Coupler Short-Range Wakefield Kicks

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Wake Fest 07, 11 December 2007

Thanks to M. Dohlus; and to Z. Li, and other participants at
C. Adolphsen's Thursday morning RF meeting at SLAC

In the ILC linac rf cavities, the fundamental mode (fm) and higher mode (hm) couplers break the structure symmetry => transverse wakefields are excited even on the cavity axis

Outline:

- Numerical calculation of the transverse wakes of the three couplers (of a cavity) in a beam pipe

- Analytical estimates (for confirmation)

- Asymptotic result after several cavities (with couplers)

- Estimate of effect on ILC beam

- Conclusions

1. Numerical Calculation

Use I. Zagorodnov's 3D time domain code, ECHO3D, to find transverse wake for couplers in a beam pipe

• Reduced mesh dispersion => short bunches in long structures ok

• 3D indirect calculation of wake => no long catch-up distance

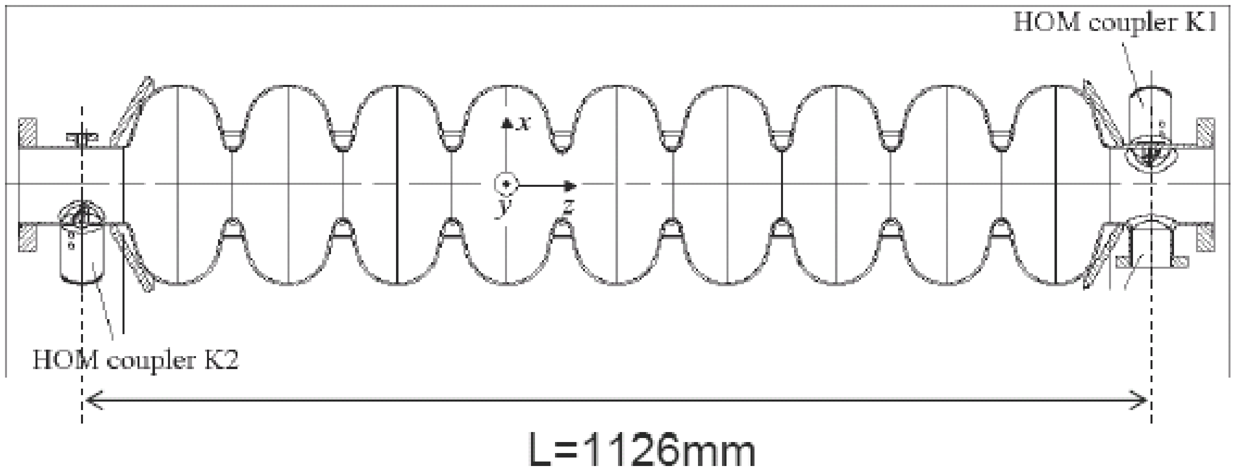
Refs:

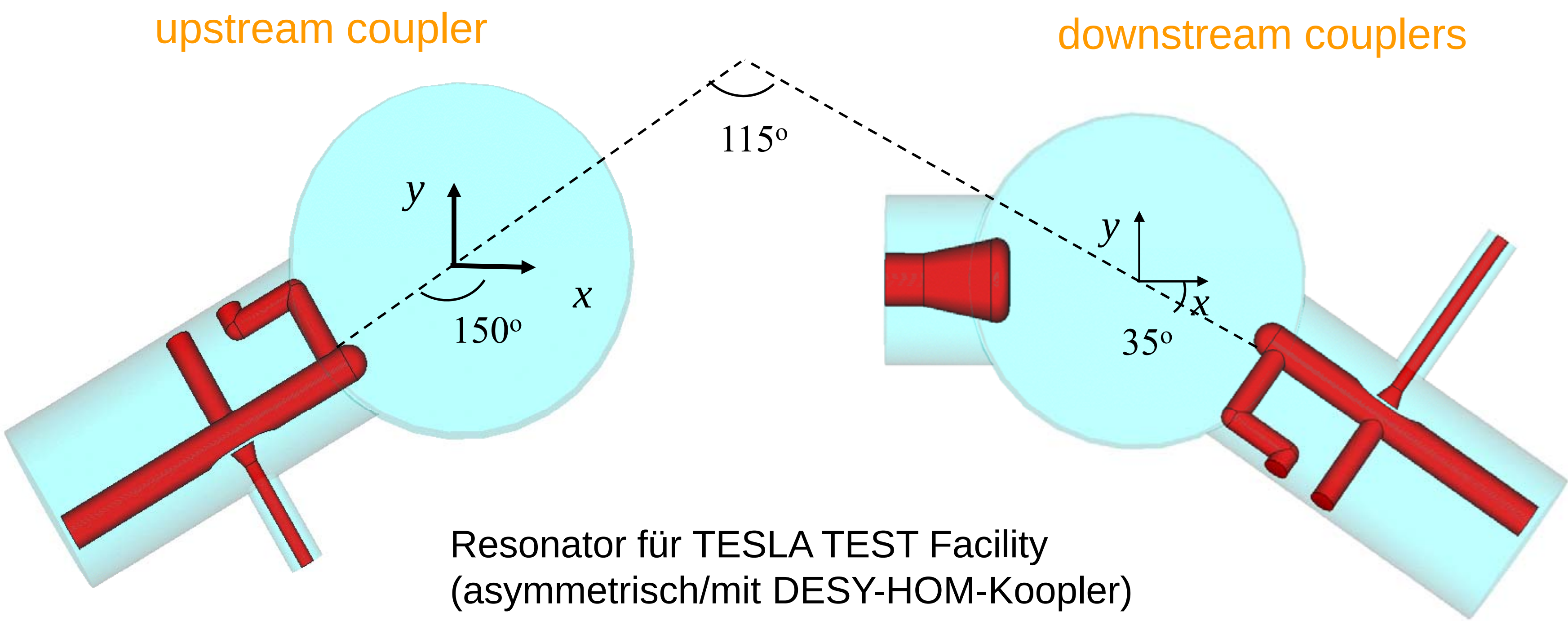
1. Zagorodnov I, Weiland T., *TE/TM Field Solver for Particle Beam Simulations without Numerical Cherenkov Radiation*// Physical Review – STAB, 8, **2005**.

