# Optical Elastic Differential Cross Section Model

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## Abstract

Differential elastic hadron-nucleus cross-sections are discussed in the framework of optical approach. The model predictions are compared with experimental data. The contribution of Coulomb scattering is discussed for charged hadrons.

# 1 History and Motivation

GEANT4 has different models for hadron-nuclear elastic scattering.

1. Historically the first model for hadron elastic scattering was the nuclear black R-disk model for neutrons (Bethe, Plazcek 1940), $k$ is the neutron wave vector:
$$\frac{d\sigma_{el}}{d\Omega} = R^2 \frac{J_1^2(k\,R\,\theta)}{\theta^2}, \quad d\Omega = 2\pi \sin\theta\, d\theta.$$
It was modified by Akhiezer and Pomerachuk (1945) for charged particles:
$$\frac{d\sigma_{el}}{d\Omega} = R^2 \left\{ \frac{J_1^2(k\,R\,\theta)}{\theta^2} + \left[\frac{2n}{kR\theta^2}\right]^2 J_0^2(k\,R\,\theta) \right\}, \quad n = \frac{\alpha Z_1 Z_2}{\beta} \ll kR,$$
where n is Zommerfeld parameter for Coulomb field, $\beta = v/c$ is the particle velocity.

2. GHEISHA elastic model is implemented in G4LElastic class using simplified parametrization (J. Ranft, 1973) in terms of invariant transfered momentum $t < 0$. For atomic weight, $A < 62$:
$$\frac{d\sigma_{el}}{d\Omega} = A^{1.63} \exp(-14.5\, A^{0.66\,t}) + 1.4\, A^{0.33} \exp(10\,t),$$

and for $A \geqslant 62$:

$$\frac{d\sigma_{el}}{d\Omega} = A^{1.33}\exp(-60.0\,A^{0.33\,t}) + 0.4\,A^{0.4}\exp(10\,t),$$

3. CHIPS approach G4QElastic is based on a more dedicated parametrization of invariant differential cross section: $d\sigma_{el}/dt$, $\sigma_{el,>t}$, and $\sigma_{el,>0} = \sigma_{el}$.
4. Coherent elastic model G4ElasticHadrNucleusHE utilizes the Glauber approach, when a nucleus is considered as a set of $\sim A$ nucleons.

The models work satisfactory, showing however sometimes accuracy and numerical problems. The contribution of Coulomb scattering is not completely clear.

It was therefore interesting to consider a model which could be simple (with Coulomb contribution), robust and universal enough for the description of hadronic calorimeters.

# 2 The model description

The model is based on optical approach when a nucleus is considered as a drop of absorptive and refractive medium. Absorption results to diffraction of projectile hadron, while refraction in surface layer provides some smoothing of diffraction picture. The differential cross section reads:

$$\frac{d\sigma_{el}}{d\Omega} = R^2 F_d^2(k\,d\,\theta)\left\{\frac{J_1^2(k\,R\,\theta)}{\theta^2} + \left[(\gamma\,k)^2 + (\delta\,k^2\,\theta)^2\right] J_0^2(k\,R\,\theta)\right\}, \quad d\Omega = 2\pi\sin\theta\,d\theta,$$

where $R$ and $d$ are nuclear geometrical parameters for the Woods-Saxon type density:

$$\rho(r) = \rho_o\left\{1+\exp\left[\frac{(r-R)}{d}\right]\right\}^{-1},$$

$k$ is the projectile wave vector (in GEANT4 $k = p/\hbar\,c$, where p is the projectile momentum multiplied by $c$), $F_d$ is the dumping factor:

$$F_d(k\,d\,\theta) = \frac{\pi\,k\,d\,\theta}{\sinh(\pi\,k\,d\,\theta)}.$$

$\gamma$ is the refraction parameter, and $\delta$ is parameter of spin-orbital interaction.

The model can be modified for charged particles taking into account that simple summation of nuclear and Coulomb cross-sections does not work:

$$\frac{d\sigma_{el}}{d\Omega} \neq \frac{d\sigma_{Coulomb}}{d\Omega} + \frac{d\sigma_{nuclear}}{d\Omega}.$$

The elastic amplitudes $f$ for nuclear and Coulomb scattering also can not be generally summarized:

$$\frac{d\sigma_{el}}{d\Omega} \neq |f_{Coulomb}(\theta) + f_{nuclear}(\theta)|^2,$$

since the nuclear amplitude $f_{nuclear}(\theta)$ depends also on Coulomb phase shifts $\delta_{Coulomb}$:

$$\frac{d\sigma_{el}}{d\Omega} = |f_{Coulomb}(\theta) + f_{nuclear}(\theta, \delta_{Coulomb})|^2.$$

However for small Coulomb scattering $\delta f_{Coulomb}(\theta) \ll f_{nuclear}(\theta)$ the amplitudes can be summarized:

$$\frac{d\sigma_{el}}{d\Omega} = |\delta f_{Coulomb}(\theta) + f_{nuclear}(\theta)|^2.$$

This case will be considered in the diffuse model.

