GenEvA: A New Framework For Event Generation

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Outline

Motivation

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- Growing need and availability of precise theoretical calculations
  - more and more NLO or even NNLO results
  - Subleading log resummation, power corrections (e.g. in SCET)
- Make these available to experiments (more easily and more quickly)
  - Need a generic way to use *inclusive parton-level* calculations to get *exclusive hadron-level* events

Partonic Calculations vs. Algorithmic Tools

- Separate partonic calculations from algorithms
  - Calculations should not have to know specifics of implementations

GenEvA: Generate Events Analytically

Goals and Benefits of GenEvA

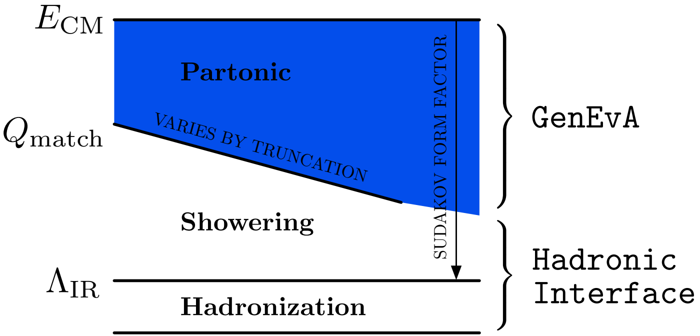
- Provide *generic* and *straightforward* framework to map inclusive parton-level calculations into exclusive hadron-level events
  - Let user think in terms of inclusive partonic cross sections
  - Avoid having to think about algorithms on case by case basis
- Use most accurate available prediction for each part of phase space

Idea behind

1. Use parton shower as phase-space generator to distribute points (events) in multiplicity, flavor and phase space
2. Determine *exact* probability $\mathrm{d}P$ for each point (event) to be generated
3. Reweight to *any desired distribution* $\mathrm{d}\sigma$ on event-by-event basis

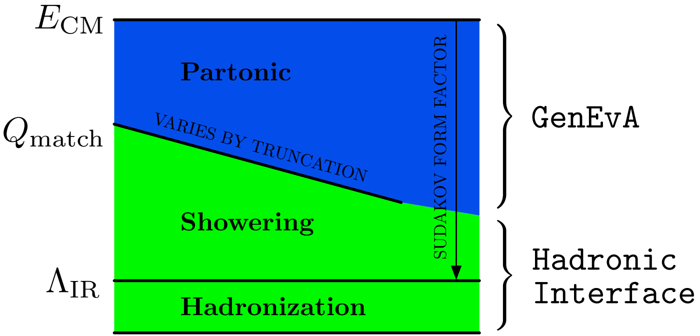
$$w = \mathrm{d}\sigma / \mathrm{d}P$$

Three Regimes of Event Generation



- Partonic Regime: Includes full field theory information
  - *Any* inclusive calculation with the notion of a matching scale

Three Regimes of Event Generation



- Partonic Regime: Includes full field theory information
  - *Any* inclusive calculation with the notion of a matching scale
- Showering Regime: Defined in terms of splitting probabilities
  - *Any* parton shower that can start showering at a given scale
- Hadronization: **Pythia**, **Herwig**

Separation between Partonic and Showering via matching scale Q_{match}

Q_{match} Dependence

Showering has logarithmic Q_{match} dependence

- Leading Q_{match} dependence is property of QCD not a specific shower
- Canceling Q_{match} dependence is theoretical not algorithmic issue
- Sufficient to include neccessary Q_{match} dependence in calculations

In **GenEvA**

- Q_{match} dependence in Partonic can be included
  - Analytically: using resummed calculation (NLO, subleading logs)
  - Numerically: on-the-fly via parton shower phase-space generation
- Value of Q_{match} is fluid and determined on event-by-event basis
  - Increase Q_{match}: Fewer final states in Partonic, more Showering
  - Decrease Q_{match}: More final states in Partonic, less Showering
- Smooth transition between Partonic and Showering