2. Zagorodnov I., *Indirect Methods for Wake Potential Integration* // Physical Review -STAB, 9, **2006.**

3. Zagorodnov I, Dohlus M, "Coupler Kick," talk given at ILC Workshop, DESY, May 2007.

Geometry





Resonator für TESLA TEST Facility
(asymmetrisch/mit DESY-HOM-Koopler)
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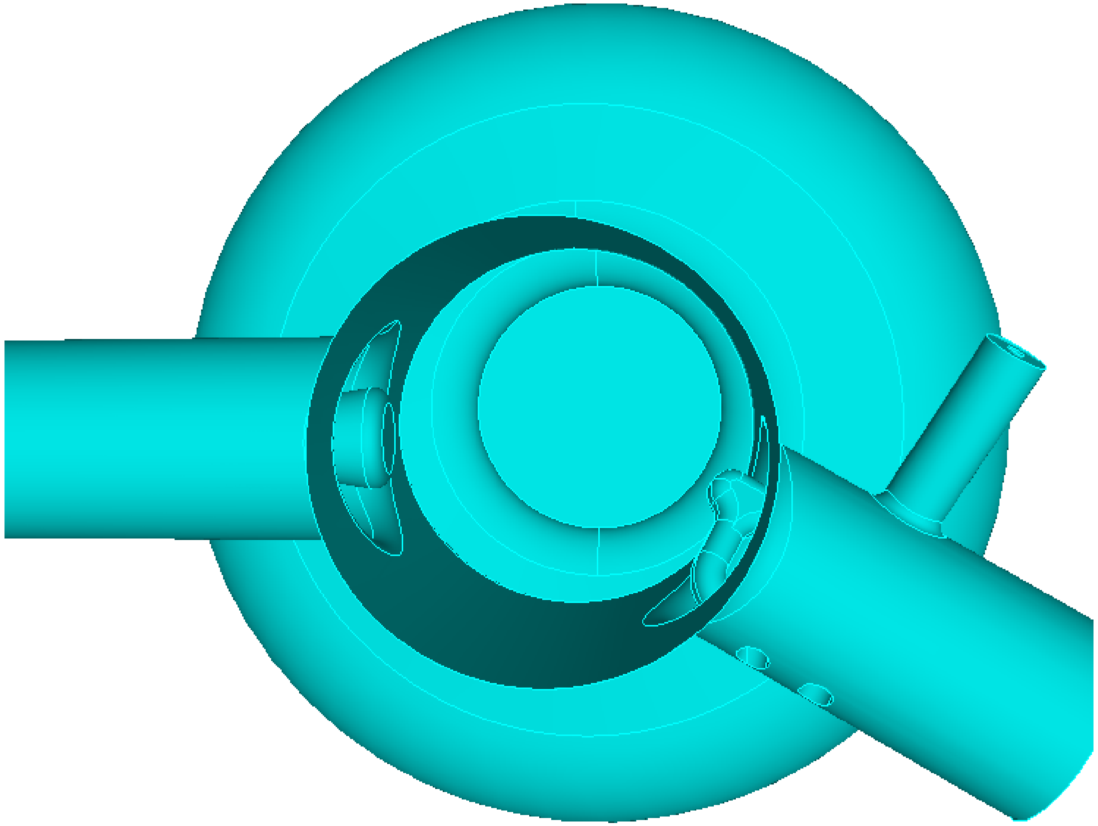
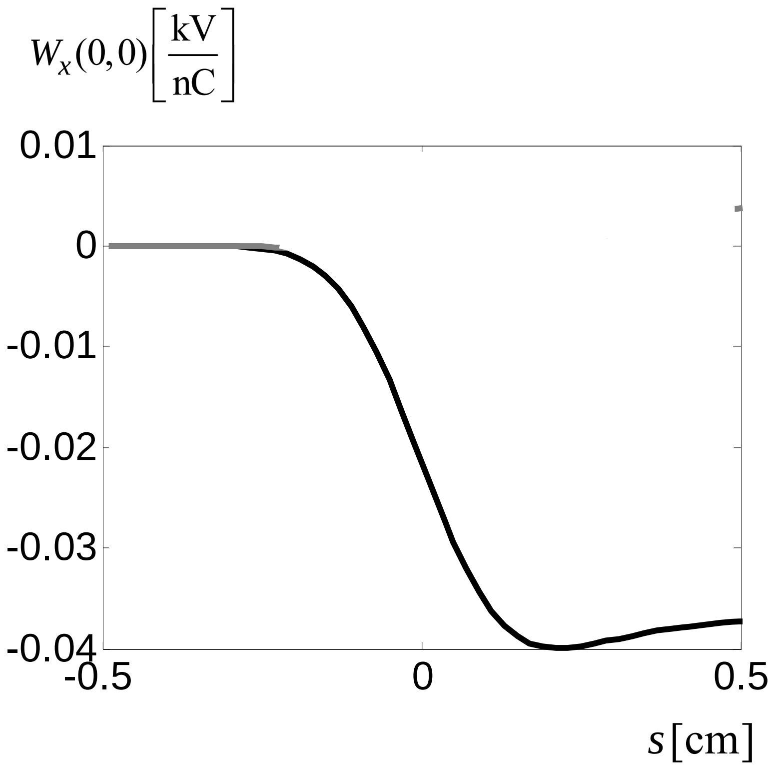
Detailed View of FM and HM Couplers

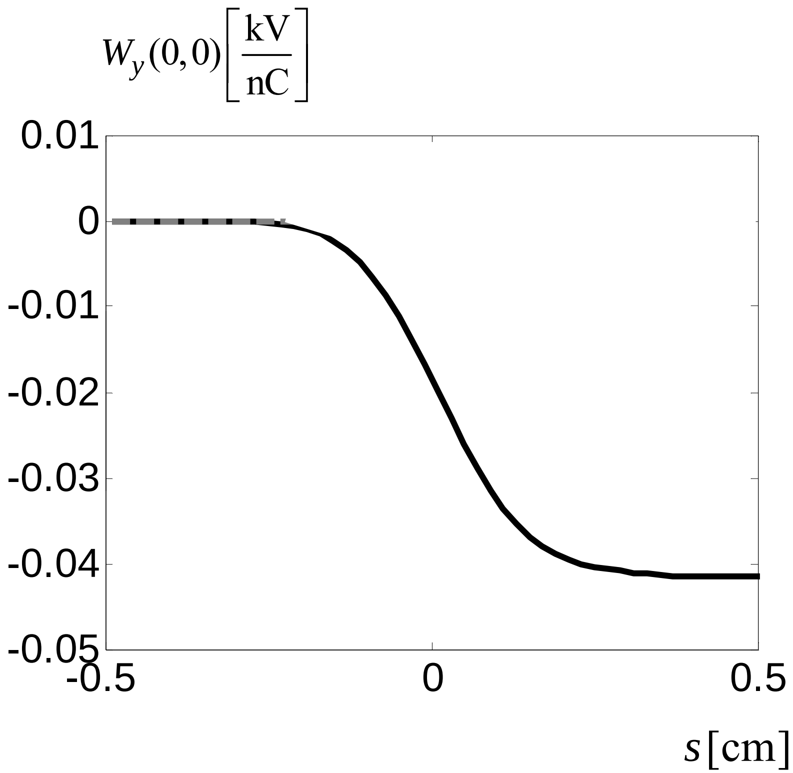


- Note complicated post-in-cavity geometry

(Zenghai Li)

