DRAFT: Alignment model of ILC LET components
– for beam dynamics simulations

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Comments:
Revision on October 3 was a result of comments from Armin Reichold and Ryuhei Sugahara. Some of suggestions by Armin could not be adopted in this version, because there were not enough information to do so.
Revision on October 15 was again a result of comments from Armin Reichold and Ryuhei Sugahara.
We still need numbers for almost all parameters.

1 Offset errors

Relevant components:

- All magnets, except weak dipole correctors
- Accelerating cavities
- BPMs

" y " below represents either horizontal or vertical position.

A. General alignment scheme

Primary reference point will be marked every about L_{pr} (2.5 km, which corresponds to distance between shafts.). Error of the point is expected to be randomly distributed, as Gaussian.
Reference point will be marked every L_r (about 50 m, 10 m or 5 m. This depends on the technique to be used. (?)). The error of the point is determined as random walk from one of the primary reference point to the next one (see below for detail).
Most of machine components are aligned on girders or cryomodules before setting in the beam line. The error of the components with respect to the girders or cryomodules is Gaussian random.
Alignment of girders, cryomodules and independent components are aligned with respect to reference points near them. The error is Gauss random.

Parameters to be defined (horizontal and vertical):

- RMS error of primary reference point.
- Parameters of random walk.
- RMS error of girders, cryomodules and independent components w.r.t. reference points
- RMS error of magnets w.r.t. girders or cryomodules.
- Initial RMS offset error of BPM w.r.t. attached magnets. (Before beam based measurement, e.g. quad-shunting, which will reduce the error.)

B. Survey line [Tunnels for Return line of RTML, BC, ML and BC], primary and normal reference points

Primary reference point will be marked every L_{pr} along a beam line (tunnel).
Offset error of the j -th primary reference point is:

$$y_{P,j} = G(a_{pr}, t_{pr}) \qquad (1\text{-}1)$$

(Tentative parameter may be a_{pr} = 2 mm.)

$G(a,t)$ is a Gaussian random with sigma a and truncated at ta .

Between two primary reference points (say between j-th and j+1-th primary points. let us call this region j-th region.), about L_{pr}/L_r reference points will be made along a survey line. At first, additional deviation from the design line will be a random walk (random offset plus random angle change) with step length l_{step} from one of the primary reference points (say j-th primary reference point). Let $y_{0,j,n}$ denote the offset at the n-th step in the j-th region and $\theta_{j,n}$ the angel of the n-th step in the j-th region, the effect of the one step can be expressed as:

$$\theta_{j,n+1} = \theta_{j,n} + G(a_\theta, t_\theta) + \theta_O$$
$$y_{0,j,n+1} = y_{0,j,n} + G(a_y, t_y) + l_{step}\theta_{j,n+1} \qquad (0 \le n \le N-1) \quad \text{(1-2)}$$
$$y_{0,j,0} = y_{P,j}$$

where a_y, t_y, a_θ and t_θ are parameters for the random walk and θ_O represents systematic error. (See reference [1].)

N is the number of steps in the j-th region, $n=0$ corresponds to the j-th primary reference point and $n=N$ corresponds the $j+1$-th primary reference point. It is natural to make L_r/l_{step} integer.

From reference [1], tentative parameters can be $a_y = 0.5\,\mu$m, $a_\theta = 0.1\,\mu$rad and $l_{step} = 4.5$ m .

Then, when the line reaches the next primary reference point, the accumulated error is corrected proportionally to the distance from the start point as follows (See the Fig. 1.).

$$y_{j,n} = y_{0,j,n} + (y_{P,j+1} - y_{0,j,N})\, s_j(0,n)/s_j(0,N) \qquad (0 \le n \le N), \qquad \text{(1-3)}$$

where $s_j(m,n)$ is distance between n-th step and m-th step, or, assuming constant l_{step},

$$s_j(m,n) = (n-m)l_{step}\,. \qquad \text{(1-4)}$$

Note that the angle error of the first step, $\theta_{j,1}$, disappeared after this correction and we can set it zero (see Appendix B).