In the case of $n/kR \ll 1$ (kind of charged lense):

$$\gamma\, k \to \gamma\, k + \frac{n}{2kR}\left[\sin^2(\frac{\theta}{2}) + A_m\right]^{-1},$$

where the parameter $A_m$ reflects the Coulomb atomic shell screening and can be estimated as:

$$A_m = \frac{1.13 + 3.76n^2}{(1.77ka_oZ^{-1/3})^2},$$

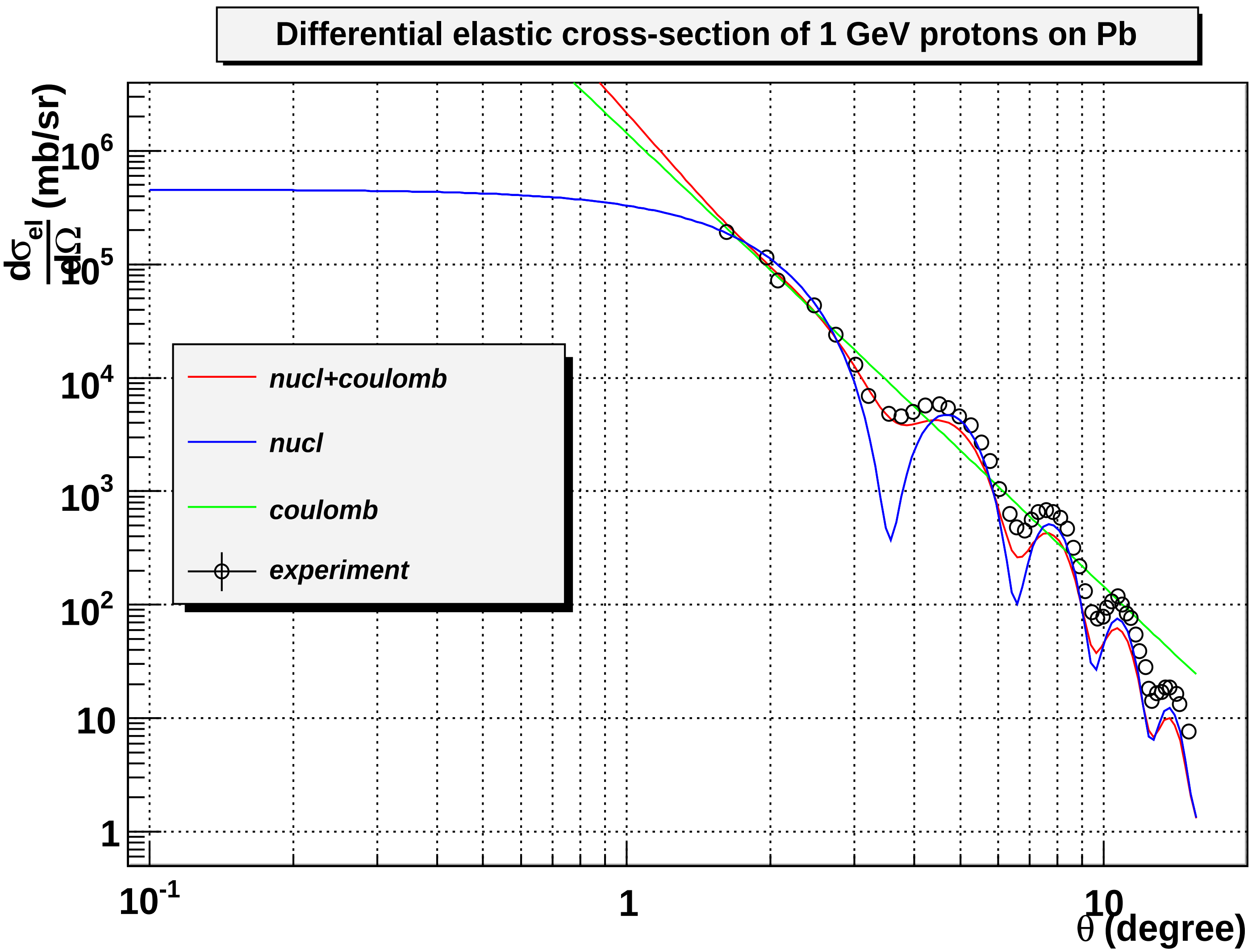
where $a_o$ is the Bohr radius, and $Z$ is the atomic number.