ECHO3D Results





Total on-axis wake for the three couplers (in beam pipe)

- Bunch length σ_z= 1 mm
- Capacitive wake: $W(s) \sim \int \lambda_z(s)\, ds \sim \text{erf}(s)$

Notation and Definitions

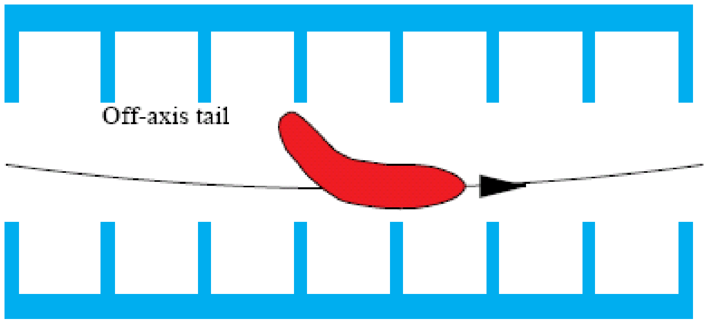
$$\lambda(s) - \text{Gaussian bunch with rms width } \sigma$$

$$k_\perp = \langle W \rangle = \int W(s)\lambda(s)ds - \text{kick factor}$$

$$k_\perp^{\text{rms}} = \left\langle (W - k_\perp)^2 \right\rangle^{0.5} = \left[\int \left(W(s) - k_\perp \right)^2 \lambda(s) ds \right]^{0.5} - \text{rms kick factor}$$

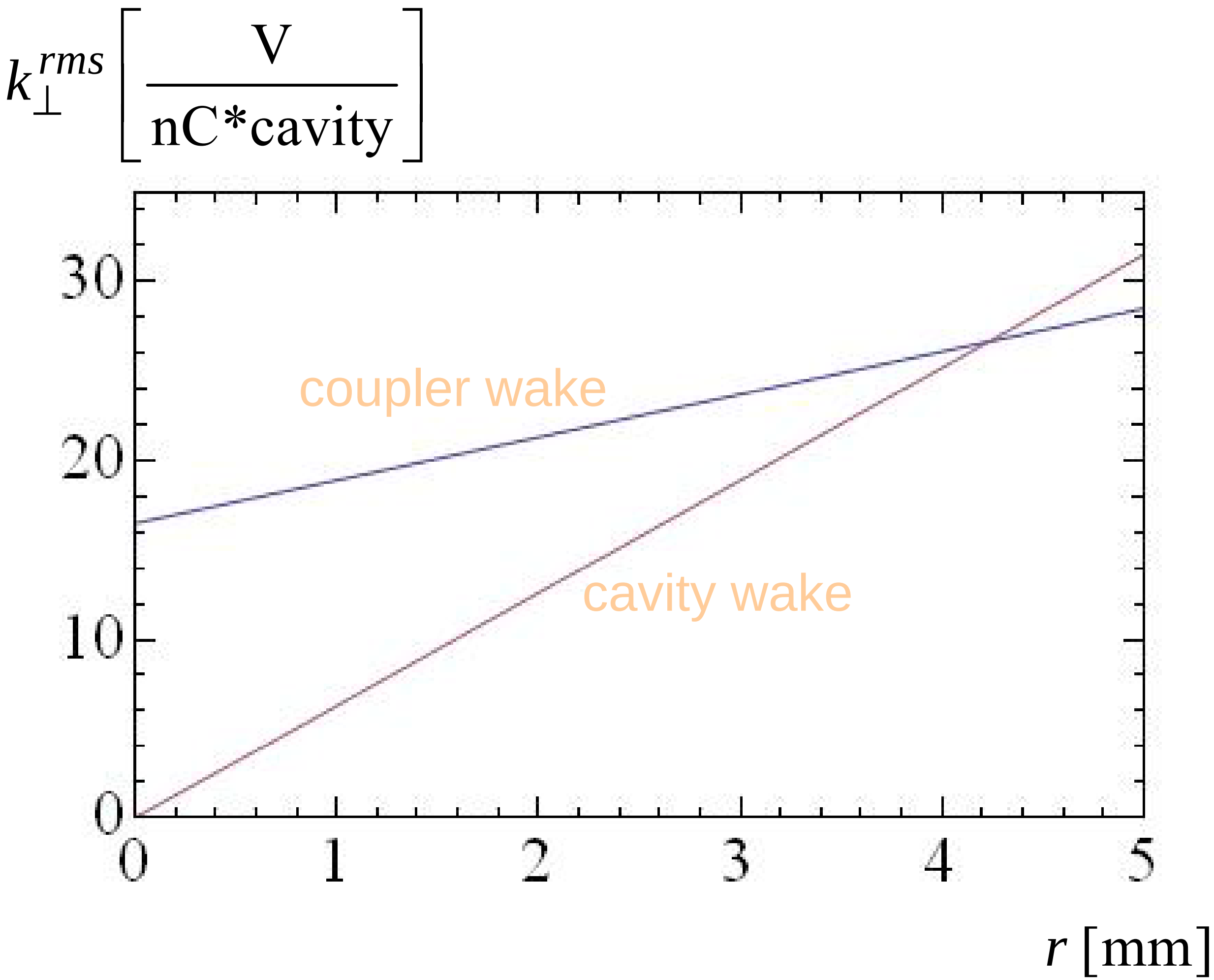
For capacitive wake $k_\perp^{rms} = k_\perp/\sqrt{3} = 0.58\, k_\perp$

