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Superconducting cavity control based on system model identification

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Abstract
A digital control system for the superconducting cavities for a linear accelerator is presented. The low level radio frequency system for the FLASH project in DESY is introduced. A field programmable gate array-based controller managed by MATLAB was developed to investigate the novel firmware implementation. An algebraic complex domain model is proposed for the system analysis. The calibration procedure of a signal path is considered for multi-channel control. For a given model structure, the input–output relation of the real plant with unknown parameters is applied. The over-determined matrix equation is created covering a long enough measurement range with the solution according to the least squares method. A base function approximation by a cubic B-spline set is applied to estimate the time-varying cavity detuning during the pulse. Control tables, feed-forward and set point, are determined for the required cavity performance, according to the recognized process. The feedback loop is tuned by fitting complex gain of the corrector unit according to the determined gain table. An adaptive control algorithm is applied for feed-forward and feedback modes. Experimental results including field measurement are presented for a cavity representative operation.

Keywords: superconducting cavity control, system identification, LLRF control

1. Introduction

The TESLA technology is based on nine-cell superconducting niobium resonators to accelerate electrons and positrons. The acceleration structure is operated in the π-mode at a frequency of 1.3 GHz. The RF oscillating field is synchronized with the motion of a particle moving at the velocity of light across the cavity (figure 1). The new low level radio frequency (LLRF) control system is under development in order to improve the regulation of accelerating fields in the resonators [1, 8]. One control section consists of many independent accelerating cavities with their own dynamics driven by klystron, the servo in the system. Fast amplitude and phase control of the cavity field are accomplished by the modulation of a signal driving the klystron through a *vector modulator*. Cavities are driven with 1.3 ms pulses to an average accelerating gradient of 25 MV m^{-1}. The cavity RF signal is *down-converted* to an intermediate frequency of 250 kHz, while preserving the amplitude and phase information. ADC and DAC converters link the analogue and digital parts of the system, respectively, with a sampling interval of 1 μs. Digital signal processing is executed in the field programmable gate array (FPGA) system to obtain field vector detection, calibration and filtering. The control feedback system regulates the vector sum of the pulsed accelerating fields in multiple cavities. The FPGA-based controller regulates the detected real (in-phase—I) and imaginary (quadrature—Q) components (I_Q) of the incident wave according to a given set point (S_P). Adaptive feed-forward (F_F) is applied to improve the compensation of repetitive perturbations induced by the beam loading and by the dynamic Lorentz force detuning. The MATLAB-based control block generates the required data for the controller. The control algorithm employs the estimated parameters of the cavity: coupling factor, half-bandwidth and time-varying

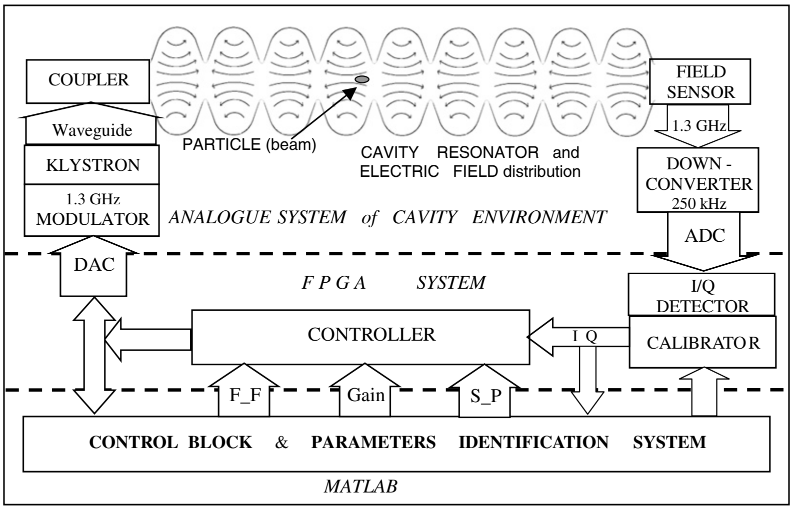

Figure 1. Functional block diagram of the LLRF control system for one cavity.

detuning. This work is a continuation of previous efforts to identify *online* the cavity parameters. It was proven that the feed-forward as a direct control method may be supplemented by the feedback operation mode, leading to system identification [2]. A wavelet method with the Haar function is applied to the system identification, for a pulsed SNS linac [3]. Alternatively, a frequency shift observer is proposed for the SNS superconducting cavity with a digital signal processor in real time [4]. An iterative learning control is another proposal to achieve a stable cavity field periodically [5]. The practical difficulties, expressed in these papers, with the *on-line* implementation of the parameters' identification algorithms have inclined the author to use the alternative approach of *off-line* analysis between pulses [7]. The presented method is useful for the repetitive, deterministic process that has been verified experimentally.

A system model was developed for investigating the efficient control method of achieving the required cavity performance: driving in resonance during *filling* and field stabilization for *flat-top* range. Initial testing of the real cavity system has focused attention on the model verification and identification of the cavity parameters. The control system was experimentally studied without beam in the first cryo-module with eight cavities—ACC1 at FLASH at DESY.

2. Outline of superconductive cavity modelling

The basic electromechanical features of a superconductive cavity resonator are considered in a model including the Lorentz force detuning and beam loading [1, 6].