Toy Version of GenEvA

Toy cross sections

$$\frac{\mathrm{d}\sigma_B}{\mathrm{d}x} = B\delta(x)\,, \qquad \frac{\mathrm{d}\sigma_V}{\mathrm{d}x} = a\left(\frac{B}{\epsilon} + V\right)\delta(x)\,, \qquad \frac{\mathrm{d}\sigma_R}{\mathrm{d}x} = aB\frac{R(x)}{x^{1+\epsilon}}$$

$$\sigma_{\mathrm{total}} = B + aV + aB\int_0^1 \mathrm{d}x\,\frac{R(x)-1}{x}$$

Toy shower

Splitting function: $Q(x) \xrightarrow{x\to 0} R(x) \xrightarrow{x\to 0} 1$

Sudakov factor: $\Delta_Q(x_1, x_2) = \exp\left(-a\int_{x_2}^{x_1} \mathrm{d}x\,\frac{Q(x)}{x}\right)$

Splitting probability: $\mathrm{d}P(x; x_{\mathrm{start}}) = a\,\frac{Q(x)}{x}\,\Delta_Q(x_{\mathrm{start}}, x)\,\mathrm{d}x$

Master Formula

$$\sigma = \sigma_p^{(0)}(x_{\rm match}^{(0)})\,\mathrm{MC}(x_{\rm match}^{(0)}) + \int_{x_{\rm match}^{(0)}}^{1} \mathrm{d}x_1\,\mathrm{d}\sigma_p^{(1)}(x_1; x_{\rm match}^{(1)})\,\mathrm{MC}(x_{\rm match}^{(1)})$$

- $\sigma_p^{(0,1)}(x_{\rm match}^{(n)})$: Analytic partonic cross sections for 0 and 1 emissions
  - $x_{\rm match}$ dependence is primarily a *scale dependence*
  - Singularities cancel analytically
- $\mathrm{MC}(x_{\rm match})$: Showering starting at $x_{\rm match}$
- $x_{\rm match}^{(0,1)}$: Matching scale for 0- and 1-parton emission
  - $x_{\rm match}^{(1)} = x_1$ to avoid dead zone
  - $x_{\rm match}^{(0)}$ some low scale (could be as low as $x_{\rm cut}$)

Partonic Cross Sections at Tree Level

Tree-level generator

$$\sigma_p^{(0)}(x_{\text{match}}) = B$$

$$\sigma_p^{(1)}(x) = aB\,\frac{R(x)}{x}$$

Total and one-emission cross sections

$$\sigma = B + aB \int_{x_{\text{match}}}^{1} \mathrm{d}x\,\frac{R(x)}{x}$$

$$\sigma^{(1)}(x) = aB\,\frac{1}{x}\begin{cases} R(x) & (x > x_{\text{match}}) \\ Q(x)\,\Delta_Q(x_{\text{match}}, x) & (x < x_{\text{match}}) \end{cases}$$

- σ correct to $\mathcal{O}(1)$, and $\sigma^{(1)}(x)$ to $\mathcal{O}(a)$
- Logarithmic x_{match} dependence in σ and $\sigma^{(1)}(x)$

Partonic Cross Sections at Tree Level

Tree-level generator Sudakov improved

$$\sigma_p^{(0)}(x_{\mathrm{match}}) = B\,\Delta_Q(1, x_{\mathrm{match}})$$

$$\sigma_p^{(1)}(x) = aB\,\frac{R(x)}{x}\,\Delta_Q(1, x)$$

Total and one-emission cross sections

$$\sigma = B + aB \int_{x_{\mathrm{match}}}^{1} \mathrm{d}x\,\frac{R(x) - Q(x)}{x}\,\Delta_Q(1, x)$$

$$\sigma^{(1)}(x) = aB\,\frac{1}{x}\begin{cases} R(x)\,\Delta_Q(1, x) & (x > x_{\mathrm{match}}) \\ Q(x)\,\Delta_Q(1, x) & (x < x_{\mathrm{match}}) \end{cases}$$

- σ correct to $\mathcal{O}(1)$, and $\sigma^{(1)}(x)$ to $\mathcal{O}(a)$
- Logarithmic x_{match} dependence in σ and $\sigma^{(1)}(x)$ cancels
  - Example of ME/PS merging, but no special algorithm needed

