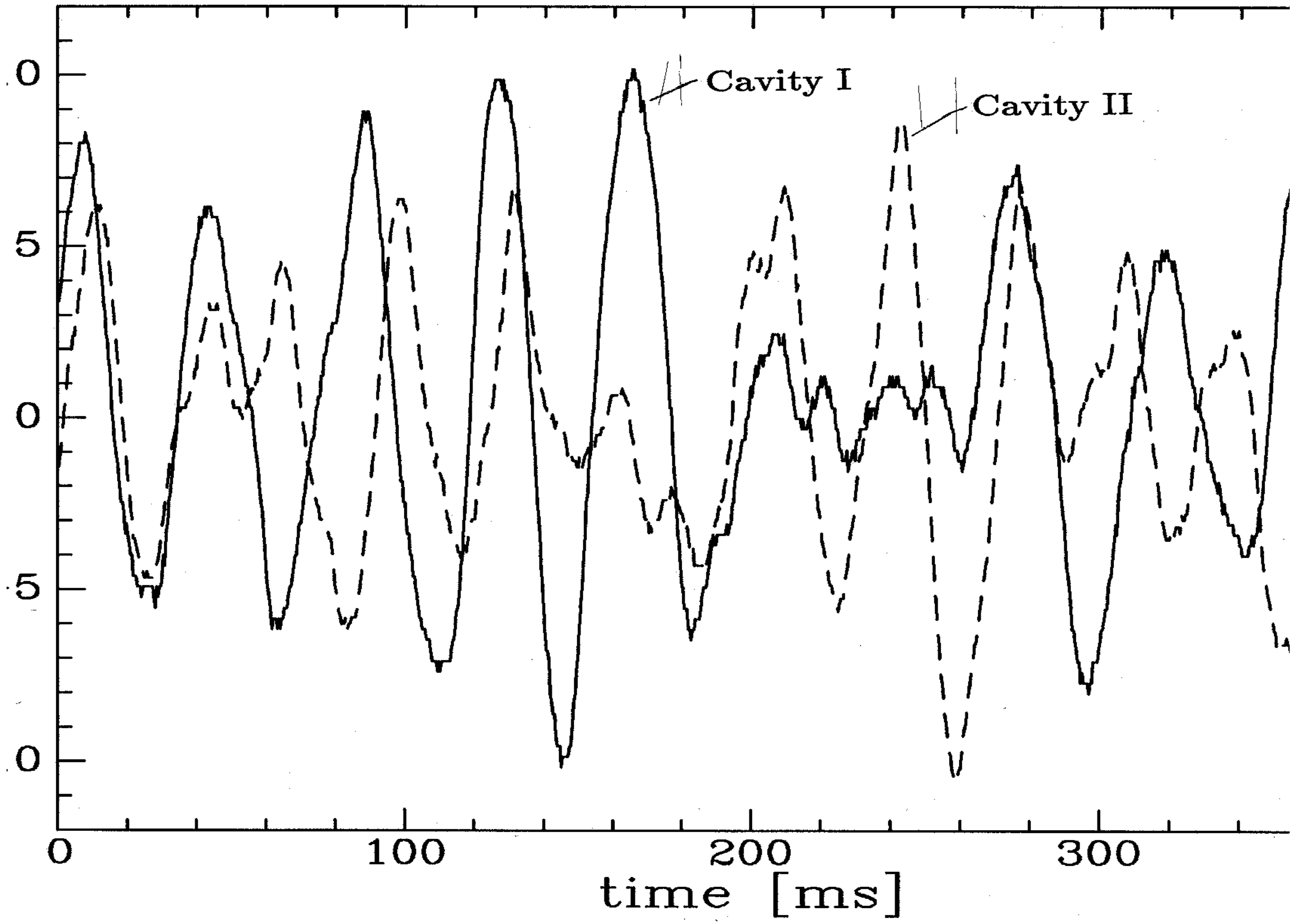
RF control requirements with vernier

RMS error	uncorrelated	correlated
σ_A	2×10^{-4}	1.1×10^{-5}
σ_f	$0.25°$	$0.13°$
σ_s	$2.6°$	∞

