

Precision Measurement of the Stop Mass at the Linear Collider

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In Collaboration with

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Publication in Preparation

Introduction

- We have previously studied the light stop, with a small mass difference to the neutralino, in an attempt to understand EW baryo-genesis the asymmetry matter anti-matter and the role of the stop in dark matter annihilation.

Phys. rev. D 72,115008(2005)

M. Carena, A. Finch, A. Freitas, C. Milstene, H. Nowak, A. Sopczak

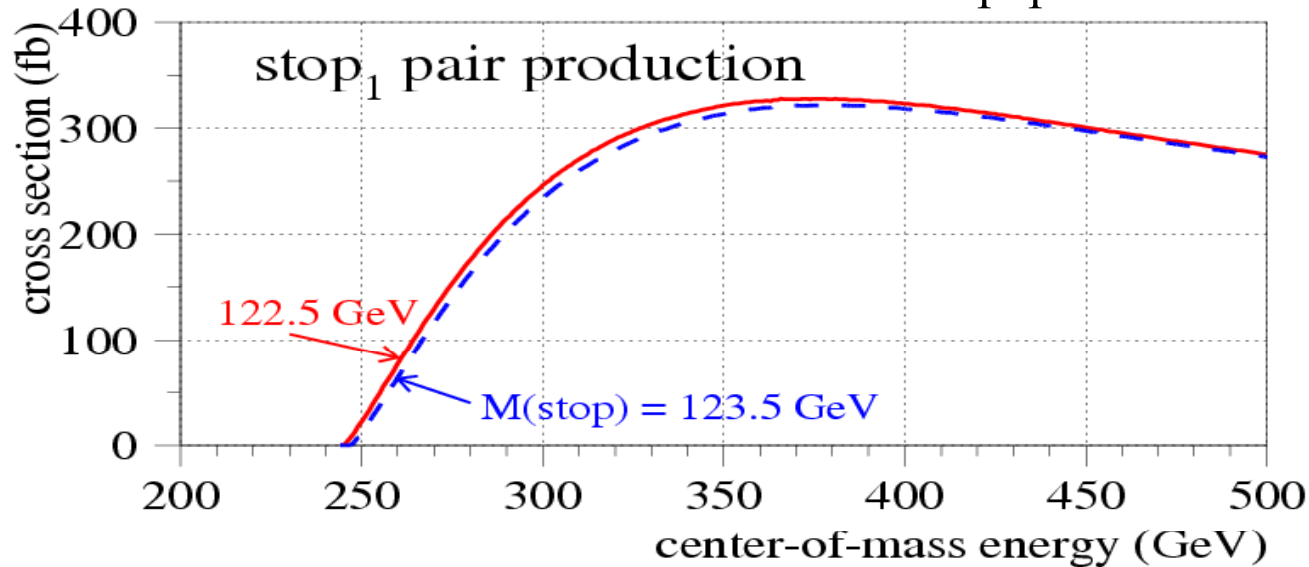
The mass precision measurement reached was $\delta m \sim 1.2 \text{ GeV}$ including theoretical errors

This analysis aims at the minimization of the systematics while using more realistic data, stop hadronization/fragmentation included. We will show that:

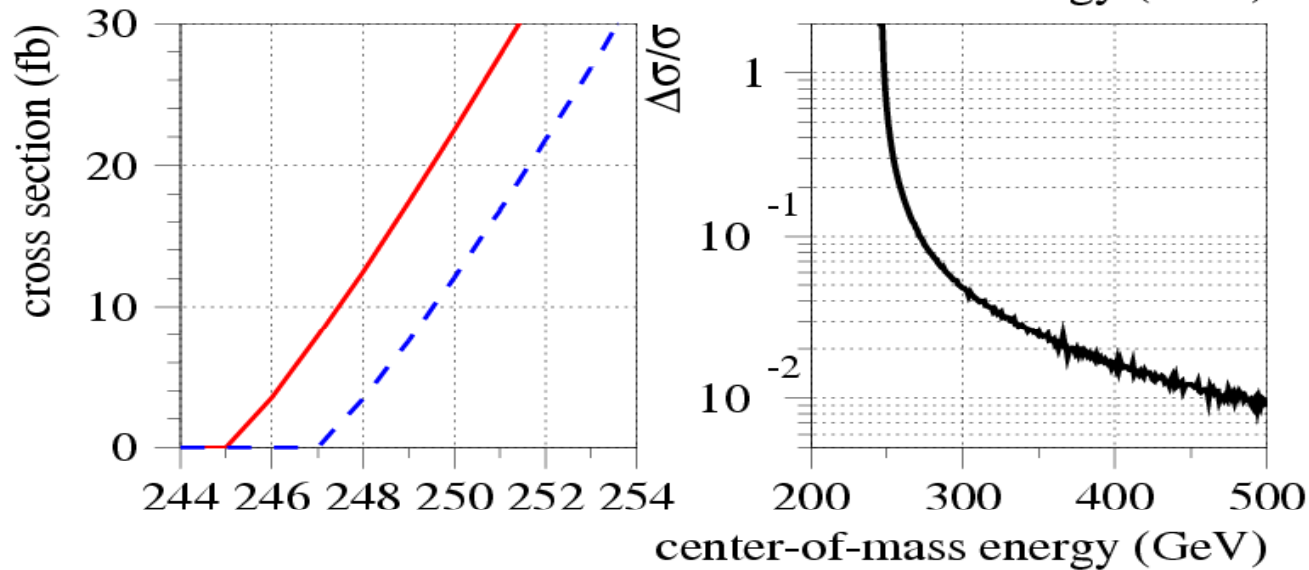
- The precision is improved in two ways:
 - a/ The systematic uncertainties are minimized by measuring the production cross-section at two energies \rightarrow cancellations .
 - b/ The 2nd energy point chosen at or close to the production energy threshold \rightarrow increased sensitivity to mass changes.
- The stop hadronization is included at production of the data \rightarrow the c quark energy is spread out in the process of hadronization. As a result:
 - the final number jets increases- the c-tagging is now necessary to identify the charm jets (bench-marking for the vertex detector)
- Two approaches are used, a cut based analysis, a multi-parameters optimization analysis IDA
- The polarization improves further the signal to background ratio

Cross-Section Precision In Production

$$e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1^*$$



*Cross-sections [fb]
calculated up to NLO
In MC software by
Freitas et al EPJ
C21(2001)361,
EPJ C34(2004)487*



The Method

$$\sigma = \frac{N - B}{\mathcal{E}L}$$
$$Y(M_x \sqrt{s_{th}}) = \frac{N_{th} - B_{th}}{N_{pk} - B_{pk}} = \frac{\sigma(\sqrt{s_{th}}) \varepsilon_{th} L_{th}}{\sigma(\sqrt{s_{pk}}) \varepsilon_{pk} L_{pk}}$$

