# Effects of theory uncertainties in Higgs coupling measurements at the ILC

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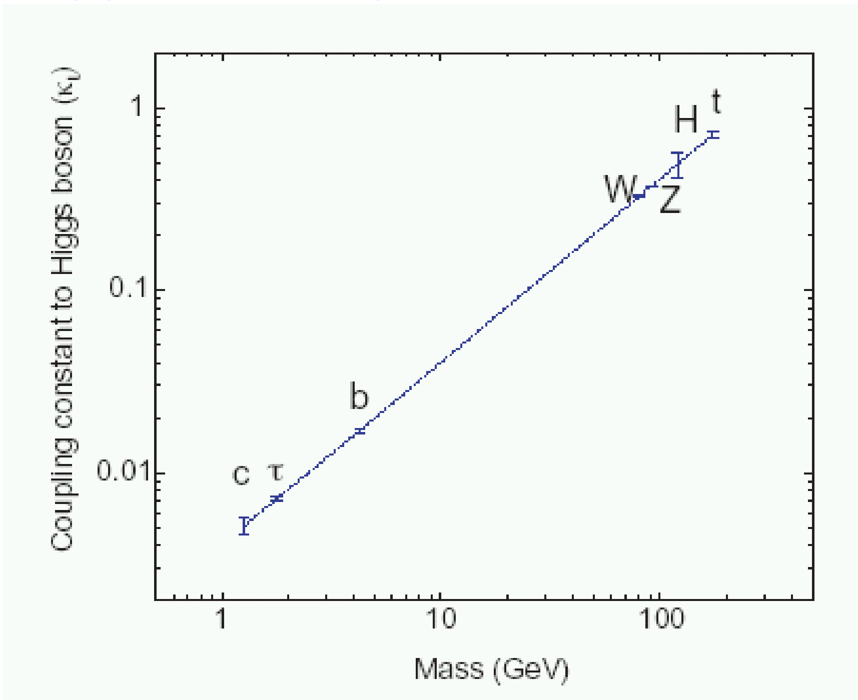
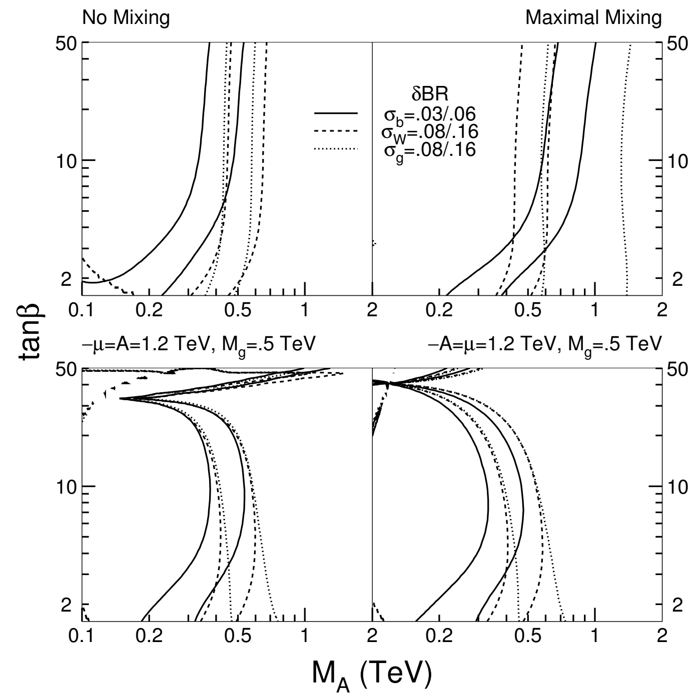
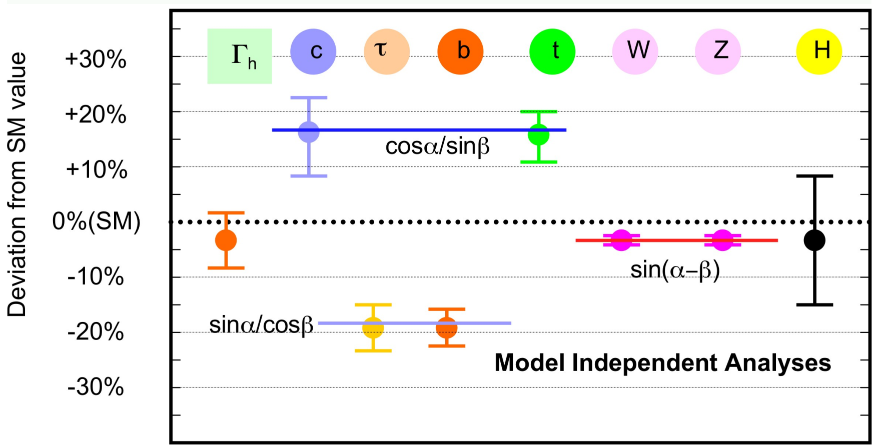
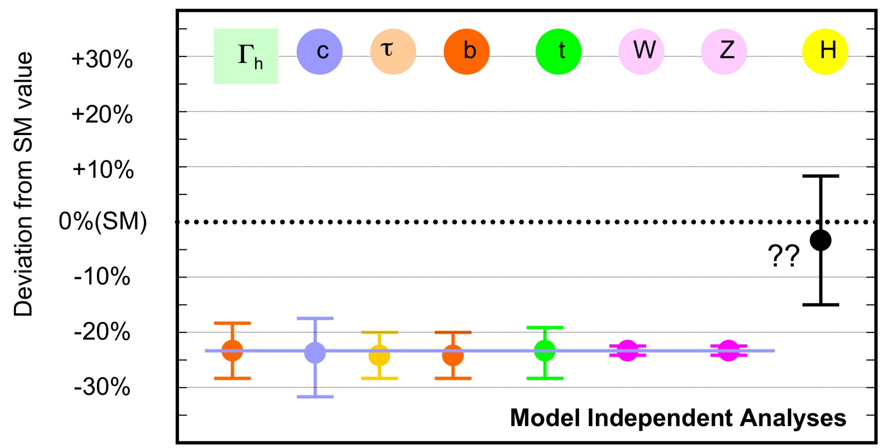
Based on A. Droll & HEL, hep-ph/0612317

# Higgs coupling measurements are a big selling point for the ILC.




# How do theory uncertainties affect this picture?

## Overview:

Theory uncertainties in Higgs couplings are around the percent-ish level.

