Emission

of low-energy photons

at the ILC collider

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OUTLOOK:

- 1. Introduction
- 2. Spectrum of low-energy photons
- 3. Beam axis displacement
- 4. Conclusions

1. Introduction

• It is well-known the importance of electromagnetic processes at linear colliders.

They are source of many problems:

electron energy losses,

background and so on.

• In particular, an emission of photons by electrons in the collective electromagnetic field of a positron bunch.

- These photons with the energy E_{γ} up to 1 eV may lead to the Compton scattering with particles of the oncoming bunch resulting in production of photons with the energy $E_{\gamma} \sim 100$ GeV.
- Thus, the important problems is the calculation of THE LOW-ENERGY PHOTON RATE
- Usually emission of such photons at linear colliders is calculated using the same approach as for synchrotron radiation.

In this case emission is called **beamstrahlung**.

This approximation is good enough for photons with the energy range from MeV to several TeV.

BUT..., at the photon energy E_{γ} less than the critical energy $E_{c} \sim 100$ keV such an approximation is invalid.

We use the following notations:

 E_e (E_p) is the energy,

 N_e (N_p) is the number of particles in the electron (positron) bunch, σ_z is the longitudinal,

 σ_x and σ_y are the horizontal and vertical transverse sizes of the bunch, $\gamma_e=E_e/(m_ec^2)$ and

 $r_e = e^2/(m_e c^2)$ is the classical electron radius.

The electric **E** and magnetic **B** fields of the positron bunch are approximately equal in magnitude,

$$|\mathbf{E}| \approx |\mathbf{B}| \sim \frac{eN_p}{(\sigma_x + \sigma_y)\sigma_z}$$
 (1)

In such fields the electron moves around a circumference of radius

$$\rho \sim \frac{\gamma_e m_e c^2}{eB}$$

and gets the deflection angle

$$\theta_e \sim \frac{\sigma_z}{\rho}$$

or

$$\theta_e \sim \frac{\eta}{\gamma_e},$$
 (2)

where the important parameter

$$\eta = \frac{r_e N_p}{\sigma_x + \sigma_y} \tag{3}$$

is introduced.

Let us consider emission of photons in the angle interval up to θ_e . The corresponding coherence length is

$$l_{\mathsf{coh}}(E_{\gamma}) \sim \frac{4\gamma_e^2 \, \hbar c}{(1 + \gamma_e^2 \theta_e^2) E_{\gamma}}.$$
 (4)

The critical photon energy $E_{\rm C}$ corresponds to the case when the coherence length is of the order of the positron bunch length

$$l_{\mathsf{coh}}(E_{\mathsf{C}}) \sim \sigma_z \,, \tag{5}$$

which leads to

$$E_{\mathsf{C}} = \frac{4\gamma_e^2 \hbar c}{(1+\eta^2)\sigma_z}.$$
 (6)

- When the photon energy is less then E_c , the number of the emitted photons is determined by coherent radiation from the electron trajectory in the whole oncoming bunch.
- On the contrary, the beamstrahlung approach is valid for the case, when the coherence length is small in comparison with the bunch length.

If η is small, $\eta \ll 1$, the number of photons dN_{γ} emitted per one bunch crossing is proportional to the number of electrons and to the square number of positrons:

$$dN_{\gamma} \propto N_e N_p^2 \frac{dE_{\gamma}}{E_{\gamma}} . {7}$$

It is **coherent bremsstrahlung** (CBS). It **differs substantially** from the beamstrahlung — see Refs.

I.F. Ginzburg, G.L. Kotkin, S.I. Polityko and V.G. Serbo,

Phys. Rev. Letters, 68 (1992) 788;

Phys. Letters, B 286 (1992) 392;

Yadernaya Fizika , 55, (1992) 3310 and 3324

For the ILC collider

Table 1: Parameters of the ILC collider

E_e (GeV)	N	$\sigma_z~(\mu { m m})$	σ_x (nm)	σ_y (nm)
250	$2 \cdot 10^{10}$	300	640	5.7

the parameter η is large,

$$\eta = 87 \tag{8}$$

and the critical energy is

$$\mathbf{E_c} = 83 \text{ keV}. \tag{9}$$

(At the KEKB collider the parameter $\eta = 2$ and $E_c = 6$ keV.)

Therefore, at the photon energy $E_{\gamma} < E_c$ one can not use formulae related to beamstrahlung or CBS.