σ - the cross-section [fb]

N- the number of selected data events

B- number of estimated background events

s -Square of the energy in center of Mass

N_{th} , B_{th} , s_{th} at or close to production threshold

N_{pk} , B_{pk} , s_{pk} , at peak value

ε_{th} , and ε_{pk} - total efficiency & acceptance

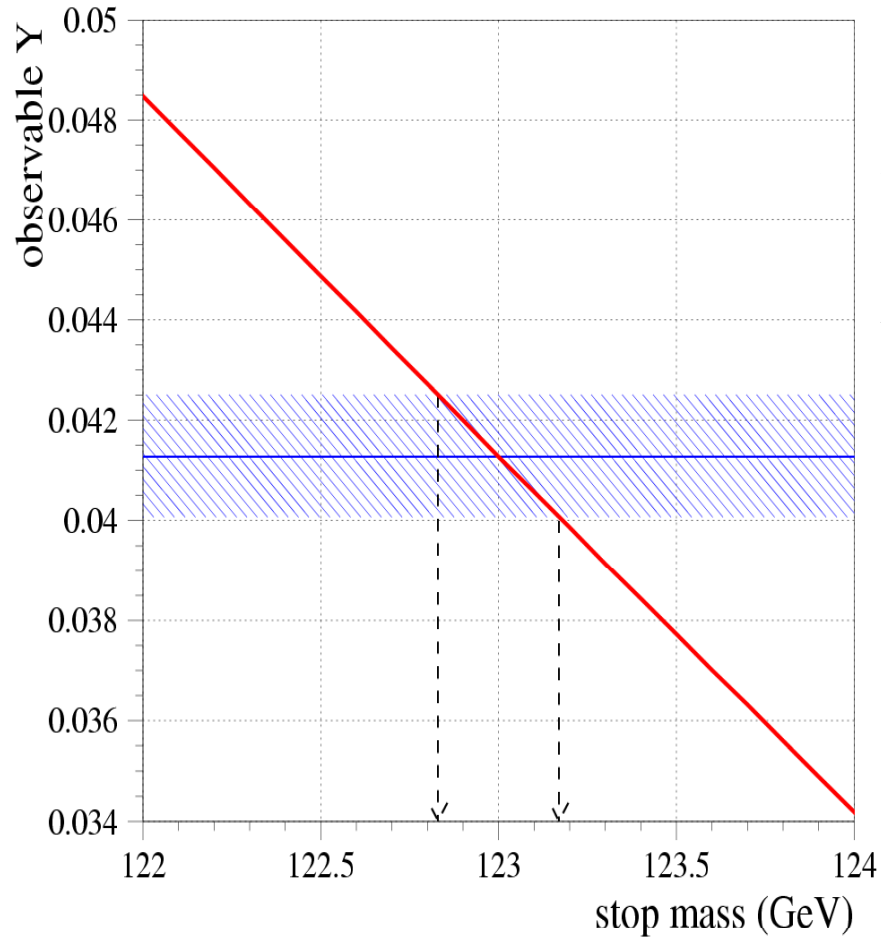
L_{th} ; and L_{pk} -Integrated luminosity

M_x : Mass to be determined with high precision.

Y- ratio of signals at threshold and peak → Allows Reduction of systematic uncertainty as well as uncertainties from L measurement.

Remark: yield close to threshold is very sensitive to M_x → choice of N_{th} and B_{th} ..

Determination of the Stop Mass



$Y=f(M_x)$ from the theoretical cross-section is been drawn in Red (NLO)
Y from the data the blue line.

As an example, Assume 3% variation of Y,
The blue hashed region \rightarrow one obtains
 \rightarrow Precision $\Delta M_x \sim \pm 0.016$, the 2 vertical arrows

The Scenario depicted:
 $E_{CM}=260\text{GeV}$ with $\sigma=9.2\text{ fb}$ and $\sigma=77\text{fb}$
at peak

Remark: Assumed luminosities
 $L_{th}=50\text{fb}^{-1}$ (260 GeV), $L_{pk}=500\text{fb}^{-1}$ (500 GeV)

$$e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1^* \rightarrow c \tilde{\chi}_0^1 \bar{c} \tilde{\chi}_0^1$$

Theoretical Motivation

- Electroweak Baryogenesis:

Sakharov Requirements:

- 1- Baryon Number Violation - (SM - Anomalous process)
- 2- C & CP violation - (SM-Quark CKM mixing)
- 3- Departure from Equilibrium - (SM-at EW phase transition)

Limitations of SM:

2) *Not Enough CP violation & 3) $\rightarrow M_{Higgs} < 40 \text{ GeV}$, LEP Bound $M_{Higgs} > 114.4 \text{ GeV}$*

\rightarrow Supersymmetry with light scalar top, below the top mass: $m_{\tilde{t}_1} < m_t$

- Dark Matter

The Supersymmetric Lightest particle (LSP), in the MSSM, the neutralino X_1^0 is a candidate

However, the annihilation cross-section $\sigma_a (X_1^0, X_1^0)$ too small

But for $m_{\tilde{t}_1} - m_{X_1^0} \sim 15\text{-}30 \text{ GeV}$, there is co-annihilation between the \tilde{t}_1 and the $X_1^0 \rightarrow \sigma_a (X_1^0, \tilde{t}_1) + \sigma_a (X_1^0, X_1^0)$ consistent with dark matter.

$$e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1^- \rightarrow c \tilde{\chi}_0^1 \bar{c} \tilde{\chi}_0^1$$

A scan in the super-symmetry parameter space

(*hep-ph/0403224v2-2004*) C. Balazs, M. Carena, C. Wagner

Baryogenesis \rightarrow ($m_{\tilde{t}_1} < m_{\text{top}}$ && $m_{\tilde{t}_1} > 120$ GeV); Higgs involved in the symmetry breaking mechanism $m_{\text{Higgs}} \leq 114.4$ GeV

\rightarrow Our points $m_{\tilde{t}_1} = 122.5$ GeV; $m_{\tilde{\chi}_0^1} = 107.2$ GeV; $\Delta m = 15.3$ GeV

