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Using TBT data at ATF DR

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  - Theoretical Background
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- Machine modeling

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Introduction

The Fourier analysis of TBT data has been first applied at LEP in 1992 as a tool for measuring the *uncoupled linear optics*.

TBT data at the j[th] BPM following a single kick in the z plane ($z \equiv x, y$)

$$z_n^j = \frac{1}{2}\sqrt{\beta_z^j}\mathrm{e}^{i\Phi_z^j} A_z \mathrm{e}^{iQ_z(\theta_j + 2\pi n)} + c.c.$$

$$\text{with} \quad n \equiv \text{ turn number} \quad A_z = |A_z|\mathrm{e}^{i\delta_z} \equiv \text{ constant of motion}$$

$$\Phi_z \equiv \mu_z - Q_z\theta \quad \text{(periodic phase function)}$$

Twiss functions:

$$\beta_z^j = |Z_j(Q_z)|^2/A_z^2 \qquad \mu_z^j = \arg\,(Z_j) - \delta_z$$

$$Z_j(Q_z) \equiv \text{ Fourier component of } z_j$$

Amplitude fit:

$$|A_z|^2 = \frac{\sum_j 1/\beta_z^{0j}}{\sum_j 1/|Z_j(Q_z))|^2}$$

Linear Coupling

Method of the variation of constants:

The general solution of the perturbed motion keeps the form of the unperturbed one with constants depending on time[a]"

Hamiltonian in presence of a perturbation, H_1,

$$H = [H_0 + H_1](q_1, ...q_n, p_1, ...p_n) = [U_0 + U_1](c_1, ...c_{2n}))$$

Equations of motion

$$\frac{dc_j}{dt} = \Sigma_m [c_j, c_m] \frac{\partial U_1}{\partial c_m}$$

When the unperturbed Hamiltonian describe the betatron motion, thus

$$\frac{dA_z}{d\theta} = i \frac{\partial U_1}{\partial A_z^*} \qquad \frac{dA_z^*}{d\theta} = -i \frac{\partial U_1}{\partial A_z}$$

[a] θ or s in our case

For perturbation fields generating linear coupling (Guignard)

$$U_1(\vec{a}) = \frac{1}{2}[C_+(\theta)a_x a_y + C_+^*(\theta)a_x^* a_y^* + C_- a_x a_y^* + C_-^* a_x^* a_y]$$

$$a_z \equiv A_z e^{iQ_z\theta}$$

where

$$C_\pm(\theta) \equiv \frac{R\sqrt{\beta_x\beta_y}}{2B\rho}\left\{\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right) + B_\theta\left[\left(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y}\right) - i\left(\frac{1}{\beta_x} \mp \frac{1}{\beta_y}\right)\right]\right\} \mathrm{e}^{i(\Phi_x \pm \Phi_y)}$$

and

$$\Phi_z \equiv \mu_z - Q_z\theta$$

"Ansatz" (Yuri Alexahin)

$$\boldsymbol{a_x(\theta) = a_{x0}(\theta) + w_-^*(\theta) a_{y0}(\theta) + w_+^*(\theta) a_{y0}^*(\theta)}$$

$$\boldsymbol{a_y(\theta) = a_{y0}(\theta) - w_-(\theta) a_{x0}^*(\theta) + w_+^*(\theta) a_{x0}^*(\theta)}$$

Inserting into the equation of motion and keeping 1^{th} order terms one finds the equations for $\boldsymbol{w_\pm}$

$$\boldsymbol{2i}\mathrm{e}^{\boldsymbol{-iQ_\pm\theta}} \frac{\boldsymbol{d}}{\boldsymbol{d\theta}} \mathrm{e}^{\boldsymbol{iQ_\pm\theta}} \boldsymbol{w_\pm(\theta) = C_\pm(\theta)}$$

The periodic solutions are

$$w_\pm(\theta) = -\int_0^{2\pi} d\theta' \frac{C_\pm(\theta')}{4\sin \pi Q_\pm} \mathrm{e}^{-iQ_\pm[\theta-\theta'-\pi \mathrm{sign}(\theta-\theta')]}$$

with

$$Q_\pm \equiv Q_x \pm Q_y$$

The functions $\tilde{w}_\pm \equiv w_\pm \mathrm{e}^{iQ_\pm\theta}$ are

- constant in coupler free regions
- experience a discontinuity $-iC_\pm \ell/2R$ at coupler locations $\Rightarrow$ diagnostics tool !
- are constant on the resonances $Q_x \pm Q_y = int$.

Minimum tune split (Guignard)

$$\Delta \equiv |\bar{C}_-^{n_-}| \qquad \bar{C}_\pm^{n_\pm} = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, C_\pm \mathrm{e}^{in_\pm\theta} = \frac{n_\pm - Q_\pm}{\pi}\int_0^{2\pi} d\theta\, w_\pm \mathrm{e}^{in_\pm\theta}$$

with

$$n_\pm \equiv \mathrm{Round}(Q_x \pm Q_y)$$

Linear coupling computation through TBT analysis

TBT beam position at the jth vertical BPM following a horizontal kick

$$y_n^j = \left[\sqrt{\beta_y^j}\left(\mathrm{e}^{-i\Phi_y^j} w_+^j - \mathrm{e}^{i\Phi_y^j} w_-^j\right)\right] A_x \mathrm{e}^{iQ_x(\theta_j + 2\pi n)} + c.c.$$

TBT beam position at the j-th horizontal BPM following a vertical kick

$$x_n^j = \left[\sqrt{\beta_x^j}\left(\mathrm{e}^{-i\Phi_x^j} w_+^j + \mathrm{e}^{i\Phi_x^j} w_-^{*j}\right)\right] A_y \mathrm{e}^{iQ_y(\theta_j + 2\pi n)} + c.c.$$

The FFT of y^j at Q_x, $Y^j(Q_x)$, for a horizontal kick ($X^j(Q_y)$ for a vertical one) is proportional to the *coupling functions* $w_\pm(\theta_j)$.