The equivalent electrical representations of the chain of nine cells of the resonator are magnetically coupled resonant circuits. The simplified version limited to one cavity as a single *LCR* circuit is quite sufficient for the purpose of cavity control modelling. However, other pass band modes may cause instability. The main parameters of the cavity model correspond to the resonant *LCR* circuit representation as follows: resonance frequency $\equiv \omega_0 = 2\pi f_0 = (LC)^{-1/2}$, characteristic resistance (normalized *shunt* impedance) $\equiv \rho = (L/C)^{1/2}$, load resistance (*shunt* impedance) $\equiv R$, loaded quality factor $\equiv Q = R/\rho$, half-bandwidth (HWHM) $\equiv 2\pi f_{1/2} = \omega_{1/2} = 1/2CR = \omega_0/2Q$.

The cavity electrical model can be represented by the transfer function as follows:

$$\begin{aligned} Z(\mathbf{s}) &= (1/R + \mathbf{s}C + 1/\mathbf{s}L)^{-1} \\ &= \omega_{1/2} \cdot R \big/ \left(\omega_{1/2} + \left(\mathbf{s}^2 + \omega_0^2\right)/2\mathbf{s}\right). \end{aligned} \tag{1}$$

Each cavity is coupled to the wave guide driven by the klystron as a power amplifier separated by a circulator. The forward power, which is provided by the wave guide, reflects partly due to the mismatched input coupler and dissipates in the circulator load. The residual transmitted power supplies cavity and beam loading. The objective of the system is stable acceleration of the beam with the best power efficiency.

The klystron is modelled as a RF current generator with the resultant value $2J_g$ after the coupler. The beam loading can be modelled as a current sink J_b fed by the electromagnetic field of the cavity. The beam current with a bunched structure has a typical charge of 8 nC, 1 MHz repetition and an average value of 8 mA.

Therefore, the resultant cavity voltage U depends on the generator current and the beam loading current and is expressed in the *Laplace space* as follows:

$$U(\mathbf{s}) = Z(\mathbf{s}) \cdot (2J_g(\mathbf{s}) - J_b(\mathbf{s})) = Z(\mathbf{s}) \cdot J(\mathbf{s}). \tag{2}$$

The essential signal modelling for the superconductive cavity resonator with its narrow bandwidth (~400 Hz) assumes a relatively slow modulation of the RF carrier with a frequency $f_g = 1.3$ GHz. Therefore, the cavity field can be modelled in the *time domain* as an *analytical signal*, according to the expression for the voltage case:

$$\mathbf{u}(t) = a(t) \cdot \mathrm{e}^{\mathrm{i}\Psi(t)} \qquad \text{for} \quad \Psi(t) = \omega_g t + \varphi(t), \tag{3}$$

where the amplitude $a(t)$ and the phase $\varphi(t)$ are slowly time varying, relative to the RF signal carrier with the frequency $\omega_g = 2\pi f_g$.

The low level frequency representation of the cavity signal in the time domain is the *complex envelope*. It is derived by the complex demodulation of the analytical signal for the given frequency ω_g, according to the expression, for the voltage case:

$$\mathbf{v}(t) = \mathbf{u}(t) \cdot \exp(-\mathrm{i}\omega_g t) = a(t) \cdot \mathrm{e}^{\mathrm{i}\varphi(t)} = I + \mathrm{i}Q, \tag{4}$$

Table 1. The main parameters of the cavity electromechanical model.

Cavity electrical parameters		Cavity mechanical modes parameters	
$f_0 = 1300$	Resonance frequency (MHz)	$\mathbf{f} = [235, 290, 450]$	Resonance frequencies vector (Hz)
$\rho = 520$	Characteristic resistance (Ω)		
$Q = 3 \times 10^6$	Loaded quality factor	$\mathbf{Q} = [100, 100, 100]$	Quality factor vector
$R = Q \cdot \rho = 1560$	Load resistance (MΩ)		
$f_{1/2} = f_0/2Q = 216$	Half band-width (Hz)	$\mathbf{K} = [0.4, 0.3, 0.2]$	Lorentz force detuning constants
$\Delta f = 390$	Pre-detuning (Hz)	vector (Hz (MV)$^{-2}$)	

where *I*—in-phase and *Q*—quadrature are the real and imaginary components of the complex envelope, respectively.

The direct relation for the complex envelope between the cavity input $\mathbf{i}(t) \leftrightarrow \mathbf{I}(\mathbf{s})$ and output $\mathbf{v}(t) \leftrightarrow \mathbf{V}(\mathbf{s})$ is obtained by the successive operations: complex modulation, cavity transfer function and complex demodulation, according to the simplified scheme in the Laplace space and time domain, respectively:

$$\mathbf{I}(\mathbf{s}) \leftrightarrow \mathbf{i}(t) * \exp(\mathrm{i}\omega_g t) \leftrightarrow \mathbf{I}(\mathbf{s} - \mathrm{i}\omega_g) * Z(\mathbf{s}) \leftrightarrow \mathbf{u}(t) * \exp(-\mathrm{i}\omega_g t) \leftrightarrow Z(\mathbf{s} + \mathrm{i}\omega_g) \cdot \mathbf{I}(\mathbf{s}) = \mathbf{V}(\mathbf{s}) \leftrightarrow \mathbf{v}(t).$$

Thus, the low-pass transformation determines the resultant cavity transfer function $Z(\mathbf{s} + \mathrm{i}\omega_g)$, which can be effectively simplified, under the condition that $|\mathbf{s}| \ll \omega_g \approx \omega_0$, as follows:

$$Z(\mathbf{s} + \mathrm{i}\omega_g) \approx \omega_{1/2} \cdot R_L/(\mathbf{s} + \omega_{1/2} - \mathrm{i}\Delta\omega), \tag{5}$$

where the cavity detuning $\Delta\omega \equiv \omega_0 - \omega_g$.

