Answers for the Exercises for the lecture "Superconducting RF-I and II"

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Exercise I

$$\xi = \sqrt{\frac{2\pi}{\phi_0} \cdot H_{C2}},$$

$$\frac{H_C}{H_{C2}} = \frac{2\pi\xi^2}{2\pi\sqrt{2}\lambda\xi} = \frac{\xi}{\sqrt{2}\lambda}$$

$$\lambda = \frac{\xi}{\sqrt{2}} \cdot \frac{H_{C2}}{H_C} = \sqrt{\frac{\phi_0}{4\pi} \cdot \frac{H_{C2}}{H_C^2}}$$

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}}, \quad H_C(T) = H_C(0) \cdot \left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$H_{C2}(T) = \frac{4\pi\lambda^2(0)H^2{}_C(0)}{\phi_0} \cdot \frac{1-\left(\frac{T}{T_C}\right)^2}{1+\left(\frac{T}{T_C}\right)^2} = H_{C2}(0) \cdot \frac{1-\left(\frac{T}{T_C}\right)^2}{1+\left(\frac{T}{T_C}\right)^2}$$

$$\xi(T) = \sqrt{\frac{\phi_0}{2\pi H_{C2}(0)}} \cdot \sqrt{\frac{1+\left(T/T_C\right)^2}{1-\left(T/T_C\right)^2}} = \xi(0) \cdot \sqrt{\frac{1+\left(T/T_C\right)^2}{1-\left(T/T_C\right)^2}}$$

$$\kappa(T) = \frac{\lambda(T)}{\xi(T)} = \frac{1}{\sqrt{2}} \cdot \frac{H_{C2}(T)}{H_C(T)} = \frac{H_{C2}(0)}{\sqrt{2}H_C(0)} \cdot \frac{1}{1+\left(T/T_C\right)^2} = \frac{\kappa(0)}{1+\left(T/T_C\right)^2}$$

$$H_C^{RF}(T) = \sqrt{2}\frac{H_C(T)}{\kappa(T)} = \frac{\sqrt{2}H_C(0)}{\kappa(0)} \cdot \left[1+\left(\frac{T}{T_C}\right)^4\right] = H_C^{RF}(0) \cdot \left[1+\left(\frac{T}{T_C}\right)^4\right]$$

Exercise II

1)

$$k=\alpha+i\beta,\quad \alpha=\omega\sqrt{\mu\varepsilon}\left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1}{2}\right]^{1/2},\quad \beta=\omega\sqrt{\mu\varepsilon}\left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1}{2}\right]^{1/2}$$

$when\ \sigma/\omega\varepsilon \gg 1$ (*very good* electric conductor case),

$$\alpha=\beta\approx\sqrt{\mu\varepsilon}\left[\frac{\sqrt{\left(\frac{\sigma}{\omega\varepsilon}\right)^2}}{2}\right]^{1/2}=\sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\therefore k\simeq(1+i)\cdot\sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\delta(\text{skin depth})\equiv\frac{1}{\beta}=\sqrt{\frac{2}{\mu\sigma\omega}}$$

$$R_S(\text{surface resistance})\equiv \text{Re(Z)=Re(}\frac{\mu\omega}{k}\text{)},$$

$$Z=\mu\omega\cdot\sqrt{\frac{2}{\mu\sigma\sigma}}\cdot\frac{1}{1+i}=\mu\omega\cdot\sqrt{\frac{2}{\mu\sigma\sigma}}\cdot\frac{1-i}{2}=(1-i)\cdot\sqrt{\frac{\mu\omega}{2\sigma}}$$

$$\text{Re}(Z)=R_S=\sqrt{\frac{\mu\omega}{2\sigma}}=\frac{1}{\sigma}\sqrt{\frac{\mu\sigma\omega}{2}}=\frac{1}{\sigma\delta}$$

2)

$$\delta=\sqrt{\frac{2}{\mu\sigma\omega}}=\sqrt{\frac{2\rho}{\mu\omega}}=\sqrt{\frac{2\times1.72\cdot10^{-8}}{4\pi\cdot10^{-7}\cdot2\pi\cdot1.3\cdot10^{9}}}=1.83\cdot E-6$$

$$\delta=1.83\mu m$$

$$R_S=\frac{1}{\sigma\delta}=\frac{\rho}{\delta}=\frac{1.72\cdot10^{-8}}{1.83\cdot10^{-6}}=9.40E-3$$

$$R_S=9.40\ m\Omega$$

3)

$$R_S(4.2K)=\sqrt{\frac{\mu\omega}{2\sigma(4.2K)}}=\sqrt{\frac{\mu\omega}{2\sigma(300K)}\cdot\frac{\sigma(300K)}{\sigma(4.2K)}}=R_S(300K)\cdot\sqrt{\frac{1}{RRR}}=\frac{9.40}{\sqrt{40}}=1.49\ m\Omega$$