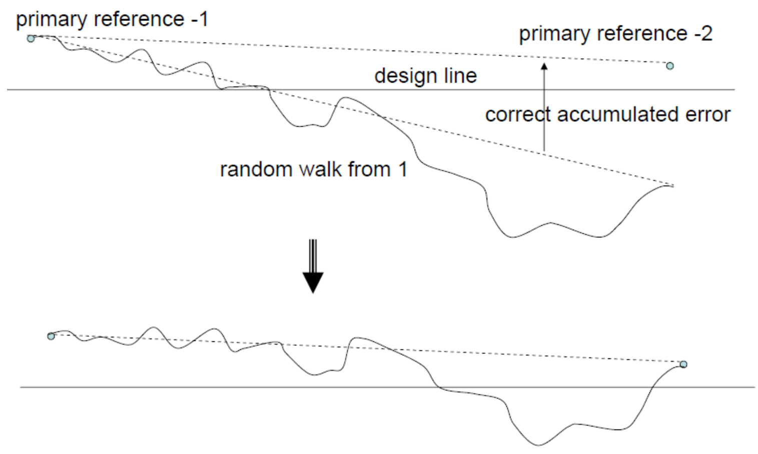

Fig.1, Correction of accumulated error of random walk.

C. Component alignment in cold regions, ML and linac parts of BC.

Cryomodules will be aligned with respect to reference points near them. So, misalignment of each cryomodule is misalignment of the reference points plus additional cryomodule alignment error which is random:

Misalignment of m-th cryomodule:

$$y_{cr,m} = y_{R(cr,m)} + G(a_{cr}, t_{cr}) \qquad (1\text{-}4)$$

(Tentative parameter may be a_{cr} = 0.1 mm.)

where $y_{R(cr,m)}$ is given from misalignment of reference points near the cryomodule.

One possible model for $y_{R(cr,m)}$ will be based on least square fitting, as follows. Assume M reference points are used for the alignment of one module. Let y_k be the offset of the k-th reference point and s_k the longitudinal position of the k-th reference point ($k = 0,1,,,M-1$) and s_{cr} the longitudinal position of the cryomodule (fidutial of the module),

$$y_{R(cr,m)} = \bar{y} + (s_{cr} - \bar{s}) \frac{\overline{sy} - \bar{s}\,\bar{y}}{\overline{s^2} - \bar{s}^2}, \qquad (1\text{-}5)$$

where bars denote average over used reference points, e.g.,

$$\bar{y} \equiv \frac{1}{M} \sum_k y_k \ . \qquad (1\text{-}6)$$

Components in each cryomodule have random misalignments with respect to the cryomodule:

Offset of i-th cavity in m-th cryomodule:

$$y_{cav,m,i} = y_{cr,m} + (s_{cav,m,i} - s_{cr,m})\theta_{cr,m} + G(a_{cav}, t_{cav}), \qquad (1\text{-}7)$$

(Tentative parameter may be a_{cav} = 0.3 mm.)

where $s_{cav,m,i}$ is the longitudinal position of the (reference of) cavity, $s_{cr,m}$ the longitudinal position of the reference of the cryomodule and $\theta_{cr,m}$ the tilt (yaw or pitch) angle of the cryomodule (see section 3).

Offset of Quad in m-th cryomodule:

$$y_{q,m} = y_{cr,m} + (s_{q,m} - s_{cr,m})\theta_{cr,m} + G(a_q, t_q), \qquad (1\text{-}8)$$

(Tentative parameter may be a_q = 0.3 mm.)

where $s_{q,m}$ is the longitudinal position of the magnet.

Assuming quad, correctors and BPM are assembled as one package, BPM is aligned with respect to the attached quad magnet:

Offset of BPM in m-th cryomodule:

$$y_{bpm,m} = y_{q,m} + G(a_{bpm}, t_{bpm}), \qquad (1\text{-}9)$$

(Tentative parameter may be a_{bpm} = 0.1 mm.)

Note that this is initial error, before beam based measurement (e.g. quad-shunting). Beam based measurements will reduce this error.

We expect offset error of dipole correctors is not relevant.

D. Component alignment in warm regions.

In warm regions, one magnet or more than one magnet will be on one girder.

