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SPATIAL RESOLUTION OF NON-INVASIVE BEAM PROFILE MONITORBASED ON OPTICAL DIFFRACTION RADIATION

A.P. Potylitsyn

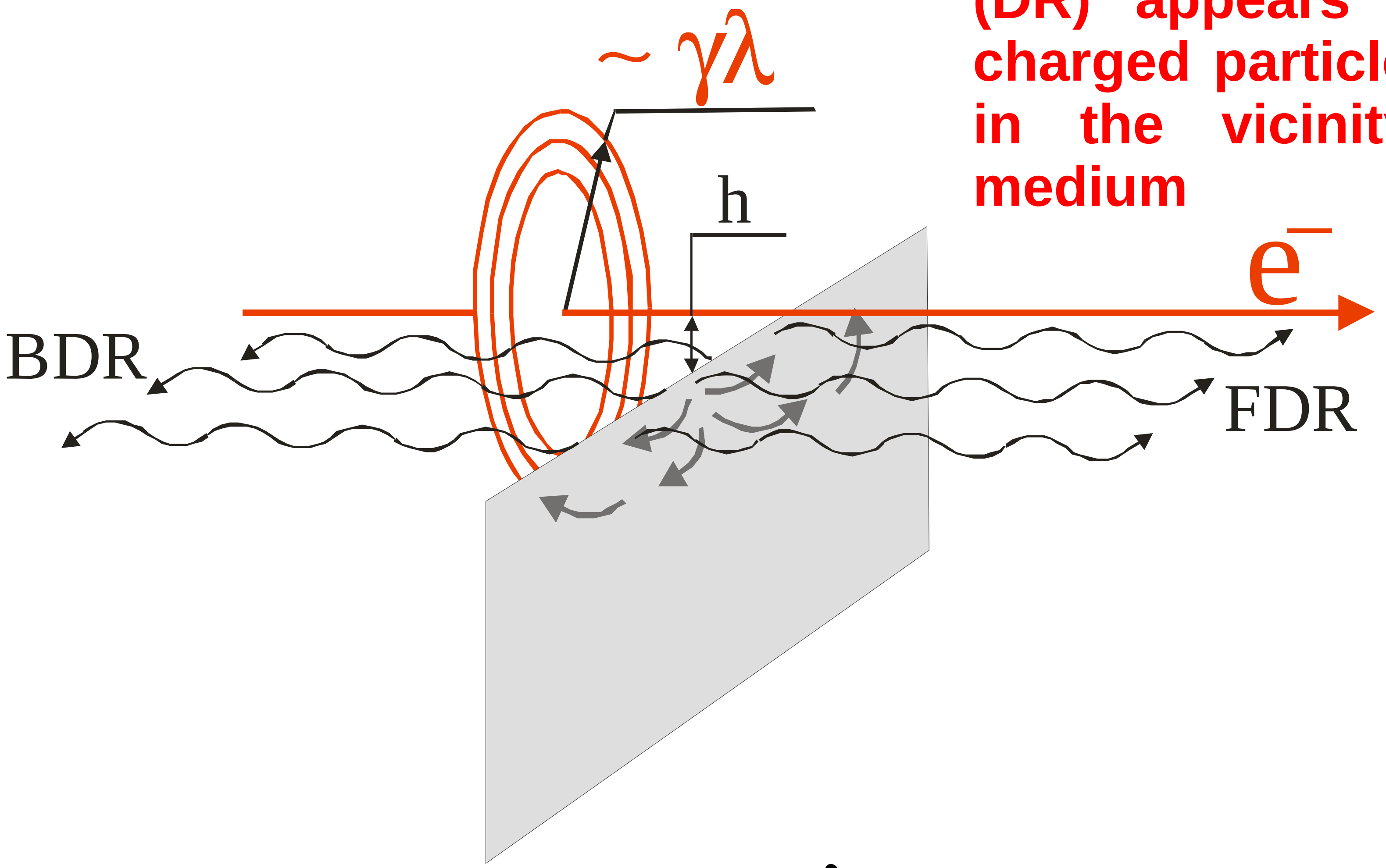
Tomsk Polytechnic University,

634050, pr. Lenina 2, Tomsk, Russia

e-mail: pap@interact.phtd.tpu.edu.ru

Diffraction radiation approach

Diffraction radiation (DR) appears when a charged particle moves in the vicinity of a medium

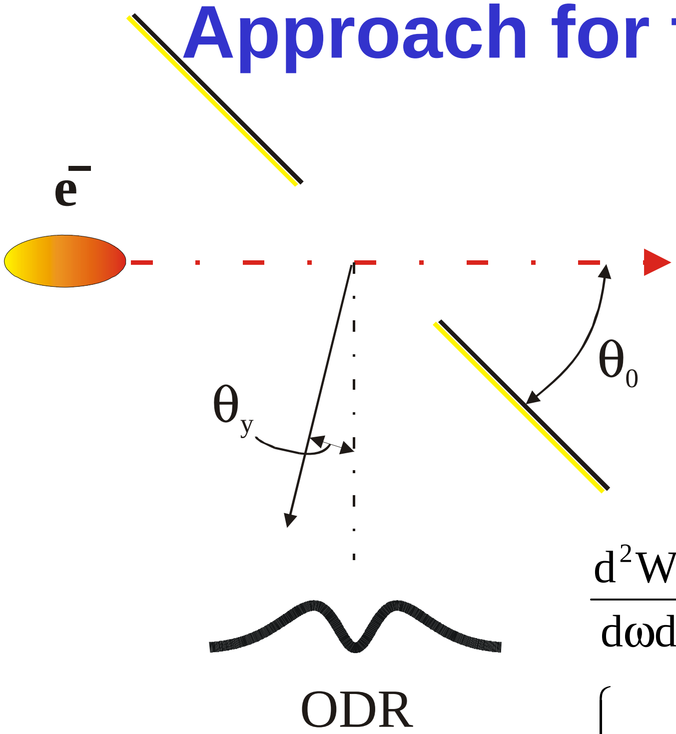


Impact parameter, h, – the shortest distance between the target and the particle trajectory

$$h \leq \frac{\gamma\lambda}{2\pi}$$

λ - observation wavelength

Approach for the beam size measurements



Assume a Gaussian beam profile

$$G\left(\overline{a}_x,\sigma_y\right)=\frac{1}{\sqrt{2\pi}\,\sigma_y}\exp\left[-\frac{\left(\overline{a}_x-a_x\right)^2}{2\sigma_y^2}\right]$$

$$\frac{d^2W_x^{slit}}{d\omega d\Omega}=\frac{\alpha\left|R_x\right|^2}{4\pi^2}\,\frac{\theta_x^2}{\gamma^{-2}+\theta_x^2}\,\frac{\exp\left(-\frac{2\pi a\sin\theta_0}{\lambda}\sqrt{\gamma^{-2}+\theta_x^2}\right)}{\gamma^{-2}+\theta_x^2+\theta_y^2}\times$$

$$\left\{\exp\left[\frac{8\pi^2\sigma_y^2}{\lambda^2}\left(\gamma^{-2}+\theta_x^2\right)\right]\cosh\left[\frac{4\pi\overline{a}_x}{\lambda}\sqrt{\gamma^{-2}+\theta_x^2}\right]+\cos\left[\frac{2\pi a\sin\theta_0}{\lambda}\theta_y+2\psi\right]\right\}$$

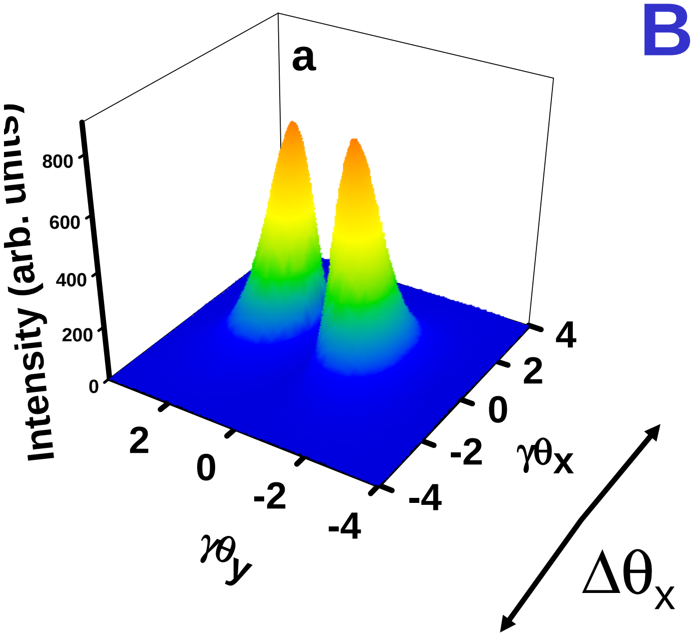
$$\frac{d^2W_y^{slit}}{d\omega d\Omega}=\frac{\alpha\left|R_y\right|^2}{4\pi^2}\,\frac{\exp\left(-\frac{2\pi a\sin\theta_0}{\lambda}\sqrt{\gamma^{-2}+\theta_x^2}\right)}{\gamma^{-2}+\theta_x^2+\theta_y^2}\times$$

$$\left\{\exp\left[\frac{8\pi^2\sigma_y^2}{\lambda^2}\left(\gamma^{-2}+\theta_x^2\right)\right]\cosh\left[\frac{4\pi\overline{a}_x}{\lambda}\sqrt{\gamma^{-2}+\theta_x^2}\right]-\cos\left[\frac{2\pi a\sin\theta_0}{\lambda}\theta_y+2\psi\right]\right\}$$

$$\psi=\arctan\left(\frac{\theta_y}{\sqrt{\gamma^{-2}+\theta_x^2}}\right)$$

, σ_y is the electron beam size and $\overline{a}_x$ is its offset with respect to the slit center

Beam size effect

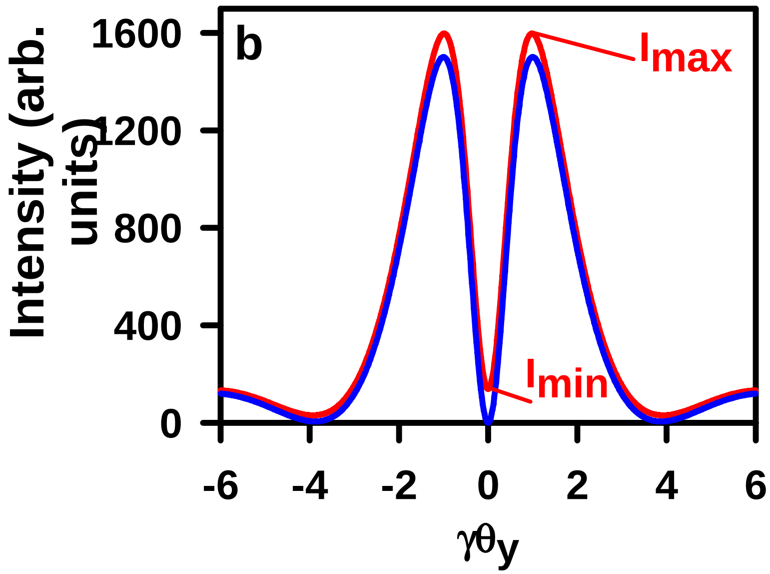


Projected vertical polarization component

$$\overline{S(\theta_y,\sigma_y)} = \int_{-\Delta\theta_x/2}^{\Delta\theta_x/2} \frac{d^2W(\theta_x,\theta_y,\sigma_y)}{d\omega d\Omega} d\theta_x$$