Exercise III

By the Two fluid mode,

$$J = \left(\frac{n_n e^2}{\nu m_e} - i \frac{n_s q_s^2}{\omega m_s} \right) E = (\sigma_n - i\sigma_s) E = \sigma E$$

$$Z = (1-i)\sqrt{\frac{\mu\omega}{2\sigma}} = (1-i)\sqrt{\frac{\mu\omega}{2(\sigma_n - i\sigma_s)}}$$

$$=(1-i)\sqrt{\frac{\mu\omega}{2}} \cdot \frac{1}{\sqrt{-i\sigma_s(1-\frac{\sigma_n}{i\sigma_s})}}$$

$$\approx (1-i) \cdot \sqrt{\frac{\mu\omega}{2}} \cdot \frac{(1+\frac{\sigma_n}{2i\sigma_s})}{\sqrt{-i\sigma_s}}, \quad \because \ \frac{\sigma_n}{\sigma_s} \gg 1$$

$$=\sqrt{\mu\omega} \cdot \left(\frac{\sigma_n}{2\sigma_s^{3/2}} + i\frac{1}{\sigma_s^{1/2}} \right) \quad \because \sqrt{i} = \sqrt{e^{-\pi/2}} = \frac{1-i}{\sqrt{2}}$$

If define the λ(London's penetration depth) as following,

$$\lambda_L \equiv \sqrt{\frac{m_s}{n_s q_s^2 \mu}} = c\sqrt{\frac{2m}{4n_s \mu e^2}} = c\sqrt{\frac{m}{2n_s \mu e^2}}$$

One can get the impedance Z as: $Z = \frac{\mu\omega\lambda_L{}^3}{\delta^2} + i\mu\omega\lambda_L = \frac{\mu^2\omega^2\lambda_L{}^3\sigma_n}{2} + i\mu\omega\lambda_L$

Z=Rs+iX,

then

$$R_S = \frac{1}{2}\sigma_n\omega^2\mu^2\lambda_L^3, \quad \mathrm{X}=\omega\mu\lambda_\mathrm{L}, \quad \sigma_n \equiv \frac{n_n e^2}{\nu m_e}$$

$$R_S =$$

$$R_S = \frac{1}{2}\sigma_n\omega^2\mu^2$$

At T=0K, all electrons become superconducting electrons (ground state).

At a finite temperature T[K], some electrons are excited from the ground state (T=0K, E=0) to the state (T=TK,E=Δ), which breaks superconducting state (normal electron). The exciting probability exciting (T, Δ) is estimated by Boltzman statistics as:

$$n_n(T,\Delta) = n_s(0,0)\exp(-\frac{\Delta}{k_B T}).$$

So one can be written as:

$$\sigma_n = \frac{e^2}{m\nu} n_s(0) e^{-\frac{\Delta}{k_B T}} .$$

Thus the Rs of RF superconductivity is given as:

$$R_S(T, f) = A \cdot f^2 \exp(-\frac{\Delta}{k_B T}) .$$

Exercise IV

This will take a time. Please try it after this summer school.

Exercise V

$$\omega_{0,1,0} = \frac{c}{\sqrt{\mu\varepsilon}} \sqrt{\frac{\rho_{0,1}^2}{a^2}} = \frac{2.405 \cdot c}{\sqrt{\mu\varepsilon} \cdot a}$$

$$a = \frac{2.405 \cdot c}{\sqrt{\mu\varepsilon} \cdot 2\pi f} = \frac{2.405 \cdot 3.00 \times 10^{10}}{2\pi \cdot 1.3 \times 10^9} = 8.83\ cm, \ \text{Damtrer=17.66}\ cm$$

$$\mu = \mu_0 = \varepsilon = \varepsilon_0 = 1 \text{ in Gauss unit,}$$

$$d = \frac{\lambda}{2} = \frac{c/f}{2} = \frac{3.00 \times 10^{10} / 1.30 \times 10^9}{2} = 11.54\ cm$$

