Exercises in the lectures on " Superconducting RF - I and - II "

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Exercise I.

Using the Abrikosov's theory:

$$H_c = \frac{\kappa}{\lambda^2}\frac{\hbar c}{\sqrt{2}e*} = \frac{\kappa}{\lambda^2}\frac{(hc/2e)}{2\pi\sqrt{2}} = \frac{\phi_0}{2\pi\sqrt{2}\lambda\xi} \quad , \quad H_{c2} = \sqrt{2}\,\frac{\lambda}{\xi}\frac{\phi_0}{2\pi\sqrt{2}\lambda\xi} = \frac{\phi_0}{2\pi\xi^2}$$

$$\phi_0 = hc/2e = 2.0678\times 10^{-7}\, Gauss\cdot cm^2$$

$$=2.0678\times 10^{-15}\ T\cdot m^2$$

1) write down ξ, λ by H_C and H_{C2},

2) get the T-dependences of ξ, H_{C2}, κ, H_C^{RF}, from the given T-dependences of λ and H_C:

$$H_C(T) = H_C(0)\left[1-(T/T_C)^2\right],\quad \lambda(T) = \frac{\lambda(0)}{\sqrt{1-(T/T_C)^4}}$$

,here H_C^{RF} is given as $H_C^{RF} = \sqrt{2}\cdot\frac{H_C}{\kappa}$.

Exercise II.

1) **Get the following formula for the surface resistance Rs for good electric conductor.**

$$R_S = \sqrt{\frac{\mu\omega}{2\sigma}} = \frac{1}{\sigma}\sqrt{\frac{\mu\sigma\omega}{2}} = \frac{1}{\sigma\delta}$$

2) **Calculate the δ and R_S for a 1300MHz copper cavity, when the σ is given as $1/\sigma = 1.72E\text{-}8\ [\Omega m]$ at $20^{O}C$.**

3) **If the RRR of the copper material is 40, calculate the Rs at 4.2K.**

Exercise III.

By the two fluid model, electric conductivity is given as the bellow:

$$\mathbf{J} = \left(\frac{n_n e^2}{\mathrm{v} m_e} - i\frac{n_s q_s^2}{\omega m_s} \right)\mathbf{E} = \sigma E, \quad \sigma = \sigma_{\mathrm{n}} - i\sigma_s$$

Put this complex electric conductivity into the formula of surface impedance: $Z=R_S+iX_S$, show the surface resistance and admittance for superconductor are:

$$R_S = \frac{1}{2}\sigma_n \omega^2 \mu^2 \lambda_L^3, \quad X_S = \omega\mu\lambda_L \ \text{ and } \ \sigma_n = \frac{n_n \cdot e^2}{\mathrm{v} \cdot m_S}$$

n_n is the number of unpaired electrons (quatsi particle), then it could be written by Boltzman statistics as:

$$\sigma_n = \frac{e^2}{m \cdot \mathrm{v}} n_s(0) e^{-\frac{\Delta}{k_B T}}$$

Show the formula of surface resistance in case of superconductor as:

$$R_S(T, f) = A(\lambda_L, \xi, \ell, T_C) \cdot f^2 \cdot \exp(-\frac{\Delta}{k_B T})$$

Exercise IV.

Get the formulas in lecture note p.65

$$\mathbf{B}_t = \frac{1}{\left(\varepsilon\mu\frac{\omega^2}{c^2} - k^2\right)}\left[\nabla_t\left(\frac{\partial B_z}{\partial z}\right) + i\varepsilon\mu\frac{\omega}{c}\mathbf{e}_z \times \nabla_t E_z\right],$$

$$\mathbf{E}_t = \frac{1}{\left(\varepsilon\mu\frac{\omega^2}{c^2} - k^2\right)}\left[\nabla_t\left(\frac{\partial E_z}{\partial z}\right) - i\frac{\omega}{c}\mathbf{e}_z \times \nabla_t B_z\right]$$

Exercise V.

Make design a 1300MHz TM_{010} – mode single cell Pill Box cavity

1.What is the diameter of the cell?

2. What is the cell length?

Exercise VI.

Superfish outputs

f_0=1293.77430MHz
Ploss=118.1551W
RsQ=265.171 Ω
Qo=28257.6
(Rsh/Q)=109.24 Ω
Hp=1753.44 A/m
Ep=0.946176 MV/m

Calculate the following cavity RF parameters from above Superfish outputs.

Rsh [Ω] =
Accelerating Voltage V [MV]=
RF wave length λ[m] =
Gradient Eacc = V/L_{eff} [MV/m]= **,defined as L_{eff} =λ/2**
Hp/Eacc[Oe/(MV/m)] = **, use 1A/m= $4\pi 10^{-3}$ Oe**
Ep/Eacc =
Eacc [MV/m] = [illegible] **Z=**
Geometrical factor Γ [Ω] =

Exercise VII.

Calculate the cable correction factors: C_{in}, C_r and C_t, when measurement results are:

p_{in}=55.5μW, p_o=50.0mW, p_r=10.72μW, p_t=3.04mW

and

p_o'=39.0mW, p_{in}'=22.6mW, p_t'=27.9mW

Exercise VIII.

Calculate βin*, βin, βt, P_{loss}[W]', Q_L, Q_{in}, Q_o, Q_t, R_s[Ω], Eacc[MV/m], E_p[MV/m], and H_p[Oe], when measure results are :

f_0=1303.590529MHz,

$\tau_{1/2}$=23.6 msec,

coupling over,

p_{in}=3.11mW, p_r=192nW, p_t=0.142mW.

For the cable correction factors, use the results of the exercise VII.

RF cavity parameters are given as following:

Γ=269Ω, Ep/Eacc=1.83, Hp/Eacc=45.2 Oe/[MV/m],

and $Eacc[MV/m] = 86.94\sqrt{P_t[W] \cdot Q_t}$