$\Delta\theta_x$ – x detector angular acceptance

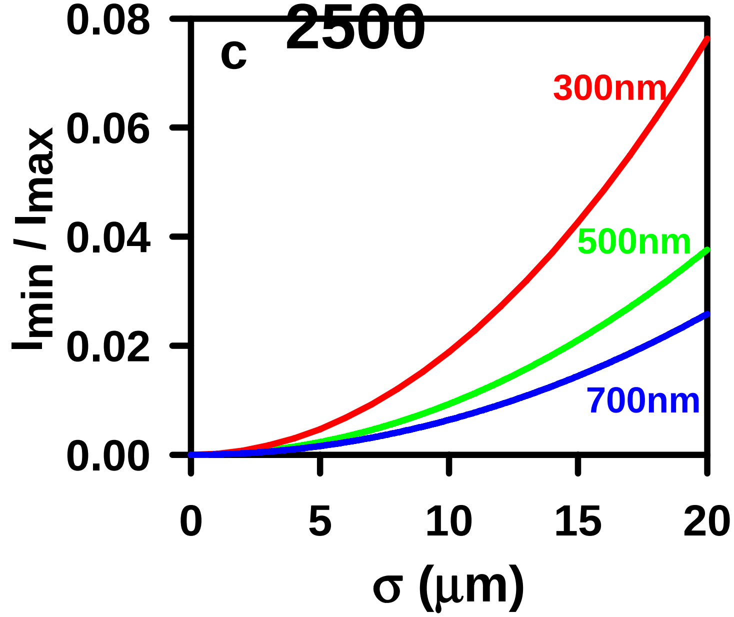
Projection

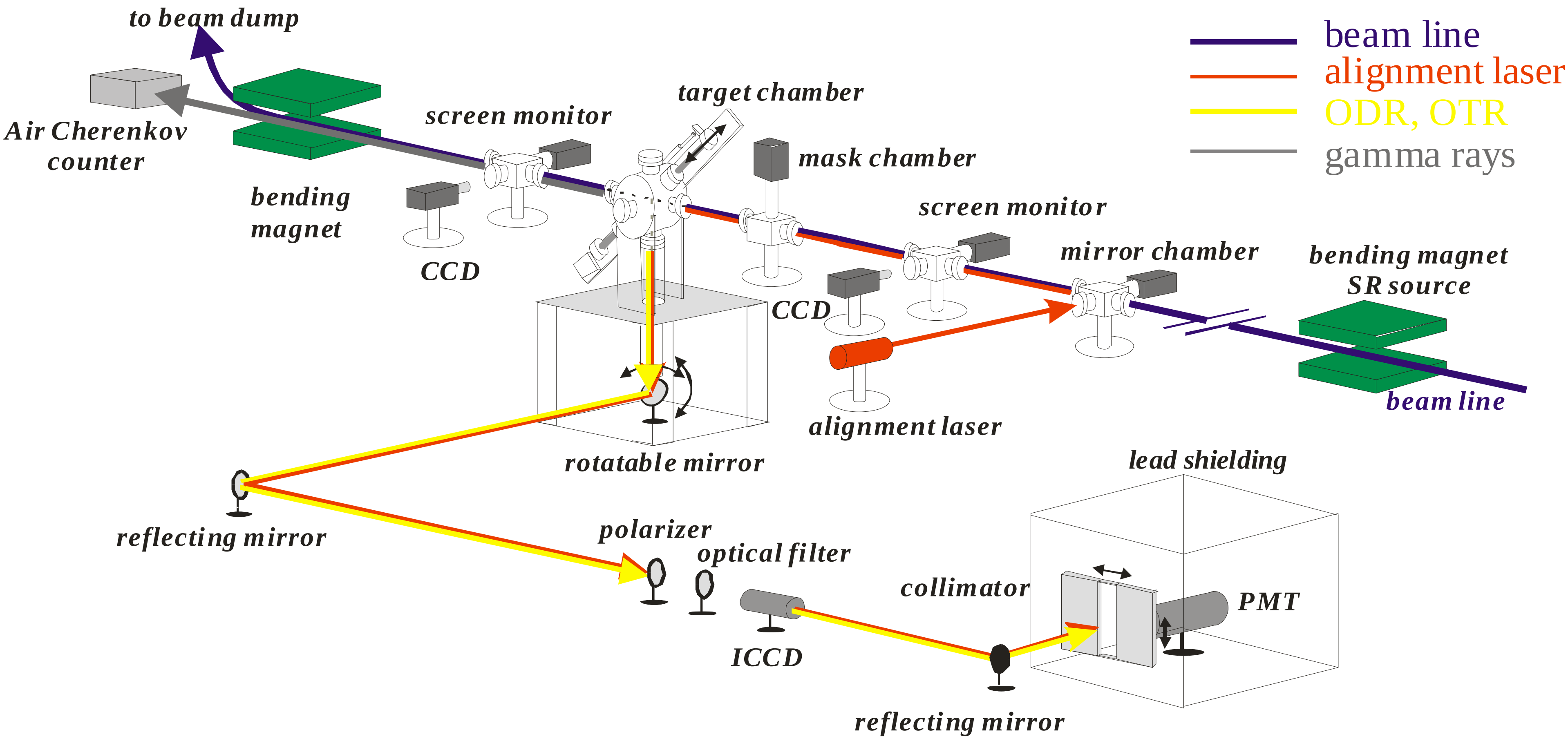


σ_y=0

σ_y=30μm

γ = 2500

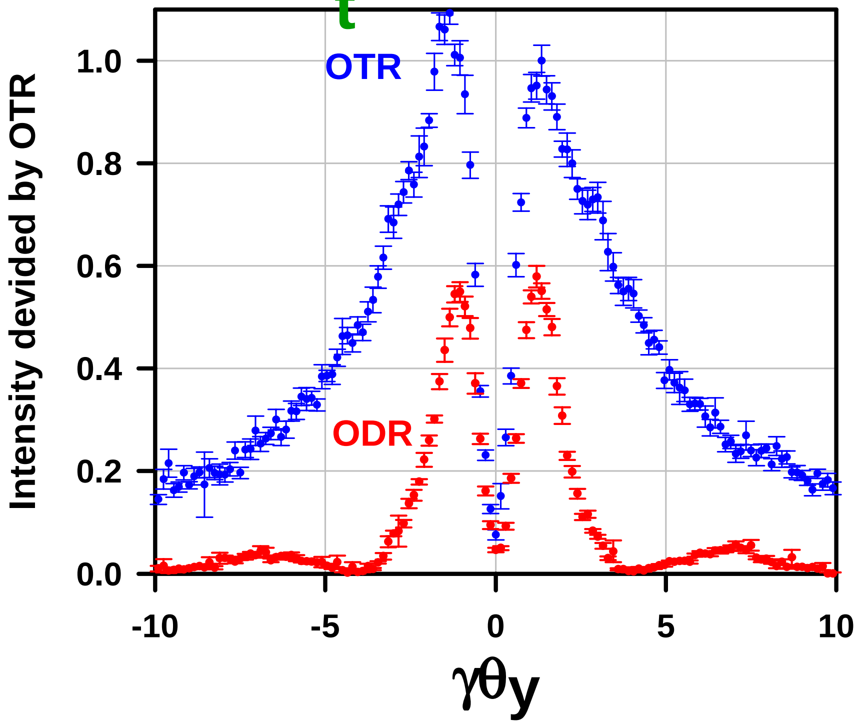




to beam dump
Air Cherenkov counter
bending magnet
screen monitor
CCD
target chamber
mask chamber
screen monitor
CCD
mirror chamber
bending magnet SR source
beam line
alignment laser
rotatable mirror
reflecting mirror
polarizer
optical filter
ICCD
collimator
lead shielding
PMT
reflecting mirror
beam line
alignment laser
ODR, OTR
gamma rays

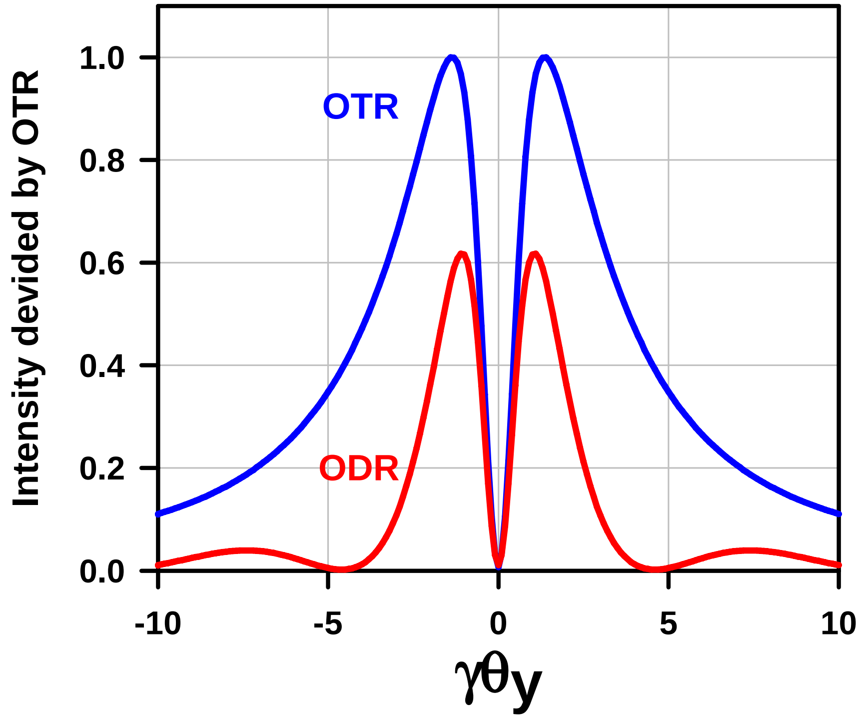
Measurements of the ODR projected vertical polarization component with PMT