Start to have a significant impact when experimental uncertainties get below the percent level.
This happens at high-energy / high-luminosity running (e.g., 1000 fb$^{-1}$ at 1000 GeV).

Most important theory uncertainties are parametric:
- $m_b$ (current uncertainty 0.95%) – feeds into $\Gamma_b$ calculation
- $\alpha_s$ (current uncertainty 1.7%) – feeds into $\Gamma_b$, $\Gamma_c$, $\Gamma_g$ calculation

## Expected experimental uncertainties

"Phase 1": 500 fb$^{-1}$ at 350 GeV, no beam polarization

| SM Higgs branching ratio uncertainties | | |
|---|---|---|
| | $m_H$ = 120 GeV | 140 GeV |
| BR($b\bar{b}$) | 2.4% | 2.6% |
| BR($c\bar{c}$) | 8.3% | 19.0% |
| BR($\tau\tau$) | 5.0% | 8.0% |
| BR($WW$) | 5.1% | 2.5% |
| BR($gg$) | 5.5% | 14.0% |

from K. Desch, hep-ph/0311092

"Phase 2": 1000 fb$^{-1}$ at 1000 GeV, $-80\%$ $e^-$ / $+60\%$ $e^+$ pol'n

| SM Higgs cross section times BR statistical uncertainties | | | |
|---|---|---|---|
| | $m_H$ = 115 GeV | 120 GeV | 140 GeV |
| $\sigma \times$ BR($b\bar{b}$) | 0.3% | 0.4% | 0.5% |
| $\sigma \times$ BR($WW$) | 2.1% | 1.3% | 0.5% |
| $\sigma \times$ BR($gg$) | 1.4% | 1.5% | 2.5% |
| $\sigma \times$ BR($\gamma\gamma$) | 5.3% | 5.1% | 5.9% |

from T. Barklow, hep-ph/0312268

## Theoretical uncertainties

| Higgs observable | Theory uncertainty |
|---|---|
| $\Gamma_{b\bar{b}}$, $\Gamma_{c\bar{c}}$ | 1% |
| $\Gamma_{\tau\tau}$, $\Gamma_{\mu\mu}$ | 0.01% |
| $\Gamma_{WW}$, $\Gamma_{ZZ}$ | 0.5% |
| $\Gamma_{gg}$ | 3% |
| $\Gamma_{\gamma\gamma}$ | 0.1% |
| $\sigma_{e^+e^-\to\nu\bar{\nu}H}$ | 0.5% |

$\Gamma_{q\bar{q}}$: N$^3$LO QCD for $m_q = 0$; NLO for $m_q \neq 0$; NNLO top-loop contrib'n; 4-loop $\overline{m_q}(m_H)$; NLO EW.

$\Gamma_{VV}$: `PROPHECY4F` full NLO off-shell $H \to 4f$.

$\Gamma_{gg}$: $m_t$-dependent NLO QCD; N$^3$LO in heavy-quark limit.

$\sigma_{e^+e^-\to\nu\bar{\nu}H}$: Full NLO EW RC's.

## Parametric uncertainties

| Parameter | Value | Percent uncertainty |
|---|---|---|
| $\alpha_s(m_Z)$ | $0.1185 \pm 0.0020$ | 1.7% |
| $\overline{m_b}(M_b)$ | $4.20 \pm 0.04$ GeV | 0.95% |
| $\overline{m_c}(M_c)$ | $1.224 \pm 0.057$ GeV | 4.7% |

$\alpha_s$: world average from PDG.

I'll address improvement of $\alpha_s$ at ILC in a little while.

$m_b$ and $m_c$: from fits to kinematic moments in inclusive semileptonic $B$ meson decays. Uncertainties dominated by theory uncertainty in QCD corrections to HQET expansions.

Other methods:

- $e^+e^- \to$ hadrons: fit to moments of $\sigma(\sqrt{s})$.

  Gaps in expt data & uncert in (large) quarkonium resonance contrib'ns

  QCD corr's to theory predictions of moments.

- Lattice QCD: fit to meson spectra.

  QCD corr's to bare lattice mass $\to$ $\overline{\text{MS}}$ conversion.

## Quantifying the impact of theory & parametric uncertainties:

- Question: "How well can you distinguish SM from BSM?"
- Choose a particular BSM model: MSSM $m_h^{\rm max}$ scenario.
- Construct a $\Delta\chi^2$ between observables in SM and in BSM model.
- Look at "reach" (e.g., in $M_A$) for a $5\sigma$ ($\Delta\chi^2 = 25$) discrepancy.

## $\chi^2$ observable

$$\chi^2 \;=\; \sum_{i=1}^{n}\sum_{j=1}^{n}(Q_i^{M_1} - Q_i^{M_2})[\sigma^2]_{ij}^{-1}(Q_j^{M_1} - Q_j^{M_2})$$

$Q_i$: the observables.
$[\sigma^2]_{ij}^{-1}$: inverse of the covariance matrix $\sigma_{ij}^2$,

$$\sigma_{ij}^2 \;=\; \delta_{ij}u_i u_j + \sum_{k=1}^{m} c_i^k c_j^k.$$

Straightforward to take into account both uncorrelated uncertainties $u_i$ and correlated uncertainties $c_i^k$.

Have to propagate the theoretical and parametric uncertainties to the observables $Q_i$.

## Propagation of theory & parametric uncertainties

Convenient to work entirely with fractional uncertainties.