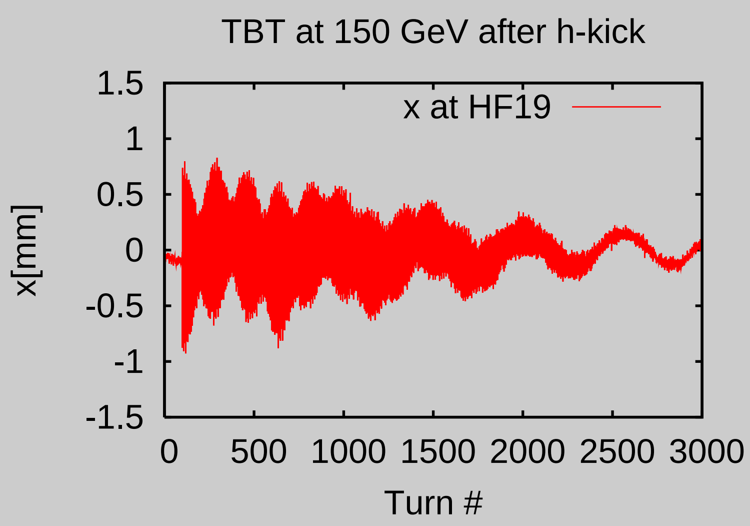
We get per each BPM 2 real equations in 4 unknowns. When between two consecutive monitors there are no strong source of coupling, the four equations can be solved in favor of $w_\pm(\theta_j) = w_\pm(\theta_{j+1})$.

Examples of Tevatron Measurements

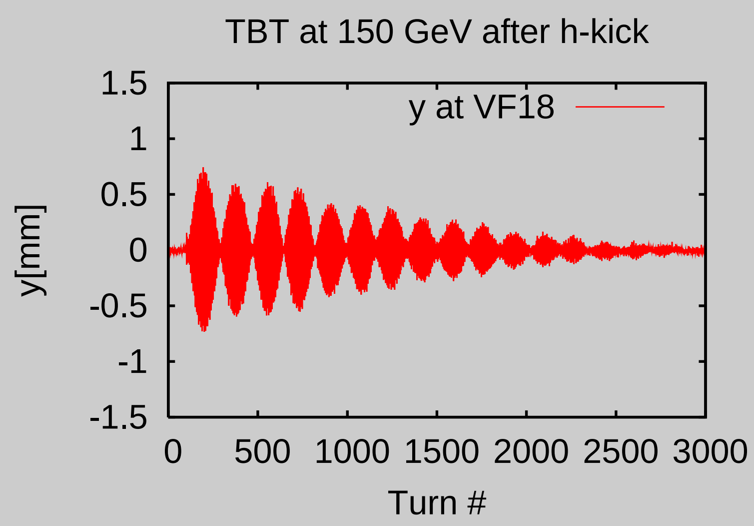
Tevatron is a $p\bar{p}$ collider working close to the $Q_x \pm Q_y$ resonances. The machine has 118 horizontal and 118 vertical BPM's. They can store 8192 positions data per BPM. The electronics upgrade allows a high resolution ($\simeq 50\ \mu$m) measurement of the TBT beam position.

Under "ideal" conditions the oscillations following a kick last some thousand turns