Experiment



$$\frac{I_{ODR}^{max}}{I_{OTR}^{max}} = 58\%$$

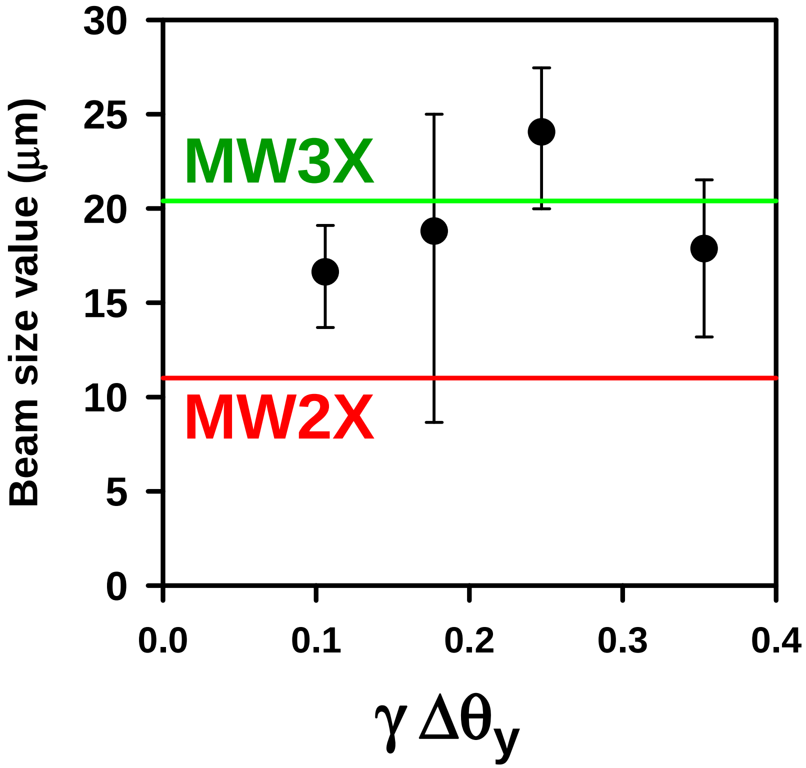
Theory

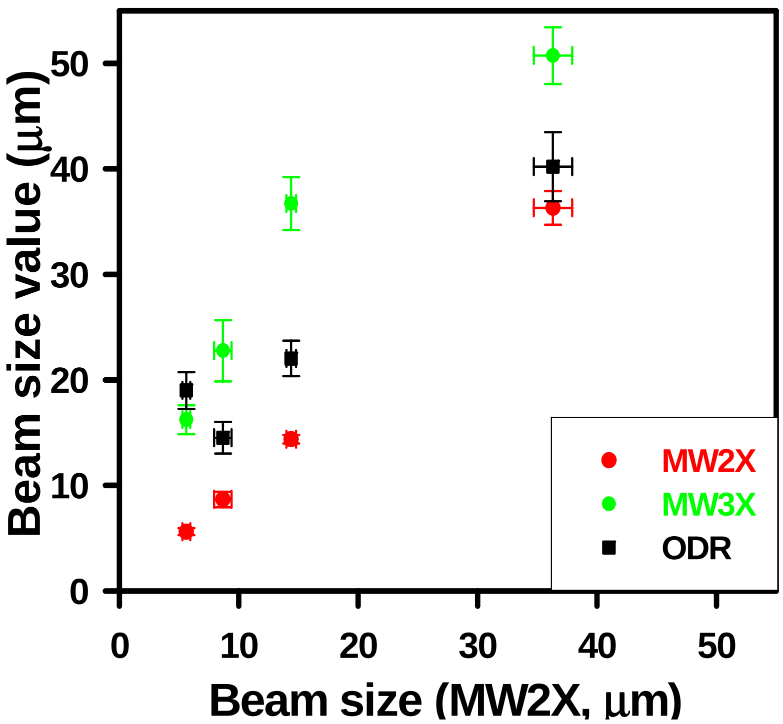


$$\frac{I_{ODR}^{max}}{I_{OTR}^{max}} = 62\%$$

Optical filter 550±20nm

Angular acceptance and beam size effect in ODR experiment

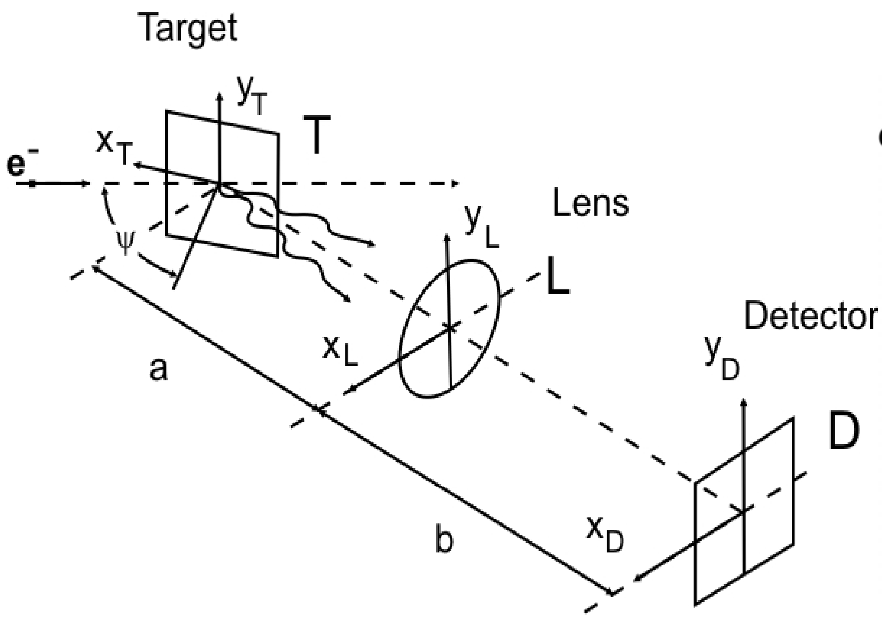


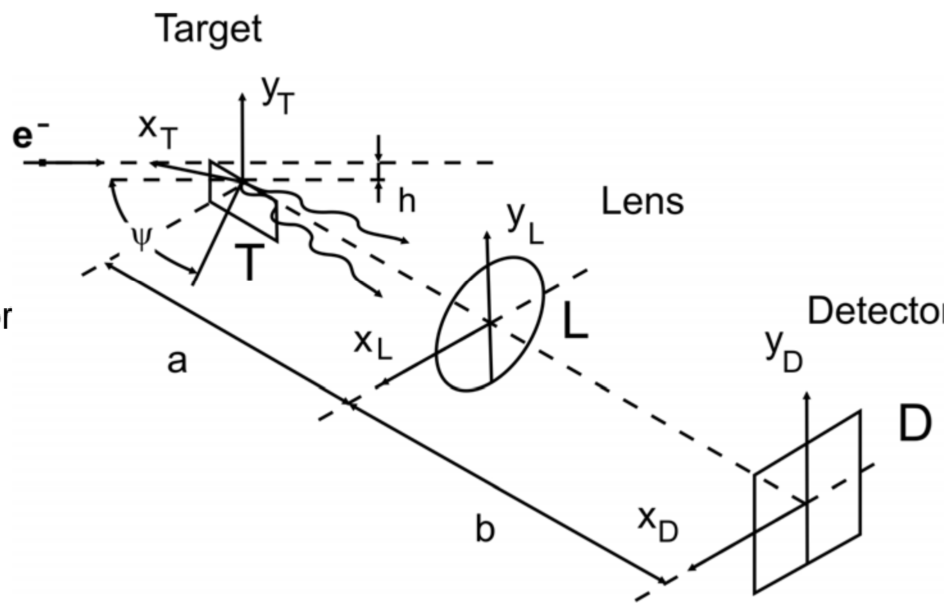


Optical transition radiation (OTR) beam size monitor

OTR monitor

ODR monitor





Pre-wave zone → $a \leq \gamma^2 \lambda$

Model

We shall introduce Cartesian coordinates on the target, lens and detector using *T, L, D* indices and use dimensionless variables

$$\begin{Bmatrix} x_T \\ y_T \end{Bmatrix} = \frac{2\pi}{\gamma\lambda}\begin{Bmatrix} X_T \\ Y_T \end{Bmatrix}, \begin{Bmatrix} x_L \\ y_L \end{Bmatrix} = \frac{\gamma}{a}\begin{Bmatrix} X_L \\ Y_L \end{Bmatrix}, \begin{Bmatrix} x_D \\ y_D \end{Bmatrix} = \frac{2\pi}{\gamma\lambda}\begin{Bmatrix} X_D \\ Y_D \end{Bmatrix}.$$

$$\begin{Bmatrix} E_x^D\left(x_D, y_D\right) \\ E_y^D\left(x_D, y_D\right) \end{Bmatrix} = const\int dx_T dy_T \int dx_L dy_L \times$$

$$\times\begin{Bmatrix} x_T \\ y_T \end{Bmatrix}\frac{K_1\left(\sqrt{x_T^2+y_T^2}\right)}{\sqrt{x_T^2+y_T^2}}\exp\left[i\left(x_T x_L + y_T y_L\right)\right]\times\exp\left[i\frac{x_T^2+y_T^2}{4\pi R}\right]\times$$

$$\times\exp\left[-i\left(x_L\frac{x_D}{M}+y_L\frac{y_D}{M}\right)\right], \quad \mathrm{R}=\frac{\mathrm{a}}{\gamma^2\lambda}, \text{ M is magnification}$$

For a rectangular lens with aperture $2x_m \times 2y_m$

$$-x_m \le x_L \le x_m \quad , \quad -y_m \le y_L \le y_m$$

One may obtain

$$\int_{-x_m}^{x_m} dx_L \int_{-y_m}^{y_m} dy_L \exp\left[-ix_L\left(x_T + \frac{x_D}{M}\right)\right]\exp\left[-iy_D\left(y_T + \frac{y_D}{M}\right)\right] =$$

$$= 4\frac{\sin\left[x_m\left(x_T + \frac{x_D}{M}\right)\right]}{x_T + \frac{x_D}{M}}\frac{\sin\left[y_m\left(y_T + \frac{y_D}{M}\right)\right]}{y_T + \frac{y_D}{M}} =$$

$$= G_x(x_T, x_D, x_m)G(y_T, y_D, y_m).$$

For M=1 fields on the detector surface may be written:

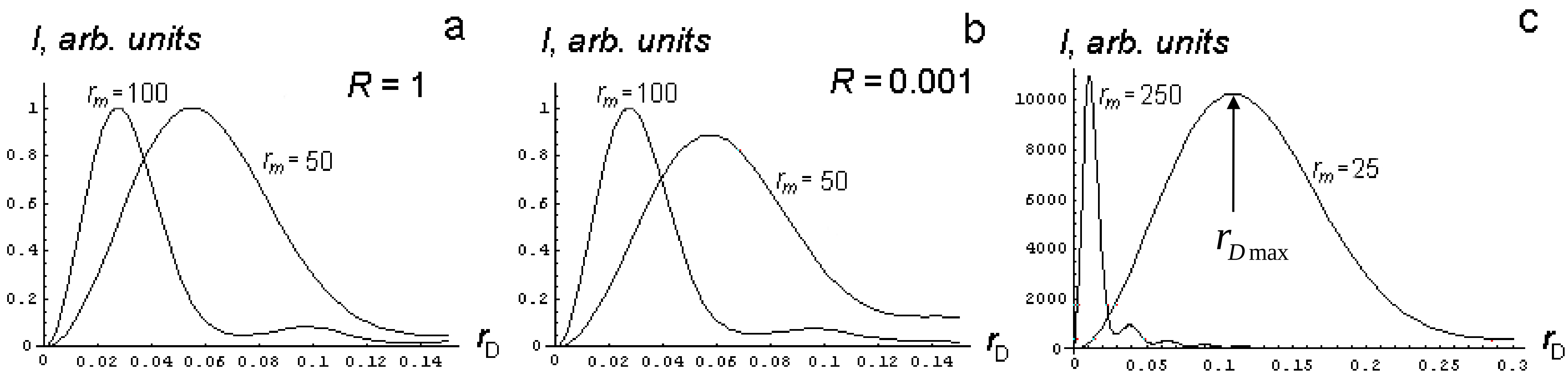
$$\begin{Bmatrix} E_x^D(x_D, y_D) \\ E_y^D(x_D, y_D) \end{Bmatrix} = const\int dx_T dy_T \begin{Bmatrix} x_T \\ y_T \end{Bmatrix} \frac{K_1\left(\sqrt{x_T^2 + y_T^2}\right)}{\sqrt{x_T^2 + y_T^2}} \times$$

$$\times \exp\left[i\frac{x_T^2 + y_T^2}{4\pi R}\right] G_x(x_T, x_D, x_m) G(y_T, y_D, y_m).$$

Intensity on the detector surface

$$I = const\left(\left|E_x^D\right|^2 + \left|E_y^D\right|^2\right)$$

OTR image from a single electron(point spread function, PSF)



a) Normalized shape of OTR source image on the detector plane for lens aperture r_m =100 (left curve) and r_m= 50 (right curve) in wave zone (*R* = 1);

b) OTR source image in «pre-wave zone» (*R* = 0.001) for the same conditions and with the same normalizing factors;

c) OTR source image at the fixed lens diameter for different distances between the target and the lens (*R* = 0.01, r_m = 250 - right curve; *R* = 0.1, r_m = 25 - left).

Spatial resolution of OTR monitor is defined by the distribution maximum position $r_{D\max}$

$$r_{D\max} \sim \frac{3}{r_m} = \frac{3}{\gamma\theta_0} \qquad \theta_0 = \frac{R_{L\max}}{a} \text{ - lens acceptance}$$

Dimension variable:

$$\sigma \approx \frac{\gamma\lambda\, r_{D\max}}{2\pi} \sim \frac{1}{2}\frac{\lambda}{\theta_0}$$

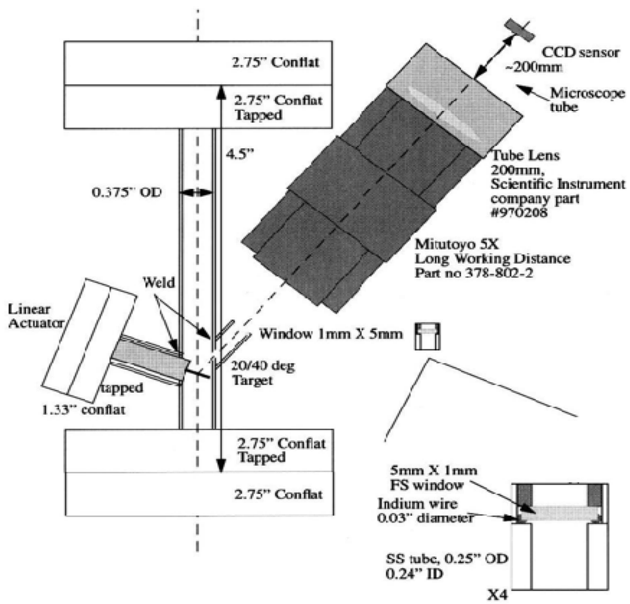
- For optical diffraction radiation (ODR) there are 2 dimension parameters – wavelength and impact parameter *h*

- Which one is defined a spatial resolution?

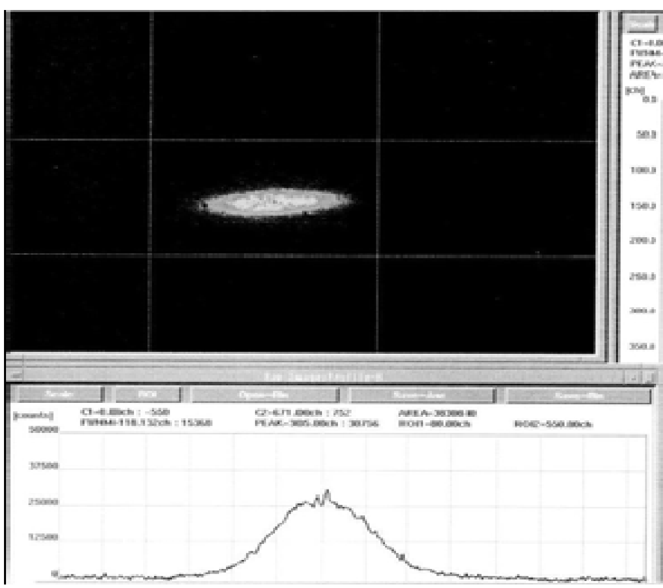
- To obtain an intensity of ODR on detector surface one have to integrate ODR field on the target surface:

$$-x_{\max} \le x_t \le x_{\max} \quad ,$$
$$-h \le y \le y_{\max}$$

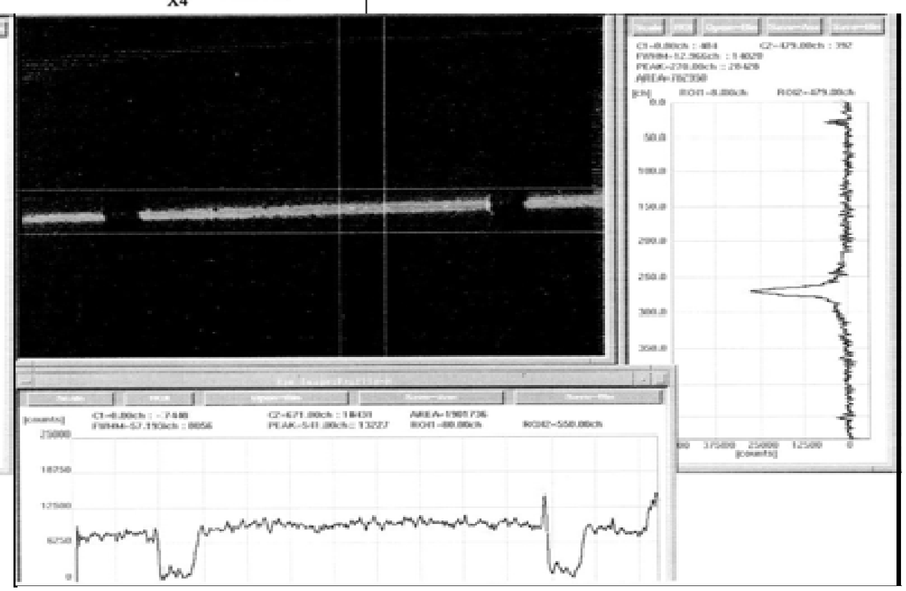
A Very High Resolution Optical Transition Radiation Beam Profile Monitor // Ross M. et al. SLAC-PUB-9280 July 2002