Partonic Cross Sections at NLO

NLO Slicing

$$\sigma_p^{(0)}(x_{\mathrm{match}}) = \tilde{B}$$

$$\tilde{B} = \sigma_{\mathrm{total}} - aB \int_{x_{\mathrm{match}}}^{1} \mathrm{d}x\, \frac{R(x)}{x}$$

$$\sigma_p^{(1)}(x) = aB\, \frac{R(x)}{x}$$

Total and one-emission cross sections

$$\sigma = \sigma_{\mathrm{total}}$$

$$\sigma^{(1)}(x) = a\,\frac{1}{x} \begin{cases} BR(x) & (x > x_{\mathrm{match}}) \\ \tilde{B}Q(x)\,\Delta_Q(x_{\mathrm{match}}, x) & (x < x_{\mathrm{match}}) \end{cases}$$

- σ and $\sigma^{(1)}(x)$ correct to $\mathcal{O}(a)$, no x_{match} dependence in σ
- Two sources of Logarithmic x_{match} dependence in $\sigma^{(1)}(x)$

Partonic Cross Sections at NLO

NLO Slicing Sudakov improved = NLO Subtraction [MC@NLO]

$$\sigma_p^{(0)}(x_{\mathrm{match}}) = \tilde{B}\,\Delta_Q(1, x_{\mathrm{match}})$$

$$\tilde{B} = \sigma_{\mathrm{total}} - aB \int_{x_{\mathrm{match}}}^{1} \mathrm{d}x\, \frac{R(x) - Q(x)}{x}$$

$$\sigma_p^{(1)}(x) = aB\,\frac{R(x) - Q(x)}{x} + a\tilde{B}\,\frac{Q(x)}{x}\,\Delta_Q(1, x)$$

Total and one-emission cross sections

$$\sigma = \sigma_{\mathrm{total}}$$

$$\sigma^{(1)}(x) = a\,\frac{1}{x}\begin{cases} BR(x) + (\tilde{B} - B)Q(x)\,\Delta_Q(1, x) & (x > x_{\mathrm{match}}) \\ \tilde{B}Q(x)\,\Delta_Q(1, x) & (x < x_{\mathrm{match}}) \end{cases}$$

- σ and $\sigma^{(1)}(x)$ correct to $\mathcal{O}(a)$, no x_{match} dependence in σ
- Two sources of Logarithmic x_{match} dependence in $\sigma^{(1)}(x)$ cancel

Partonic Cross Sections at NLO

Simple NLO [POWHEG inspired]

$$\sigma_p^{(0)}(x_{\mathrm{match}}) = \sigma_{\mathrm{total}}\,\Delta_T(1, x_{\mathrm{match}})$$

$$\sigma_p^{(1)}(x) = a\,\sigma_{\mathrm{total}}\,\frac{T(x)}{x}\,\Delta_T(1, x)$$

$$T(x) = \frac{B}{\sigma_{\mathrm{total}}}\,R(x) \qquad \Delta_T = \exp\biggl(-a\int \mathrm{d}x\,\frac{T(x)}{x}\biggr)$$

Total and one-emission cross sections

$$\sigma = \sigma_{\mathrm{total}}$$

$$\sigma^{(1)}(x) = a\,\frac{1}{x}\begin{cases} B R(x)\,\Delta_T(1, x) & (x > x_{\mathrm{match}}) \\ \sigma_{\mathrm{total}}\,Q(x)\,\Delta_T(1, x_{\mathrm{match}})\Delta_Q(x_{\mathrm{match}}, x) & (x < x_{\mathrm{match}}) \end{cases}$$

- σ and $\sigma^{(1)}(x)$ correct to $\mathcal{O}(a)$
- Still no logarithmic x_{match} dependence in $\sigma^{(1)}(x)$

More Partonic Cross Sections

Everything boils down to calculating partonic cross sections.