Girders will be aligned with respect to the nearest reference points. Misalignment of each girder is misalignment of the reference points plus some additional random error:

Misalignment of l-th girder:

$$y_{g,l} = y_{R(g,l)} + G(a_g, t_g) \ , \qquad (1\text{-}10)$$

(Tentative parameter may be a_g = 0.1 mm.)

where $y_{R(g,l)}$ is given from misalignment of reference points near the girder, as same as $y_{R(cr,m)}$.

Offset of i-th magnet, magnet type-k, which is not on a girder:

$$y_{k,i} = y_{R(k,i)} + G(a_k, t_k) \ , \qquad (1\text{-}11)$$

(Tentative parameter may be a_k = 0.1 mm.)

where $y_{R(k,i)}$ is given from misalignment of reference points near the magnet as same as $y_{R(cr,m)}$ and

$y_{R(g,l)}$.

Offset of i -th magnet, magnet type- k , on l -th girder:

$$y_{k,l,i} = y_{g,l} + (s_{k,l,i} - s_{g,l})\theta_{g,l} + G(a_k, t_k), \qquad (1\text{-}12)$$

where $s_{k,l,i}$ is the longitudinal position of the (reference of) magnet, $s_{g,l}$ the longitudinal position of the reference of the girder and $\theta_{g,l}$ the tilt (yaw or pitch) angle of thegirder (see section 3, where $\theta_{g,l}$ is denoted as $\theta_{k,i}$).

Offset of BPM which is attached to i -th magnet:

$$y_{bpm,l,i} = y_{k,l,i} + G(a_{bpm,k}, t_{bpm,k}). \qquad (1\text{-}13)$$

(Tentative parameter may be $a_{bpm,k}$ = 0.1 mm.)

Note that this is initial error, before beam based measurement (e.g. quad-shunting). Beam based measurement will reduce this error.

The parameters will depend on type of the magnets. So we write these with index k .

2 Rotation error (rotation around beam axis)

Relevant components:

- All magnets
- BPMs

Parameters to be defined:

- RMS rotation of cryomodule.
- RMS rotation of quads w.r.t. cryomodules.
- RMS error of cold BPM and correctors w.r.t. attached quads.
- RMS error of magnets in warm region
- RMS error of warm BPM w.r.t. attached magnets.

A. Cold region

We assume each cryomodule has independent rotation:

$$\phi_{cr,m} = G(a_{\phi,cr}, t_{\phi,cr}). \qquad (2\text{-}1)$$

Cold components are aligned w.r.t. the cryomodule.

Rotation of quad magnet in m -th cryomodule:

$$\phi_{q,m} = \phi_{cr,m} + G(a_{\phi,q}, t_{\phi,q}). \qquad (2\text{-}2)$$

Assuming quad, correctors and BPM are assembled as one package, correctors and BPM are aligned with respect to the attached quad magnet.

Rotation of dipole corrector for x and y in m -th cryomodule:

$$\phi_{dx,m} = \phi_{q,m} + G(a_{\phi,dx}, t_{\phi,dx}), \qquad (2\text{-}3)$$

$$\phi_{dy,m} = \phi_{q,m} + G(a_{\phi,dy}, t_{\phi,dy}). \qquad (2\text{-}4)$$

Rotation of BPM in m -th cryomodule:

$$\phi_{bpm,m} = \phi_{q,m} + G(a_{\phi,bpm}, t_{\phi,bpm}). \qquad (2\text{-}5)$$

B. Warm region

We assume each warm magnet has independent rotation error.

Rotation of i -th magnet, magnet type- k :

$$\phi_{k,i} = G(a_{\phi,k}, t_{\phi,k}). \qquad (2\text{-}6)$$

Rotation of BPM which is attached to i -th magnet:

$$\phi_{bpm,i} = \phi_{k,i} + G(a_{\phi,bpm,k}, t_{\phi,bpm,k}) . \qquad (2\text{-}7)$$

3 Tilt error (yaw and pitch)

Relevant components:

- Girders (for offset error of components on them)
- Long bending magnets (yaw for vertical bends, pitch for horizontal bends)
- Solenoid magnets
- Accelerating cavities

Parameters to be defined (yaw and pitch) :

- RMS tilt of cryomodule w.r.t. loacal survey line (reference points near the module).
- RMS tilt of cavities w.r.t. cryomodules.
- RMS tilt of magnets in warm region

"θ" below represents yaw or pitch.