$FWHM = 10\mu$



$FWHM = 5.8\mu$



Near-field imaging of optical diffraction radiation generated by a 7-GeV electron beam

A. H. Lumpkin, W. J. Berg, N. S. Sereno, D.W. Rule,* and C.-Y. Yao

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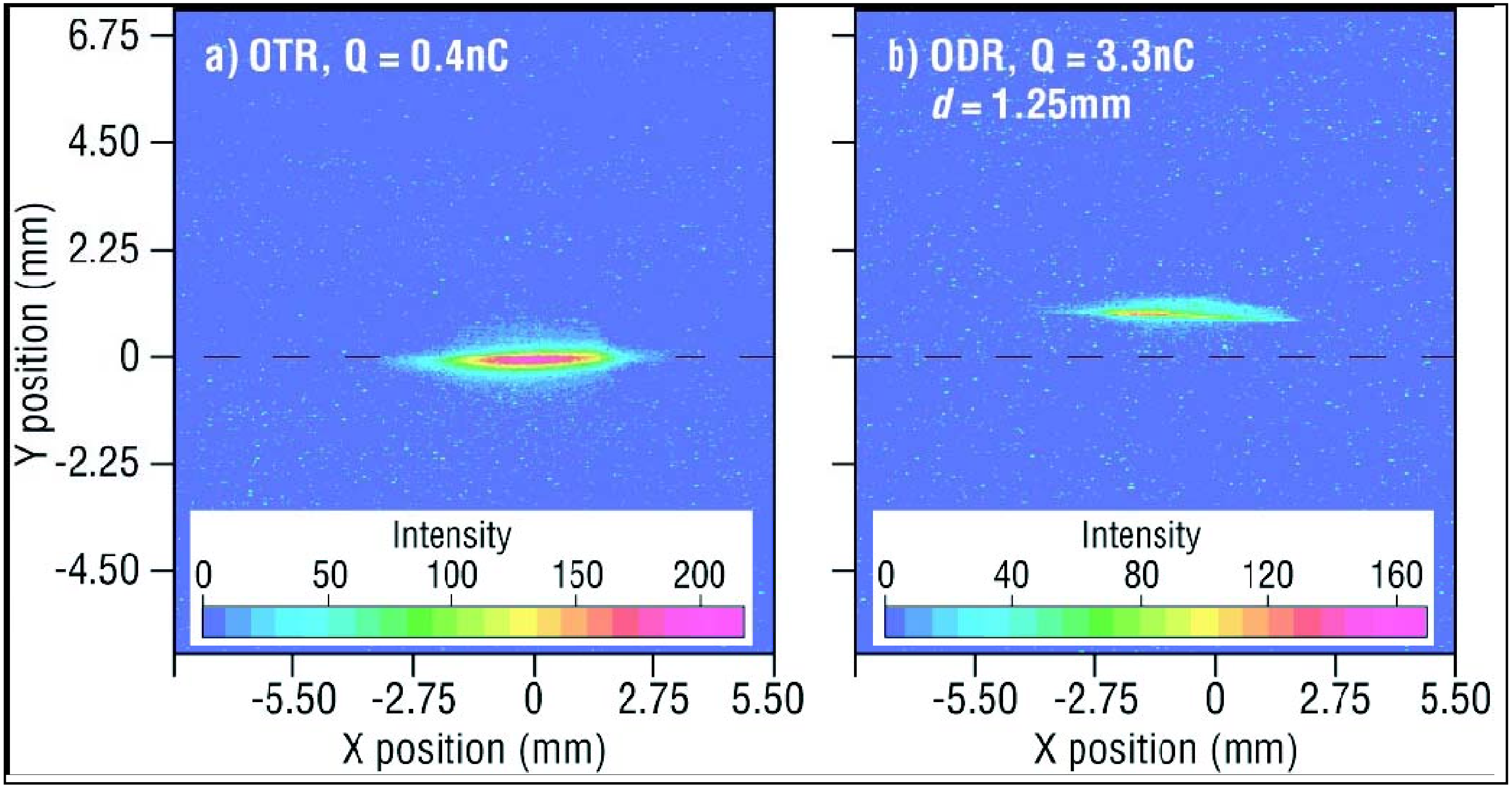
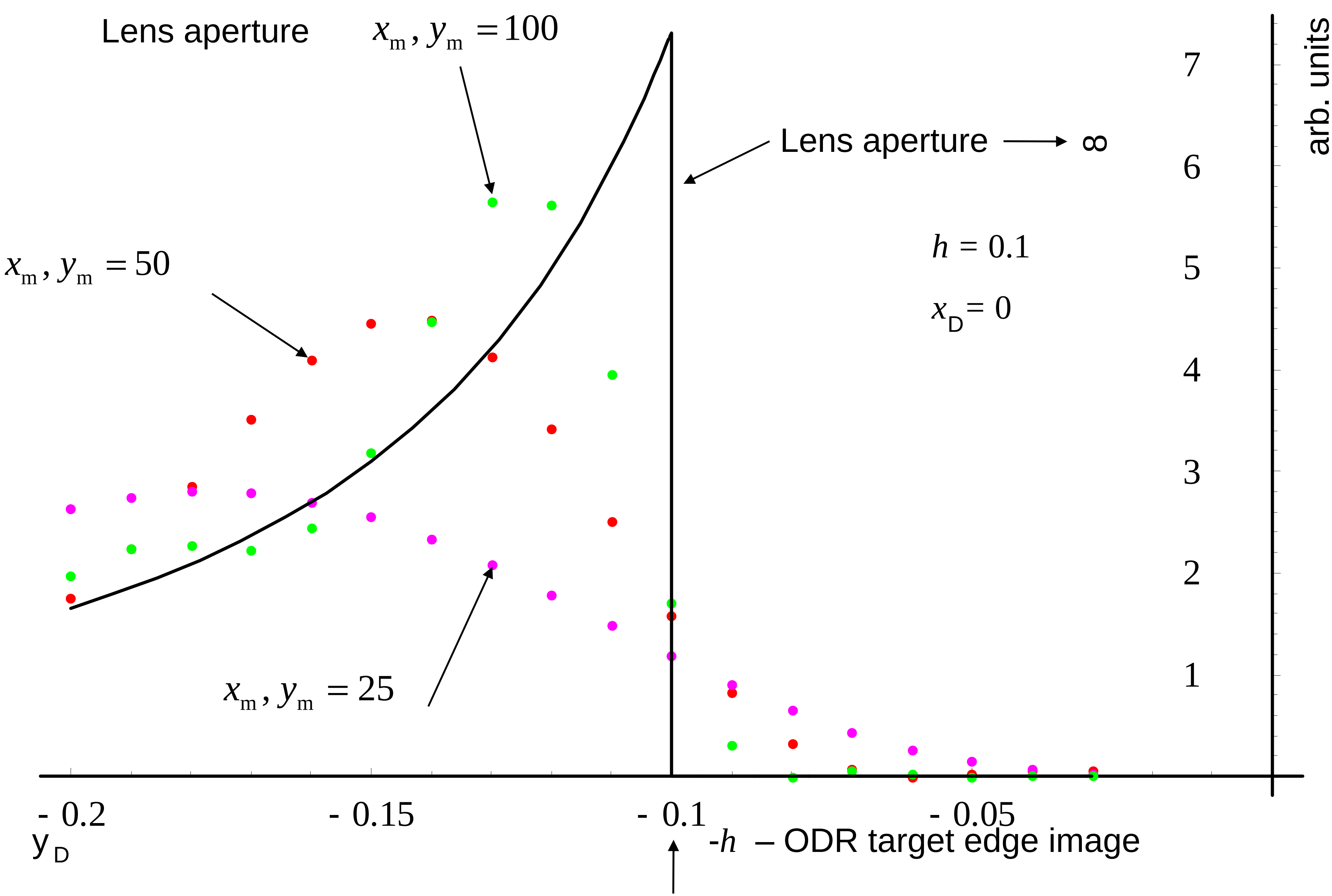
E = 7 GeV λ = 0.8 μm h = 1.25 mm $\sigma_x = 1375\mu m$ $\sigma_y = 200\mu m$

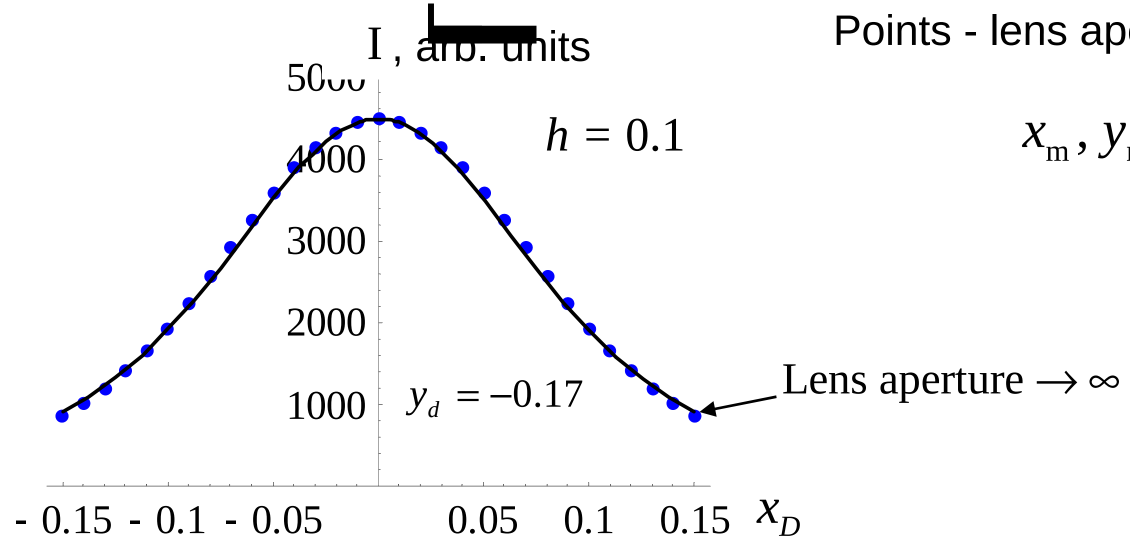


PSF for ODR case (perpendicular to target edge)

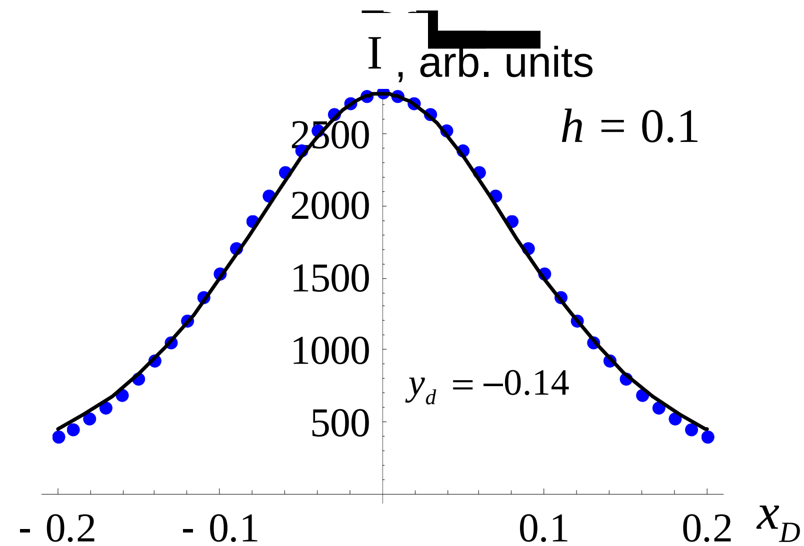


PSF for ODR case (parallel to target edge)