TBT position after a horizontal kick

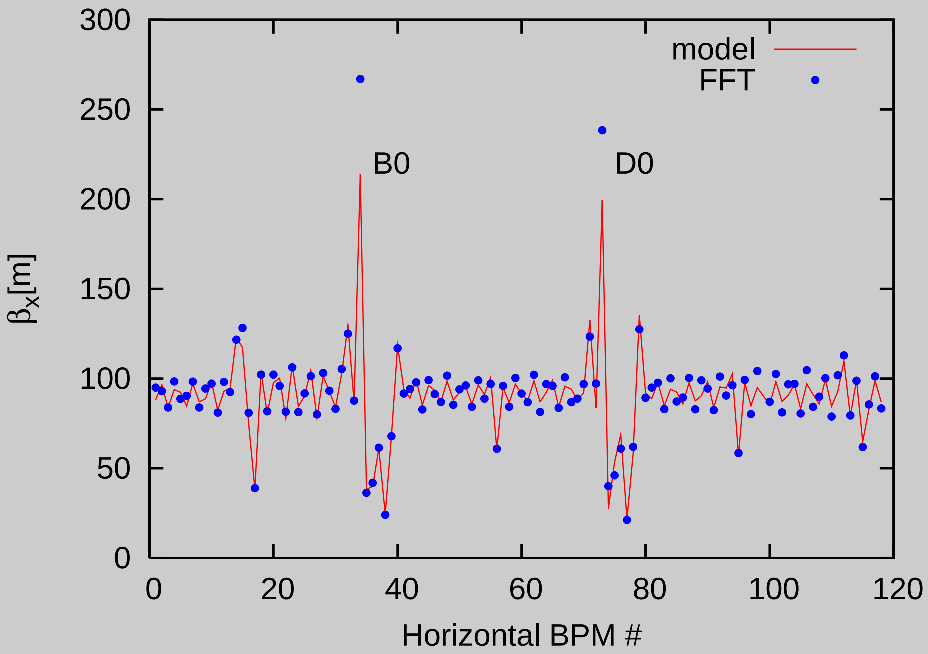


TBT position at HF19

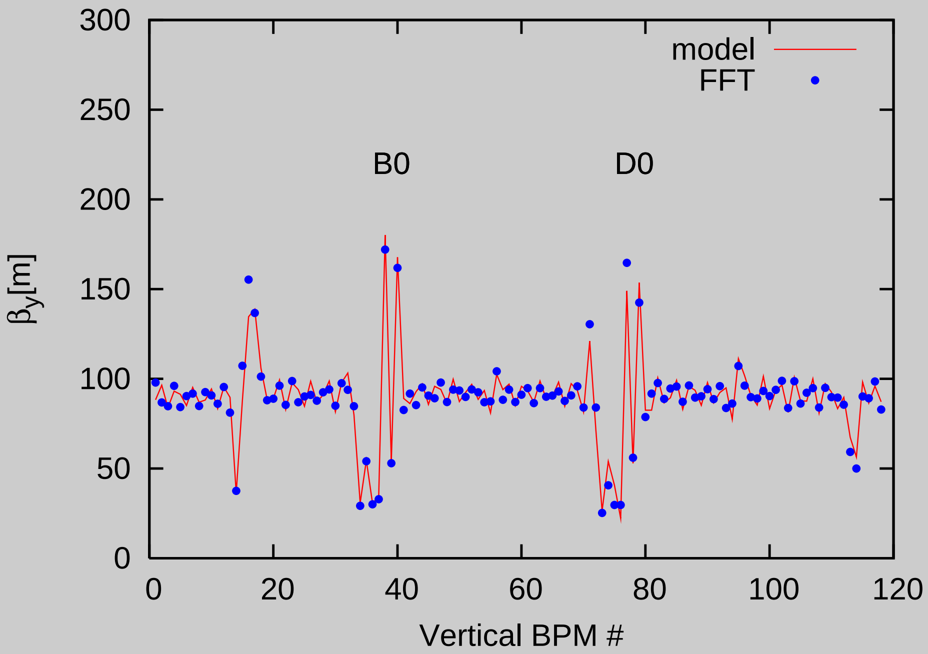


TBT position at VF18

Reconstructed Injection Optics (November 2005 data)