Uncertainty in $\mathrm{BR}_i$ due to theoretical uncertainty in $\Gamma_k$:

$$c_i^k = \frac{\Gamma_k}{\mathrm{BR}_i}\frac{\partial \mathrm{BR}_i}{\partial \Gamma_k}\sigma_{\Gamma_k} \quad \text{where} \quad \frac{\Gamma_k}{\mathrm{BR}_i}\frac{\partial \mathrm{BR}_i}{\partial \Gamma_k} = \begin{cases} -\mathrm{BR}_k & \text{for } i \neq k \\ (1-\mathrm{BR}_k) & \text{for } i = k. \end{cases}$$

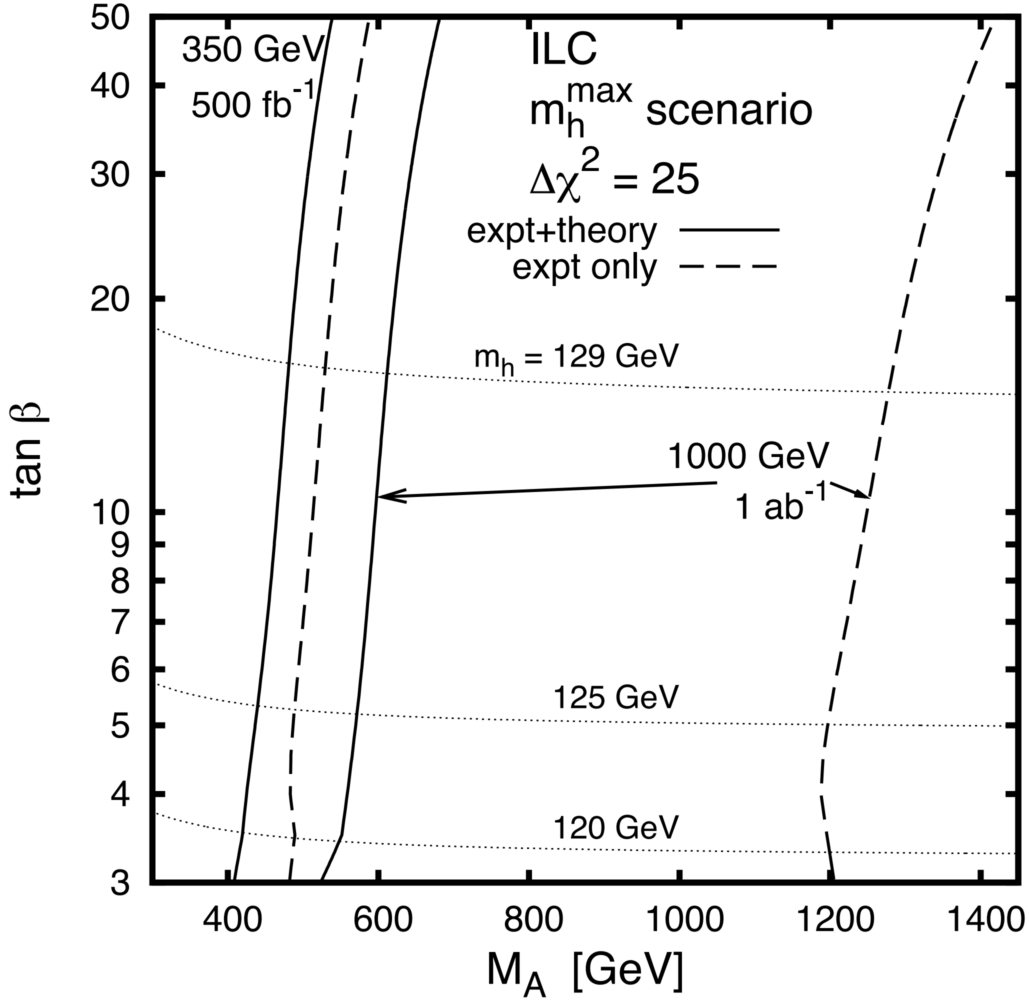
Uncertainty in $\mathrm{BR}_i$ due to parametric uncertainty in input $x_j$:

$$c_i^{x_j} = \frac{x_j}{\mathrm{BR}_i}\frac{\partial \mathrm{BR}_i}{\partial x_j}\sigma_{x_j} = \sum_{k=1}^{n}\left[\frac{\Gamma_k}{\mathrm{BR}_i}\frac{\partial \mathrm{BR}_i}{\partial \Gamma_k}\right]\left[\frac{x_j}{\Gamma_k}\frac{\partial \Gamma_k}{\partial x_j}\right]\sigma_{x_j}$$

Normalized derivatives $(x/\Gamma)(\partial\Gamma/\partial x)$:

| Normalized derivatives of Higgs partial widths | | | | | | |
|---|---|---|---|---|---|---|
| | $\alpha_s(m_Z)$ | | $\overline{m_b}(M_b)$ | | $\overline{m_c}(M_c)$ | |
| $m_H$ | 120 GeV | 140 GeV | 120 GeV | 140 GeV | 120 GeV | 140 GeV |
| $\Gamma_{b\bar b}$ | $-1.177$ | $-1.217$ | 2.565 | 2.567 | 0.000 | 0.000 |
| $\Gamma_{c\bar c}$ | $-4.361$ | $-4.400$ | $-0.083$ | $-0.084$ | 3.191 | 3.192 |
| $\Gamma_{gg}$ | 2.277 | 2.221 | $-0.114$ | $-0.112$ | $-0.039$ | $-0.032$ |

## Results




Phase 1: Reach $\sim 500$ GeV without thy/param uncerts.
Reduced by about 10% by including thy/param uncerts.

Phase 2: Reach $\sim 1200$ GeV without thy/param uncerts.
Reduced by about $2\times$ to $\sim 600$ GeV including thy/param uncerts.

Phase 1:

Parametric and theoretical uncertainties make all the measurements a little worse.