This method corresponds to elastic Coulomb-Wentzel scattering recently implemented in electromagnetic physics:

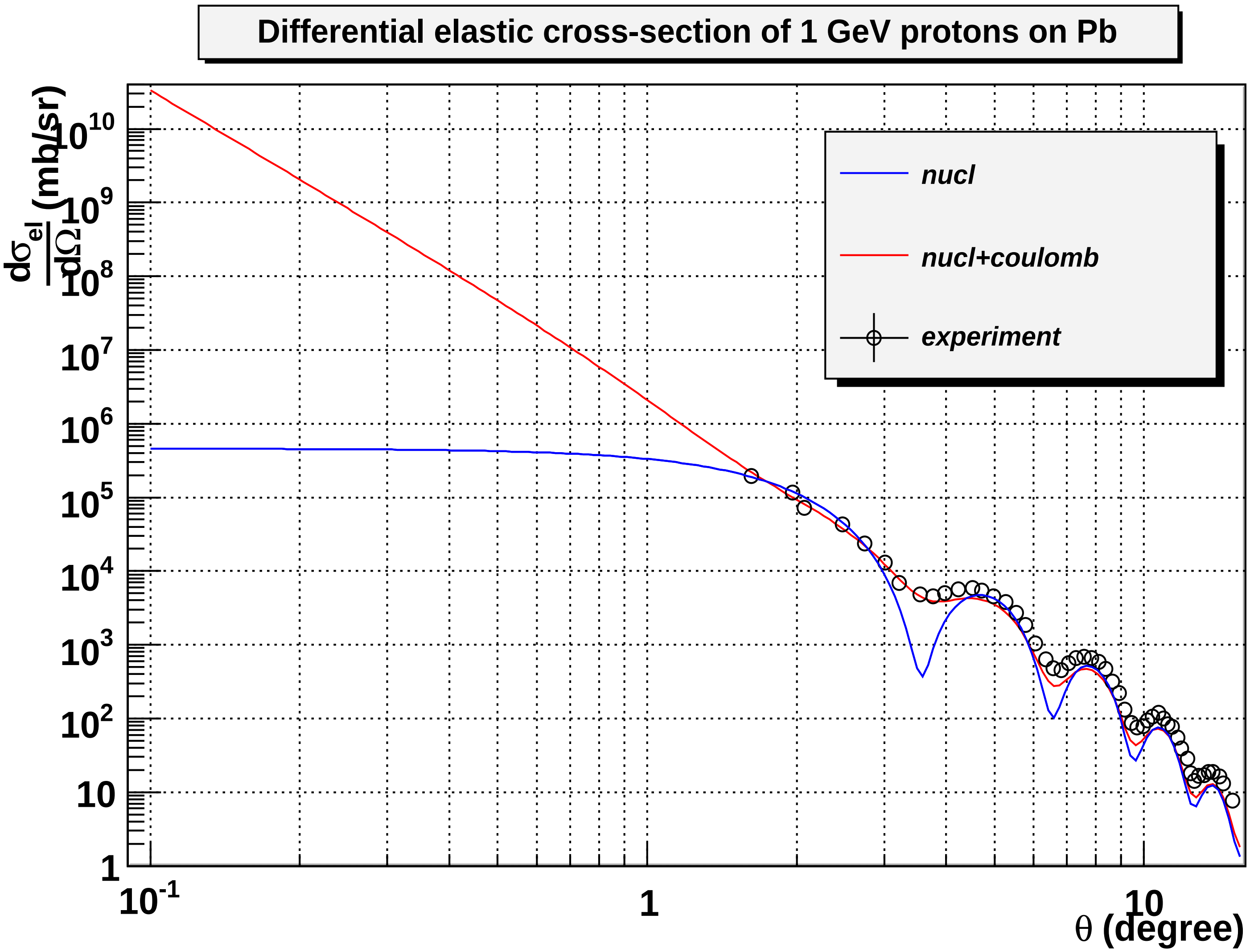
$$\frac{d\sigma_{el}^{cw}}{d\Omega} = \frac{n^2}{4k^2}\left[\sin^2(\frac{\theta}{2}) + A_m\right]^{-2}, \quad V(r) = \frac{e^2 Z_1 Z_2}{r^2}\exp(-r/R), \quad A_m = (2kR)^{-2}.$$