In Ref.

N.F. Shulga, D.N. Tyutyunik. JETP Lett. 78 (2003) 1212

it was pointed out that **the adequate approach** to calculation of emission in this range of the photon energy can based on **the approximation of classical currents**.

Our assumptions:

- (i) We consider emission of photons by electrons;
- (ii) Electron and positron bunches perform the head-on collisions;
- (iii) Transverse distribution of particles in the positron bunch does not change during collisions;
- (iv) Bunches in the interacting region have Gaussian particle distributions with the transverse density $n(\varrho)$ of the form

$$n(\boldsymbol{\varrho}) = \int_{-\infty}^{+\infty} n(\boldsymbol{\varrho}, z) dz = \frac{N}{2\pi\sigma_x \sigma_y} \exp\left[-\frac{\varrho_x^2}{2\sigma_x^2} - \frac{\varrho_y^2}{2\sigma_y^2}\right]. \tag{10}$$

2. Spectrum of low-energy photons

Let an electron with the impact parameter ϱ deflects in the electromagnetic field of the positron bunch and acquires the transverse momentum \mathbf{p}_{\perp} :

$$\xi(\varrho) = \frac{\mathbf{p}_{\perp}}{2m_e c} = -r_e \int \frac{\varrho - \varrho'}{(\varrho - \varrho')^2} n_p(\varrho') d^2 \varrho'. \tag{11}$$

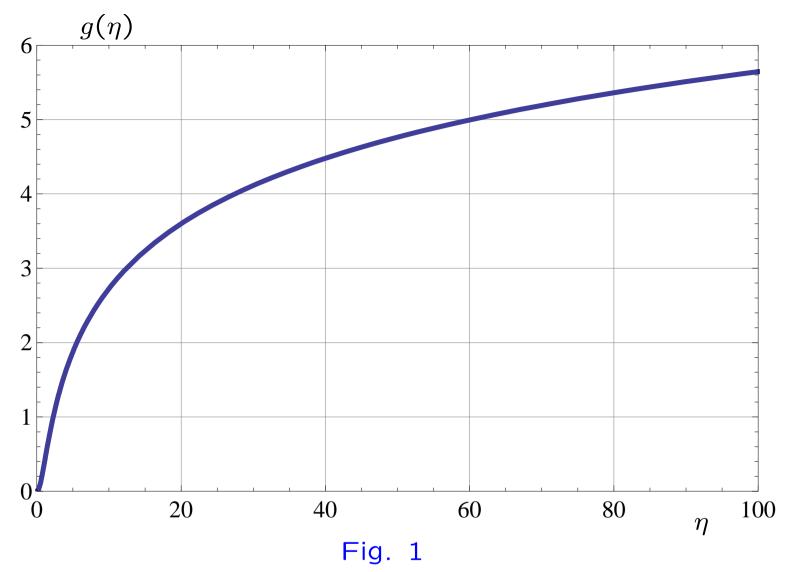
A probability to emit low-energy photon is

$$dw = \alpha F(\xi) \frac{dE_{\gamma}}{E_{\gamma}}, \qquad (12)$$

$$F(\xi) = \frac{2}{\pi} \left[\frac{2\xi^2 + 1}{\xi\sqrt{\xi^2 + 1}} \ln(\xi + \sqrt{\xi^2 + 1}) - 1 \right]. \tag{13}$$

The number of low-energy photons, **produced by an electron bunch per one bunch crossing is then**

$$dN_{\gamma} = N_e \alpha g \frac{dE_{\gamma}}{E_{\gamma}}, \quad g = \langle F(\xi) \rangle = \int F(\xi) \frac{n_e(\varrho)}{N_e} d^2\varrho.$$
 (14)



The function $g(\eta)$ for identical beams

For **small** parameter $\eta \ll 1$

$$g = \frac{8}{3\pi} \langle \xi^2 \rangle \approx 0.5 \,\eta^2 \propto N_p^2 \,. \tag{15}$$

This result coincides with that obtained in the CBS approximation.

For large parameter $\eta \gg 1$

$$g = \frac{2}{\pi} \left[\langle \ln (4\xi^2) \rangle - 1 \right] \approx \frac{2}{\pi} \left(\ln \eta^2 - 0.34 \right),$$
 (16)

in this case the dependence on N_p becomes weaker:

$$g \propto \ln\left(N_p^2\right)$$
. (17)

We have calculated that for ILC

$$g = 5.5$$
. (18)

Let us compare this result with the corresponding one in the beamstrahlung case.