Points - lens aperture

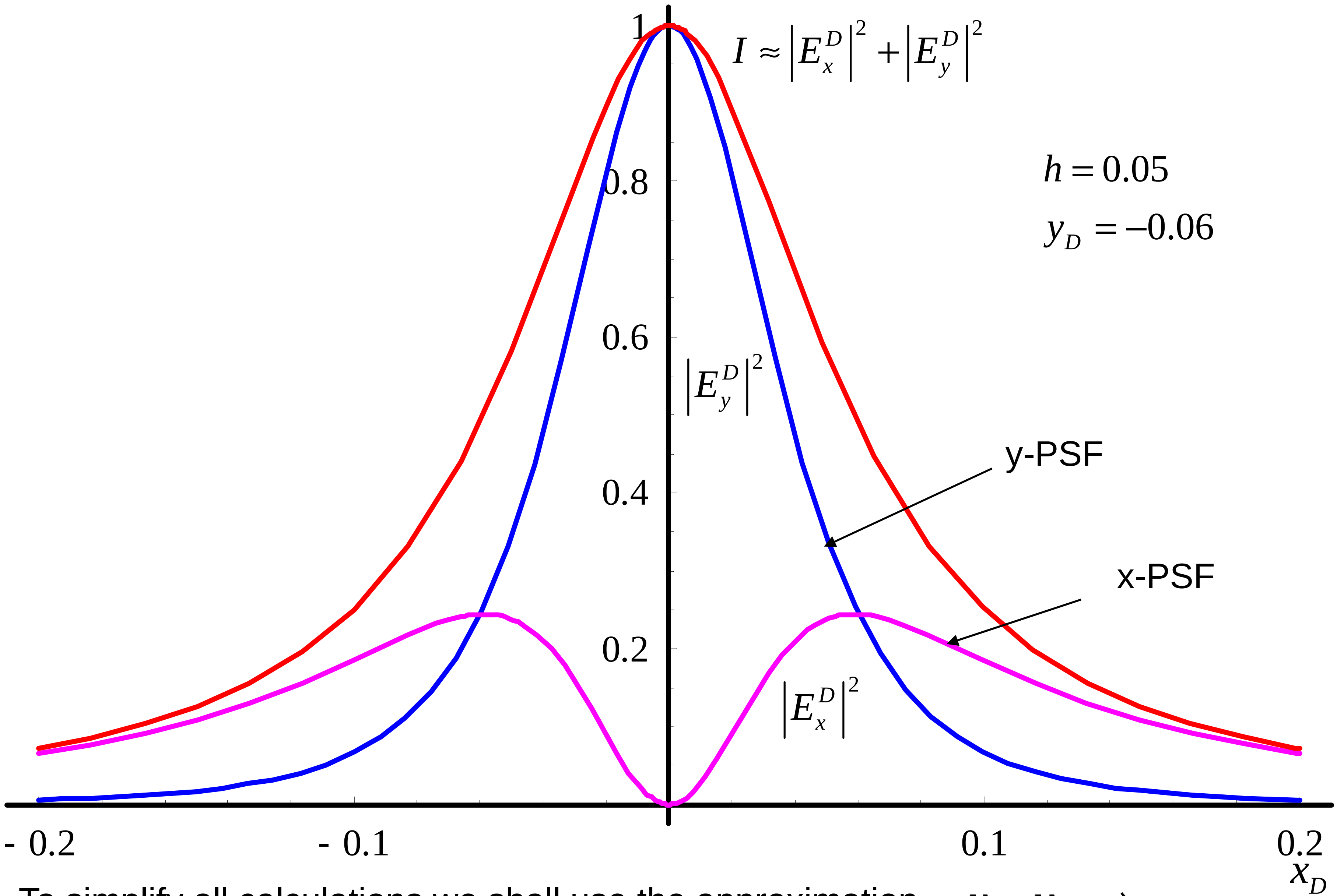


$x_m, y_m = 25$



$x_m, y_m = 50$

PSF for both ODR polarized components

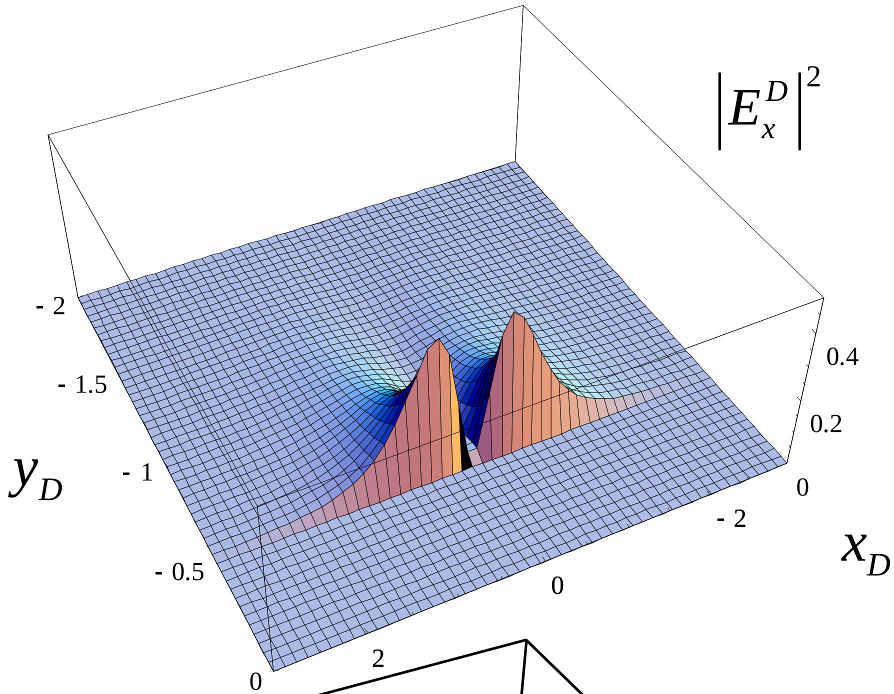


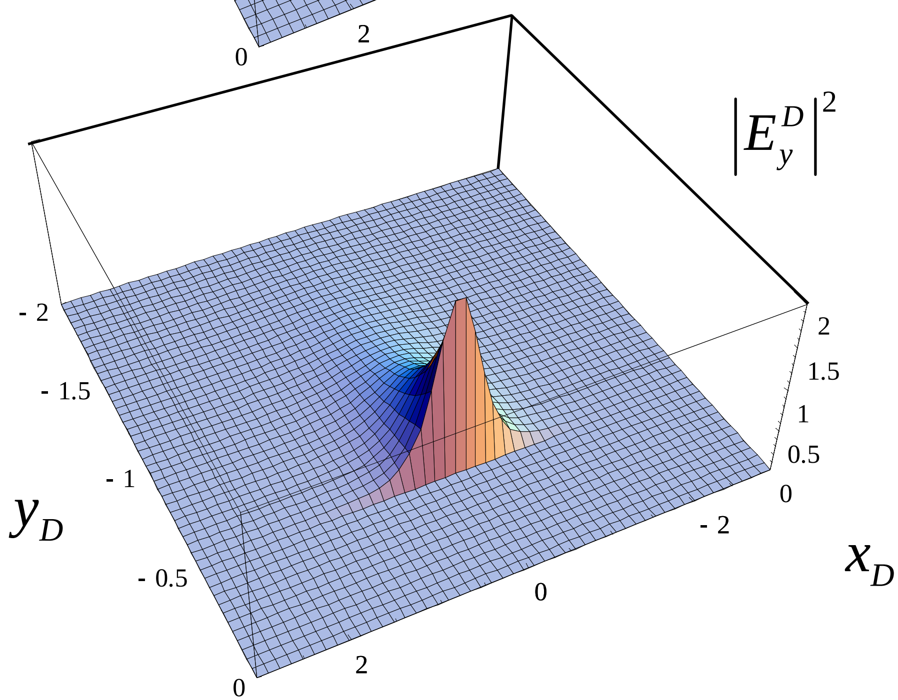
To simplify all calculations we shall use the approximation $x_m, y_m \to \infty$

2D PSF for ODR case

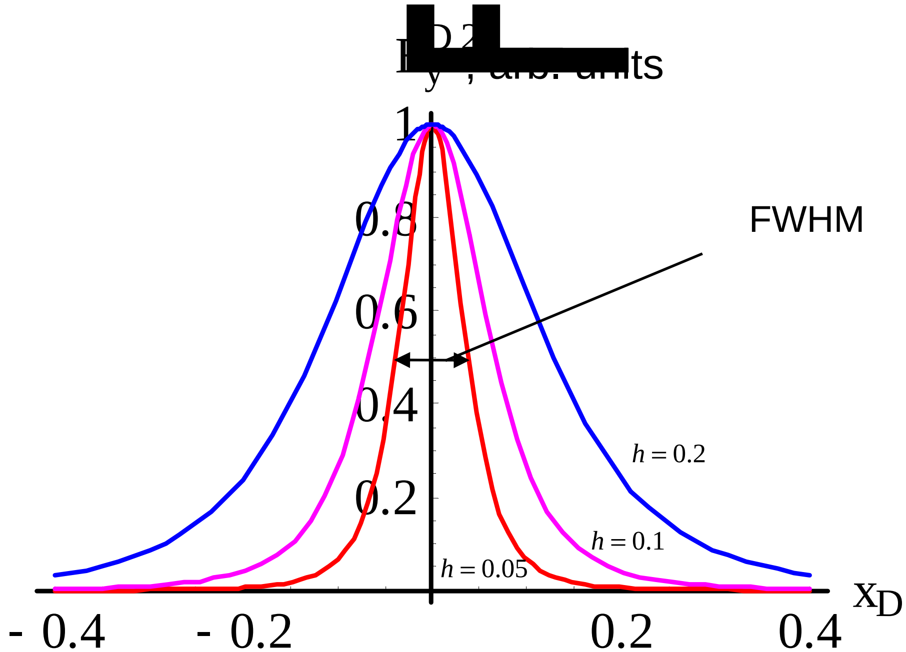
$h = 0.5$

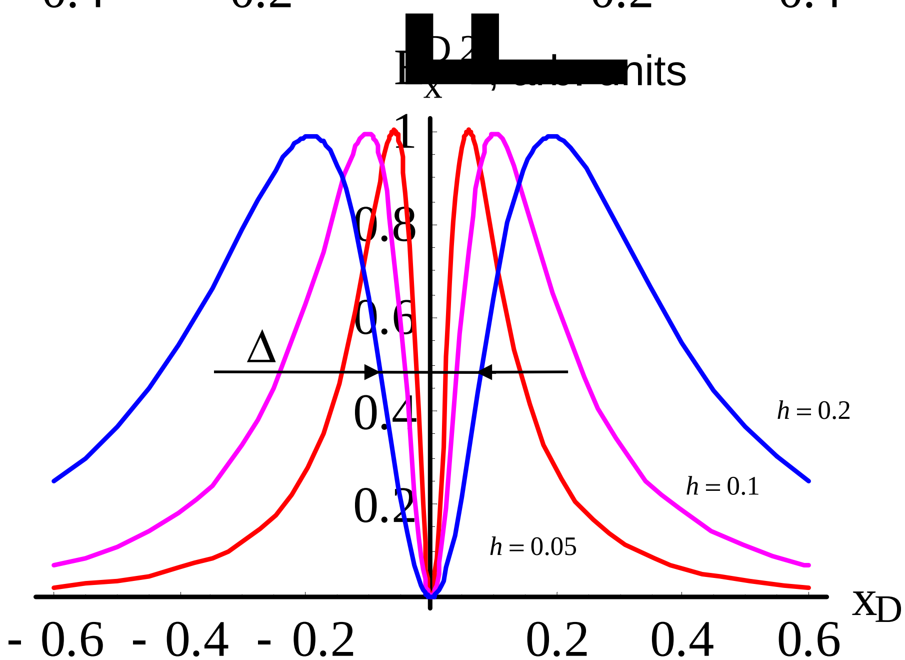
Significant broadening of deep (for $\left|E_x^D\right|^2$) and peak (for $\left|E_y^D\right|^2$) is observed with increasing of impact parameter h