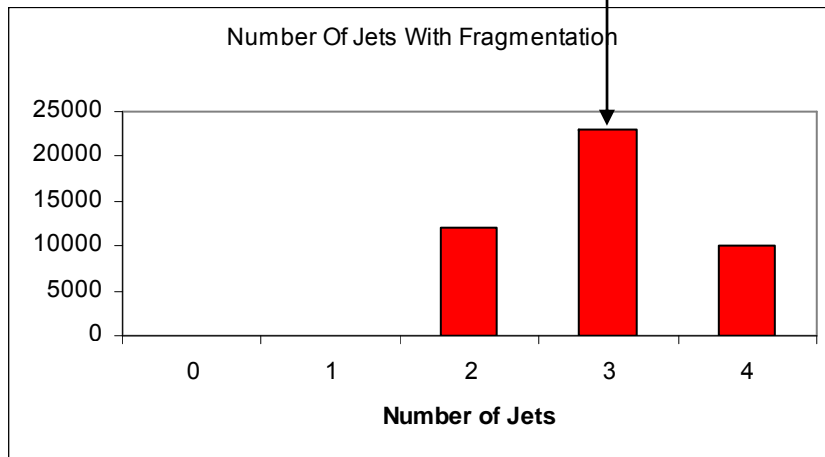
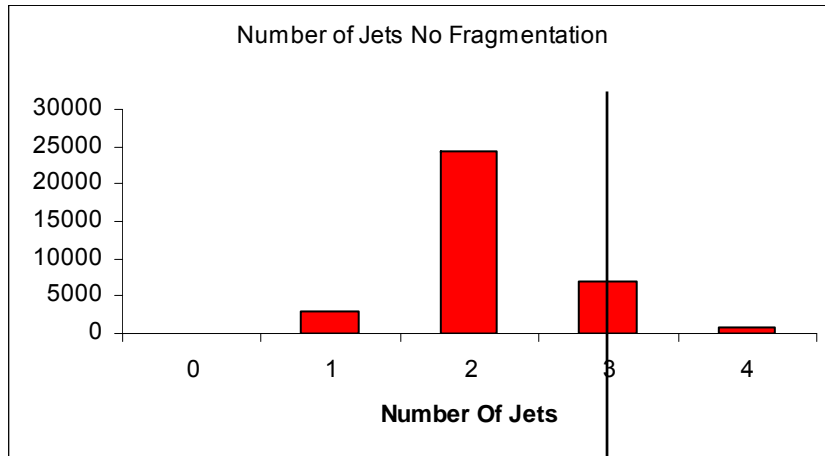
Events Final State :

- Stop Hadronization \rightarrow the final state jets smeared :
due to Radiation + Fragmentation
- Soft Multi-jets in the final state
- Stop Hadronization \rightarrow the final state jets smeared :
due to gluon radiation + fragmentation
- At ECM=260 GeV mostly 2 jets, carry the charm.
- At ECM=500 GeV 2 jets \rightarrow 2,3,4 jets (more energy available in the CM)
 \rightarrow the Charm tagging (*T. Kuhl*) a necessary tool
to identify the charm jets (Vertex bench-marking)
- Analysis uses N-tuple tool incorporating jet finding algorithm (*T. Kuhl*)

Simulation Characteristics

- Signal and Background generated with: Pythia (6.129)
Simdet (4-0-3)– Circe(1.0)
 - Hadronisation and fragmentation of the \tilde{t} and the fragmentation of the c quark from the Lund string fragmentation Pythia uses Peterson fragmentation
(*Peterson et al PR D27:105*)
 - The \tilde{t} fragmentation is simulated using Torbjorn 's code
//http://www.thep.lu.se/torbjorn/pythia/main73.f
- The \tilde{t}_1 quark is **set stable** until **after fragmentation** where it is Allowed to **decay again** as described in (*Kraan, EPJ C37:91*)
- Signal and Background are generated in each channel for the given luminosity in conjunction to the cross-sections

Jet Multiplicity – Without/With Fragmentation



- Stop fragmentation simulated using Torbjorn code
[//http://www.thep.lu.se/torbjorn/pythia/main73.f](http://www.thep.lu.se/torbjorn/pythia/main73.f)
- The stop fragmentation parameter is set relative to the bottom fragmentation parameter

$$\tilde{\epsilon}_t = \epsilon_b \cdot m_b^2 / m_t^2$$
 And $\epsilon_b = -0.0050 \pm 0.0015$ following (OPAL, EPJ C6:225)
- The jet Multiplicity without Fragmentation
 Upper figure
 ~ 70% 2 jets
- The jet Multiplicity with $\tilde{\epsilon}$ Fragmentation
 Lower Figure
 ~ 50% 3 jets
 & bigger admixture of 4jets

The cross-sections

Process	$\sigma[\text{pb}]$ at ECM=260GeV			$\sigma[\text{pb}]$ at ECM=500GeV		
P(e-)/ P(e+)	0/0	-80%/+60%	+80%/-60%	0/0	-80%/+60%	+80%/-60%
$\tilde{t}_1 \tilde{t}_1^*$	0.032	0.017	0.077	0.118	0.072	0.276
W W	16.9	48.6	1.77	8.6	24.5	0.77
Z Z	1.12	2.28	0.99	0.49	1.02	0.44
Wenu	1.73	3.04	0.50	6.14	10.6	1.82
eeZ	5.1	6.0	4.3	7.5	8.5	6.2
qq, qq \neq tt	49.5	92.7	53.1	13.1	25.4	14.9
tt	0.0	0.0	0.0	0.55	1.13	0.50
2 γ ($p_t > 5$ GeV)	786			936		

Table 1

A. Freitas et al EPJ C21(2001)361, EPJ C34(2004)487 and GRACE and COMPHEP -Next to leading order, assuming a stop mixing angle (0.01)

Pre-Selection Cuts

- A short list of the sequential cuts applied as a pre-selection first, allowed larger samples to be produced
- The pre-selection cuts are the same at the 500 and 260 GeV unless listed in parenthesis for 500 GeV

Pre-selection: 260GeV ;(500 GeV)

- $4 < \text{Number of Charged tracks} < 50$
- $P_t > 5 \text{ GeV}$
- $\cos\theta_{\text{Thrust}} < 0.8$
- $|P_l / P_{\text{tot}}| < 0.9$
- $E_{\text{vis}} < 0.40 \text{ ECM}; (E_{\text{vis}} < 0.76 \text{ ECM})$
- $M(\text{inv}) < 200 \text{ GeV}$