A rough estimation of the beamstralung rate can be performed using the known formulae for the synchrotron radiation. The electron moving through the uniform magnetic field $B_{\rm ext}$ emits $dn_{\gamma}^{\rm SR}$ photons in the time Δt

$$dn_{\gamma}^{SR} = \frac{dI}{E_{\gamma}} \Delta t = \frac{4}{9} \alpha F(E_{\gamma}/E_c^{SR}) \frac{dE_{\gamma}}{E_{\gamma}} \frac{eB_{\text{ext}}}{m_e c} \Delta t .$$
 (19)

Here dI is the classical synchrotron radiation intensity per unit time, E_c^{SR} is the critical synchrotron energy and $F(y) \approx 1.33 \, y^{1/3}$ at $y \ll 1$.

In our case

$$E_c^{\text{SR}} \sim \frac{15}{12} \hbar \frac{e^2 N_p \gamma_e^2}{m_e c \sigma_z (\sigma_x + \sigma_y)},$$
 (20)

and $\Delta t \sim \sigma_z/c$.

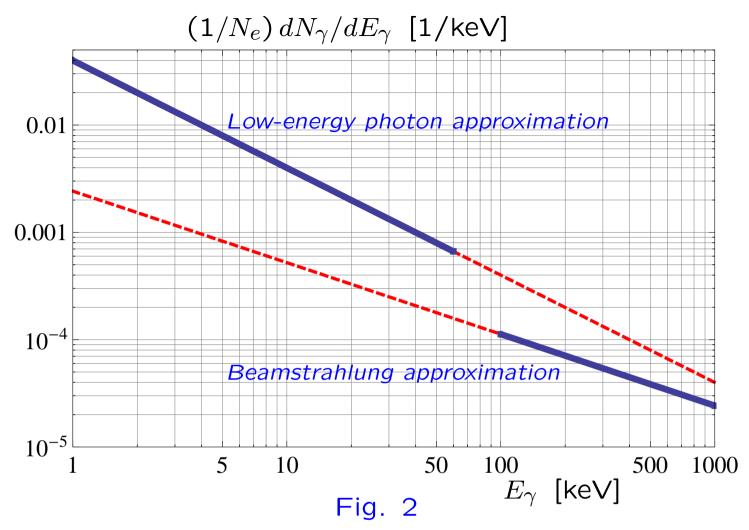
As a result, we obtain the estimate for the number of beamsttrahlung photns emitted per one bunch collision

$$dN_{\gamma}^{BS} \approx N_e \, \alpha \, \eta \, \left(\frac{E_{\gamma}}{E_{\rm c}^{\rm SR}}\right)^{1/3} \, \frac{dE_{\gamma}}{E_{\gamma}}$$
 (21)

with $E_c^{SR} = 18$ GeV for ILC.

We compare our estimation with the result obtained by V. Telnov using his simulation code for the processes on linear colliders. There is a good agreement!

For the photon energy less then $E_c=83$ keV, the ration $dN_{\gamma}^{\rm BS}/dN_{\gamma}$ is considerable less than unit (see Fig. 2



Spectrum of photons at ILC produced by one electron in the low-energy photon approximation and in beamstrahlung approximation

2. Beam axis displacement

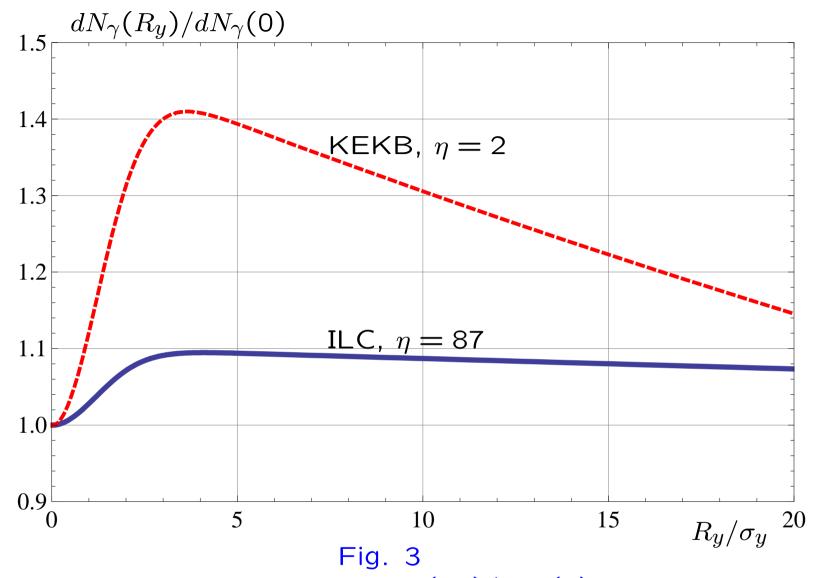
If the electron bunch axis is shifted in the vertical direction by a distance R_y , the luminosity decreases very quickly