Sample point on experimental uncert only $\Delta\chi^2 = 25$ contour:

| Phase 1 sample point: $M_A = 537.6$ GeV, $\tan\beta = 20$ | | | | | | |
|---|---|---|---|---|---|---|
| Observable | Shift | Expt uncert | Pull | Thy+par uncert | Total uncert | Pull |
| BR($b\bar{b}$) | 8.1% | 2.5% | 3.25 | 1.6% | 3.0% | 2.71 |
| BR($c\bar{c}$) | −12.0% | 13.2% | −0.90 | 16.1% | 20.8% | −0.57 |
| BR($\tau\tau$) | 10.0% | 6.4% | 1.56 | 1.8% | 6.6% | 1.51 |
| BR($WW$) | −11.6% | 3.9% | −2.96 | 1.8% | 4.3% | −2.68 |
| BR($gg$) | −14.7% | 9.4% | −1.56 | 5.8% | 11.1% | −1.33 |

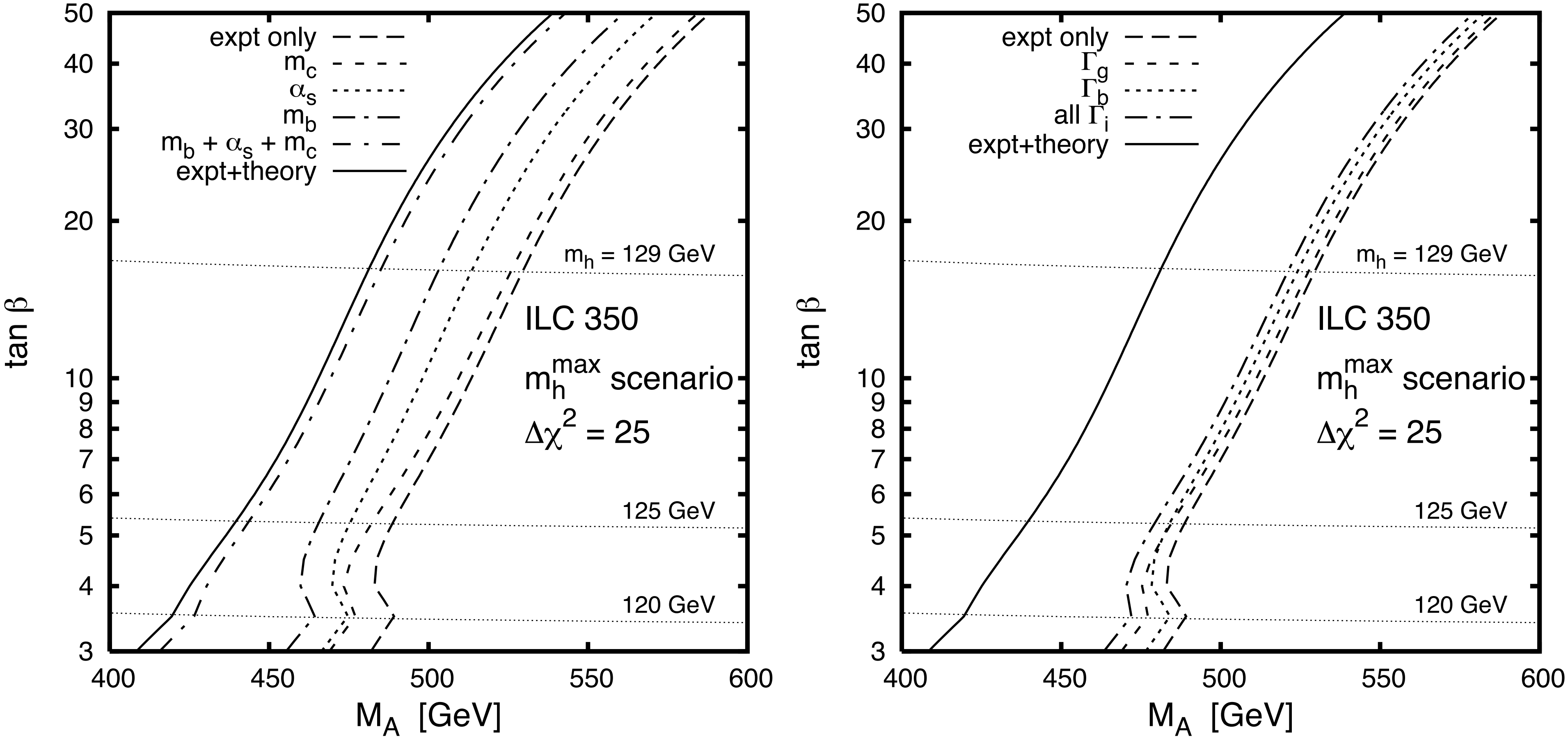
$\sum(\text{Pull})^2$:

25 with experimental uncertainties only

18.9 summing "Total uncert" pulls above

17.4 including correlations

## Phase 1:




Effect is mostly due to $m_b$ and $\alpha_s$ input uncertainties.

## Phase 2:

Parametric and theoretical uncertainties have a huge impact on the measurements, especially the most precise Phase 2 rates.

Sample point on experimental uncert only $\Delta\chi^2 = 25$ contour:

| Phase 2 sample point: $M_A = 1302.4$ GeV, $\tan\beta = 20$ | | | | | | |
|---|---|---|---|---|---|---|
| Observable | Shift | Expt uncert | Pull | Thy+par uncert | Total uncert | Pull |
| $\sigma \times$ BR($b\bar{b}$) | 1.7% | 0.45% | 3.72 | 1.7% | 1.8% | 0.93 |
| $\sigma \times$ BR($WW$) | −2.1% | 0.93% | −2.22 | 1.9% | 2.1% | −0.98 |
| $\sigma \times$ BR($gg$) | −4.6% | 2.0% | −2.32 | 5.8% | 6.2% | −0.74 |
| $\sigma \times$ BR($\gamma\gamma$) | 0.27% | 5.5% | 0.05 | 1.9% | 5.8% | 0.05 |
| BR($b\bar{b}$) | 1.7% | 2.5% | 0.67 | 1.7% | 3.0% | 0.55 |
| BR($c\bar{c}$) | −2.5% | 13.3% | −0.19 | 16.1% | 20.8% | −0.12 |
| BR($\tau\tau$) | 2.1% | 6.4% | 0.34 | 1.8% | 6.6% | 0.32 |
| BR($WW$) | −2.1% | 3.9% | −0.53 | 1.8% | 4.3% | −0.48 |
| BR($gg$) | −4.6% | 9.4% | −0.48 | 5.8% | 11.1% | −0.41 |