Straightforward to generalize

- More emissions at tree level
- Combining NLO and additional tree level emissions, e.g.
  - 2+3 jet: analytic NLO matrix element
  - 4 jet: **MadGraph**
  - 5 jet: **AMEGIC++**
  - 6 jet: **ALPGEN**
  - 7 jet: **O'Mega**
  - 8+ jets: Parton Shower with quantum interference [Nagy, Soper]

In principle also straightforward, modulo technical details

- NNLO (+ additional tree level emissions)
- Resummed expressions (NLL, NNLL)

The Parton Shower as Phase Space Generator

Advantages

- Automatically covers all of multiplicity, flavor, phase space
- Automatically has the right singularity structure
- As a by-product, resums leading logarithms
- It's fast!

Challenges

- Need to know precise probability distribution $\mathrm{d}P$

  Use analytic parton shower algorithm [Christian Bauer, FT, arXiv:0705.1719]

  - Momentum conservation at each vertex
  - Full analytic control over $\mathrm{d}P$

- Must take into account that phase space is covered multiple times by parton shower

The Event Weight

$$w = \frac{\mathrm{d}\sigma}{\mathrm{d}P}$$

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- $\mathrm{d}\sigma = \sigma(\Phi)\,\mathrm{d}\Phi$ is function of Lorentz invariant phase space Φ

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$$w = \frac{\sigma(\Phi)\mathrm{d}\Phi}{\mathcal{P}(\Sigma)\mathrm{d}\Sigma}$$

- $\mathrm{d}\sigma = \sigma(\Phi)\,\mathrm{d}\Phi$ is function of Lorentz invariant phase space Φ
- $\mathrm{d}P = \mathcal{P}(\Sigma)\,\mathrm{d}\Sigma$ is function of full parton shower history $\Sigma = \{t_i, z_i\}$

The Event Weight

$$w \equiv w(\Phi) = \frac{\sigma(\Phi)}{\mathcal{P}[\Sigma(\Phi)]J[\Sigma(\Phi)]}$$

- $\mathrm{d}\sigma = \sigma(\Phi)\,\mathrm{d}\Phi$ is function of Lorentz invariant phase space Φ
- $\mathrm{d}P = \mathcal{P}(\Sigma)\,\mathrm{d}\Sigma$ is function of full parton shower history $\Sigma = \{t_i, z_i\}$
- Mapping $\Sigma \to \Phi \equiv \Phi(\Sigma)$, Jacobian $J(\Sigma) = \mathrm{d}\Sigma/\mathrm{d}\Phi$

The Event Weight

$$w \equiv w(\Phi) = \frac{\sigma(\Phi)}{\sum_i \mathcal{P}[\Sigma_i(\Phi)] J[\Sigma_i(\Phi)]}$$

- $d\sigma = \sigma(\Phi)\, d\Phi$ is function of Lorentz invariant phase space Φ
- $dP = \mathcal{P}(\Sigma)\, d\Sigma$ is function of full parton shower history $\Sigma = \{t_i, z_i\}$
- Mapping $\Sigma \to \Phi \equiv \Phi(\Sigma)$, Jacobian $J(\Sigma) = d\Sigma/d\Phi$
- Each Φ can have multiple $\Sigma_i(\Phi)$ that map to it
  - Have to sum over all parton shower histories $\Sigma_i(\Phi)$ that map to the same point Φ in phase space.

Overcounting

$$w \equiv w(\Phi) = \frac{\sigma(\Phi)}{\sum_i \mathcal{P}[\Sigma_i(\Phi)] J[\Sigma_i(\Phi)]}$$

Summing over $\mathcal{P}[\Sigma_i(\Phi)]$ is hard

- Requires to explicitly construct all $\Sigma_i(\Phi)$ for given Φ
- Same problem in subtraction methods

Overcounting

$$w \equiv w(\Sigma) = \frac{\sigma(\Sigma)}{\mathcal{P}(\Sigma)}$$

Instead make weight function of Σ

Overcounting

$$w \equiv w(\Sigma) = \frac{\sigma[\Phi(\Sigma)]}{\mathcal{P}(\Sigma)J(\Sigma)}\,\hat{\alpha}(\Sigma)$$