The Coulomb-Wentzel cross-section reads:

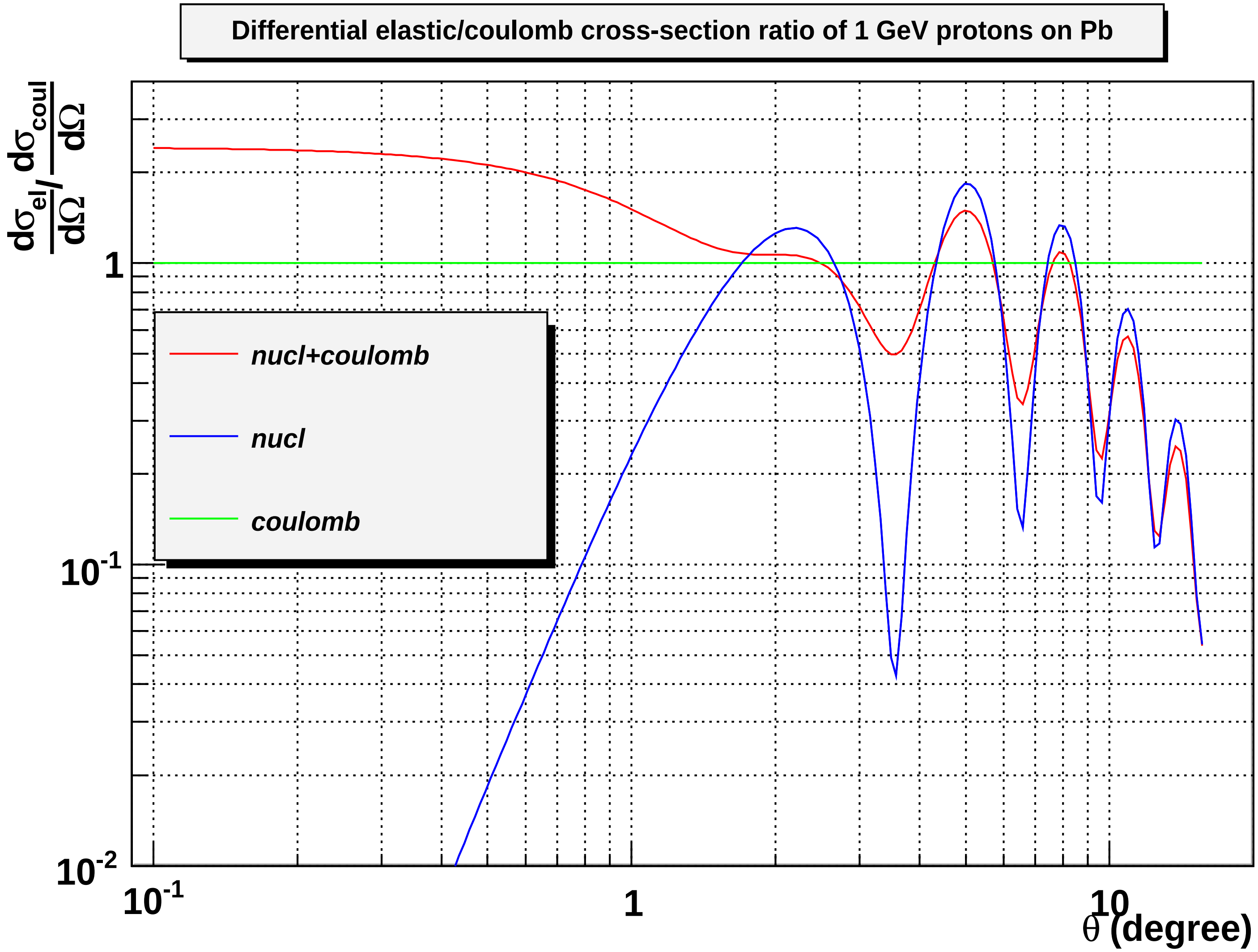
$$\sigma_{el}^{cw} = \frac{n^2}{k^2}\frac{\pi}{A_m(1+A_m)}, \quad \sigma_{el}^{cw}(\theta_1,\theta_2) = 2\pi\frac{n^2}{k^2}\frac{\cos\theta_1 - \cos\theta_2}{(1-\cos\theta_1+2A_m)(1-\cos\theta_2+2A_m)}.$$