Exercise VI

$$R_{sh}[\Omega] = 3.087 M\Omega$$

$$V[MV] = \sqrt{R_{sh} \cdot P_{loss}} = \sqrt{3.0868 \times 10^6 \times 1118.1551} = 0.05875\ MV$$

$$\lambda[m] = 3.0 \times 10^8 / 1.2937743 \times 109 = 0.23188\ m$$

$$Eacc[MV/m] = \frac{V}{\lambda/2} = \frac{0.05875}{0.23188/2} = 0.5067 MV/m$$

$$Hp / Eacc = 1753.44 \times 4\pi \times 10^{-3} / 0.5067 = 43.46\ Oe / [MV/m]$$

$$Ep / Eacc = 1.867$$

$$Eacc[V/m] = \frac{\sqrt{R_{sh}/Q_0}}{L_{eff}} \sqrt{P_{loss} \cdot Q_0} = \frac{\sqrt{\frac{3.087 \times 10^6}{28257.6}}}{0.23188/2} \cdot \sqrt{P_{loss} \cdot Q_0} = 90.15 \cdot \sqrt{P_{loss} \cdot Q_0}$$

$$\Gamma[\Omega] = Q_0 \cdot R_s = 265.171\ [\Omega]$$

Exercise VII

$$C_{in} = \left(\frac{p_0}{p_{in}}\right)\cdot\sqrt{\frac{p'_{in}}{p_0}} = \left(\frac{50.0\times10^{-3}}{55.5\times10^{-6}}\right)\cdot\sqrt{\frac{22.6\times10^{-3}}{39.0\times10^{-3}}} = 685.8$$

$$C_r = \left(\frac{p_0}{p_r}\right)\cdot\sqrt{\frac{p'_0}{p'_{in}}} = \frac{50.0\times10^{-3}}{10.72\times10^{-6}}\cdot\sqrt{\frac{39.0\times10^{-3}}{22.6\times10^{-3}}} = 6127.1$$

$$C_t = \left(\frac{p_0}{p_t}\right)\cdot\sqrt{\frac{p'_0}{p'_t}} = \frac{50.0\times10^{-3}}{3.04\times10^{-3}}\cdot\sqrt{\frac{39.0\times10^{-3}}{27.9\times10^{-3}}} = 19.45$$

Exercise VIII

$$P_{in} = 685.8\times3.11\times10^{-3} = 2.133\ W$$

$$P_r = 6127.1\times192\times10^{-9} = 1.176\ mW$$

$$P_t = 19.45\times0.142\times10^{-3} = 2.762\ mW$$

$$P_{loss} = 2.133 - 1.176\times10^{-3} - 2.762\times10^{-3} = 2.129\ W$$

$$\beta^* = \frac{1+\sqrt{\frac{P_r}{P_{in}}}}{1-\sqrt{\frac{P_r}{P_{in}}}} = \frac{1+\sqrt{5.5157\times10^{-4}}}{1-\sqrt{5.5157\times10^{-4}}} = \frac{1+0.02349}{1-0.02349} = 1.048 \beta_t = \frac{P_t}{P_{loss}} = \frac{2.762\times10^{-3}}{2.129} = 1.297\times10^{-3}$$

$$\beta_{in} = (1+\beta_t)\cdot\beta_{in}* = (1+1.297\times10^{-3})\times1.048 = 1.049$$

$$Q_L = \frac{2\pi f \tau_{1/2}}{\ln(2)} = \frac{2\times3.1416\times1.303590529\times10^9\times23.6\times10^{-3}}{\ln(2)} = \frac{193.30\times10^6}{0.69315} = 2.789\times10^8$$

$$Q_0 = (1+\beta_{in}+\beta_t)\cdot Q_L = (1+1.049+1.297\times10^{-3})\times2.789\times10^8 = 2.0503\times2.789\times10^8 = 5.718\times10^8$$

$$Q_{in} = (1+\beta_t)\cdot\beta*_{in} = (1+1.297\times10^{-3})\times1.048 = 1.049$$

$$Q_t = \frac{Q_0}{\beta_t} = \frac{5.718\times10^8}{1.297\times10^{-3}} = 4.408\times10^{11}$$

$$R_s = \frac{\Gamma}{Q_0} = \frac{265.17}{5.718\times10^8} = 463.75\times10^{-9} = 463.75\ n\Omega$$

$$E_{acc} = Z\cdot\sqrt{P_{loss}\cdot Q_0} = 90.148\times\sqrt{2.129\times5.718\times10^8} = 3.145\times10^6\ V/m = 3.145\ MV/m$$

$$E_p = (E_p/E_{acc})\times E_{acc} = 1.867\times3.145 = 5.872\ MV/m$$

$$H_p = (H_p/E_{acc})\times E_{acc} = 43.46\times3.145 = 136.68\ \text{Oe}$$