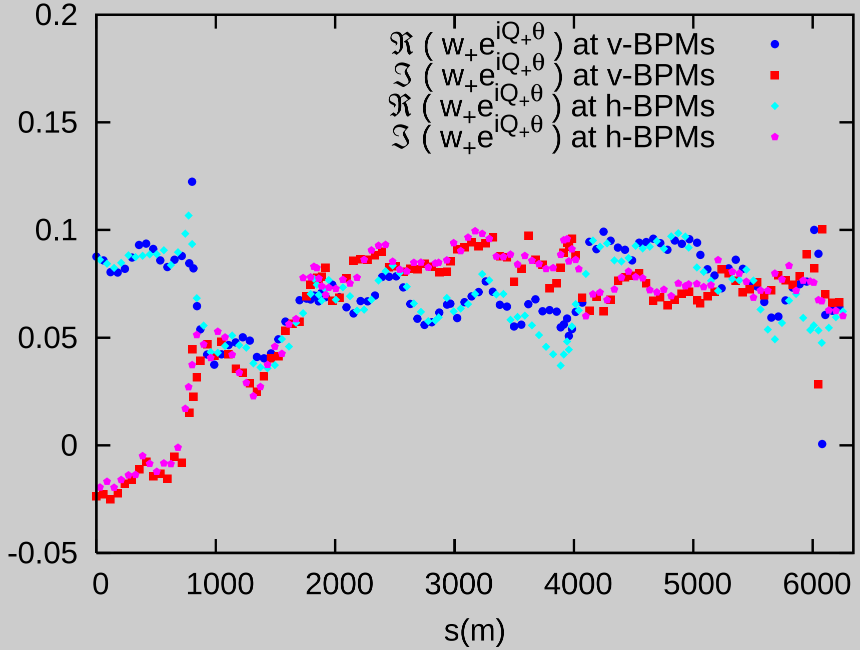


Horizontal

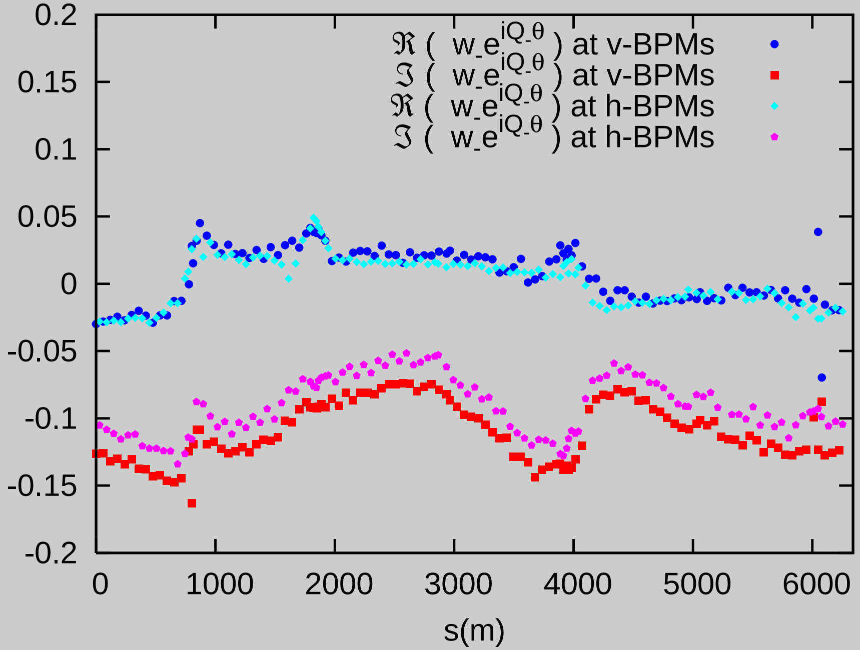


Vertical

Coupling functions (November 2005 data)



$\tilde{w}^+$

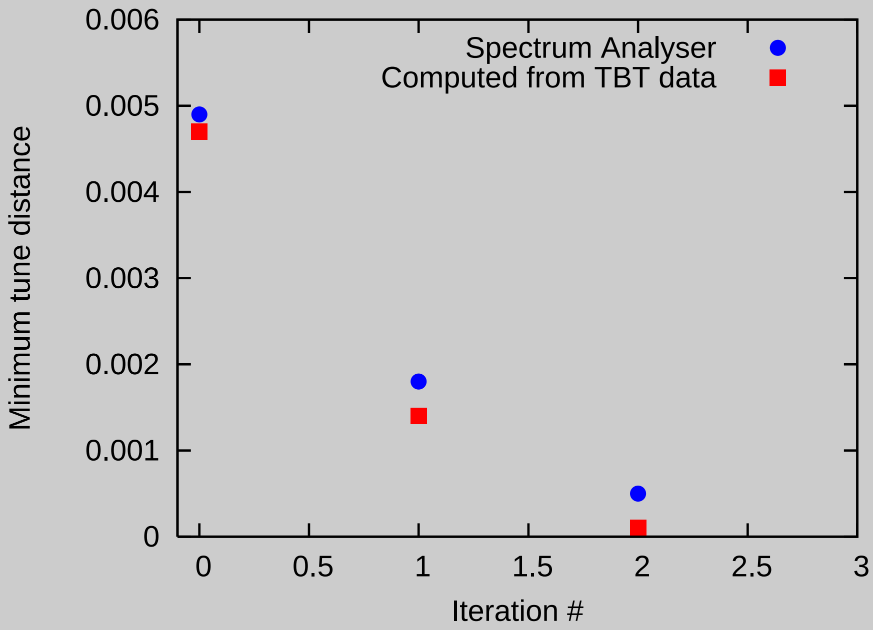


$\tilde{w}^-$

Jumps visible around 1000 (SQA0), 1500 (A38) and 4000 (D16) meters.

An application program for the TBT analysis has been integrated in the TEVATRON control system and is used routinely at shot set up for correcting the minimum tune split $\Delta \equiv |\bar{C}_-|$ with two skew quadrupole circuits. TEVATRON being a fast ramping machine (83 seconds from 150 to 980 GeV), the TBT analysis is a very practical method for measuring optics and coupling also during acceleration.