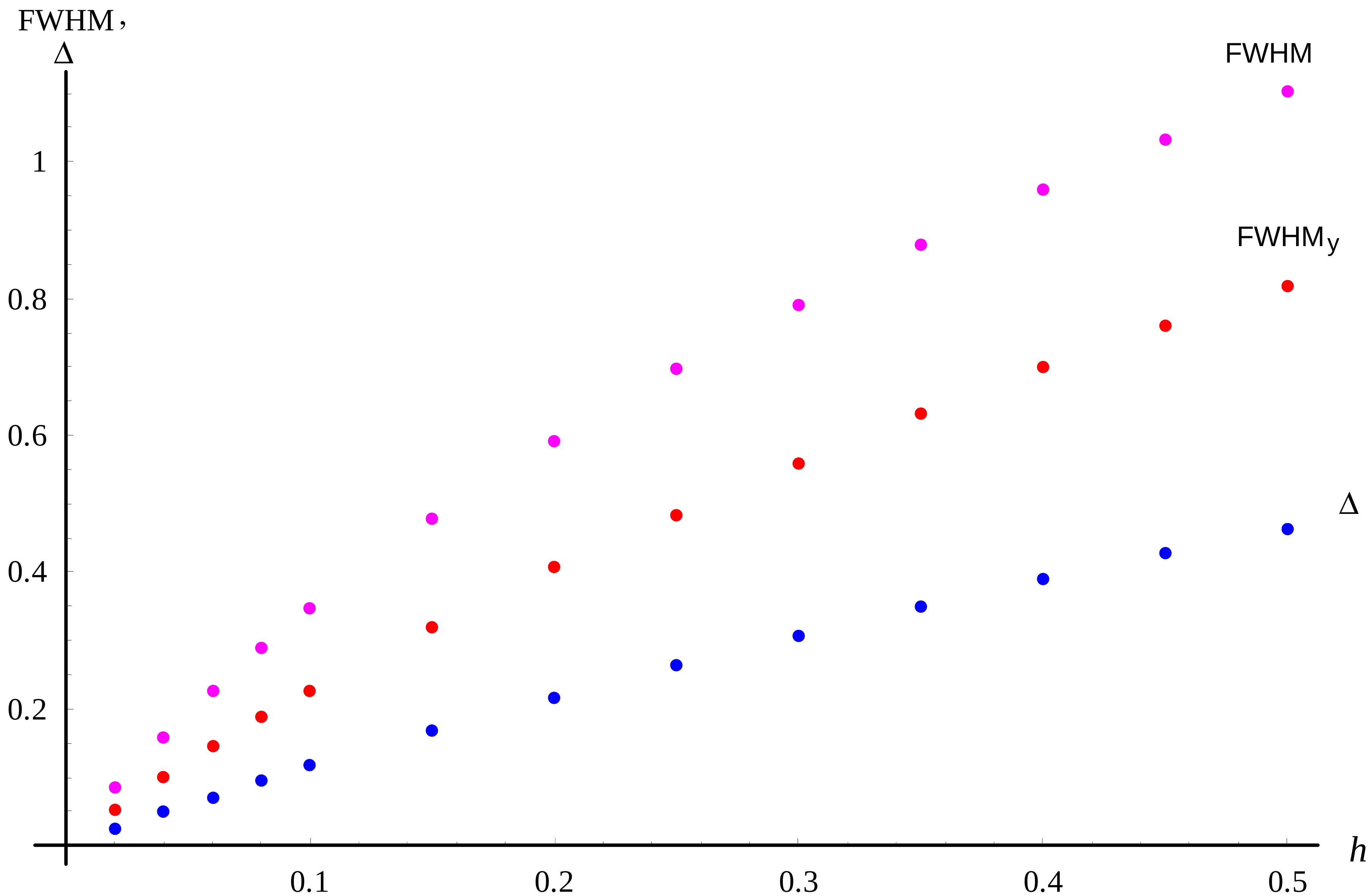


Characteristics of y-PSF and x-PSF





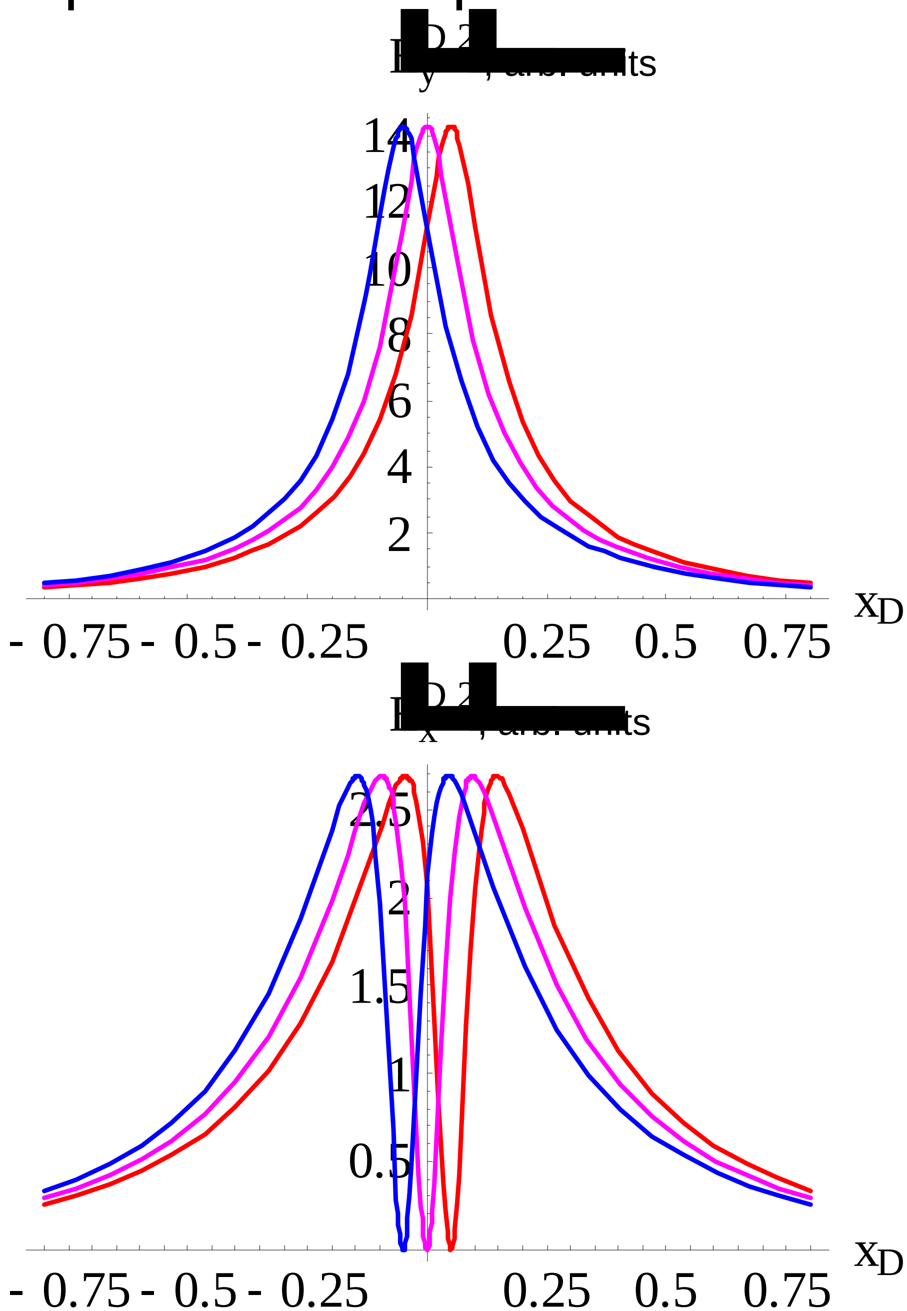
Dependence on impact parameter



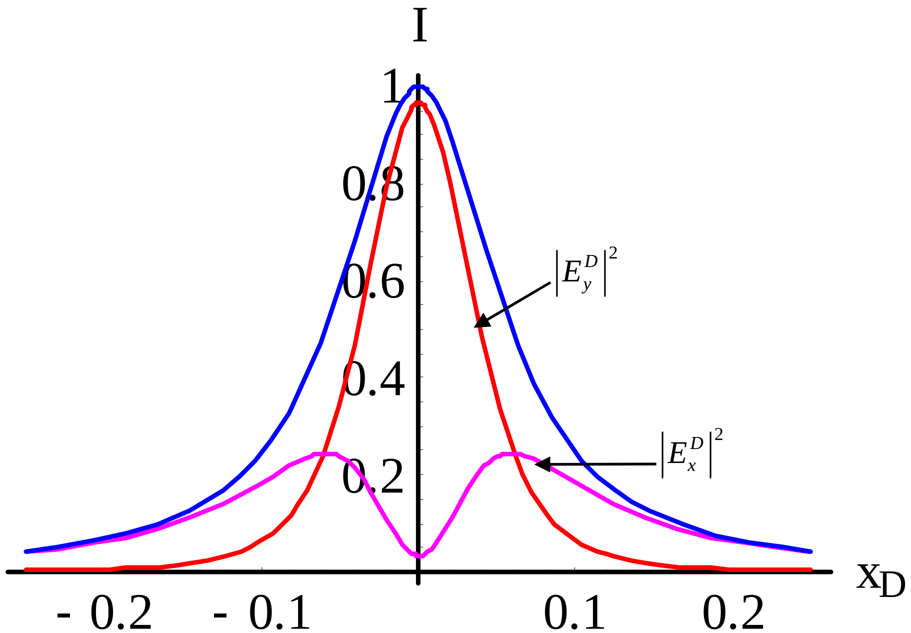
Almost linear dependence of FWHM and Δ on impact parameter