The cuts were refined further at Selection as shown next

Selection Cuts at $E_{\text{CM}}=260, 500 \text{ GeV}$

Variable	ECM 260 GeV	ECM 500 GeV
Number of jets	$N_{\text{jets}} \geq 2$	$N_{\text{jets}} \geq 2 \ \& \ E_n < 25 \text{ GeV}$ $n=3,4$
Number of charged tracks	$5 \leq N_{\text{tracks}} \leq 25$	$5 \leq N_{\text{tracks}} \leq 20$
Transverse Momentum p_t	$15 < p_t < 45 \text{ GeV}$	$22 < p_t < 50 \text{ GeV}$
Thrust T	$0.77 < T < 0.97$	$0.55 < T < 0.90$
Longitudinal Momentum	$ p_L / p_{\text{tot}} < 0.85$	$ p_L / p_{\text{tot}} < 0.85$
Visible Energy E_{vis}	$0.1 < E_{\text{vis}} / \text{ECM} < 0.3$	$0.1 < E_{\text{vis}} / \text{ECM} < 0.3$
Acoplanarity Φ_{acop}	$ \cos(\text{acop}) < 0.9$	$ \cos(\text{acop}) < 0.9$
Invariant mass of jet pair m_{jj}	$m_{\text{jj}}^2 < 5500 \text{ GeV}^2$ or $m_{\text{jj}}^2 > 8000 \text{ GeV}^2$	$m_{\text{jj}}^2 < 5500 \text{ GeV}^2$ or $m_{\text{jj}}^2 > 10000 \text{ GeV}^2$
Charm tagging likelihood P_c	$P_c > 0.6$	$P_c > 0.6$
<i>Signal Efficiency</i>	<i>0.340</i>	<i>0.212</i>

Table 2

In order to optimize the cancellation of the systematics we aim to have a selection as similar as possible at the two energies. (cancellation in $Y=(N_{\text{th}}-B_{\text{th}})/(N_{\text{pk}}-B_{\text{pk}})$)
The two-photons background did require a 5GeV p_t pre-selection cut.

Events Generated and After Sequential cuts

	L=50fb ⁻¹ at ECM=260GeV			L= 500fb ⁻¹ at ECM=500GeV		
P (e-)/ P(e+)	Generated	0/0	+80%/-60%	Generated	0/0	+80%/-60%
$\tilde{t}_1 \tilde{t}_1^*$	50000	543	1309	50000	12514	29270
WW	180000	38	4	210000	91	8
ZZ	30000	8	7	30000	90	81
Wenu	210000	208	60	210000	18540	5495
eeZ	210000	2	2	210000	<18	<15
qq, q≠t	350000	42	45	350000	37	43
tt	-	0	0	180000	18	17
2-Photons	1.6 10 ⁶	53	53	8.5x10 ⁶	31	31
<u>Total backgrd</u>	-	<u>351</u>	<u>171</u>	-	<u>18807</u>	<u>5781</u>
<u>S/B</u>		1.5	7.6		0.7	5.2

0/0 polarization beam → Unambiguous discovery

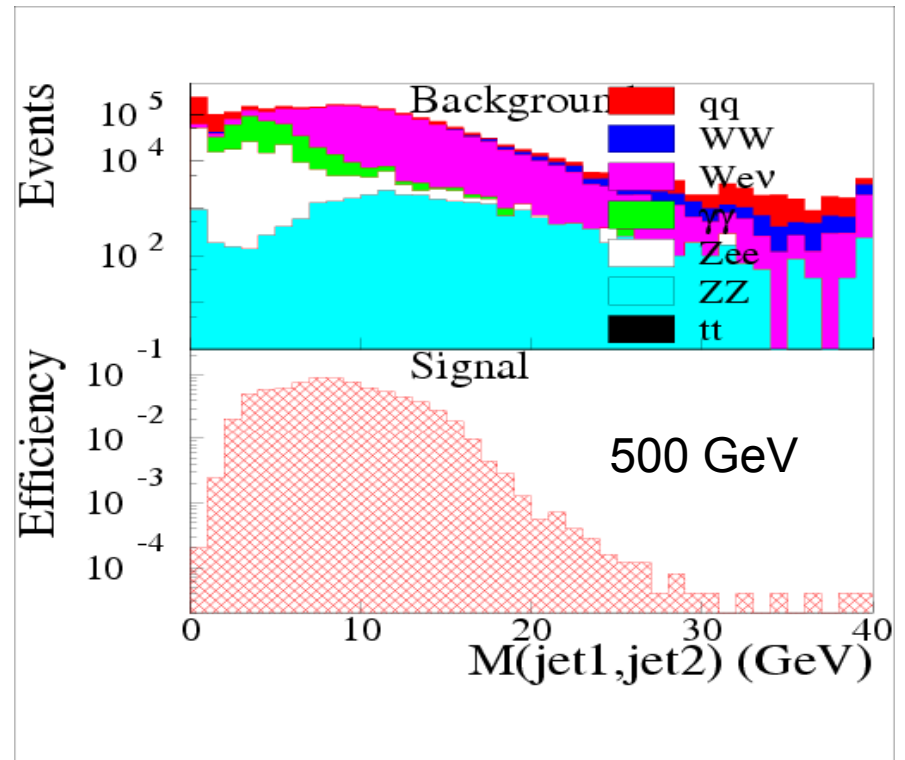
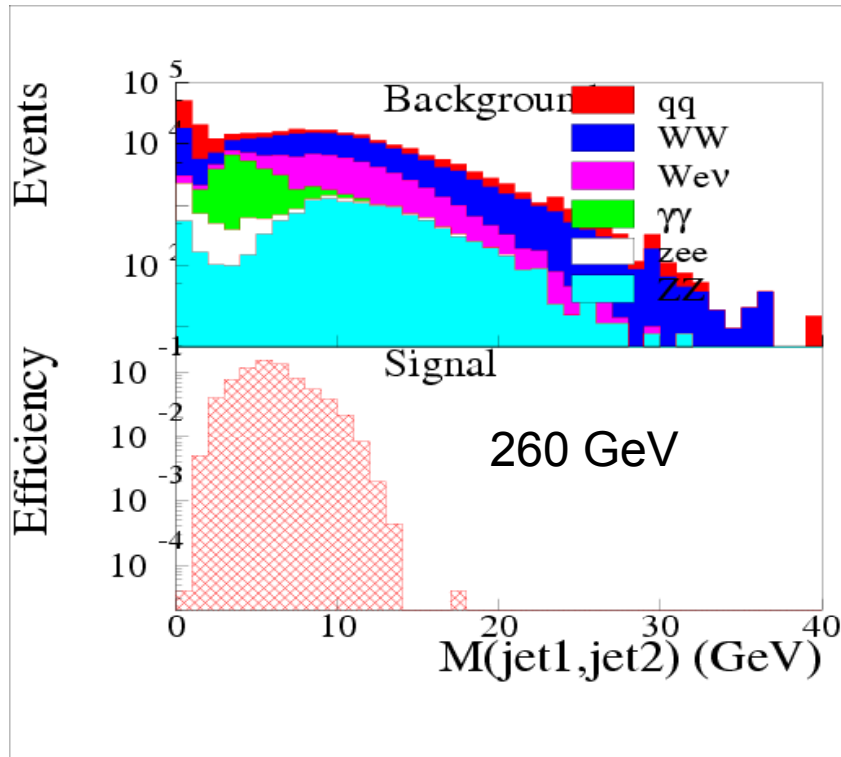
+80%/-60% polarization → Precision Measurement