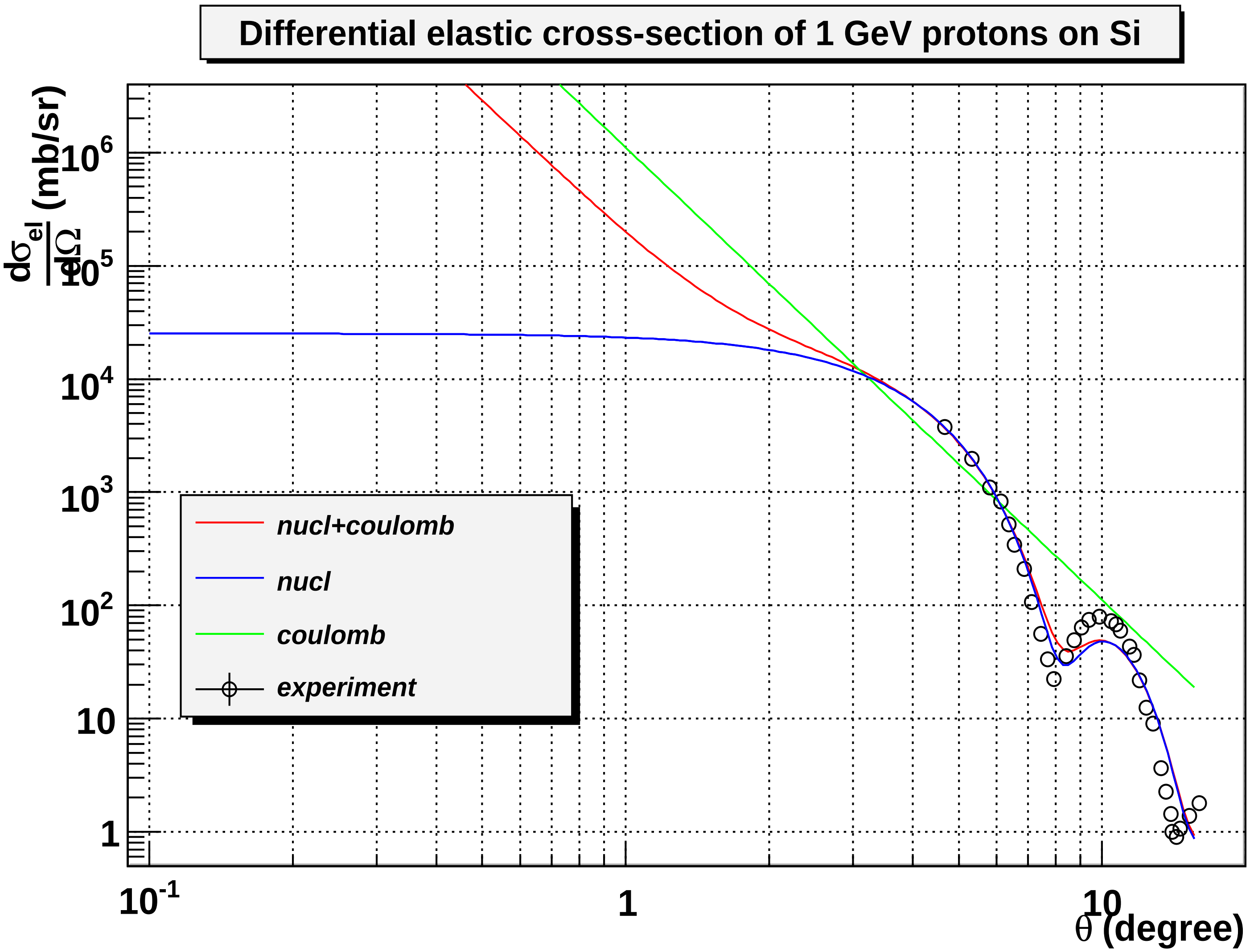



$\sigma_{el}^{nucl} = 1210$ mb, with $\lambda = 250550$ micron ( 25 cm), $\sigma_{el}^{cw}(2^0, 16^o) = 642.65$ mb, with $\lambda = 471742$ micron (47 cm).