PSF dependence on particle offset

For small offset relative to optical axes distributions remain the same



“Smoothing” of PSF due to beam size

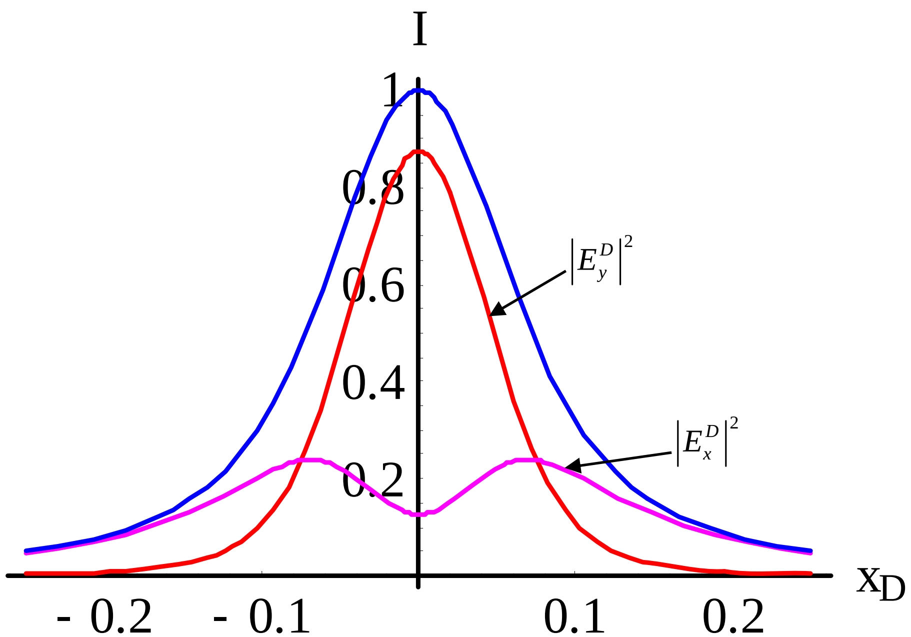


$\lambda = 800nm$

$\gamma = 13700$

$h = 100\mu$

$\sigma_x = 20\mu$



$\lambda = 800nm$

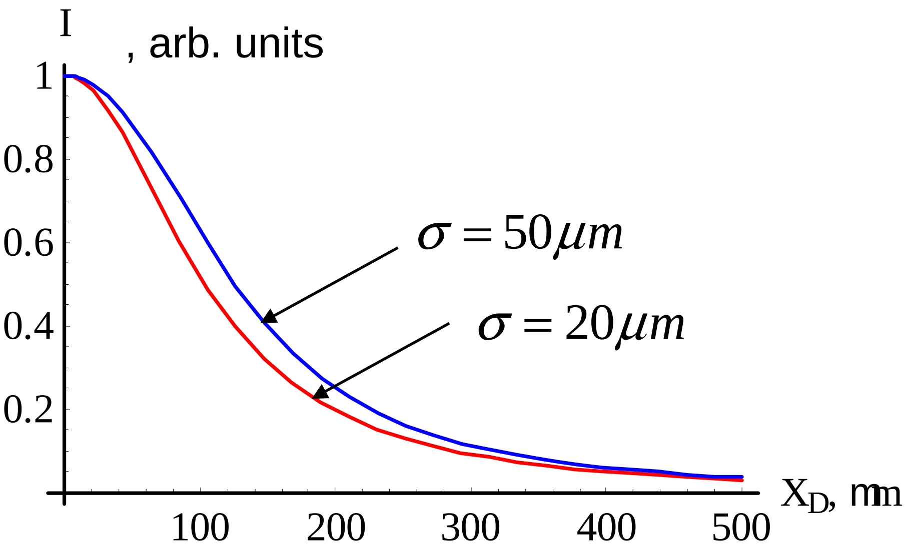
$\gamma = 13700$

$h = 100\mu$

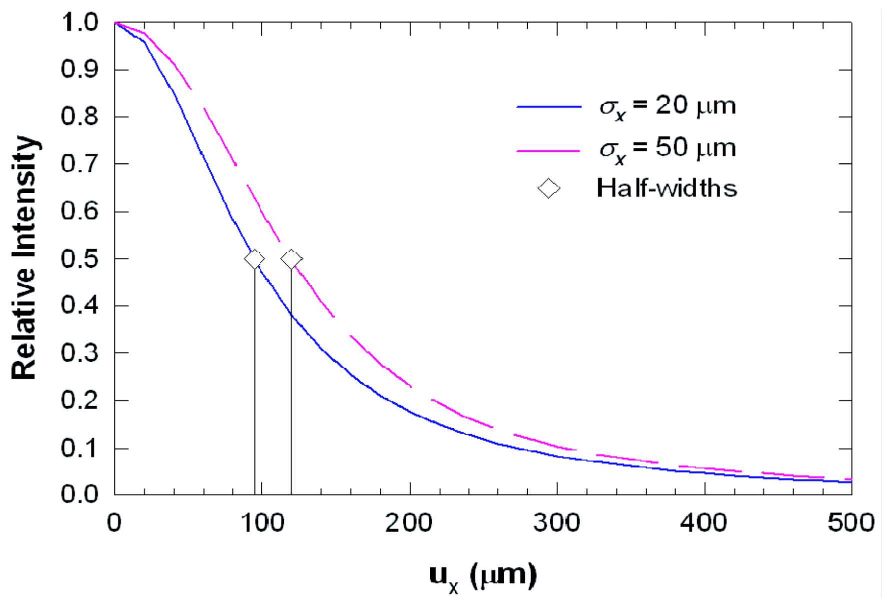
$\sigma_x = 50\mu$

Dependence of smoothed PSF on beam size

Developed model



E = 7 GeV
λ = 0.8 μ
h = 100 μm



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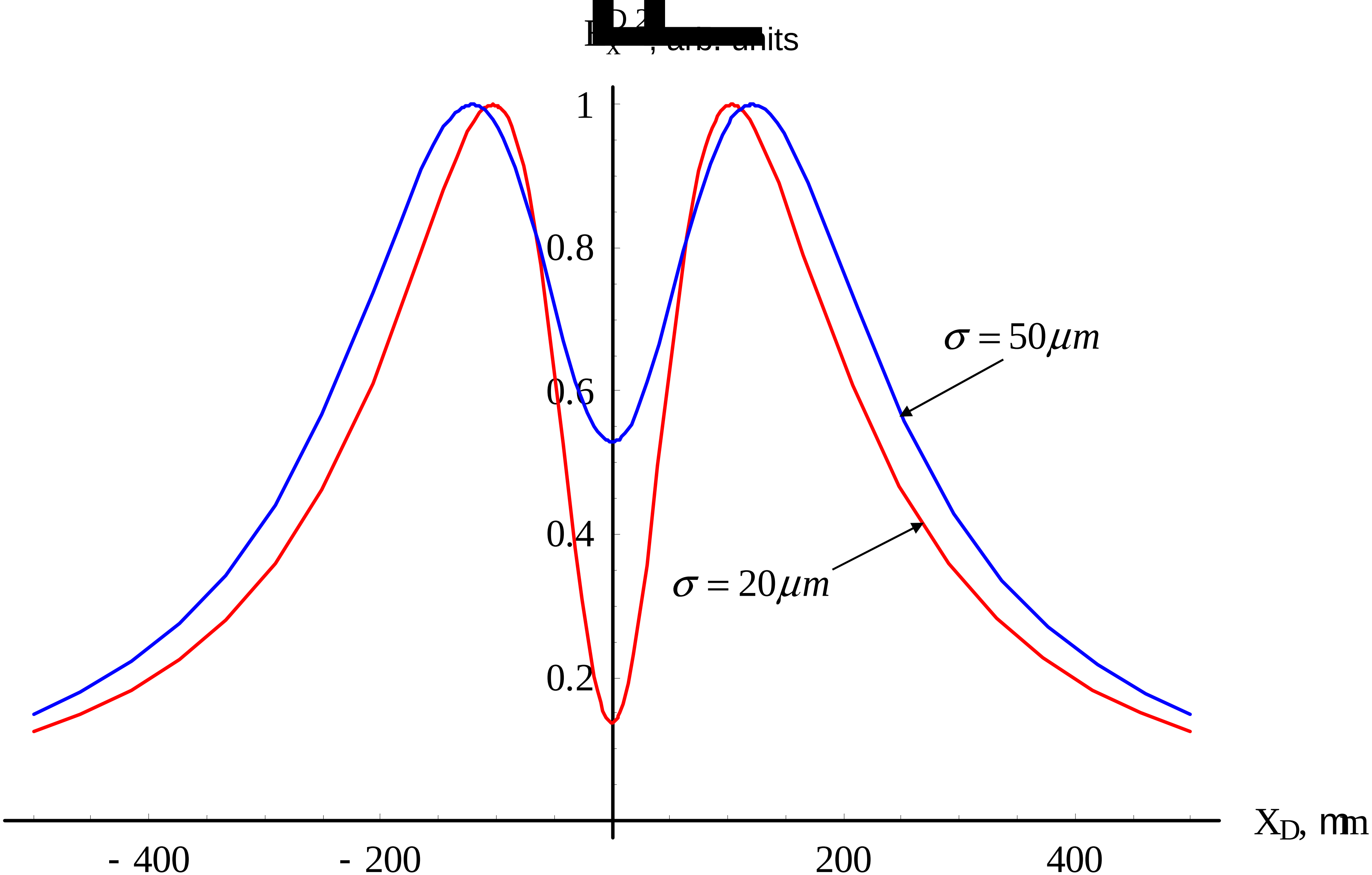
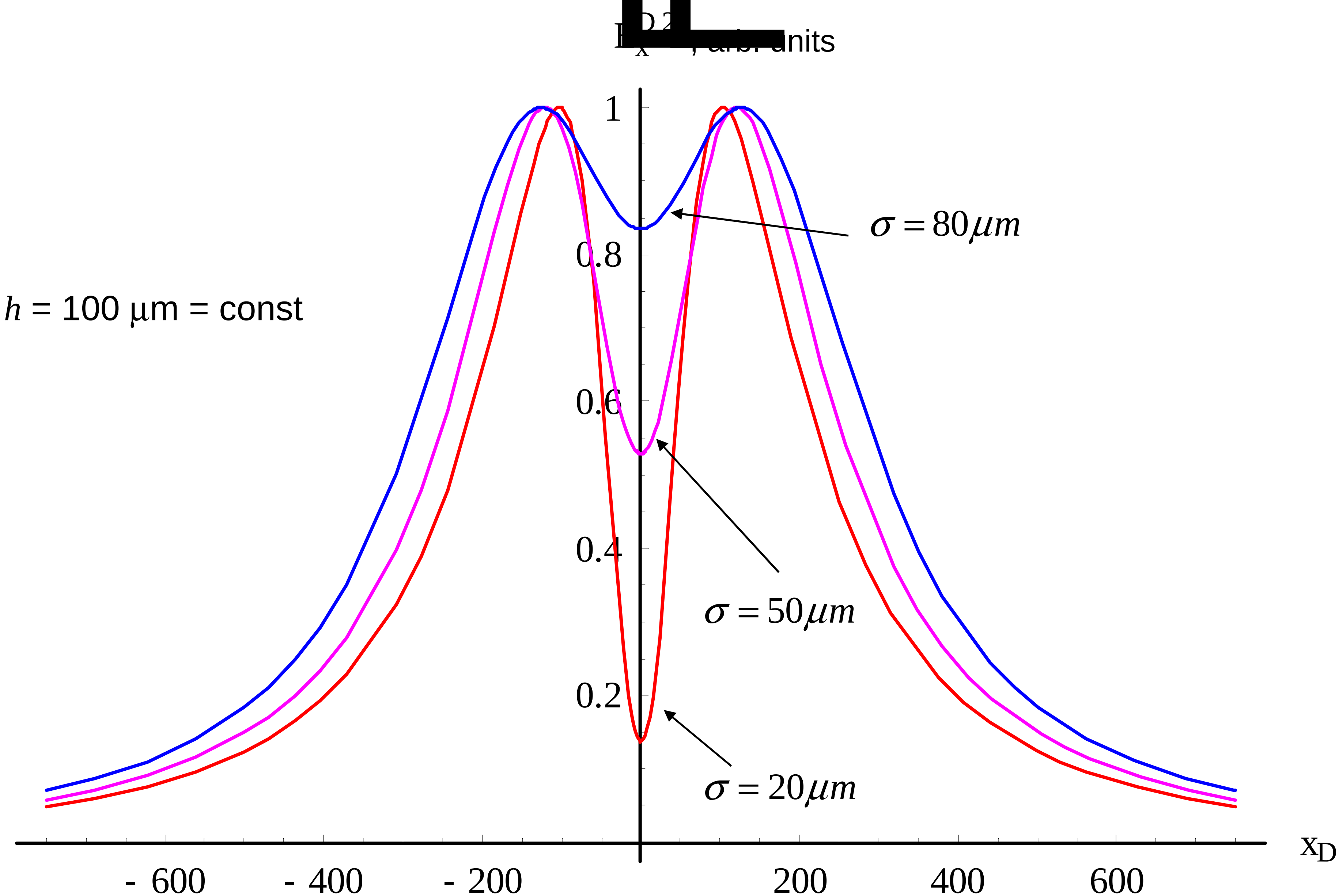
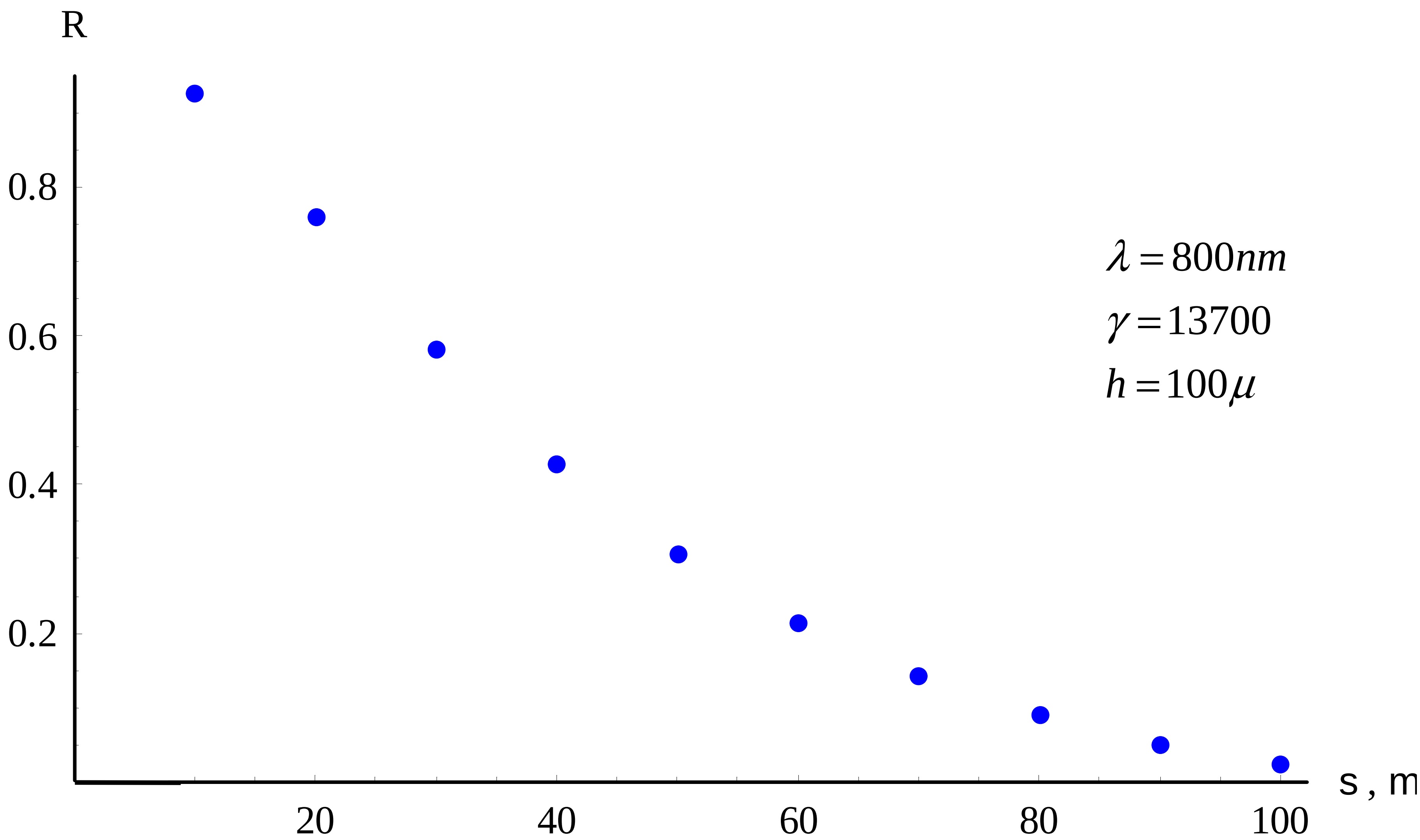
Smoothed x-PSF for different beam sizes