Remark: \tilde{t}_1 fragmentation → the separation from the Wenu more difficult

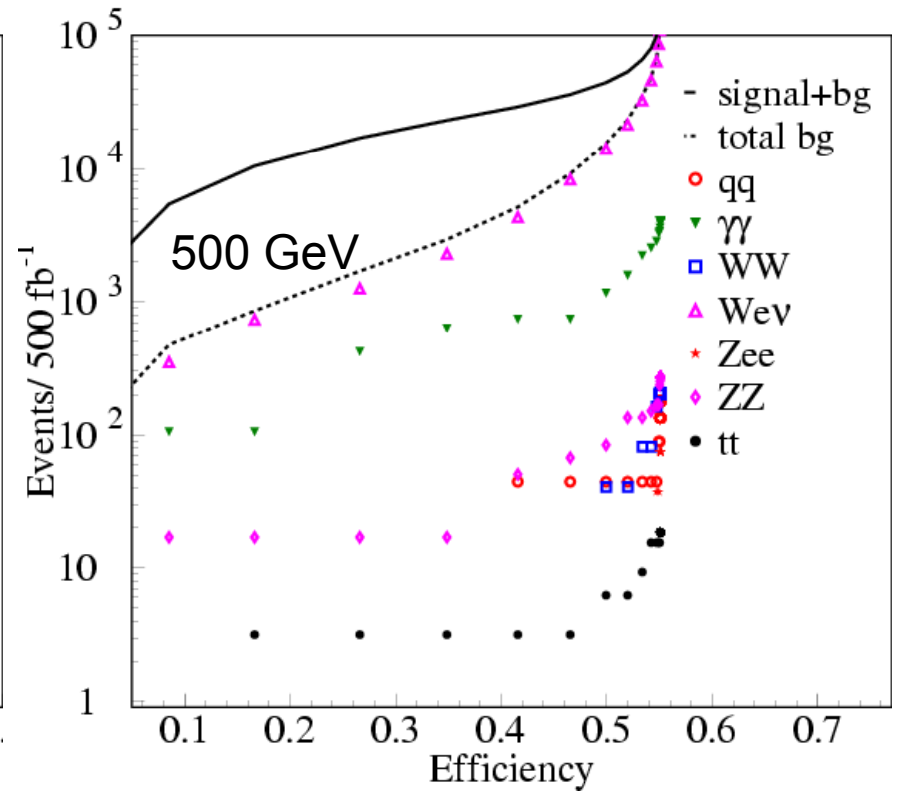
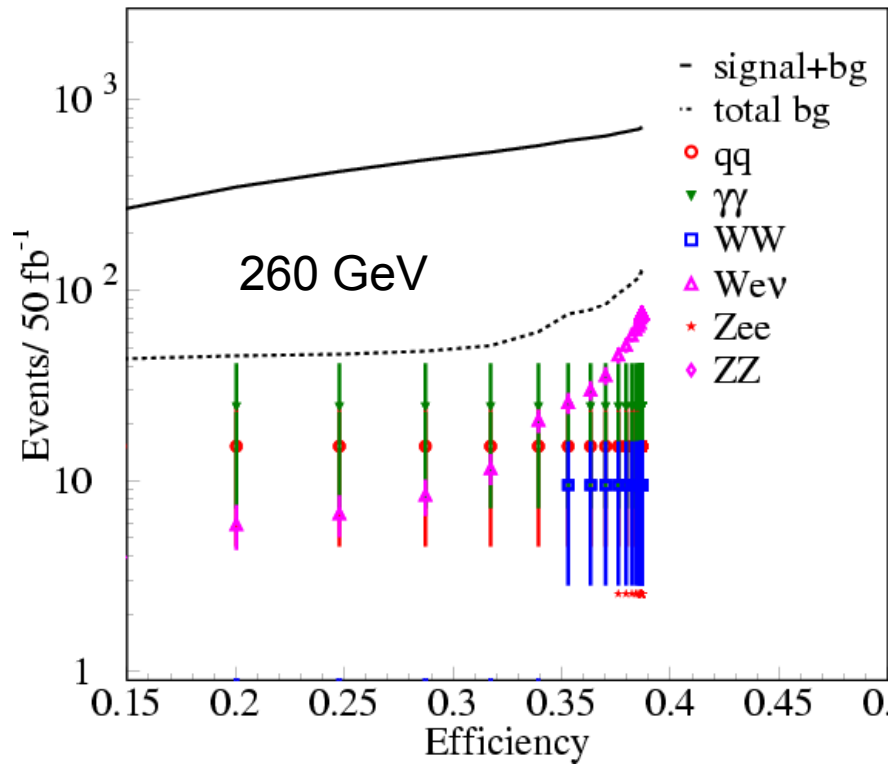
Iterative Discriminant Analysis (IDA)

- A NN approach was also used the Iterative Discriminant Analysis (IDA) . (modified Fisher Disc. Analysis)
- IDA combines the kinematic variables in parallel. The same kinematical variables we used in the cut based analysis . A non linear discriminant function followed by iterations are enhancing the separation between signal and background.
- Both the signal and background have been divided in two equally sized samples, one sample is used for training, the other as data.
- Two IDA steps have been performed, with a cut after the 1st IDA iteration keeping 99% of the signal efficiency.
- The performance is shown in the two next figures at 260 and 500 GeV.

Invariant Mass Di-Jets 1 Step Before Final IDA



IDA Performance



Systematic Uncertainty in Kinematics Cuts Variables

Variables	Error on Variable	Relative shift On signal eff $\sqrt{s} = 260 \text{ GeV}$	Relative shift On signal eff $\sqrt{s} = 500 \text{ GeV}$	Error on Y
energy scale	1%	3.7% (3.4%)	3.1% (1.3%)	<1% (2.1%)
N_{tracks}	0.5%	Negligible	Negligible	
Charm tagging	0.5%			
Luminosity	-	0.4%	0.2%	0.4%
Charm frag	0.011	0.3% (0.1%)	0.8% (0.6%)	<1%
Stop frag.	0.0015	2.4% (1.2%)	1.0% (3.5%)	2.7% (2.8%)

All cuts are applied to hadronic and jet observables → Calibration quantities are jet energy scale & jet angle.