Minimum tune split measured with S.A. and computed from TBT data



- Spectrum Analyser
- TBT Analysis

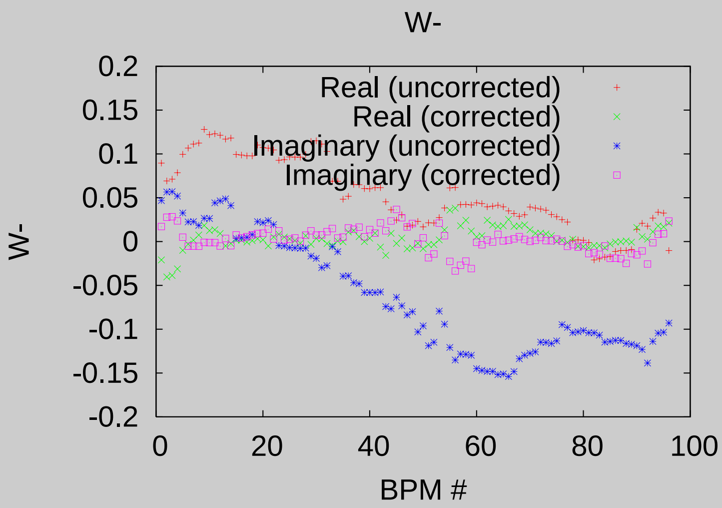
Simulations for ATF DR

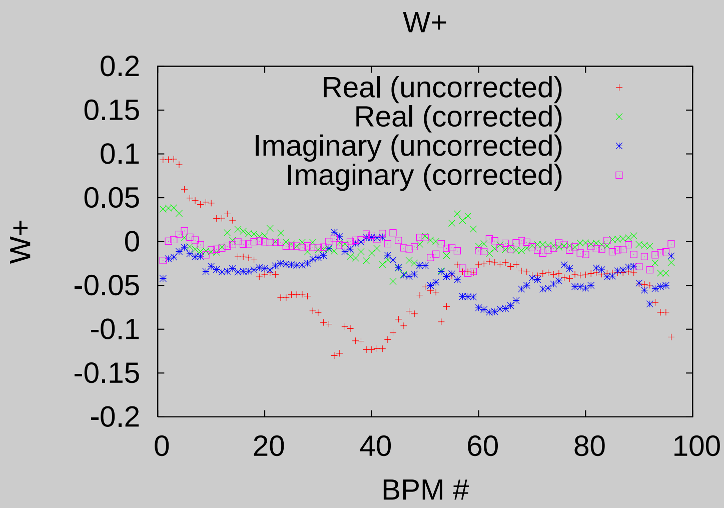
Main goal: preserve design small vertical emittance, beside optics correction.

Therefore one must correct betatron coupling *and* spurious vertical dispersion.

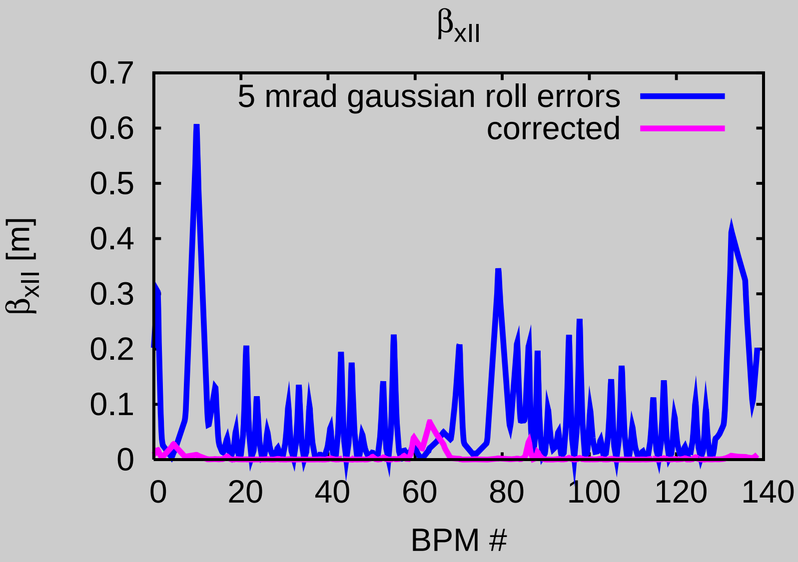
Error simulation[a]: gaussian random roll errors (rms value: 5 mrad) for all normal quadrupoles (ideal model from M.Woodley).

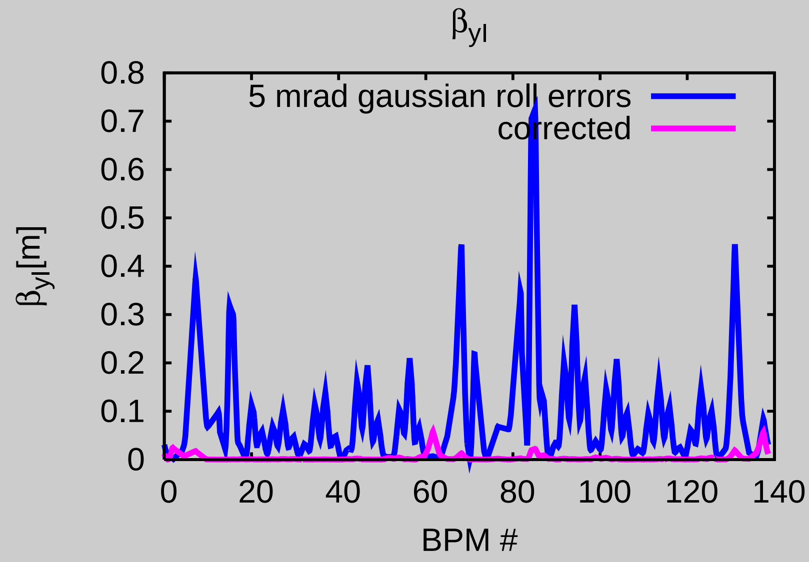
Correction: simultaneous correction of $w^{\pm}$ and spurious vertical dispersion using all skew quadrupoles and assuming 96 BPMs.