Moving to the time domain yields the *state space* equation with $\mathbf{v}(t)$ as the state phasor of the cavity electrical model:

$$\mathrm{d}\mathbf{v}(t)/\mathrm{d}t = \mathbf{A}_e \cdot \mathbf{v}(t) + \omega_{1/2} \cdot R_L \cdot \mathbf{i}(t), \tag{6}$$

where the phasor $\mathbf{A}_e = -\omega_{1/2} + \mathrm{i}\Delta\omega$ for the complex representation.

The superconducting resonator has an extremely high loaded quality factor $Q \sim 3 \times 10^6$ and a narrow bandwidth of about 430 Hz (FWHM). Hence, the cavity is very sensitive to the mechanical distortion caused by microphonics and the Lorentz force, changing the resonator frequency. The cavity model is non-stationary with the time-varying detuning $\Delta\omega$. The cavity detuning value can be comparable to the cavity bandwidth under the real operation condition. This cavity parameter has two dominant deterministic components: the Lorentz force detuning and the initial pre-detuning. The mechanically biased pre-detuning attempts to compensate the EM-forced detuning factor, during the operational condition of the cavity.

The mechanical model of the superconductive cavity has been created for simulation purposes. The model describes the Lorentz force detuning, which is a function of the square of the time-varying field gradient. It is based on the heuristic relationship for the independent mechanical modes of the cavity with the resonance frequency and the mechanical quality factor for the given mode. Three dominating resonance frequencies are considered in the model, and the superposition of all modes and initial pre-detuning yields the resultant detuning.

The main parameters of the cavity electromechanical model, for simulation purposes, are combined in table 1.

The discrete model of the cavity behaviour has been developed for digital implementation of the cavity model. The discrete state space equations, for the parallel electrical and mechanical processing, have been solved iteratively by MATLAB with time interval $T = 1\ \mu$s.

The simulation results for the cavity real operational condition are presented in figure 2. The cavity is driven in a pulsed mode forced by the control feedback supported by feed-forward. During the first stage of operation, the cavity *fills* with constant forward power, resulting in an exponential increase of the electromagnetic field, according to its natural behaviour under the resonance condition. When the cavity gradient has reached the required final value, the beam loading current is injected, resulting in a steady-state *flat-top* operation. Turning off the generator and the beam current, at the end of the RF pulse, yields an exponential *decay* of the cavity field.

3. Cavity control system modelling

3.1. Multi-channel system modelling

A discrete-time model in the complex domain is introduced to analyse the LLRF digital control system (figure 3). A signal is modelled by a complex envelope called a 'phasor'. Modules of the system form a phasor according to their characteristics described by complex factors. In a practical application of a linear accelerator, one klystron drives many cavities. Therefore, a multi-channel system, driven with a common klystron *phasor* $\mathbf{u}$, for vector sum control is considered.

The initial considerations are for a single cavity without a beam. The electrical model of the single cavity is assumed as the only dynamic part of the *i*th channel. The discrete model is based on a difference equation of first order for the output *phasor* $\mathbf{v}^i$, driven with the input *phasor* $\mathbf{u}$. The recursive equation of the cavity model, with the sampling interval T, is expressed by a complex form, for step k:

$$\mathbf{v}^i_{k+1} = \mathbf{E}^i_k \cdot \mathbf{v}^i_k + \mathbf{A}^i \cdot \mathbf{u}_k, \tag{7}$$

where the input coupling is described by $\mathbf{A}^i$ and the system dynamics (time varying) are described by $\mathbf{E}^i_k = \left(1 - T\omega^i_{1/2}\right) + T\Delta\omega^i_k \cdot \boldsymbol{i}$, with the *i*th cavity parameters: constant half-bandwidth $= \omega^i_{1/2}$ and time-varying detuning $= \Delta\omega^i_k$.

The input coupling factors $\mathbf{A}^i$ are responsible for power distribution and phase alignment of the cavities. The static but nonlinear klystron unit (common for all cavities) is modelled by the time varying factor $\mathbf{D}_k$. The static and linear measurement channel, as an output unit, is represented by the constant factor $\mathbf{B}^i$ for the *i*th channel. A calibration of the measurement path is essential for cavity control, and is required in a multi-channel case. A calibration of a single channel is accomplished by means of a calibrator unit $\mathbf{C}^i$