Instead make weight function of Σ

- Define $\sigma(\Sigma) = \dfrac{\sigma[\Phi(\Sigma)]}{J(\Sigma)}\,\hat{\alpha}(\Sigma)$ with $\hat{\alpha}(\Sigma) = \dfrac{\alpha(\Sigma)}{\sum_i \alpha(\Sigma_i)}$
- $\hat{\alpha}(\Sigma)$ distributes $\sigma(\Phi)$ among $\Sigma_i(\Phi)$, can be chosen freely
  - Trivial but inefficient: $\alpha(\Sigma) = 1$
  - Ideal but hard: $\alpha(\Sigma) = \mathcal{P}(\Sigma)J(\Sigma)$
  - General: Pick $\alpha(\Sigma_i) \approx \mathcal{P}(\Sigma)J(\Sigma)$ and compute $\sum_i \alpha(\Sigma_i)$ using **ALPHA** algorithm [ALPGEN]

Proof of Concept Version

Already works

- $e^+e^- \to$ jets (jet $= u,\ d,\ s,\ c$)
- Use **MadGraph** matrix elements (currently up to 6 final states)
  - Pure matrix elements
  - Sudakov improved resummed matrix elements
- Showering regime covered by underlying analytic parton shower

On paper, but not in C++ yet

- NLO matrix elements
  - resummed NLO (one-loop 2 jet + tree-level 3 jet)
  - + additional resummed tree-level matrix elements

Comparison with MadGraph/MadEvent

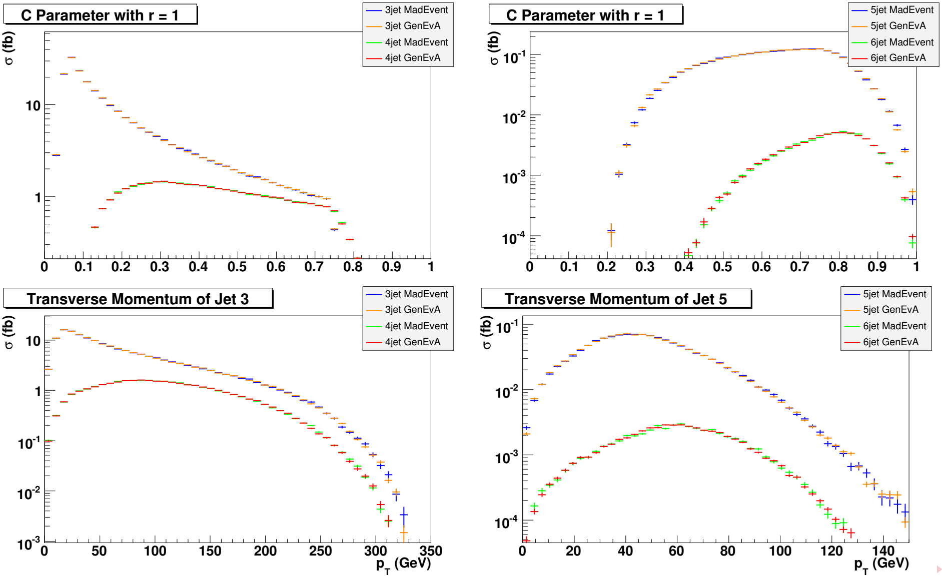
Reweight to pure **MadGraph** ME and compare with **MadEvent**

Cross section in ab for

- $E_{\mathrm{CM}} = 1000\,\mathrm{GeV}$
- $Q_{\mathrm{cut}} = 100\,\mathrm{GeV}$

process	MadEvent	GenEvA
$4j$	36483 ± 49	36439 ± 69
$u\bar{u}gg$	14055 ± 32	14003 ± 44
$d\bar{d}gg$	3490 ± 9	3498 ± 22
$u\bar{u}c\bar{c}$	283.4 ± 1.3	273 ± 7
$u\bar{u}d\bar{d}$	175.9 ± 0.9	184 ± 6
$u\bar{u}u\bar{u}$	131.9 ± 0.9	135 ± 4
$5j$	2540.5 ± 3.3	2550 ± 6
$u\bar{u}ggg$	909.8 ± 2.1	916 ± 3
$d\bar{d}ggg$	227.4 ± 1.0	229 ± 2
$u\bar{u}c\bar{c}g$	54.44 ± 0.31	54 ± 1
$u\bar{u}d\bar{d}g$	33.96 ± 0.31	35 ± 1
$u\bar{u}u\bar{u}g$	25.41 ± 0.16	25 ± 1