Based on LEP, we assume 1% energy scale, 1 deg for jet angle

Effect on signal efficiency: Partial cancellation between 260 and 500 GeV

We assume cancellation in total luminosity in Y between 260&500GeV

In parenthesis IDA's values if different

Effect of Stop and Charm Fragmentation

Comparison of the signal generated with and without gluon radiation

→ The signal efficiency changes due to jet number cut is 2.5%

→ We assume an error of 1% for the number of jets

Charm fragmentation parameters assumed as precise as for LEP/OPAL

→ $\varepsilon_c = -0.0031 \pm 0.0011$

Stop fragmentation is set relative to bottom fragmentation, $\varepsilon_{\tilde{t}1} = \varepsilon_b (m_b/m_t)^2$

$\varepsilon_{\tilde{t}1} = -0.0050 \pm 0.0015$

They don't cancel between the 2 energies but are small

Including the effects of the fragmentation at both energy points

$\delta\varepsilon_c = \pm 35\% \rightarrow \text{Error } \delta Y = 0.2\%$

$\delta\varepsilon_{\tilde{t}1} = \pm 30\% \rightarrow \text{Error } \delta Y = 2.4\%$

→ contribute an error $O(\text{few}\%)$

Theoretical Uncertainties

- Precise cross-section calculations are needed
- $\tilde{t}1$ production receives large corrections from QCD gluon exchange
Between the final state $\tilde{t}1$ (bigger @Threshold) \rightarrow Coulomb corr.
- NLO- QCD corrections $\sim 100\%$ @threshold down to 10% at high energies are included here
- NNLO-QCD corrections are expected of to be same order than NLO based on the results for the top quark. The missing higher order correction $\sim 7\%$ @260GeV, 2.5% @500 GeV
- It is expected that theoretical uncertainties can be brought down by a factor 2
- Here we assume an uncertainty of 3.5% @260GeV and 1% @500 GeV
- The EW corrections : NLO \sim several %, the NNLO $\sim 1\%$
- Combined $\rightarrow \sim 4\%$ @260 GeV and 1.5% @500GeV $\rightarrow \delta Y = 5.5\%$

Combined Statistical and Systematic Errors

Error source for Y	Sequential Cuts	IDA- method
Statistical	3.1%	2.7%
Detector effects	1.0%	2.1%
Charm fragmentation	0.5%	0.5%
Stop fragmentation	2.7%	2.8%
Sum Exp systematics	3.0%	3.6%
Statistical systematics	4.3%	4.5%
Theory for Signal σ	5.5%	5.5%
Theory for BG σ	0.5%	1.7%
Total error δY	7.3%	7.2%

Results

Combining the statistical and systematic errors Table 6(*)

$\delta Y = 7.3\% \rightarrow \delta m_{\tilde{\tau}_1} \sim 0.45 \text{ GeV}$ – a factor 4 better (*Phys. rev. D 72,115008(2005)*)

(dominated by the theory, expected to improve for signal and background)

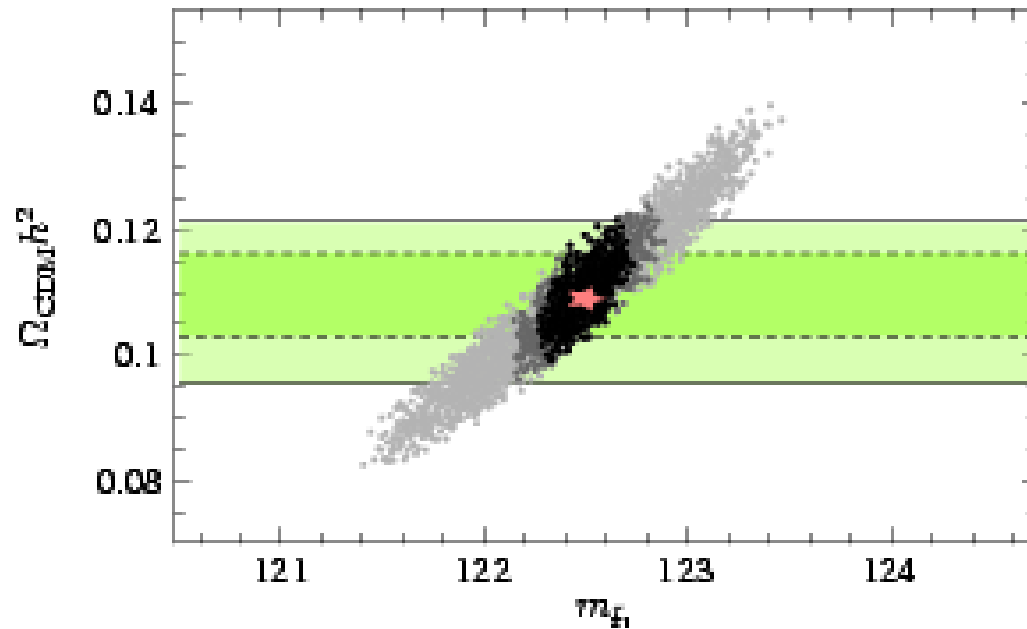
$\delta Y = 4.3\% \rightarrow \delta m_{\tilde{\tau}_1} \sim 0.26 \text{ GeV}$ (cut based experimental errors alone)

$\delta Y = 4.5\% \rightarrow \delta m_{\tilde{\tau}_1} \sim 0.28 \text{ GeV}$ (experimental errors alone & IDA)

→ Improvements in dark matter relic density due to improvement in $\delta m_{\tilde{\tau}_1}$ is shown in the next figure.

Other limiting factors start to interplay, e.g. the precision on the neutralino mass $\delta m_{\chi_1^0} \sim 0.3 \text{ GeV}$,(hep-ph/0608255, M.Carena, A.Freitas)