[a]MADX-PTC used for generating trajectories and computing the Mais-Ripken coupled twiss functions


βxII
5 mrad gaussian roll errors
corrected
βxII [m]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
BPM #


βyI
5 mrad gaussian roll errors
corrected
βyI[m]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
BPM #

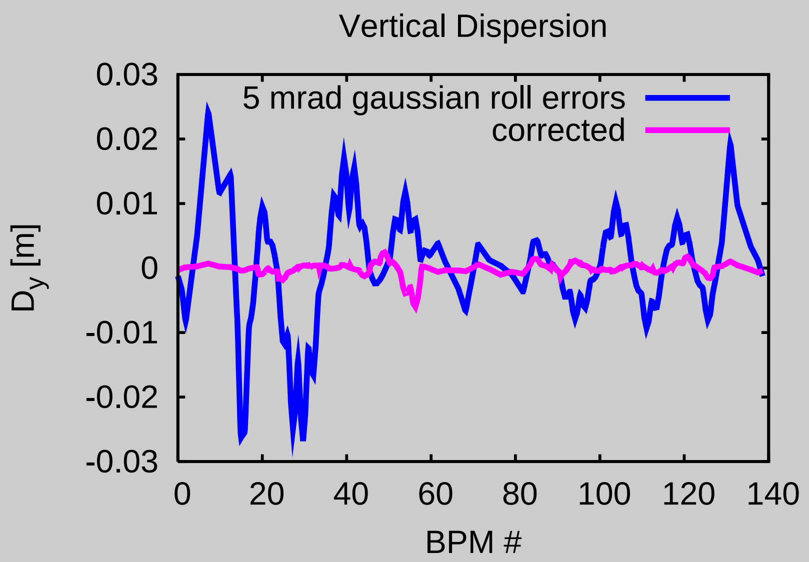

Vertical Dispersion
5 mrad gaussian roll errors
corrected
Dy [m]
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
0
20
40
60
80
100
120
140
BPM #

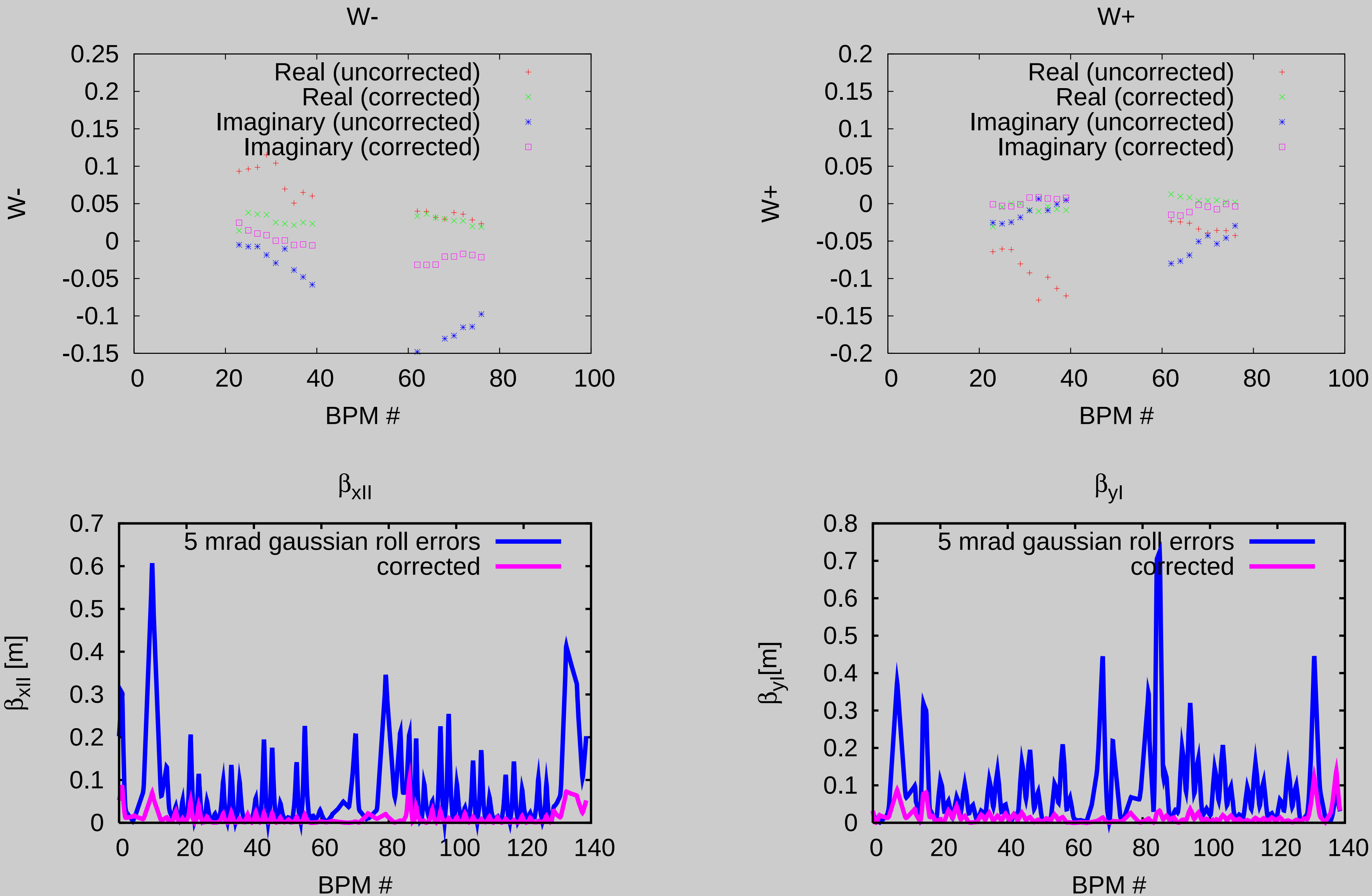
Table 1: Transverse Emittance

	ε_x (nm)	ε_y (nm)
Nominal	0.973	0.000
with errors	0.971	0.042
β-tron coupling correction	0.973	0.012
D_y correction	0.970	0.013
correcting both	0.973	0.001

Currently 20 BPMs are available: what can we expect?