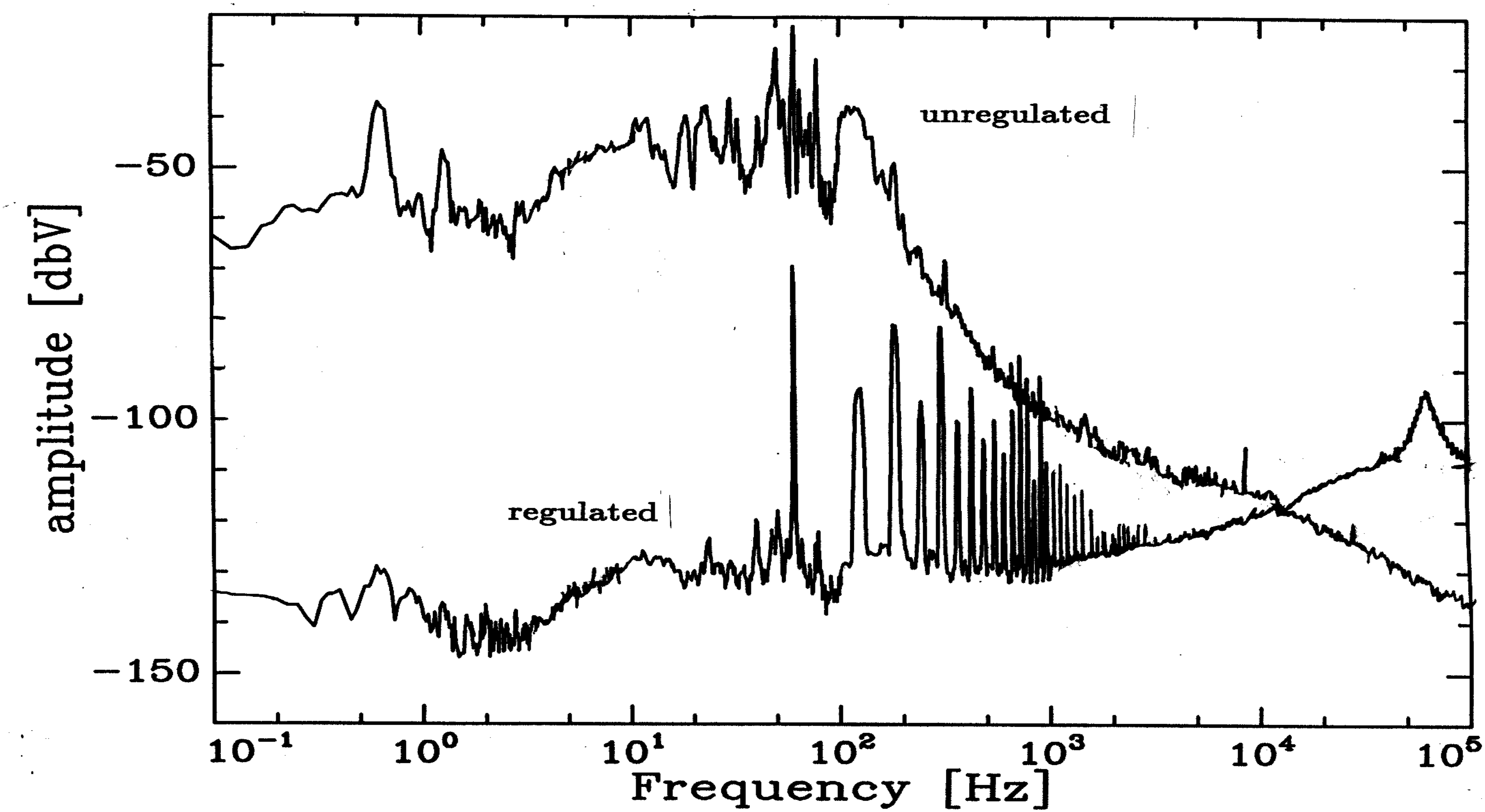
σ_A : relative RMS amplitude error
σ_f : fast RMS phase error
σ_s : slow RMS (along linac) phase error