Comparison with MadGraph/MadEvent

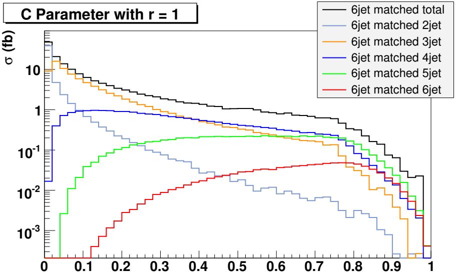


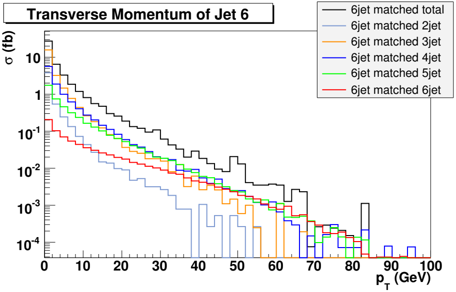
Fully Inclusive Sample

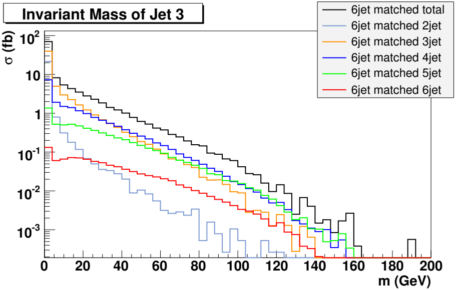
Match to MadGraph ME

Contributions of **2, 3, 4, 5, 6** jet matrix elements for

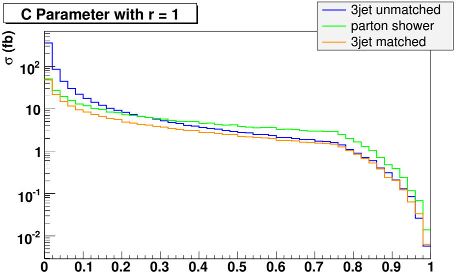
- $E_{\mathrm{CM}} = 1000\,\mathrm{GeV}$
- $Q_{\mathrm{match}} = 50\,\mathrm{GeV}$
- $Q_{\mathrm{cut}} = 10\,\mathrm{GeV}$

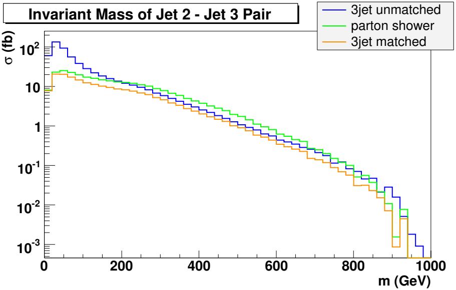






Matching Matrix Elements and Parton Shower





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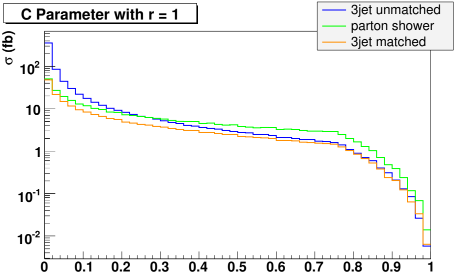
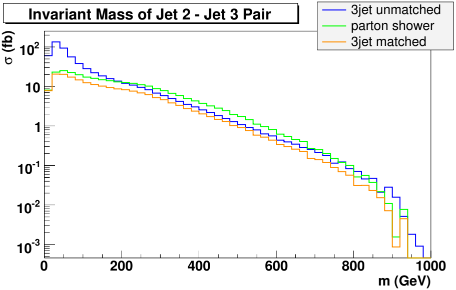
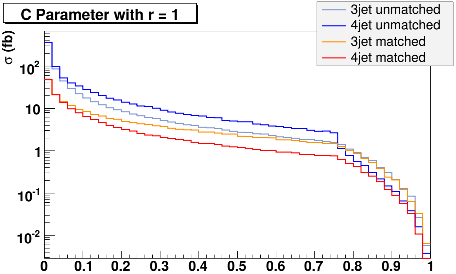
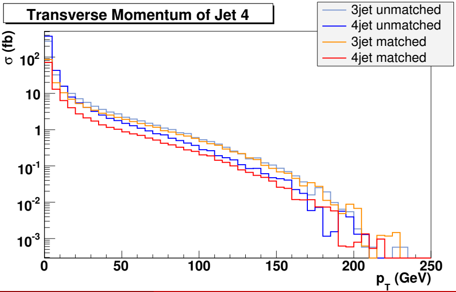