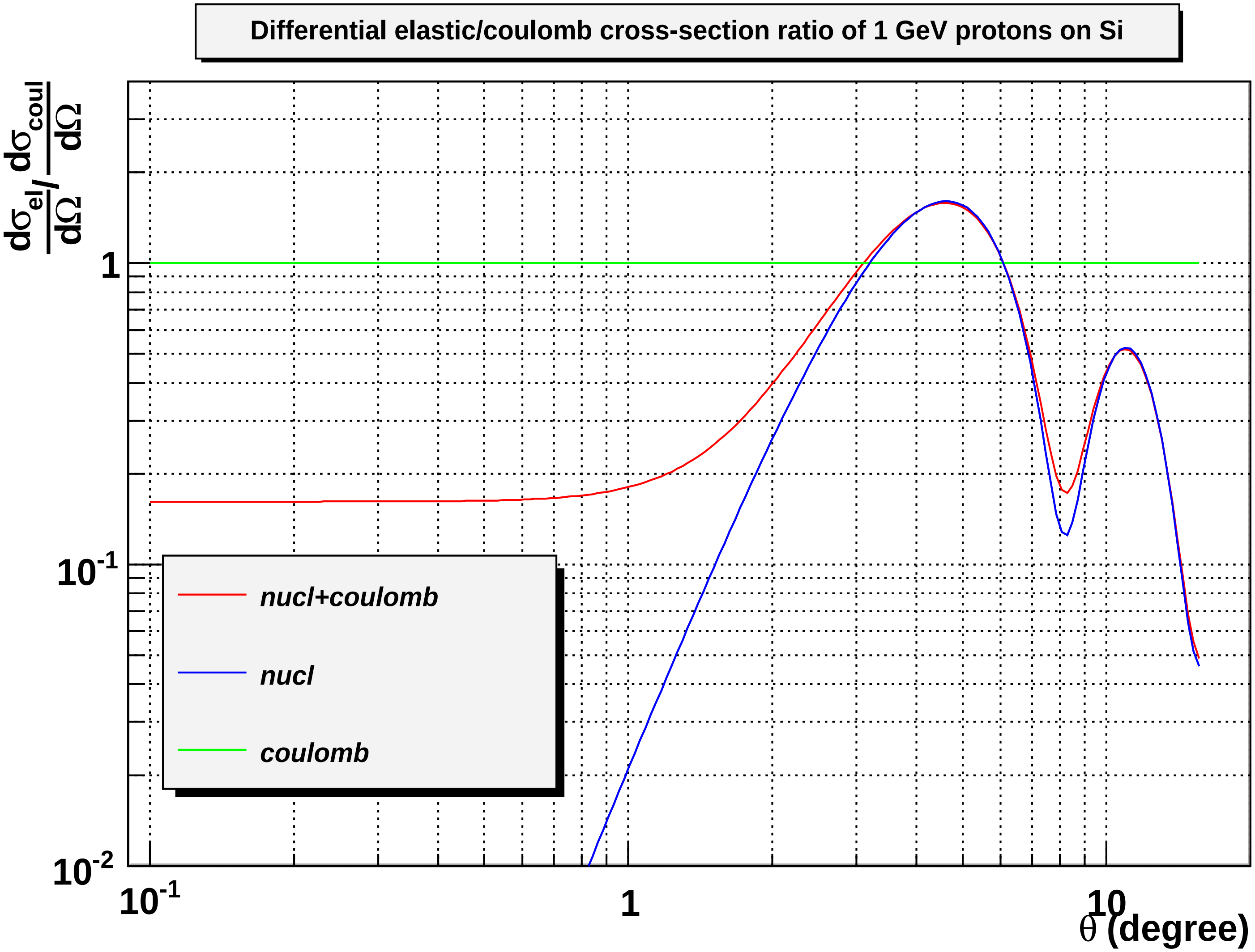



$$\sigma_{el}^{cw}(0,1^o) \sim \sigma_{el}^{cw} = 2.36423 \cdot 10^9 \text{ mb, with } \lambda = 0.12823 \text{ micron}$$


Differential elastic/coulomb cross-section ratio of 1 GeV protons on Pb
$\frac{d\sigma_{el}}{d\Omega} / \frac{d\sigma_{coul}}{d\Omega}$
1
$10^{-1}$
$10^{-2}$
$10^{-1}$
1
10
θ (degree)
nucl+coulomb
nucl
coulomb




$$\sigma_{el}^{cw}(0, 1^o) \sim \sigma_{el}^{cw} = 5.46779 \cdot 10^8 \text{ mb, with } \lambda = 0.366068 \text{ micron}$$