A. Cavities

We assume each cryomodule has independent tilt with respect to local survey line at the module:

$$\theta_{cr,m} = \theta_{ref,cr,m} + G(a_{\theta,cr}, t_{\theta,cr}) , \qquad (3\text{-}1)$$

where $\theta_{ref,cr,m}$ is the angle of local reference line, which is determined by position errors of reference points near the module.

One possible model for $\theta_{ref,cr,m}$ will be based on least square fitting, as follows. Assuming M reference points are used for the alignment. Let y_k be the offset of the k-th reference point and s_k the longitudinal position of the k-th reference point ($k = 0,1,,, M-1$),

$$\theta_{ref,cr,m} = \frac{\overline{sy} - \bar{s}\,\bar{y}}{\overline{s^2} - \bar{s}^2} , \qquad (3\text{-}2)$$

where bars denote average over used reference points, e.g.,

$$\bar{y} \equiv \frac{1}{M} \sum_k y_k . \qquad (3\text{-}3)$$

Cold components are aligned w.r.t. the cryomodule.
Tilt of i-th cavity in m-th cryomodule:

$$\theta_{cav,m,i} = \theta_{cr,m} + G(a_{\theta,cav}, t_{\theta,cav}) . \qquad (3\text{-}4)$$

B. Girders and Long Magnets

We assume each girder or long magnet has independent tilt error with respect to local survey line.
Tilt of i-th girder or magnet, type-k :

$$\theta_{k,i} = \theta_{ref,k,i} + G(a_{\theta,k}, t_{\theta,k}) , \qquad (3\text{-}5)$$

$\theta_{ref,k,i}$ is the angle of local reference line, which is determined by offset of reference points near the girder or the magnet, which will be determined as same as $\theta_{ref,cr,m}$.

APPENDIX A

Though the equations (1-2) and (1-3) will be enough for simulations, for further considerations, we can express the offset as:

$$y_{j,n} = \left(1-\frac{n}{N}\right)y_{P,j} + \frac{n}{N}y_{P,j+1} + \left(1-\frac{n}{N}\right)\sum_{i=1}^{n}\Delta_{y,i} - \frac{n}{N}\sum_{i=n+1}^{N}\Delta_{y,i} + l_{step}\frac{n(N-n)}{2}\theta_O$$
$$+ l_{step}\left(1-\frac{n}{N}\right)\sum_{i=1}^{n}(1-i)\Delta_{\theta,i} - l_{step}\frac{n}{N}\sum_{i=n+1}^{N}(N+1-i)\Delta_{\theta,i} \qquad (1 \le n \le N) \quad , \text{(A-1)}$$

where $\Delta_{y,i}$ is the random number ($G(a_y, t_y)$) for offset at the i –th step and $\Delta_{\theta,i}$ the random number ($G(a_\theta, t_\theta)$) for angle change at the i –th step.

Assuming the all random numbers are independent and without truncations ($t_{pr}, t_y, t_\theta = \infty$) it can be shown after a little manipulations that the variance of the offset at the n -th offset will be as follows.

$$\sigma_{y,n}^2 = \frac{(N-n)^2+n^2}{N^2}a_{pr}^2 + \frac{n(N-n)}{N}a_y^2 + \left(\frac{n(N-n)}{2}\right)^2 (l_{step}\theta_O)^2$$
$$+ \left\{\left(1-\frac{n}{N}\right)^2 \frac{n(n+1)(2n+1)}{6} + \left(\frac{n}{N}\right)^2 \frac{(N-n)(N-n+1)(2N-2n+1)}{6}\right\}(l_{step}a_\theta)^2 \qquad \text{(A-2)}$$

$\frac{(N-n)^2+n^2}{N^2}$, $\frac{n(N-n)}{N^2}$ and

$\left\{\left(1-\frac{n}{N}\right)^2 \frac{n(n+1)(2n+1)}{6} + \left(\frac{n}{N}\right)^2 \frac{(N-n)(N-n+1)(2N-2n+1)}{6}\right\} \Big/ N^3$ are shown in Fig. 2 as functions of n/N, assuming $N = 500$. Note that their behaviors almost do not depend on N, if N is large. $\sigma_{y,n}$ is shown in Fig. 3 as a function of n/N, assuming $N = 500$, $\theta_O = 0$ $a_y = 0.5\,\mu$m, $a_\theta = 0.1\,\mu$rad and $l_{step} = 4.5$ m. Two lines are from different assumptions; $a_{pr} = 1$ mm and $a_{pr} = 0$.