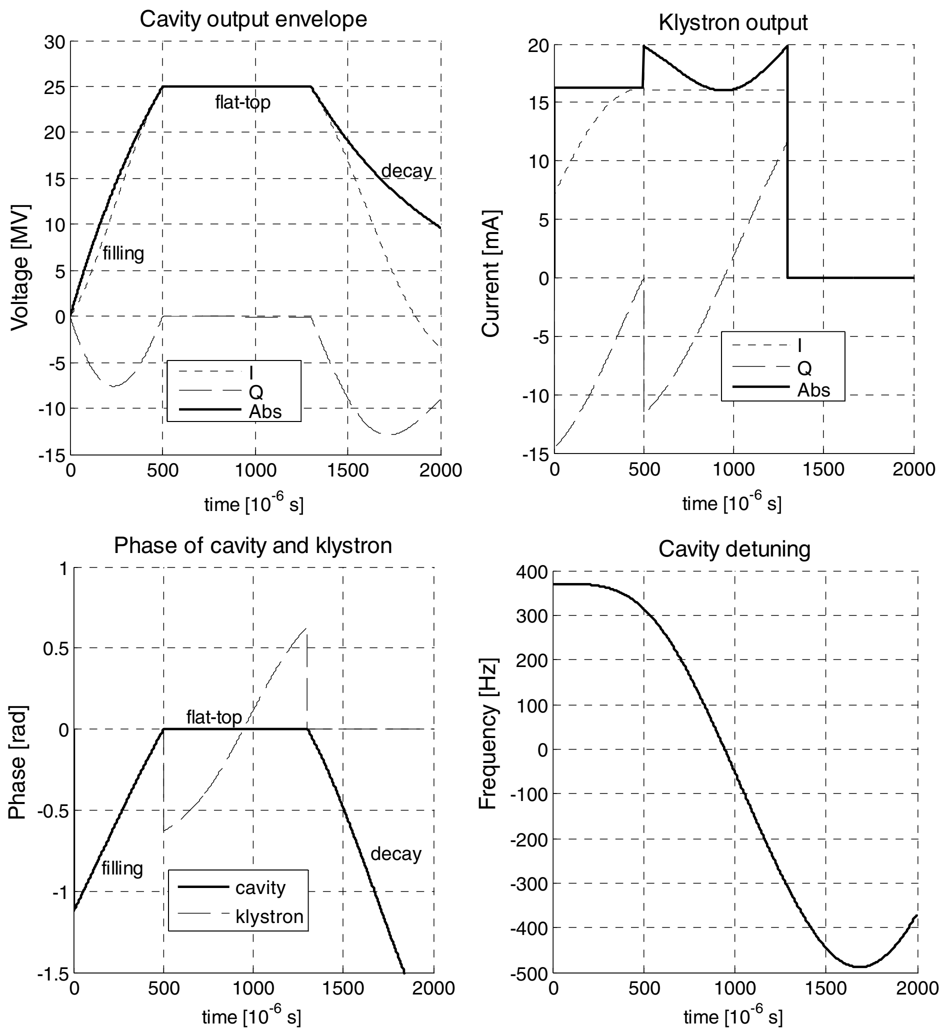

Figure 2. The MATLAB results of simulation for the cavity real operation condition.

implemented inside the FPGA controller area, so $\mathbf{B}^i \cdot \mathbf{C}^i = c \approx$ const. Scaling of each channel is performed according to the cavity field gradient taken from an outside measurement system. After phasing of each channel, an average *flat-top* phase is equal to the given value matched to the beam. However, the current consideration is without a beam.

Consequently, a vector sum for all channels is considered. Summarizing equation (7) for all channels and introducing new variables yields, for a step k:

$$\mathbf{v}_{k+1} = \mathbf{E}_k \cdot \mathbf{v}_k + \mathbf{F} \cdot \mathbf{u}_k, \tag{8}$$

where the vector sum is $\mathbf{v}_k = \sum_i \left(\mathbf{v}_k^i\right)$, the resultant input factor is $\mathbf{F} = \sum_i (\mathbf{A}^i)$ and equation $\mathbf{E}_k \cdot \mathbf{v}_k = \sum_i \left(\mathbf{E}_k^i \cdot \mathbf{v}_k^i\right)$ defines a weighted average system factor $\mathbf{E}_k = (1 - T\omega_{1/2}) + T\Delta\omega_k \cdot \boldsymbol{i}$, with the resultant half-bandwidth $= \omega_{1/2}$ and resultant detuning $= \Delta\omega_k$.

The resulting, multi-cavity model, introduced by equation (8), has the same structure as a single channel model (7), but with new parameters, in the case of diverse operational condition for cavities.

Including the klystron unit in the cavity system, the ultimate model, driven with the controller *phasor* $\mathbf{x}_k$, is given by

$$\mathbf{v}_{k+1} = \mathbf{E}_k \cdot \mathbf{v}_k + \mathbf{H}_k \cdot \mathbf{x}_k, \tag{9}$$

with the input coupling complex factor $\mathbf{H}_k = \mathbf{F} \cdot \mathbf{D}_k$.

3.2. Controller modelling

The FPGA-based controller executes the procedure of feed-forward driving supported by feedback according to prearranged control tables: $\mathbf{FF}_k$, $\mathbf{SP}_k$, $\mathbf{G}_k$. A multi-cavity phasor $\mathrm{c} \cdot \mathbf{v}_k$ is compared to the reference phasor $\mathbf{SP}_k$ (set point) creating an error phasor. The error phasor is multiplied by the complex gain $\mathbf{G}_k$ of the corrector unit, producing a feedback phasor. Superposition of a feedback phasor and a compensating phasor $\mathbf{FF}_k$ (feed-forward) results in the controller phasor $\mathbf{x}_k$. Consequently, the controller model is expressed, for step k:

$$\mathbf{x}_{k+1} = \mathbf{FF}_k + \mathbf{G}_k \cdot (\mathbf{SP}_k - \mathrm{c} \cdot \mathbf{v}_k). \tag{10}$$

Control tables are determined for the required cavity performance, according to the control algorithm based on estimated parameters of the cavity system.