Differential elastic/coulomb cross-section ratio of 1 GeV protons on Si
dσ_el/dΩ / dσ_coul/dΩ
1
10^-1
10^-2
10^-1
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θ (degree)
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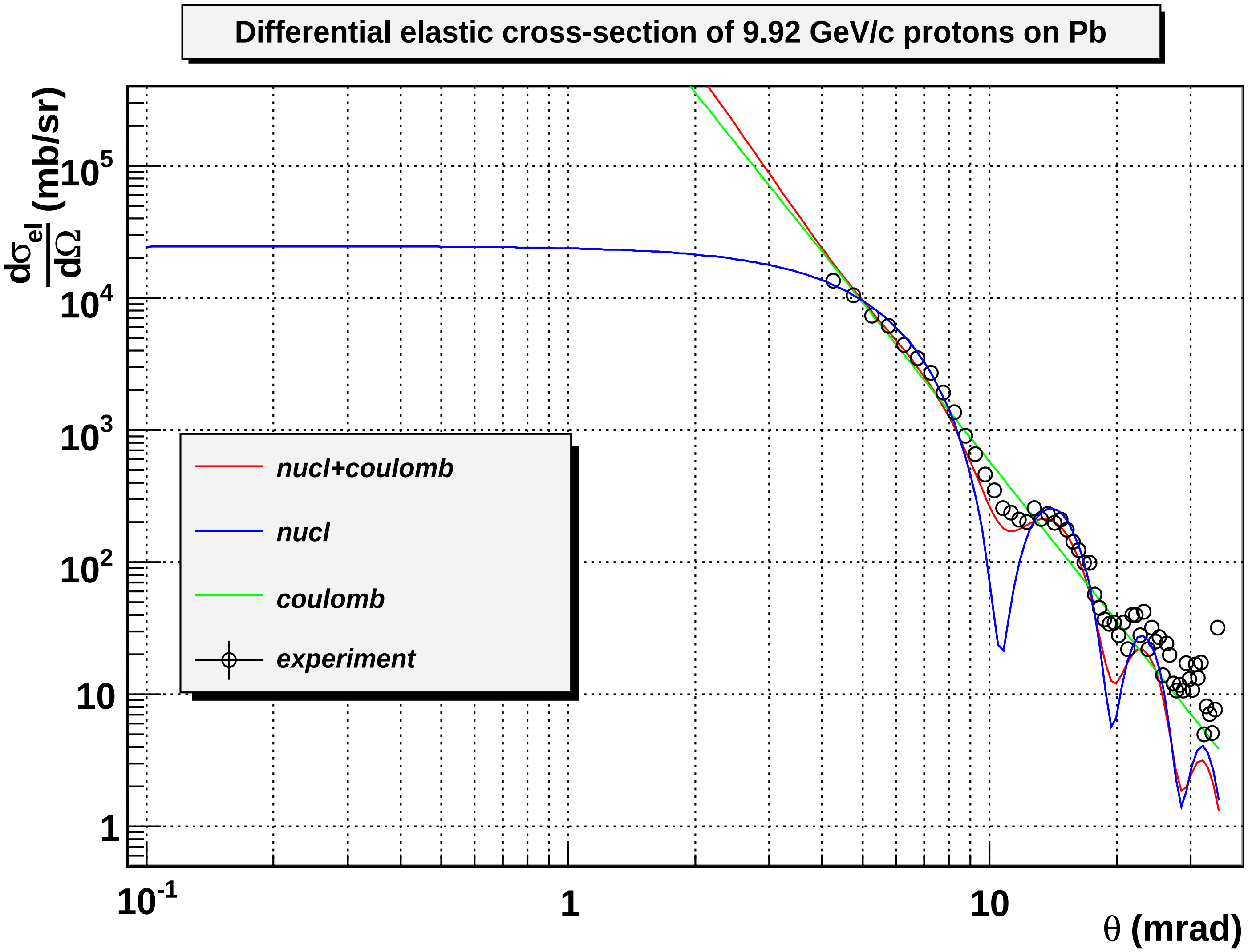
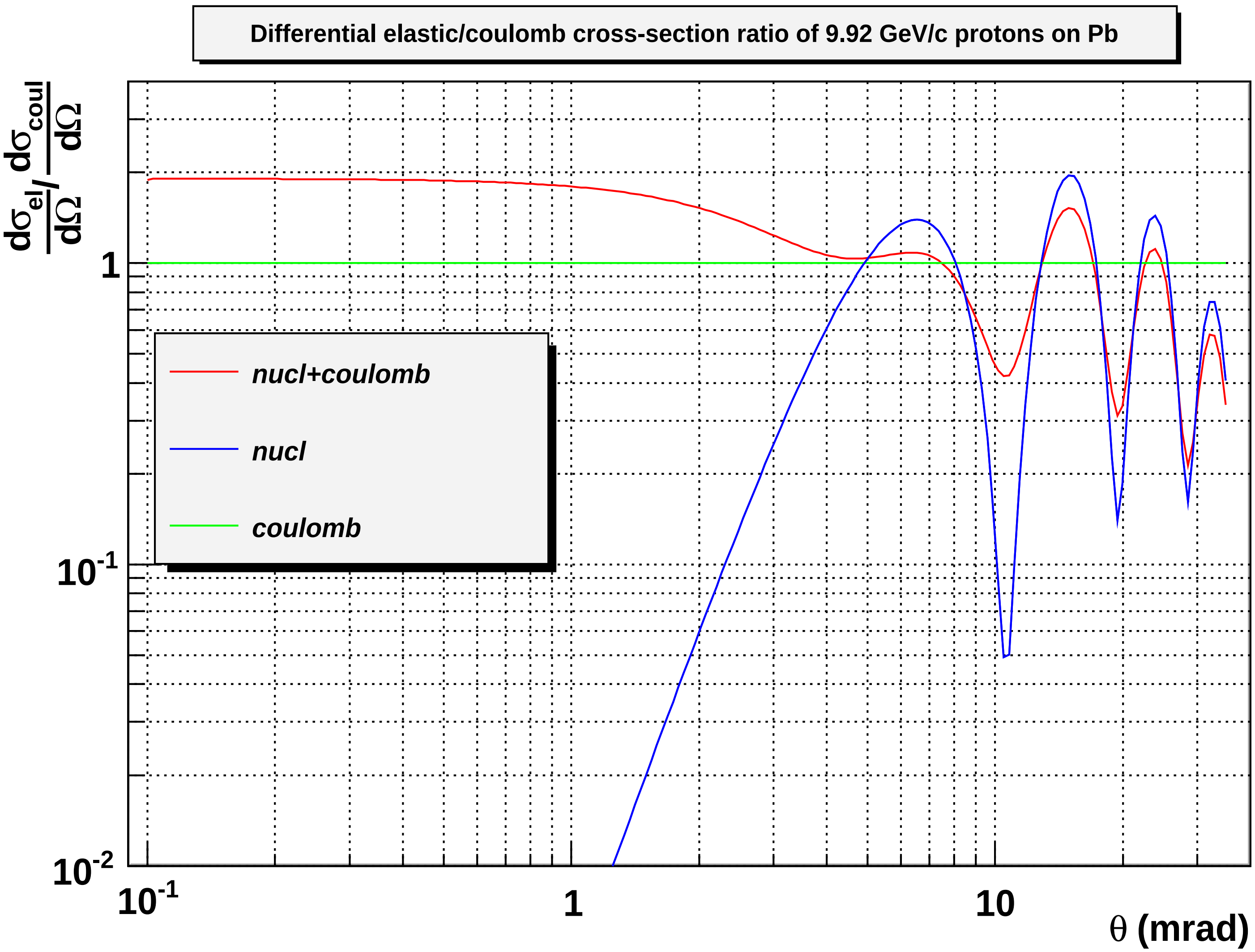
nucl
coulomb




$$\sigma_{el}^{cw}(0, 4\ mrad) \sim \sigma_{el}^{cw} = 2.11854 \cdot 10^9 \text{ mb, with } \lambda = 0.143101 \text{ micron.}$$

$$\sigma_{el}^{cw}(4\ mrad, 40\ mrad) = 919.547 \text{ mb, with } \lambda = 329689 \text{ micron (33 cm).}$$


Differential elastic/coulomb cross-section ratio of 9.92 GeV/c protons on Pb
dσ_el/dΩ / dσ_coul/dΩ
1
10^-1
10^-2
10^-1
1
10
θ (mrad)
nucl+coulomb
nucl
coulomb

# 3 Conclusions and ToDo

1. The simple optical model for differential elastic cross-section was implemented based on diffraction-refraction approach, G4DiffuseElastic class
2. The model shows satisfactory agreement with experimental data even without parameter fitting.
3. Some parameter fitting is needed to improve the model predictions. It can be applied for heavy nuclei.
4. Coulomb scattering contributes a lot in the region of the first diffraction maximum-minimum region. It should be taken into account in experimental data fitting (affects MSC, NIEL).
5. The model can generalized for big Coulomb scattering (low energy, ions) taking into account rainbow and glory modes.