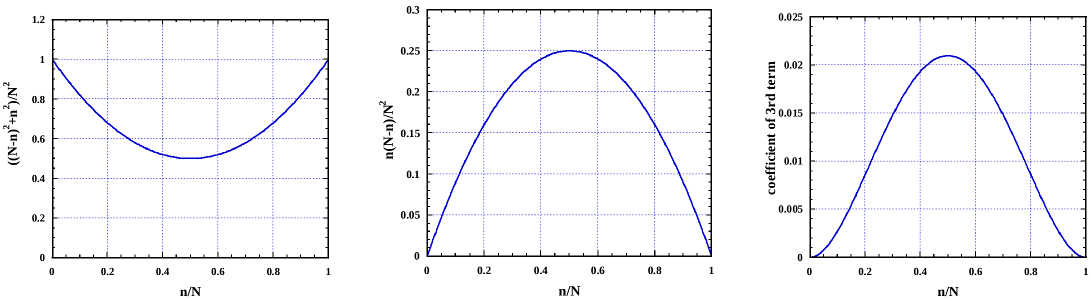

Fig. 2, Coefficients of three terms in Eq. (6) as function of n/N

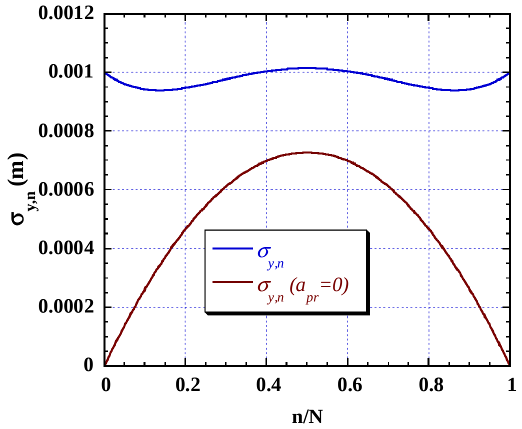

Fig. 3, $\sigma_{y,n}$ (a_{pr} =1mm) and $\sigma_{y,n}$ (a_{pr} = 0) as function of n/N

APPENDIX B

Here, we show the effect of the initial angle of the random walk, $\theta_{j,1}$, disappears after the correction of the accumulated error.

From Eq. (1-2), looking at the term includes $\theta_{j,m}$ in $y_{0,j,n}$, ($m = 1,2,3,,,n$)

$$y_{0,j,n} = l_{step} \sum_{m=1}^{n} (n - m + 1)\theta_{j,m} + \text{other terms} \qquad \text{(B-1)}$$

For $n = N$,

$$y_{0,j,N} = l_{step} \sum_{m=1}^{N} (N - m + 1)\theta_{j,m} + \text{other terms} \qquad \text{(B-2)}$$

Then, from the Eq. (1-3) and (1-4),

$$\begin{aligned} y_{j,n} &= l_{step} \sum_{m=1}^{n} (n - m + 1)\theta_{j,m} - l_{step} \frac{n}{N} \sum_{m=1}^{N} (N - m + 1)\theta_{j,m} + \text{other terms} \\ &= -l_{step} \frac{N-n}{N} \sum_{m=1}^{n} (m - 1)\theta_{j,m} - l_{step} \frac{n}{N} \sum_{m=n+1}^{N} (N - m + 1)\theta_{j,m} + \text{other terms} \end{aligned} \qquad \text{(B-3)}$$

Now it has been shown that the coefficient of $\theta_{j,1}$ vanishes, looking at he term of $m = 1$ in Eq. (B-3).

Reference

[1] http://www-pnp.physics.ox.ac.uk/~licas/page_talks/IWAA2004/iwaa2004_gg_talk_v1.pdf and http://iwaa2004.web.cern.ch/IWAA2004/subsite/PDF/20041007_TS10-3_Grzegorz-Grzelak.pdf

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