$$L(R_y) = L(0) \exp\left(-\frac{R_y^2}{4\sigma_y^2}\right), \qquad (22)$$

where L(0) is the luminosity without displacement.

On the contrary, the number of low-energy photons increases by about 10 % at $R_y = 4 \sigma_y$.

After that, the rate of photons decreases, but even at $R_y = 20 \sigma_y$ it is larger by 7 % then at $R_y = 0$ (see Fig. 3).



Normalized photon emission rate $dN_{\gamma}(R_y)/dN_{\gamma}(0)$ in dependence on the displacement R_y of the bunch axes in the vertical direction

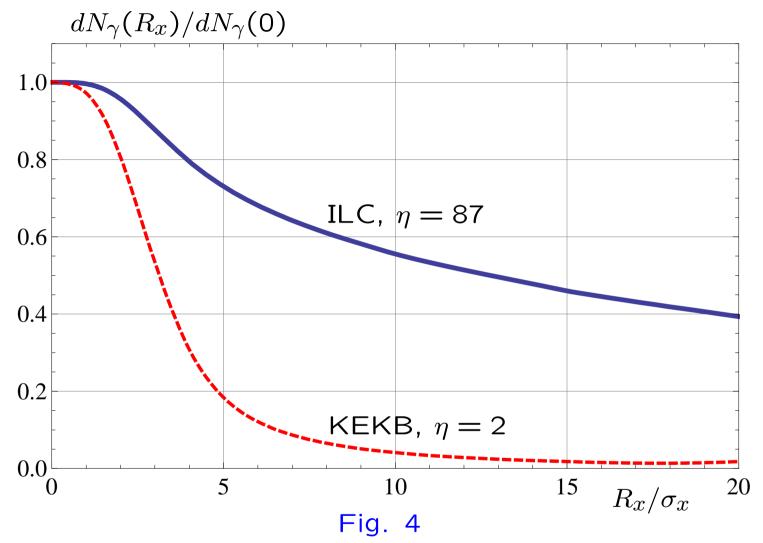
At $R_y = 0$ a considerable portion of the electrons moves in the region of small impact parameters, where the electric and magnetic fields of the positron bunch are small.

For R_y in the range of $\sigma_y^2 \ll R_y^2 \ll \sigma_x^2$ the electrons fly through a stronger electromagnetic field of the positron bunch, therefore, the number of the emitted photons increases.

Shifting the electron bunch axis in the horizontal direction, the positron bunch fields immediately become weaker and the photon rate decreases.

However, this decrease is not so strong as that of the luminosity.

For example, $dN_{\gamma}(R_x)/dN_{\gamma}(0) = 0.7$ at $R_x = 5 \sigma_x$, but the luminosity drops by three orders of magnitude.



Normalized photon emission rate $dN_{\gamma}(R_x)/dN_{\gamma}(0)$ in dependence on the displacement R_x of the bunch axes in the horizontal direction

Conclusions-I

- Electromagnetic processes at linear colliders are source of many problems related to energy losses and background
- In particular, we certainly should know photon spectrum up to energy of 1 eV
- Beamstrahlung approach for calculation the rate of photons is invalid at the photon energy E_{γ} less than 100 keV

Conclusions-II

- In this report we present results of our calculation for this region of photon energy based on approximation of classical currents
- For the ILC collider the number of photons per one electron is of the order of

$$rac{dN_{\gamma}}{N_e}pprox 5lpha rac{dE_{\gamma}}{E_{\gamma}}$$

Conclusions-III ???...

• These photons can inform us about

BUNCH SIZES JUST FROM THE INTERACTION POINT

• We find out that there is a specific dependence of dN_{γ} on the impact parameter between the beam axes. It is interesting to discuss a possibility to use this feature for

A FAST CONTROL OVER BEAM AXIS DISPLACEMENT