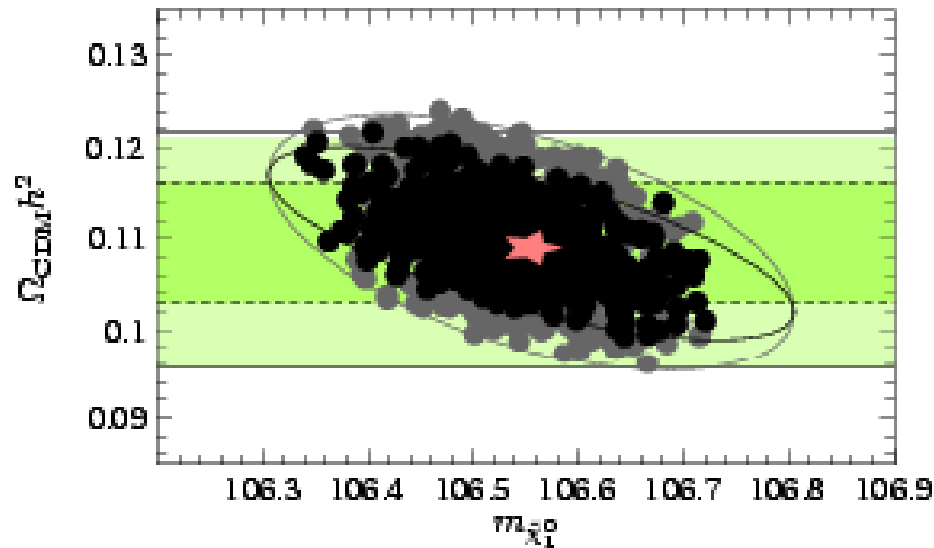
Dark Matter Relic Abundance=f ($m_{\tilde{\tau}_1}$)



Dark Matter relic density accounting
 The estimated experimental errors
 For stop, Chargino, neutralino and
 Higgs sector –(scan over 1σ)
 versus $m_{\tilde{\tau}_1}$ for
 $\delta m_{\tilde{\tau}_1} = 1.2$ GeV light gray dot
 Previous study
 $\delta m_{\tilde{\tau}_1} = 0.44$ GeV dark gray dot
 Now this study
 $\delta m_{\tilde{\tau}_1} = 0.26$ GeV black dots
 Expected this study
 with seq. cuts

$\delta m_{\tilde{\tau}_1} = 0.44$ GeV $\rightarrow \Omega_{\text{CDM}} h^2 = 0.109 + 0.0014 - 0.011$ Exp. Err.+ Th. Err.
 $\delta m_{\tilde{\tau}_1} = 0.26$ GeV $\rightarrow \Omega_{\text{CDM}} h^2 = 0.109 + 0.0013 - 0.0010$ Exp. Err. Seq. cuts
 WMAP: $\Omega_{\text{CDM}} h^2 = 0.1106 + 0.0056 - 0.0075$

Relic Abundance as Function of $m_{\chi_1^0}$



Dark Matter relic density as a function of the neutralino mass accounting for the estimated experimental errors as before but as function of the Lightest neutralino mass $m_{\chi_1^0}$

- Gray dots for $\delta m_{\tilde{\tau}_1} = 0.3$ This study Errors from Experiment+theory
- Black dots for $\delta m_{\tilde{\tau}_1} = 0.17$ This Study Experiment. Err. and IDA

$$\delta m_{\tilde{\tau}_1} = 0.44$$

$$\text{GeV} \rightarrow \Omega_{\text{CDM}} h^2 = 0.109 + 0.0014 - 0.011 \text{ Exp. Err. + Th. Err.}$$

$$\text{WMAP: } \Omega_{\text{CDM}} h^2 = 0.1106 + 0.0056 - 0.0075$$

Conclusion

- More realistic data were produced including hadronization/fragmentation
- The precision, however, improved by a factor three on our previous analysis with $\delta m_{\tilde{\tau}_1} = 0.44$ GeV
- This method could be applied to other particles e.g. to measure the Higgs mass
- The method improves the precision to the mass determination in two ways
a/ by reducing the systematics in Y- cancellation between the two energy points.
b/ by choosing the energy at threshold, Y extremely sensitive to the mass
- The polarization separates the right-handed signal $\tilde{\tau}_1$ from background.
- Due to hadronization and fragmentation the c-tagging was a necessary tool to identify the charm jets at $E_{CM} = 500$ GeV (benchmark for the vertex detector)
- IDA and the sequential cuts give almost identical results. IDA gives better statistical uncertainties but worse systematics $\delta m_{\tilde{\tau}_1} = 0.17$ GeV
- Progress in the theoretical calculations is expected and partly accounted for
- With that precision we become limited by other factors.
- With this mass precision, the calculated relic density is in accordance with WMAP and SLOAN ,
 $\delta m_{\tilde{\tau}_1} = 0.15$ GeV $\rightarrow \Omega_{CDM} h^2 = 0.109 + 0.0011 - 0.009$
WMAP: $\Omega_{CDM} h^2 = 0.1106 + 0.0056 - 0.0075$

Backup slides

A Sample Parameter Point

- $m_{\tilde{U}_3}^2 = -99^2 \text{ GeV}^2$
- $A_t = -1050 \text{ GeV}$
- $M_1 = 112.6 \text{ GeV}$
- $M_2 = 225 \text{ GeV}$
- $|\mu| = 320 \text{ GeV}$
- $\Phi_\mu = 0.2$
- $\tan \beta = 5$

Which gives:

$$m_{\tilde{t}_1} = 122.5 \text{ GeV}; m_{\tilde{t}_2} = 4203 \text{ GeV};$$

$$m_{\tilde{\chi}_1^0} = 107.2 \text{ GeV}; m_{\tilde{\chi}_1^+} = 194.3 \text{ GeV}; m_{\tilde{\chi}_2^0} = 196.1 \text{ GeV}$$

$$m_{\tilde{\chi}_3^0} = 325.0 \text{ GeV}; m_{\tilde{\chi}_2^+} = 359.3 \text{ GeV}$$

$$\cos\theta_{\tilde{t}} = 0.0105 \sim \tilde{t} \text{ right-handed}$$

$$\rightarrow \Delta m = 15.2 \text{ GeV}$$

Events Generated and After IDA Selection

	L=50fb ⁻¹ at ECM=260GeV		L= 500fb ⁻¹ at ECM=500GeV	
P (e-)/ P(e+)	0/0	+80%/-60%	0/0	+80%/-60%
$\tilde{t}_1 \tilde{t}_1^*$	618	1489	24538	57394
WW	11	1	<20	<2
ZZ	<2	<2	51	46
Wenu	68	20	4262	1263
eeZ	3	2	<18	<15
qq, q≠t	16	17	45	52
tt	0	0	3	3
2-Photons	<25	<25	772	772
<u>Total background</u>	<u>125</u>	<u>67</u>	<u>5133</u>	<u>2136</u>
S/B	4.9	22	4.7	27

The efficiencies improves from 34% ,21.2% cut based → 38.7% ,41.6% IDA, while the background is of the same order of magnitude.