For measuring D_y the TBT capability is not needed.

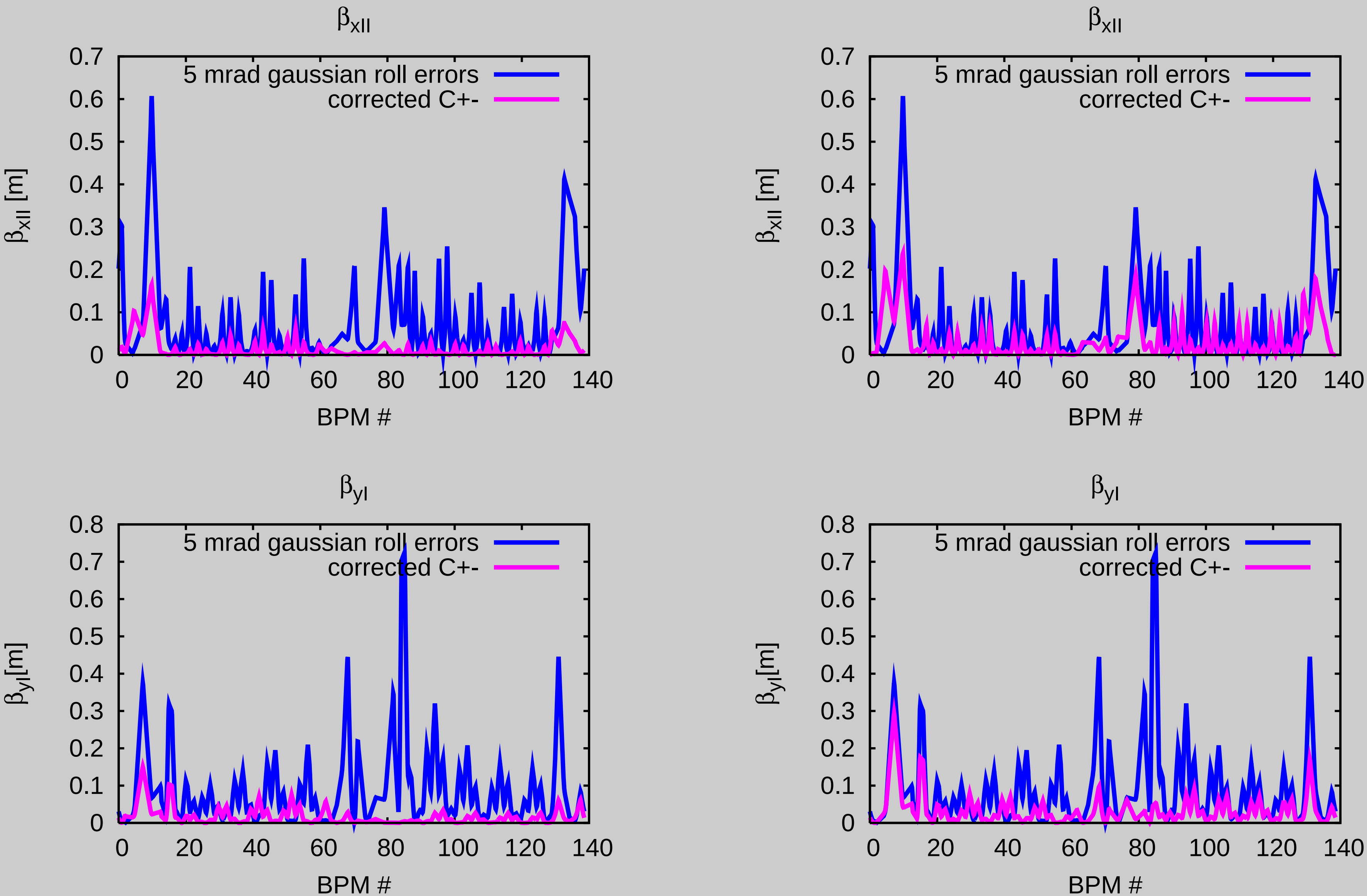
We can compensate the betatron coupling where the values of $w^{\pm}$ are known and correct the spurious vertical dispersion at all BPMs.



ε_y=0.022 nm (only betatron coupling correction)

ε_y=0.004 nm (betatron coupling + dispersion correction)

We can also correct just the coupling coefficients $C^{\pm}$, together with D_y



96 BPMs ε_y=0.006 nm

20 BPMs ε_y=0.013 nm

Summary

- the simultaneous correction of betatron coupling and spurious vertical dispersion presented here looks promising
- also with 20 BPMs we should be able to see the effect of the skew quadrupoles on the coupling functions and eventually correct the machine linear coupling
- for localizing coupling sources a larger number of observation points (BPMs) is needed
- caution with numbers: results are just qualitative, no statistics, and in particular no quadrupole misalignement considered!