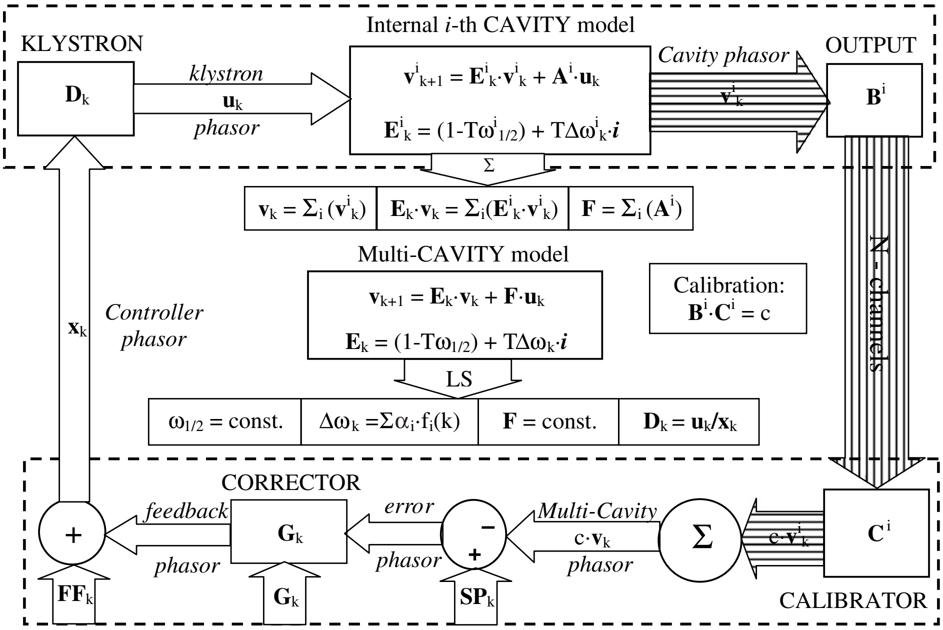

Figure 3. Algebraic model of the LLRF control system.

4. Parameter identification of the cavity system

The control algorithm is based on the cavity system model and requires therefore the identification of the process parameters [3]. The two complex time varying factors $\mathbf{E}_k$ and $\mathbf{H}_k$, respectively, should be recognized in equation (9). The least square (LS) method is proposed for parameter estimation under a noisy and non-stationary condition [7].

The first stage of the estimation procedure is based on the linear part of the model expressed by equation (8), where the factor $\mathbf{F}$ has a constant scalar value F after proper calibration. The complex equation (8) is expanded to a scalar form, applying the real (v_r, u_r) and imaginary (v_i, u_i) components of the cavity and klystron phasor respectively, for steps k and k+1, as follows:

$$(\mathrm{v}_r)_{k+1} = (1 - T\omega_{1/2}) \cdot (\mathrm{v}_r)_k - T\Delta\omega_k \cdot (\mathrm{v}_i)_k + F \cdot (\mathrm{u}_r)_k \qquad (11)$$

$$(\mathrm{v}_i)_{k+1} = (1 - T\omega_{1/2}) \cdot (\mathrm{v}_i)_k + T\Delta\omega_k \cdot (\mathrm{v}_r)_k + F \cdot (\mathrm{u}_i)_k. \qquad (12)$$

Three scalar unknowns include two stable parameters: the half-bandwidth $\omega_{1/2}$ and factor F, and the time varying detuning $\Delta\omega_k$. These parameters should be estimated for each kth step of the process, described by two equations (11) and (12) involving measured input and output data, respectively. The time-varying detuning $\Delta\omega_k$ can be approximated for each kth step by L-order series of chosen base functions $\{f_j\}$ with unknown, but constant coefficients α_j, as follows:

$$T\Delta\omega_k = \sum_{j=1}^{L} \alpha_j f_j(k) = \mathbf{f}_k \cdot \boldsymbol{\alpha}, \qquad (13)$$

where $\boldsymbol{\alpha}$ is the column vector of L coefficients, $\mathbf{f}_k$ is the row vector of L values of base functions for the kth step.

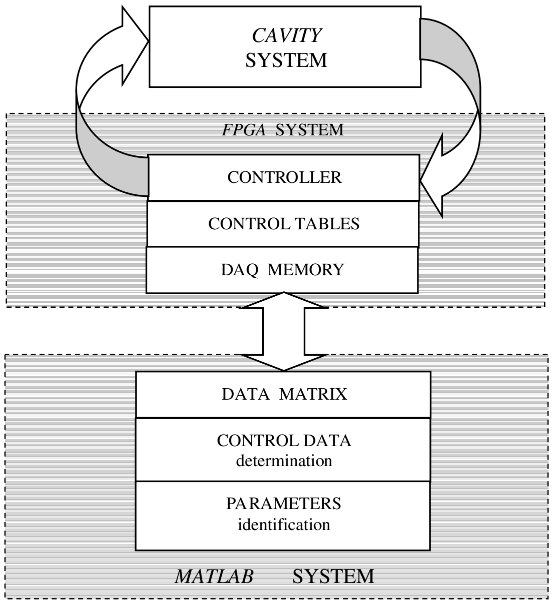

Figure 4. Adaptive control process for the cavity system driving.