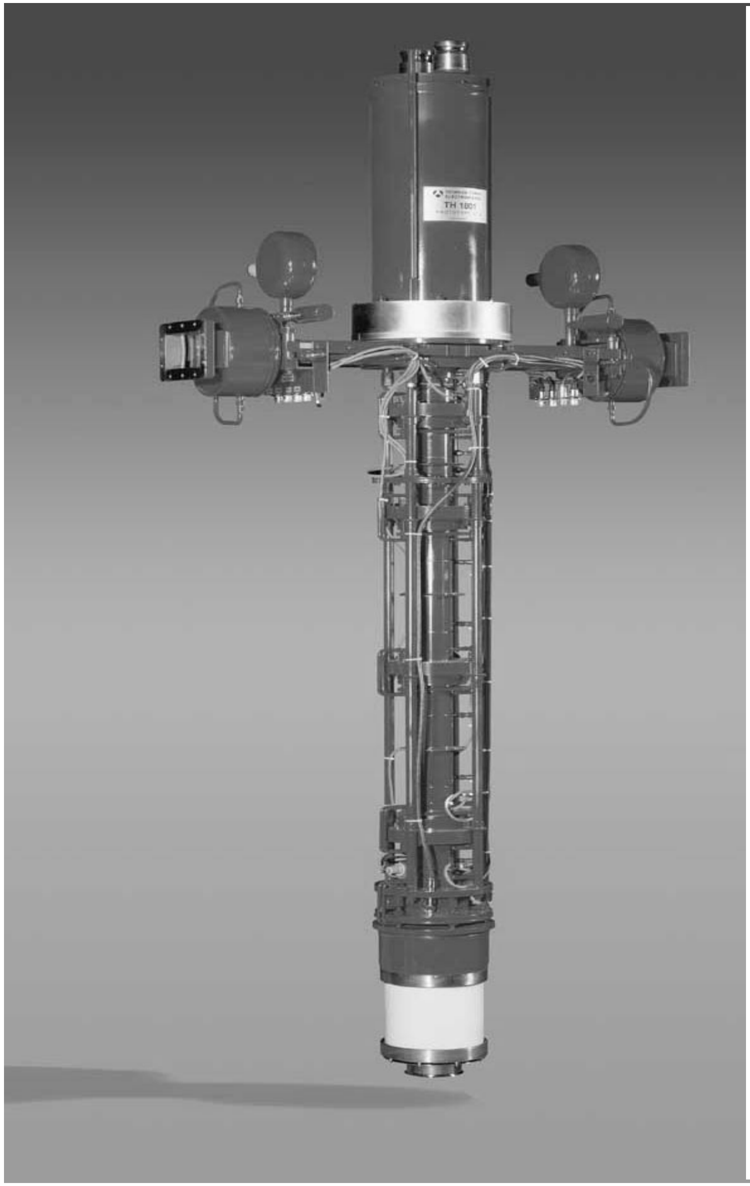
Microphonics Noise (JLAB)