Machine Modeling using TBT Data

The Fourier analysis[a] of the measured TBT data

$$\begin{aligned} x_n = & A_I\sqrt{\beta_{xI}}\cos(\phi_{xI}+\delta_I+2\pi n Q_I) + \\ & A_{II}\sqrt{\beta_{xII}}\cos(\phi_{xII}+\delta_{II}+2\pi n Q_{II}) \\ y_n = & A_I\sqrt{\beta_{yI}}\cos(\phi_{yI}+\delta_I+2\pi n Q_I) + \\ & A_{II}\sqrt{\beta_{yII}}\cos(\phi_{yII}+\delta_{II}+2\pi n Q_{II}) \end{aligned}$$

gives the coupled Mais-Ripken twiss functions $\beta_{zI,II}$ and $\phi_{zI,II}$ $(z \equiv x, y)$, a part for the constants of motion $A_{I,II}$ and $\delta_{I,II}$.

[a] there are other ways of analysing the TBT data, such as MIA and ICA

The eigenvectors of the coupled transport matrix are related to the Mais-Ripken twiss functions

$$\begin{aligned}
V_{11} &\equiv \sqrt{\beta}_{xI}\cos\phi_{xI} & V_{12} &\equiv \sqrt{\beta}_{xI}\sin\phi_{xI} \\
V_{13} &\equiv \sqrt{\beta}_{xII}\cos\phi_{xII} & V_{14} &\equiv \sqrt{\beta}_{xII}\sin\phi_{xII} \\
V_{31} &\equiv \sqrt{\beta}_{yI}\cos\phi_{yI} & V_{32} &\equiv \sqrt{\beta}_{yI}\sin\phi_{yI} \\
V_{33} &\equiv \sqrt{\beta}_{yII}\cos\phi_{yII} & V_{34} &\equiv \sqrt{\beta}_{yII}\sin\phi_{yII}
\end{aligned}$$

Taking into account that the BPMs may have (unknown) calibration errors and may be tilted around the longitudinal axis[a] the actual eigenvector components are related to the measured ones, $\bar{V}^i_{lk}$ ($i \equiv$ BPM index), by

$$
\begin{aligned}
\frac{1}{A_I}[\cos(\delta_I)\bar{V}^i_{11} + \bar{V}^i_{12}\sin(\delta_I)] &= \frac{1}{r_i}V^i_{11} + \frac{\chi_i}{r_i}V^i_{31} \\
\frac{1}{A_I}[-\sin(\delta_I)\bar{V}^i_{11} + \bar{V}^i_{12}\cos(\delta_I)] &= \frac{1}{r_i}V^i_{12} + \frac{\chi_i}{r_i}V^i_{32} \\
\frac{1}{A_{II}}[\cos(\delta_{II})\bar{V}^i_{13} + \bar{V}^i_{14}\sin(\delta_{II})] &= \frac{1}{r_i}V^i_{13} + \frac{\chi_i}{r_i}V^i_{33} \\
\frac{1}{A_{II}}[-\sin(\delta_{II})\bar{V}^i_{13} + \bar{V}^i_{14}\cos(\delta_{II})] &= \frac{1}{r_i}V^i_{14} + \frac{\chi_i}{r_i}V^i_{34} \\
\frac{1}{A_I}[\cos(\delta_I)\bar{V}^i_{31} + \bar{V}^i_{32}\sin(\delta_I)] &= \frac{1}{r_i}V^i_{31} - \frac{\chi_i}{r_i}V^i_{11} \\
\frac{1}{A_I}[-\sin(\delta_I)\bar{V}^i_{31} + \bar{V}^i_{32}\cos(\delta_I)] &= \frac{1}{r_i}V^i_{32} - \frac{\chi_i}{r_i}V^i_{12} \\
\frac{1}{A_{II}}[\cos(\delta_{II})\bar{V}^i_{33} + \bar{V}^i_{34}\sin(\delta_{II})] &= \frac{1}{r_i}V^i_{33} - \frac{\chi_i}{r_i}V^i_{13} \\
\frac{1}{A_{II}}[-\sin(\delta_{II})\bar{V}^i_{33} + \bar{V}^i_{34}\cos(\delta_{II})] &= \frac{1}{r_i}V^i_{34} - \frac{\chi_i}{r_i}V^i_{14}
\end{aligned}
$$

[a] The BPM reading is related to the actual beam position by

$$
x^{meas} = \frac{x + y\tan\chi}{r_x} \qquad y^{meas} = \frac{y - x\tan\chi}{r_y}
$$

with $\chi \equiv$ BPM tilt and $r_z \equiv z/z^{meas}$ ($z \equiv x, y$).

Goal: adjust

- quadrupole gradient and tilt
- BPMs calibration and tilt
- $A_{I,II}$ and $\delta_{I,II}$

in order to fit the values of the eigenvectors measured at the BPMs.

Data taking being very fast, this approach could be a good alternative to time consuming Orbit Response Matrix methods.

Application to Tevatron

- Number of observation points: 2×118
- Current Tevatron model (A.Valishev): 216 normal and 216 skew thin quadrupoles to simulate gradient and tilt errors. We must add the unknown BPM calibrations and tilts (with the additional condition $< r_i >=1$) and the oscillation amplitude and phase.

Attempts of using MAD-X for fitting have failed (too slow, no convergence when applied to real data).

Project for the immediate future: write our own task-optimised minimisation code.

It could be of interest for ATF DR too. Larger the number of observation points, better constrained is the problem, especially for finding out *the* model: as much as possible BPMs should be upgraded for such an application.

Acknowledgements: Yuri Alexahin