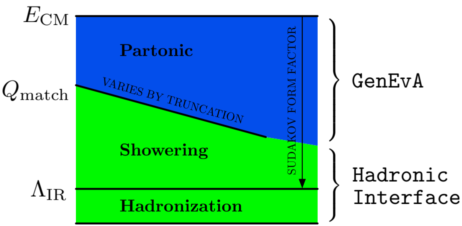


Conclusions

GenEvA is a *framework* to turn theory calculations into exclusive events

- Any matrix element with the notion of a matching scale
- Any parton shower that can start showering at a given scale
- Simple matching, independent of jet algorithm and parton shower
- Analytic parton shower can be used as efficient phase space generator

Need help from experts to extend philosophy to the full range of interesting processes (pp, $W/Z/t$ production, ...)



Outline

Backup Slides

Weighted vs. Unweighted samples

$$N_{\text{eff}} = \frac{\left[\sum_n w_n\right]^2}{\sum_n w_n^2} \qquad \text{vs.} \qquad N_{\text{unw}} = \frac{\sum_n w_n}{w_{\text{max}}}$$

- N_{unw}: expected number of events after unweighting
  - Poor efficiency from rare events with large weights
  - Systematic error due to unknown true w_{max} or truncation
- N_{eff}: measures statistical power of weighted sample
  - Number of events of a *statistically equivalent* unweighted sample
  - If N_{eff}/N too small, partially unweight until $N_{\text{eff}} \lesssim N$
- Always $N_{\text{eff}} > N_{\text{unw}}$, usually $N_{\text{eff}} \gg N_{\text{unw}}$

$e^+e^- \to 5j$	N_{eff}/N	N_{unw}/N	
MadEvent	0.89	0.11	(0.5 truncated)
GenEvA	0.40	0.03	
	0.89	0.12	(partially unweighted)

Master Formula Generalized

$$\sigma = \sigma_p^{(0)}(x_{\mathrm{match}}^{(0)})\,\mathrm{MC}(x_{\mathrm{match}}^{(0)}) + \int_{x_{\mathrm{match}}^{(0)}}^{1} \mathrm{d}x_1\, \mathrm{d}\sigma_p^{(1)}(x_1; x_{\mathrm{match}}^{(1)})\,\mathrm{MC}(x_{\mathrm{match}}^{(1)})$$
$$+ \int_{x_{\mathrm{match}}^{(0)}}^{1} \mathrm{d}x_1 \int_{x_{\mathrm{match}}^{(1)}}^{x_1} \mathrm{d}x_2\, \sigma_p^{(2)}(x_1, x_2; x_{\mathrm{match}}^{(2)})\,\mathrm{MC}(x_{\mathrm{match}}^{(2)}) + \cdots$$

- $\sigma_p^{(n)}(x_1, \ldots, x_n; x_{\mathrm{match}}^{(n)})$: Analytic partonic cross sections for n emissions
  - $x_{\mathrm{match}}^{(n)}$ dependence is primarily a *scale dependence*
  - Singularities cancel analytically
- $\mathrm{MC}(x_{\mathrm{match}})$: Showering starting at x_{match}
- $x_{\mathrm{match}}^{(n)} \equiv x_{\mathrm{match}}^{(n)}(x_n)$: Matching scale for n-parton emission
  - $n = n_{\max}$: $x_{\mathrm{match}}^{(n)} = x_n$ to avoid dead zone
  - $n < n_{\max}$: $x_{\mathrm{match}}^{(n)} < x_n$ arbitrary, could be as low as x_{cut}

Matching Dependence