Substituting equation (13) into (11) and (12), all unknowns are extracted as follows:

$$\mathbf{v}_{k+1} = \mathbf{w}_k \cdot \mathbf{z}, \qquad (14)$$

where $\mathbf{z} = [(1 - T\omega_{1/2}), \boldsymbol{\alpha}, F]$ is the resultant column vector of unknown $L + 2$ values,

$$\mathbf{v}_k = [(\mathrm{v}_r)_k, (\mathrm{v}_i)_k], \qquad \mathbf{w}_k = [\mathbf{v}_k, \mathbf{v}'_k \cdot \mathbf{f}_k, \mathbf{u}_k]$$

for

$$\mathbf{v}'_k = [-(\mathrm{v}_i)_k, (\mathrm{v}_r)_k] \qquad \text{and} \qquad \mathbf{u}_k = [(\mathrm{u}_r)_k, (\mathrm{u}_i)_k].$$

In a practical application, equation (14) is considered for N steps of an approximation range, creating 2N over-determined

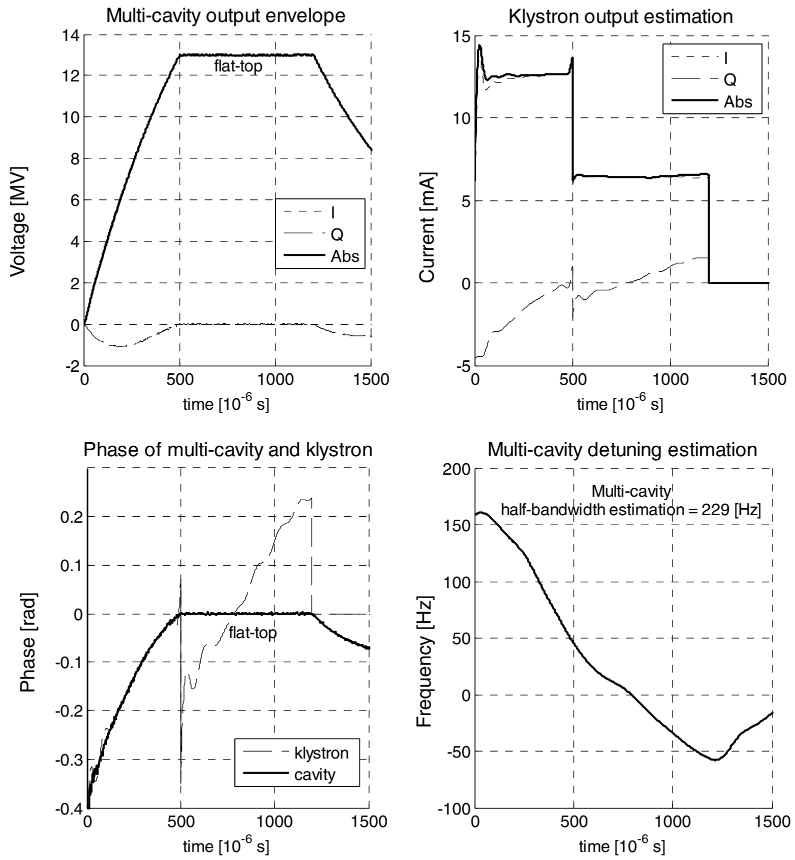

Figure 5. Vector sum control of an eight-cavity module—feed-forward and feedback driving (loop gain = 100). The experimental results of adaptive control without a beam.

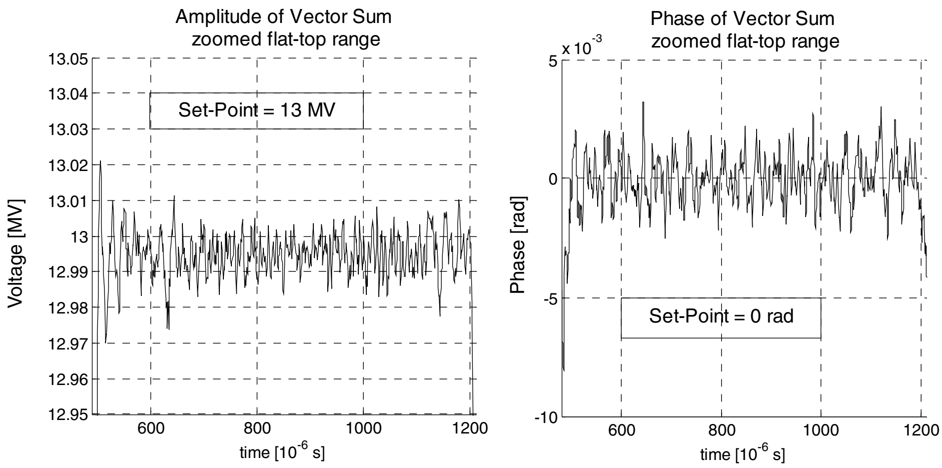

Figure 6. Amplitude and phase of vector sum control of an eight-cavity module—zoomed flat-top range for figure 5. Field stabilization: amplitude relative accuracy = 4.7×10^{-4}, phase accuracy = 8.2×10^{-4} rad (standard deviation).

equations ($2N \gg L+2$), expressed by the matrix form, as follows:

$$\mathbf{V} = \mathbf{W} \cdot \mathbf{z}, \tag{15}$$

where $\mathbf{V}$ is the total output vector (dim = $2N \times 1$) and $\mathbf{W}$ is the total matrix of the model structure (dim = $2N \times L + 2$).