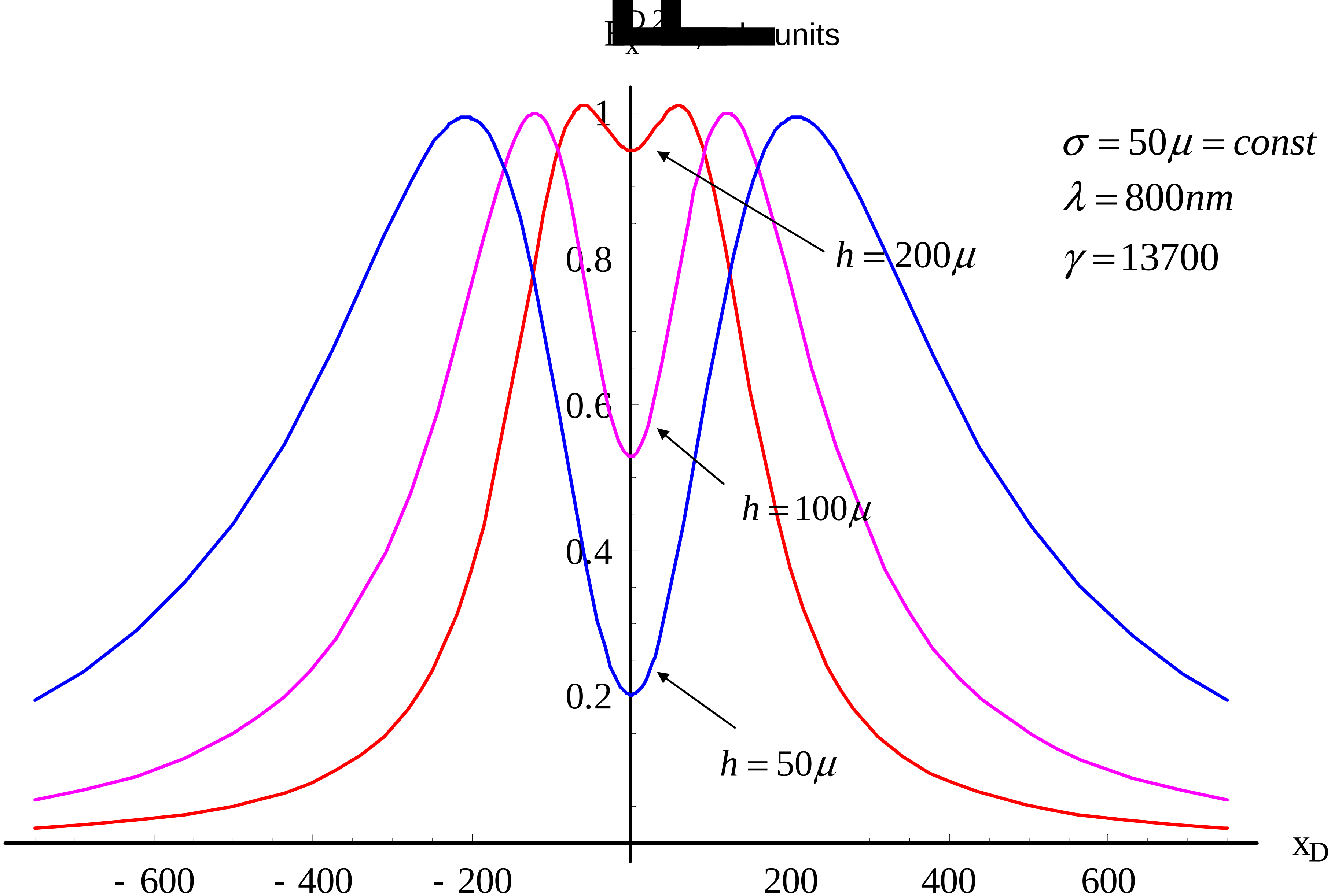
Smoothed of d[redacted] for different σ



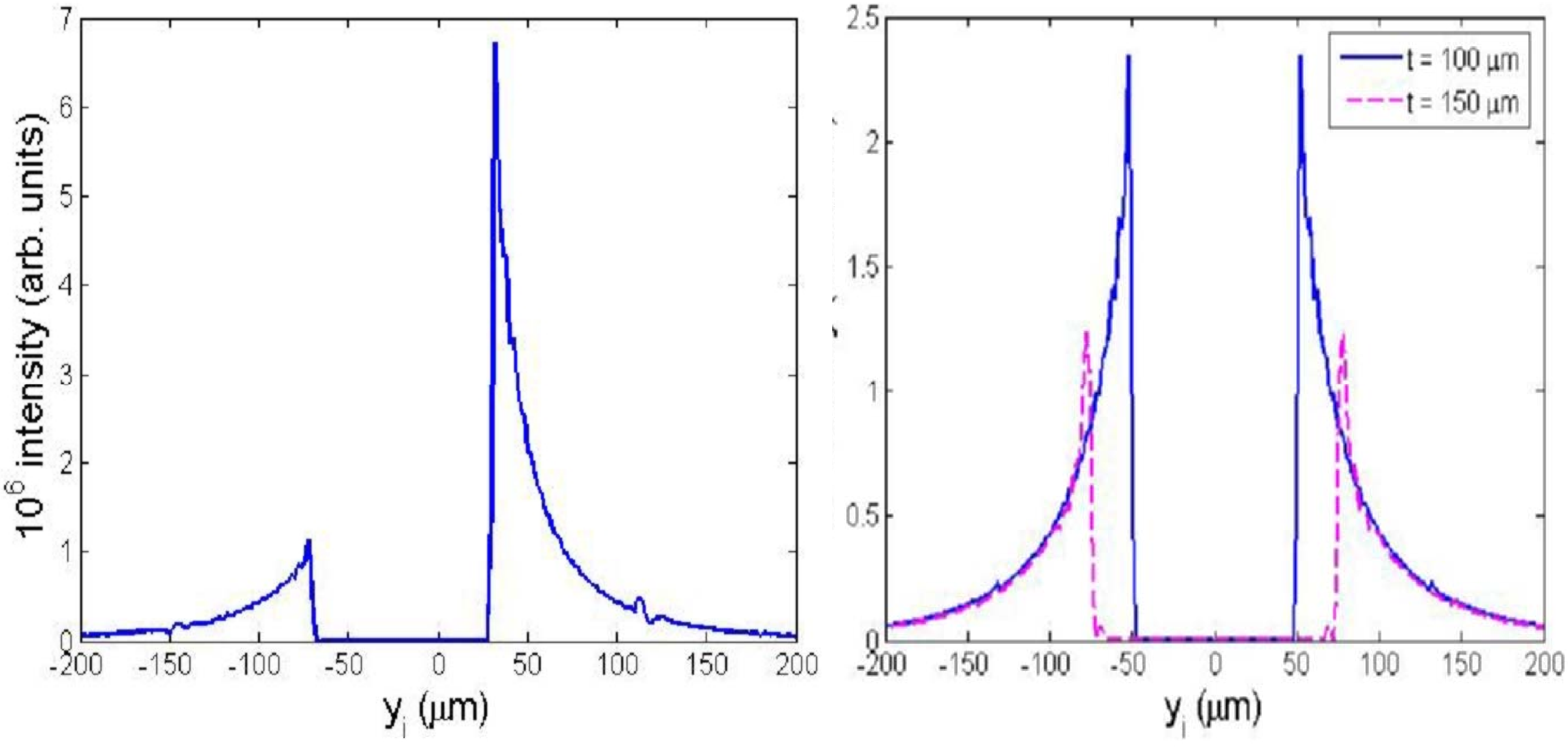
Ratio $R = \frac{I^x_{\max} - I^x_{\min}}{I^x_{\max} + I^x_{\min}}$ as a tool for beam size measurements



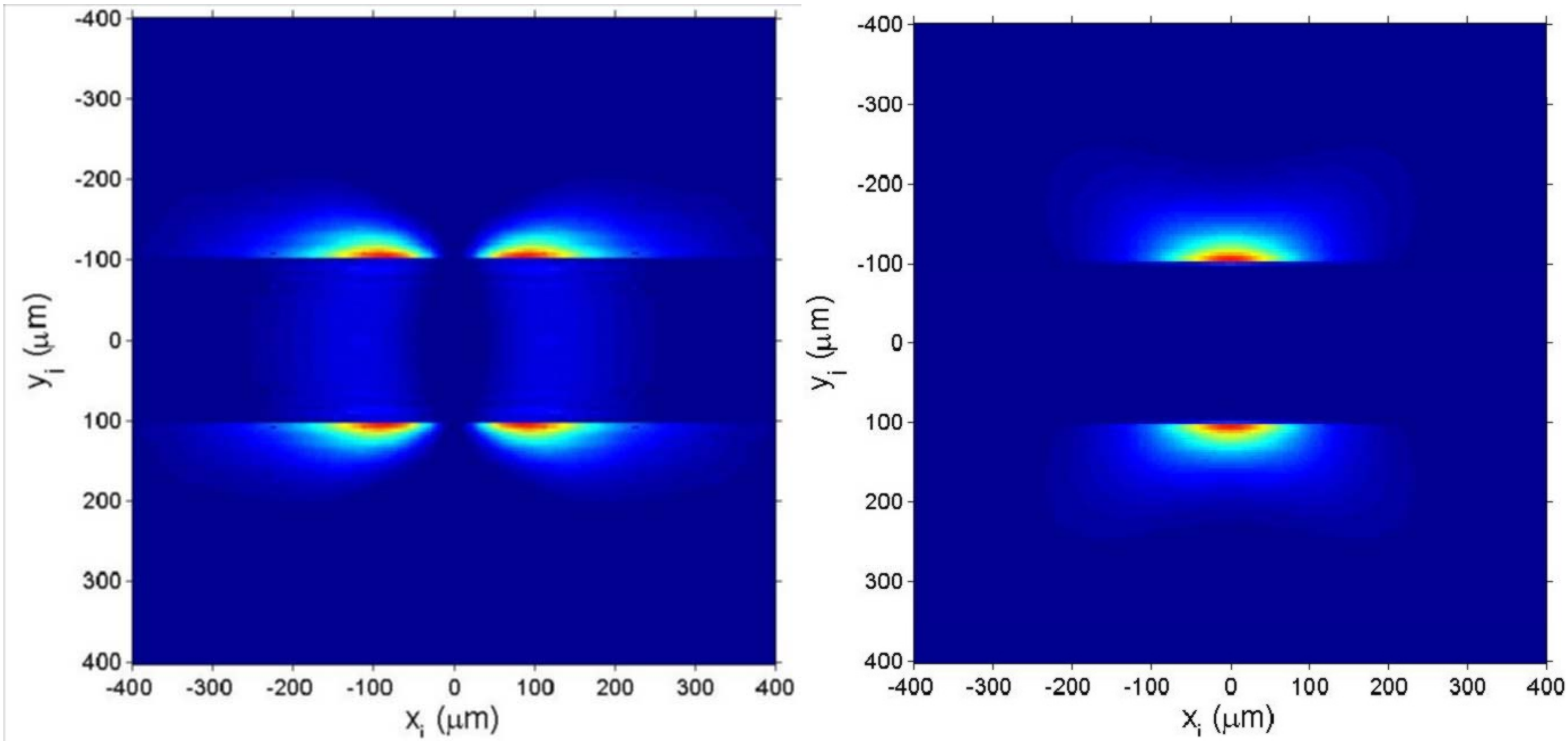
Dependence of x-compon[redacted]age on impact parameter



ODR imaging of a slit



PSF – x, PSF-y for slit



Conclusion

- 1.The model developed allows to obtain 2D- and 1D-distributions of an ODR intensity on detector as well as distributions of both polarized components for beam with finite transversal sizes.
- 2.The ODR distribution from single electron (point spread function, PSF) is defined by impact parameter h only (if $h >> \lambda$).
- 3.Polorized components of PSF may provide higher spatial sensitivity in contrast with total PSF.
- 4.The deep shape of polarized ODR x-component depends on a transverse beam size along target edge.
- 5.Measurements of deep "smoothing" allows to achieve a spatial resolution $\sigma_x \approx 0.2H$ (i.e. $\sigma_x \approx 10\ \mu m$ for $H \approx 50\ \mu m$)
- 6.For impact parameter $h \approx 0.1\gamma\lambda / 2\pi$ there may be lost ~ 60% of a total ODR intensity measuring ODR X-component only.

Thanks for your attention!