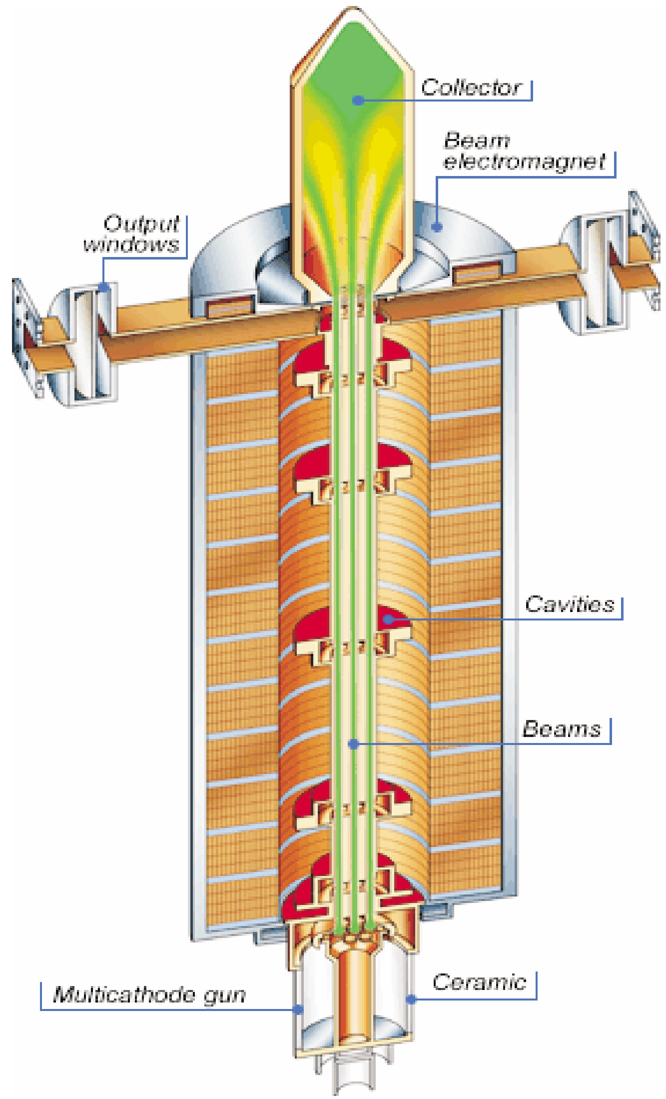


RF Control Performance at JLAB



Multibeam Klystron

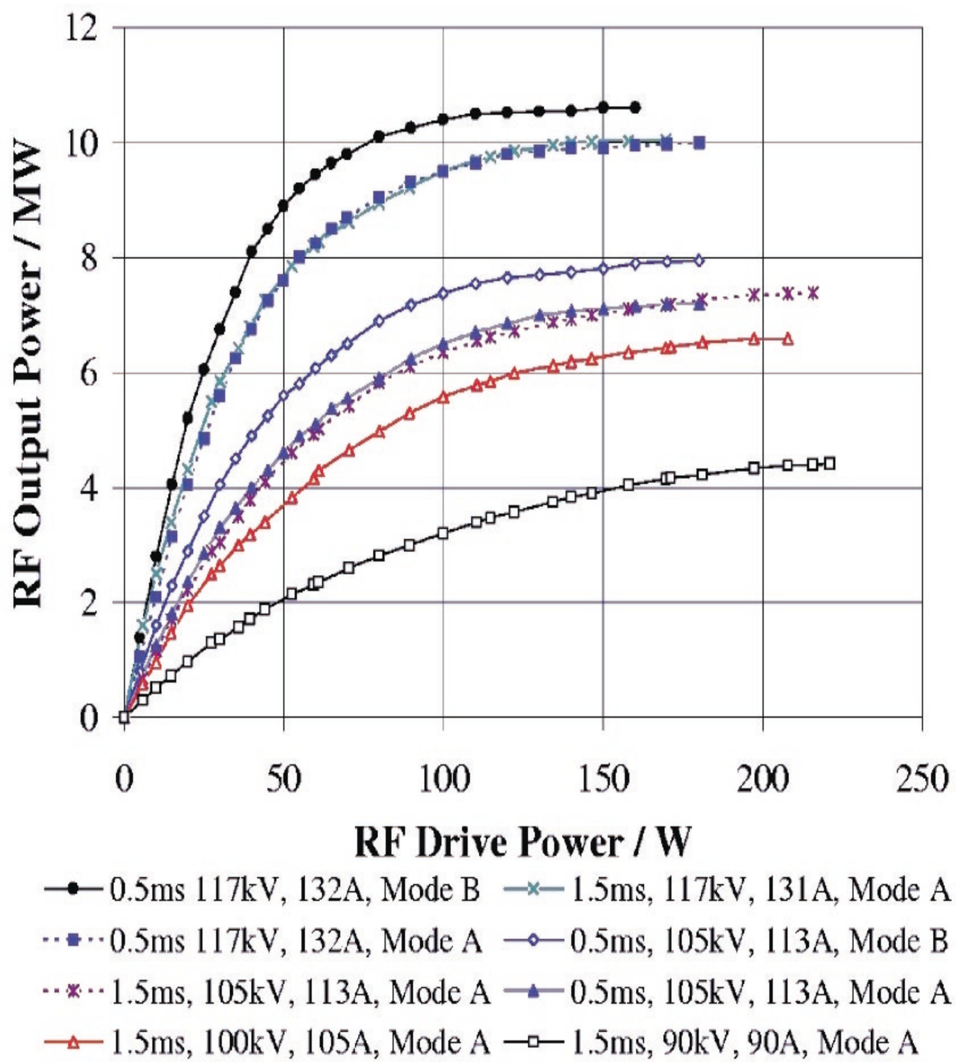




Multibeam Klystron

The TH1801

	Design
Operation Frequency	1300MHz
RF Pulse Duration	1.5ms
Repetition Rate	10Hz
Cathode Voltage	110kV
Beam Current	130A
HV Pulse Duration	1.7ms
No. of Beams	7
Total Perveance	$3.5 \cdot 10^{-6} A/V^{3/2}$
No. of Cavities	6
Max. RF Peak Power	10MW
RF Average Power	150kW
Efficiency	70% goal
Gain	48dB
Solenoid Power	4kW goal



Conclusion

- State space models for superconducting multicell cavities are available
  - Electrical models describe interaction with klystron and beam
  - Physical parameters of mechanical model not completely understood
- Model based analysis of feedback/feedforward systems agrees well with measurements
- Synthesis of optimal controller including feedforward and feedback remains to be done.
- Control of cavity detuning induced by microphonics is still an open issue.