Multiplying the two sides of equation (15) by matrix transposition $\mathbf{W}^T$, the solution for the vector $\mathbf{z}$ is given by

$$\mathbf{z} = (\mathbf{W}^T \cdot \mathbf{W})^{-1} \cdot \mathbf{W}^T \cdot \mathbf{V}. \tag{16}$$

It is the unique solution, according to the LS method for the measured data of vector $\mathbf{V}$ and structure matrix $\mathbf{W}$. An algorithm for the identification of cavity parameters was implemented in the Matlab system applying the cubic B-spline set of functions for the linear decomposition of the time-varying detuning [7].

The second stage of the estimation procedure is based on the nonlinear part of the model described by the complex equation $\mathbf{u}_k = \mathbf{D}_k \cdot \mathbf{x}_k$ for the klystron unit (figure 3). Finally,

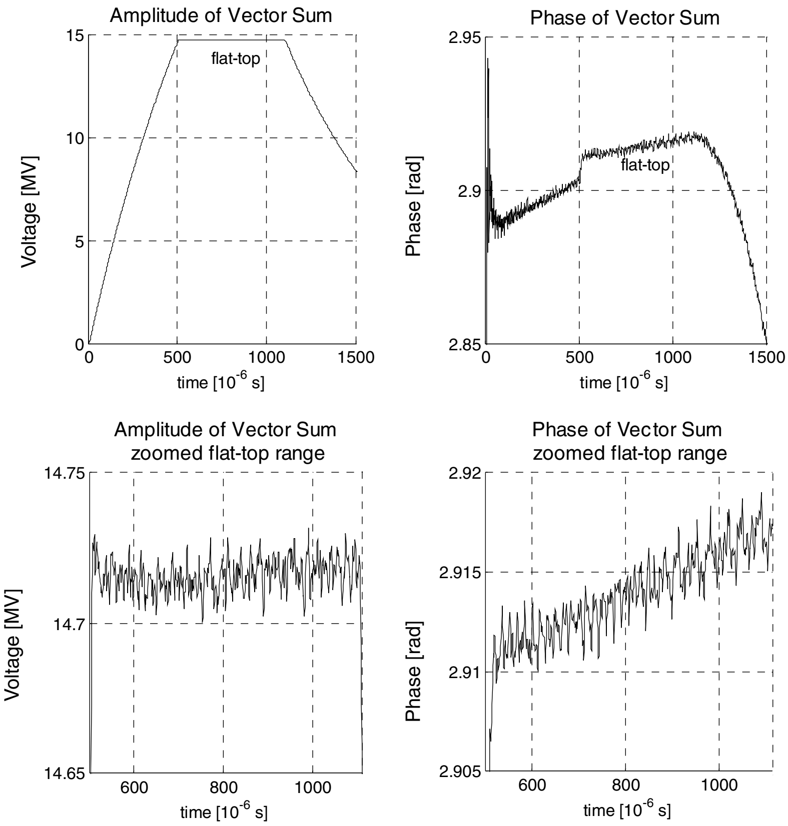

Figure 7. Amplitude and phase of vector sum control of an eight-cavity module. Data taken from the DOOCS system at DESY. Field stabilization: amplitude relative accuracy $= 3.7 \times 10^{-4}$, phase accuracy $= 2.1 \times 10^{-3}$ rad (standard deviation).

Table 2. The estimated formulas for the control tables.

	Table		
Range	Feed-forward	Set-point	Gain
Filling	$\mathbf{FF}_k = T\omega_{1/2} \cdot \mathrm{v} \cdot \exp(\mathrm{i}\varphi_{k+1})/\mathbf{H}_{k+1}$ $\varphi_{k+1} = \varphi_k + \mathrm{T}\Delta\omega_k$	$\mathbf{SP}_k = v \cdot [1 - \exp(-k \cdot T\omega_{1/2})] \cdot \exp(\mathrm{i}\varphi_k)$	$\mathbf{G}_k = L \cdot T\omega_{1/2}/\mathbf{H}_{k+1}$ $L = \mathrm{const.}$
Flat-top	$\mathbf{FF}_k = \mathbf{V} \cdot T(\omega_{1/2} - \Delta\omega_{k+1} \cdot \mathrm{i})/\mathbf{H}_{k+1}$	$\mathbf{SP}_k = \mathbf{V} = \mathrm{V} \cdot \exp(\mathrm{i}\Phi)$	$\mathbf{G}_k = (L+1) \cdot T(\omega_{1/2} - \Delta\omega_{k+1} \cdot \mathrm{i})/\mathbf{H}_{k+1}$

the factor $\mathbf{H}_k$ from equation (9) is estimated, as follows: $\mathbf{H}_k = \mathbf{F} \cdot \mathbf{u}_k/\mathbf{x}_k$. The prior estimation (filtering), applying the cubic B-spline set of functions, for measured data cavity, controller and klystron phasor, was performed for off-line identification of the parameters presented in this paper.