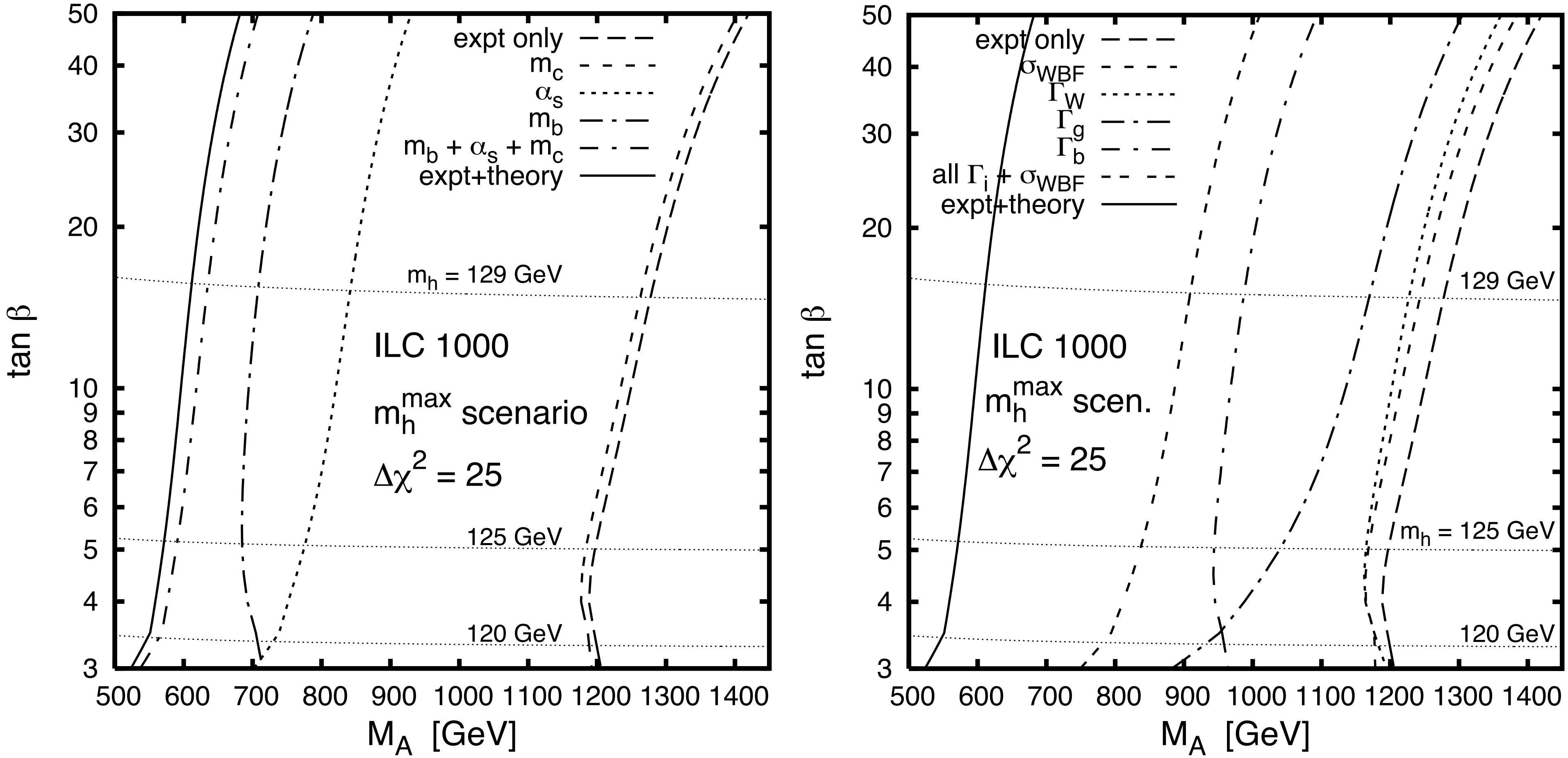
$\sum(\text{Pull})^2$:

25 with experimental uncertainties only

3.2 summing "Total uncert" pulls above

1.7 including correlations

## Phase 2:




Effect is again mostly due to $m_b$ and $\alpha_s$ uncertainties.

Theory uncertainty in $\Gamma_b$ (and $\Gamma_g$ at low $\tan\beta$) also moderately important.