Stop Discovery Reach Snowmass 2005

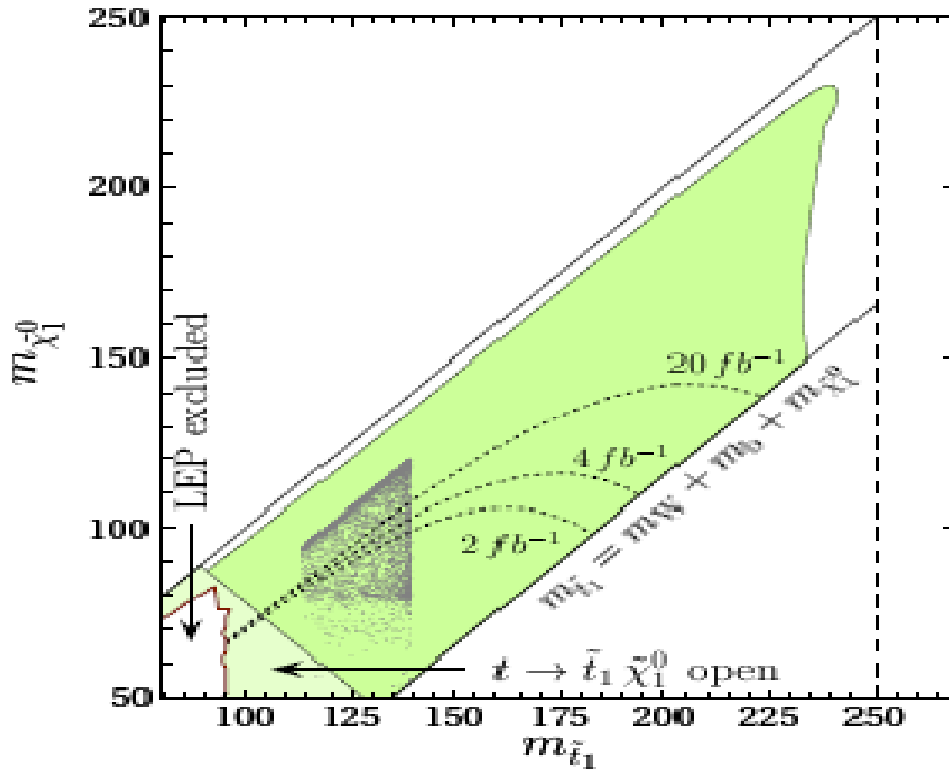


Fig 4a-Luminosity: 500 fb⁻¹
E_{cm}=500 GeV

From Simulations:
strong green region:

$$e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1^* \rightarrow c \tilde{\chi}_0^1 \bar{c} \tilde{\chi}_0^1$$

And Significance:

$$(S/\sqrt{S+B}) > 5$$

Background B

Signal S= $\epsilon\sigma$ L

For ϵ , Signal efficiency

For σ , Theoretical

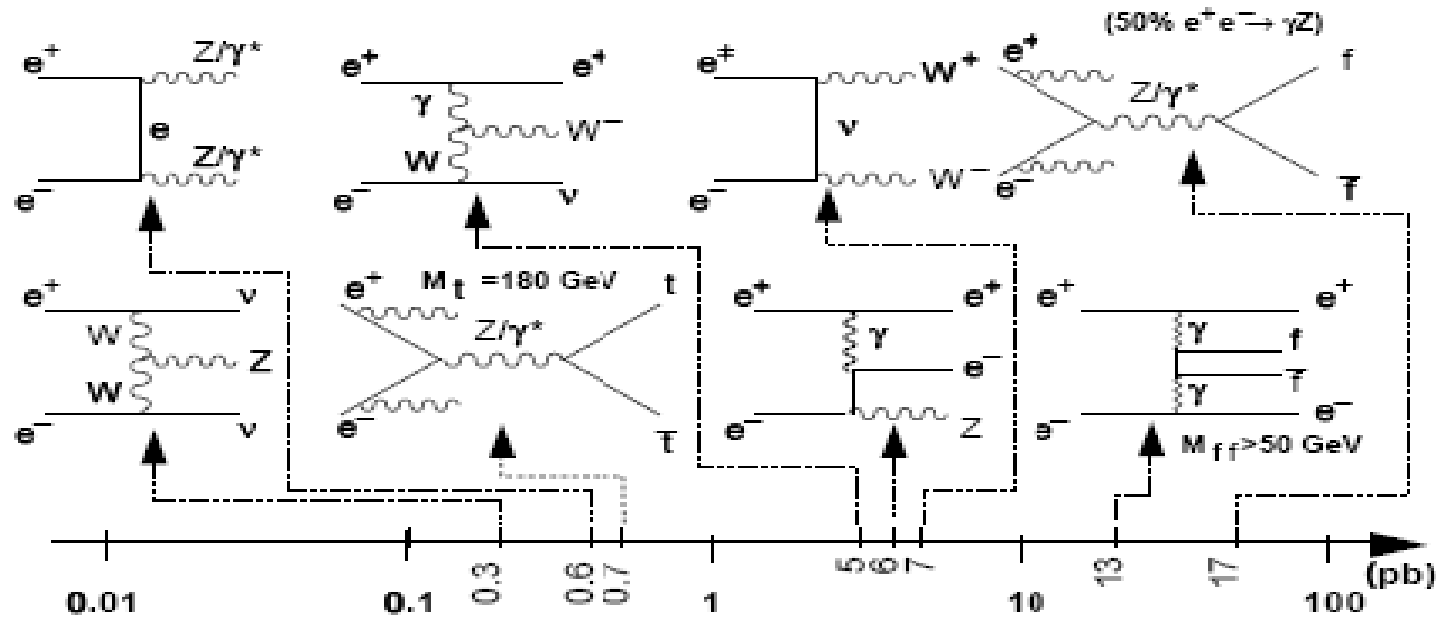
cross-section

dark gray region:

Consistent with DM

And Baryogenesis

Background- Channels @500 GeV



Z Phys. C 76 (1997) 549- A.Bartl, H. Eberl, S. Kraml, W.Majerotto, W.Porod, A. Sopczak

C-Tagging — The Data Samples

- Neural Network (NN):

data used: 255000 stops, $M_{\text{stop}}=120-220$; $D_m=5, 10, 20$
GeV

240000 W_{ev} , the most resilient background

Systematic Uncertainty in Kinematics Cuts Variables

Variable	Error on variable	Error on Y
p_t	2%	0.28%
$\cos\theta_{\text{Thrust}}$	1.8%	0.18%
E_{vis}	2%	0
Φ_{acop}	1%	0.08%
m_{jj}	4%	0.61%

Table 5

- All cuts are applied to hadronic and jet observables→ Calibration quantities are jet energy scale & jet angle.
- Based on LEP, we assume 2% calibration error for jets, 1 deg for jet angle
- Effect on signal efficiency: Partial cancellation between 260 and 500 GeV
- We assume cancellation in total luminosity in Y between 260&500GeV