5. Control of the cavity system

5.1. Feed-forward and feedback driving

The required cavity performance is driving in resonance during *filling* and field stabilization for the *flat-top* range. The cavity is driven in feed-forward and feedback modes to fulfil a desired operation condition. Combining equations (9) and (10) yields a resulting equation for the cavity control system, as follows:

$$\mathbf{v}_{k+1} = \mathbf{E}_k \cdot \mathbf{v}_k + \mathbf{H}_k \cdot [\mathbf{FF}_{k-1} + \mathbf{G}_{k-1} \cdot (\mathbf{SP}_{k-1} - \mathrm{c} \cdot \mathbf{v}_{k-1})]. \tag{17}$$

The required control tables, feed-forward—$\mathbf{FF}_k$, set point—$\mathbf{SP}_k$ and complex gain—$\mathbf{G}_k$, should be determined for the given cavity system factors: $\mathbf{E}_k$ and $\mathbf{H}_k$. The required cavity phasor $\mathbf{v}_k$ can be achieved by ideal feed-forward compensation according to the recognized system model. However, the feedback mode compensates stochastic disturbances and a model discrepancy.

Let us consider equation (17), separately for the two modes of operation, reduced as

$$\mathbf{v}_{k+1} = \mathbf{E}_k \cdot \mathbf{v}_k + \mathbf{U}_k, \tag{18}$$

where $\mathbf{U}_k = \mathbf{H}_k \cdot \mathbf{FF}_{k-1}$ represents the feed-forward and $\mathbf{U}_k = \mathbf{H}_k \cdot \mathbf{G}_{k-1} \cdot (\mathbf{SP}_{k-1} - \mathrm{c} \cdot \mathbf{v}_{k-1})$ represents the feedback.

The solution of equation (18) gives the estimated formulae presented in table 2, for two modes of operation, within two ranges.

- Filling range
  Equation (18) is driven with the phasor $\mathbf{U}_k$ with a constant amplitude and modulated phase φ_k, so the RF signal tracks the time-varying resonance frequency of the cavity. In response, the cavity phasor $\mathbf{v}_k$ has the same phase and exponential growth of amplitude, according to natural behaviour, under the resonance condition. The initial phase φ_1 and asymptotic amplitude v are determined to

fulfil the condition for a final filling phasor $\mathbf{v}_k$, which is equal to the required flat-top phasor $\mathbf{V}$.

- Flat-top range
- Equation (18) is driven with the phasor $\mathbf{U}_k$ compensating the step varying factor $\mathbf{E}_k$, so the cavity phasor $\mathbf{v}_k = \mathbf{V}$ is stabilized. The complex gain $\mathbf{G}_k$ and factor c are determined for the given loop gain L assumed as a constant scalar parameter for the steady-state cavity phasor $\mathrm{c} \cdot \mathbf{V} = L \cdot (\mathbf{SP}_k - \mathrm{c} \cdot \mathbf{V})$. The loop gain L value is limited due to the stability condition.

5.2. *Adaptive control process*

Superconducting cavity control is carried out according to the scheme of figure 4. Control data, generated by the Matlab system, are loaded into the internal FPGA memory and actuate the controller. The input and output data of the cavity are acquired to another area of the memory during pulse operation. The acquired data are conveyed to the Matlab system, for parameter identification processing, between pulses. Estimated cavity parameters are considered as actual values for the required cavity performance and are applied to create the control tables for the next pulse. But new control tables modify the trajectory of the nonlinear process and again new parameters are estimated. This iterative processing quickly converges to the desired state of the cavity, assuming deterministic conditions for successive pulses. Stochastic fluctuations of the required trajectory are reduced by the averaging processing (filtering) for successive pulses.

The experimental results of the adaptive control, under real operation conditions, are presented in figures 5 and 6 for feed-forward and feedback driving. The cavity is activated with a pulse of 1.2 ms duration and repetition of 10 Hz. During the first stage of the operation (~0.5 ms), the cavity is *filling* with constant forward power resulting in an exponential increase of the field under the resonance condition. When the cavity phasor has reached the required final value, the steady state is forced during the flat-top range (~0.7 ms). The initial pre-detuning attempts to compensate the Lorentz force detuning and balances the required power consumption during pulse operation of the cavity. Switching off the klystron power yields an exponential decay of the cavity field. For a comparison to the presently working controller, figure 7 shows data taken from the measurement and control system DOOCS at DESY.

6. Conclusions

The cavity control system for the superconducting linear accelerator project is introduced in this paper. Digital control of the superconductive cavity has been performed by applying the FPGA technology system in DESY. These experiments focused attention on the general recognition of the cavity characteristics and projected control methods. Calibration and correction of the signal path are considered for the efficient driving of a cavity. Identification of the resonator parameters has been proven to be a successful approach in achieving the required performance, i.e. driving on resonance during *filling* and field stabilization during *flat-top* time, while requiring reasonable levels of power consumption. Feed-forward and feedback modes were successfully applied in operating the cavity. Representative results of the experiments are presented for the typical operational condition. Preliminary application tests of the FPGA controller have been carried out using the superconducting cavities in the ACC1 module of the FLASH laser setup at DESY. The proposed control algorithm is still under development, and it is too early for an ultimate assessment and comparison. The achieved amplitude stabilization is comparable to the present controller, but phase stabilization is much better (figures 6 and 7).

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