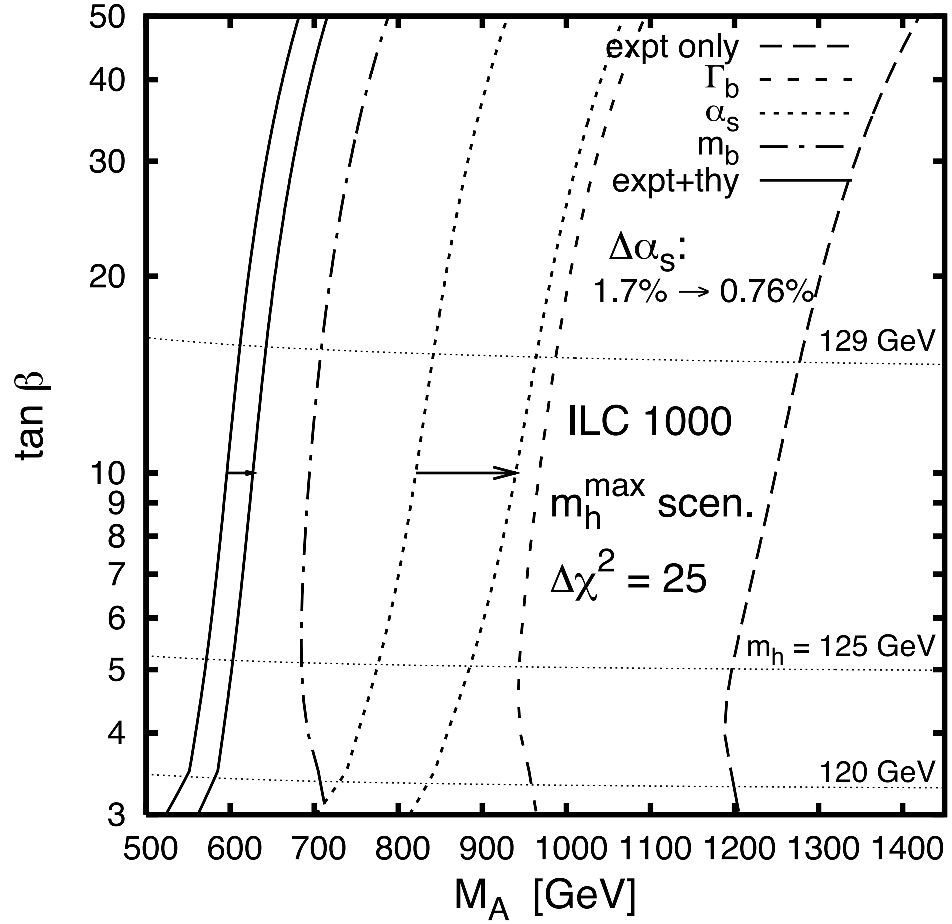
## Outlook: $\alpha_s$

ILC measurements will improve the precision on $\alpha_s(m_Z)$ by $\gtrsim 2\times$:

- Event shape observables
- $\sigma_{t\bar{t}}/\sigma_{\mu^+\mu^-}$ above $2m_t$
- $\Gamma_Z^{had}/\Gamma_Z^{lept}$ at $Z$ pole (GigaZ option)

Effect of improving $\Delta\alpha_s(m_Z)$ from 0.0020 (1.7%) [current PDG] to 0.0009 (0.76%) [Tesla TDR] (includes GigaZ).

Not much impact unless $\Delta\overline{m_b}(M_b)$ is also improved.



## Outlook: other observables

Phase 2 experimental precision dominated by three channels:
$\sigma \times \text{BR}(b\bar{b})$, $\sigma \times \text{BR}(gg)$: suffer directly from large par/thy uncerts.
$\sigma \times \text{BR}(WW)$: affected indirectly through Higgs total width.

"Error ellipsoid" is wide in some directions, narrow in others.
Choosing a model chooses a slice through the error ellipsoid.

A brief foray into the MSSM:
Study characteristic features of MSSM Higgs couplings:

$$\frac{g_{h^0\bar{t}t}}{g_{H_{\text{SM}}\bar{t}t}} = \frac{g_{h^0\bar{c}c}}{g_{H_{\text{SM}}\bar{c}c}} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)$$

$$\frac{g_{h^0\bar{b}b}}{g_{H_{\text{SM}}\bar{b}b}} = \frac{g_{h^0\tau\tau}}{g_{H_{\text{SM}}\tau\tau}} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)$$

$$\frac{g_{h^0WW}}{g_{H_{\text{SM}}WW}} = \frac{g_{h^0ZZ}}{g_{H_{\text{SM}}ZZ}} = \sin(\beta-\alpha)$$

Interested in the approach to decoupling:

$$\cos(\beta-\alpha) \simeq \frac{1}{2}\sin 4\beta\frac{m_Z^2}{M_A^2} \longrightarrow 0 \text{ for } M_A \gg m_Z$$

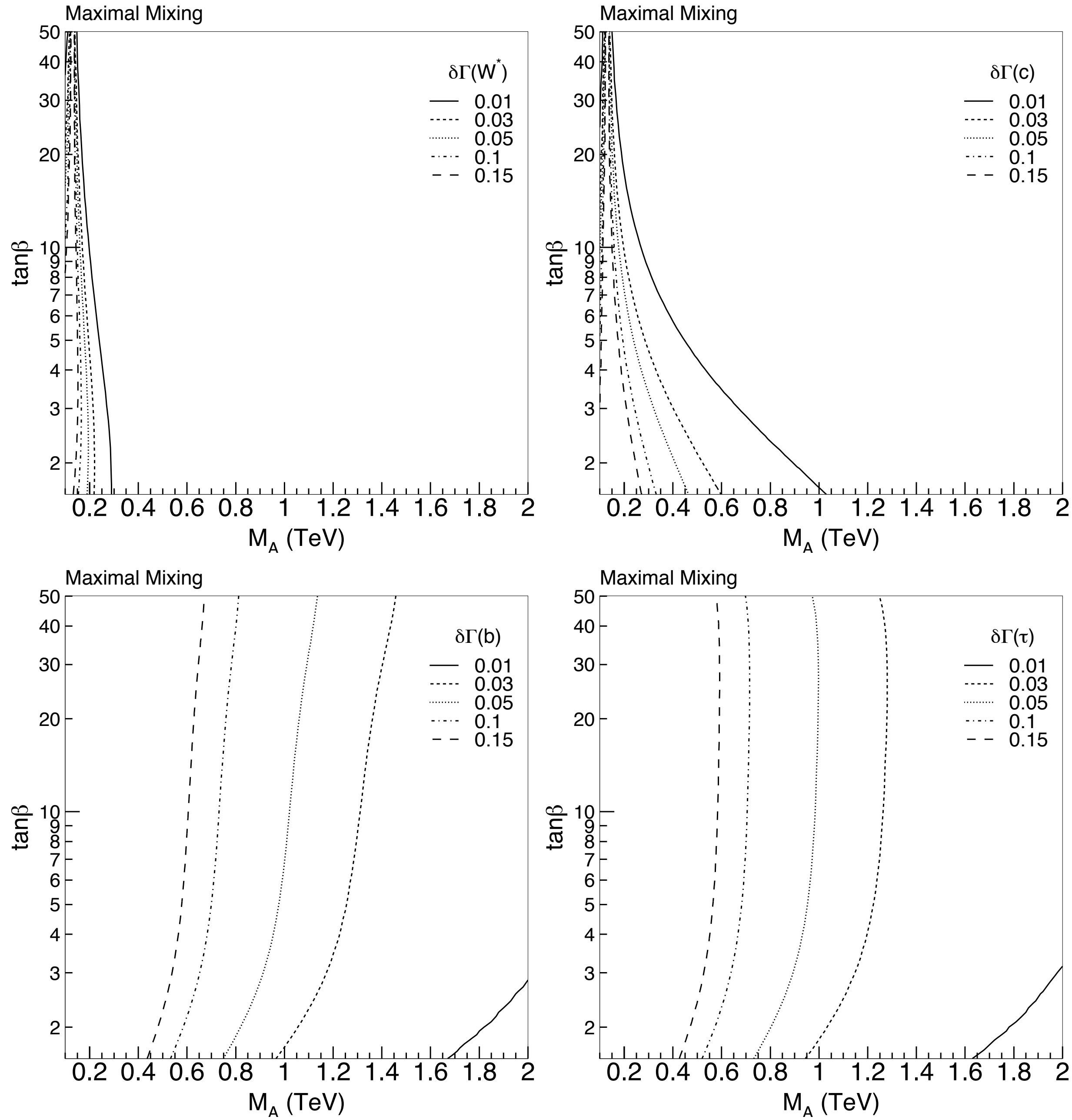
Plug in and keep leading term in $m_Z^2/M_A^2$:

$$\begin{aligned} \frac{\delta\Gamma_W}{\Gamma_W} = \frac{\delta\Gamma_Z}{\Gamma_Z} &\simeq -\frac{1}{4}\sin^2 4\beta \frac{m_Z^4}{M_A^4} \simeq -4\cot^2\beta \frac{m_Z^4}{M_A^4} \\ \frac{\delta\Gamma_b}{\Gamma_b} \simeq \frac{\delta\Gamma_\tau}{\Gamma_\tau} &\simeq -\tan\beta \sin 4\beta \frac{m_Z^2}{M_A^2} \simeq +4\frac{m_Z^2}{M_A^2} \\ \frac{\delta\Gamma_c}{\Gamma_c} &\simeq \cot\beta \sin 4\beta \frac{m_Z^2}{M_A^2} \simeq -4\cot^2\beta \frac{m_Z^2}{M_A^2} \end{aligned}$$

(Last equality: used large $\tan\beta$ approximation $\sin 4\beta \simeq -4\cot\beta$.)

Biggest deviations from SM are in $\Gamma_b$ and $\Gamma_\tau$.
Shifts in $\Gamma_c$ and $\Gamma_g$ are $\cot\beta$ suppressed.
Shifts in $\Gamma_W$ and $\sigma_{\nu\bar{\nu}H}$ are typically quite small: $\sim (m_Z/m_A)^4$.

This picture is not dramatically altered by radiative corrections.




[Carena, Haber, H.L., Mrenna, hep-ph/0106116]

Parametric & theoretical uncertainties are washing out sensitivity to shift in $\Gamma_b$ relative to $\Gamma_W$!

Want another non-hadronic final state to restore sensitivity. $\sigma \times \mathrm{BR}(\tau\tau)$ would be perfect.

Sensitivity would come from the ratio:

$$\frac{\sigma \times \mathrm{BR}(\tau\tau)}{\sigma \times \mathrm{BR}(WW)} = \frac{\Gamma_\tau}{\Gamma_W}$$

- $m_b$, $\alpha_s$, QCD uncertainties in total width cancel.
- Ratio $\Gamma_\tau/\Gamma_W$ exhibits large deviation from SM.

Using covariance matrix in $\Delta\chi^2$ means we don't need to play with ratios: everything is automatic.

Going from Phase 1 to Phase 2, expt precision on key final states improves:

$b\bar{b}$: 5–6$\times$ $\quad$ $WW$: 4–5$\times$ $\quad$ $gg$: 3.5–5.5$\times$

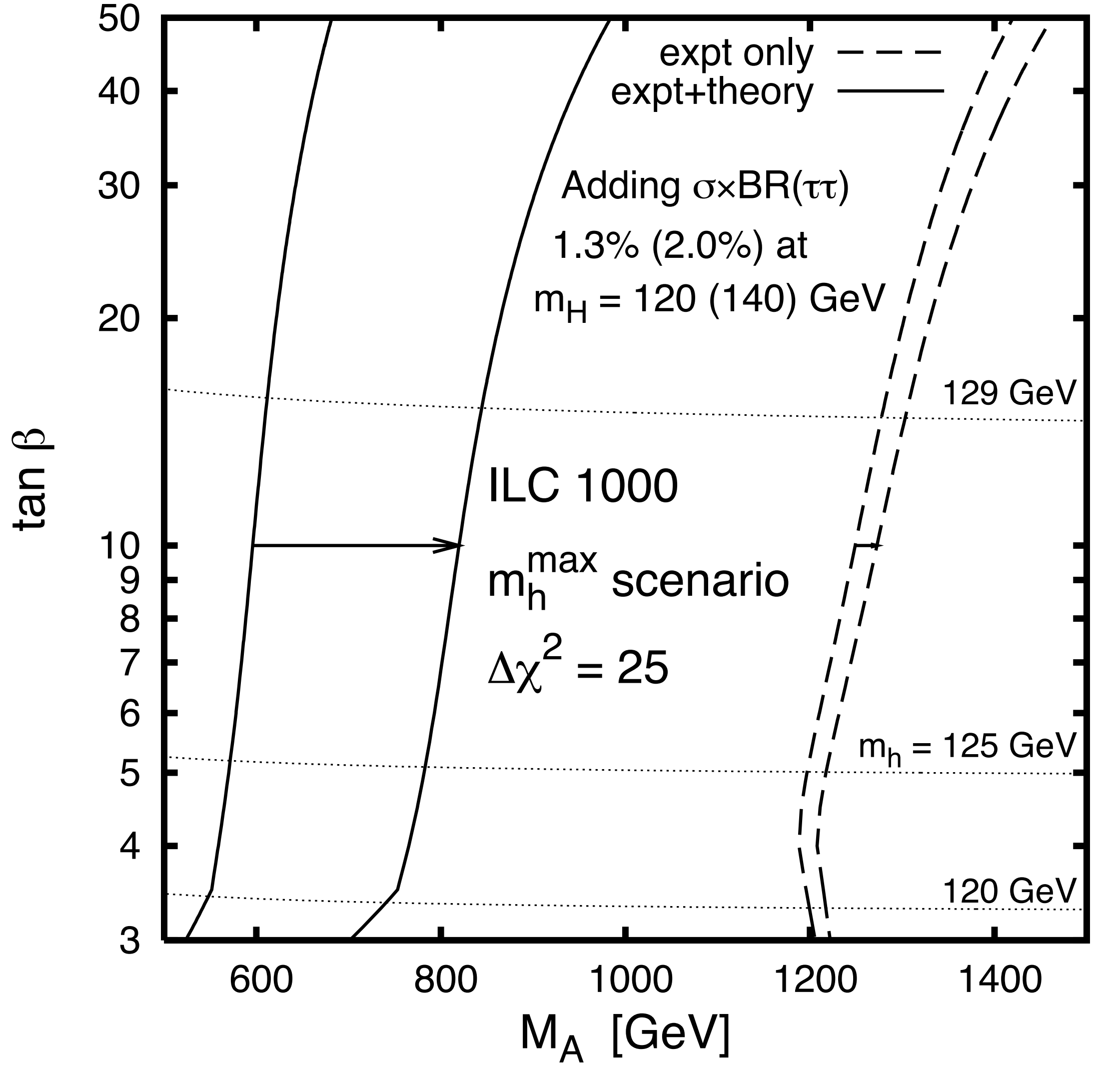
"Reasonable" to expect similar improvement in $\tau\tau$: assume 4$\times$ and see what happens.

"Phase 2": 1000 fb$^{-1}$ at 1000 GeV, $-80\%$ $e^-$ / $+60\%$ $e^+$ pol'n

| SM Higgs cross section times BR statistical uncertainties | | | |
|---|---|---|---|
| | $m_H = 115$ GeV | 120 GeV | 140 GeV |
| $\sigma \times \mathrm{BR}(b\bar{b})$ | 0.3% | 0.4% | 0.5% |
| $\sigma \times \mathrm{BR}(WW)$ | 2.1% | 1.3% | 0.5% |
| $\sigma \times \mathrm{BR}(gg)$ | 1.4% | 1.5% | 2.5% |
| $\sigma \times \mathrm{BR}(\gamma\gamma)$ | 5.3% | 5.1% | 5.9% |
| $\sigma \times \mathrm{BR}(\tau\tau)$ | – | 1.3% | 2.0% |

Original selection required $\sum \mathrm{vis} = m_H$; have to change this for $\tau\tau$.

Effect of adding a measurement of $\sigma \times \mathrm{BR}(\tau\tau)$ in Phase 2:

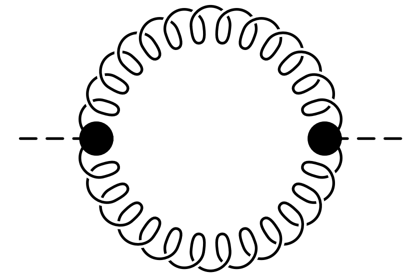



Not a big effect on expt-only reach.

Much bigger effect once param/theory uncertainties are included.

## Outlook: a worry about $b$, $c$, $g$ separation

QCD corr's to $H \to gg$ are calculated using dispersion: $\Im$m part of forward scattering, with everything possible in the loop.



This includes $g$ splitting to $q\bar{q}$: needed to cancel IR divergence in quark bubble in $g$ leg.

But $H \to gg \to q\bar{q}g$ could be tagged as $H \to q\bar{q} \to q\bar{q}g$!
How to separate these is a question of expt cuts.

`HDECAY`'s approach: switch for including/excluding heavy flavors in gluon splitting.

| `NF-GG` | $\Gamma_{gg}$ | $\Gamma_{b\bar{b}}$ | $\Gamma_{c\bar{c}}$ |
|---|---|---|---|
| 5 | – | – | – |
| 4 | $-9\%$ | $+1\%$ | – |
| 3 | $-12\%$ | – | $+30\%$ |

numbers for $m_H = 120$ GeV

## Conclusions (1/2)

Theory uncertainties are at the level of a couple of percent.

Start to have a significant impact when experimental uncertainties get below the percent level – big impact on Phase 2.

Most important theory/parametric uncertainties are:
$m_b$ (current uncertainty 0.95%) – feeds into $\Gamma_b$ calculation
- Improving this is important!
- Need more QCD theory work on semileptonic B decay spectra.

$\alpha_s$ (current uncertainty 1.7%) – feeds into $\Gamma_b$, $\Gamma_c$, $\Gamma_g$ calculation
- Will improve by $\gtrsim 2\times$ at ILC. GigaZ valuable here.

Understanding the pattern of theory/parametric uncertainties points out the most valuable new experimental channels.
Adding $\sigma \times \mathrm{BR}(\tau\tau)$: small impact with only experimental uncerts; huge impact after theory & parametric uncertainties included.

## Conclusions (2/2)

### Wish list:

[expt] $\sigma \times \mathrm{BR}(H \to \tau\tau)$ at 1 TeV.

[expt] Quantify correlations among experimental uncertainties.

[thy] Better $m_b$ extraction from existing data.

[thy/expt] How to deal with gluon splitting to heavy quarks.