It gives an estimate for the head-tail difference in the kick (banana shape)



The picture from
R. Wanzenberg,
Review of beam dynamics …,
Linac Conference, 1996

Numerical Results Summary



$\sigma = 300\ \mu\text{m}$

(coupler wake
For σ= 1 mm)

2. Analytical Approximation

• High frequency impedance of an abrupt transition in a beam pipe, e.g. an iris or the ILC couplers can be obtained using the *Optical Approximation*

• Result depends only on cross-sectional shape of object (and the beam pipe shape)

• This is a lower order effect than the *Diffraction Model* (high frequency impedance of a cavity with beam pipes) => for the ILC couplers, the cavities around the posts can be ignored

Refs:

1. Stupakov G., Bane K.L.F., Zagorodnov I., *Optical approximation in the theory of geometric impedance*, Physical Review--STAB, **10**: 054401, 2007.

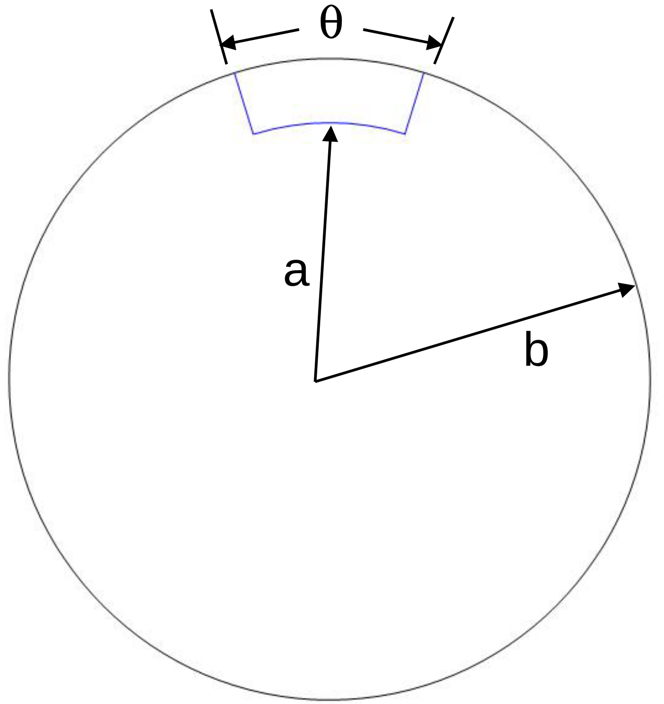
2. Bane K.L.F, Stupakov G., Zagorodnov I., *Impedance Calculations of Non-Axisymmetric Transitions Using the Optical Approximation*, Physical Review--STAB, **10**: 074401, 2007.

• In optical approx. transverse $Z \sim 1/\omega$ and wake $W \sim \int \lambda(s)\, ds$, independent of bunch length; need to find constant of proportionality

• For beam near axis at offset y: $Z_y \approx Z_{y0} + y\, Z_{y1}$; with $2\pi\omega Z_{y0} = \int_C \phi_d \mathbf{n}\cdot\nabla\phi_m dl$, where integral is over (projected) edge of coupler, and ϕ_d (ϕ_m) are solutions to Poisson's equation in beam pipe, with a dipole (monopole) source on axis

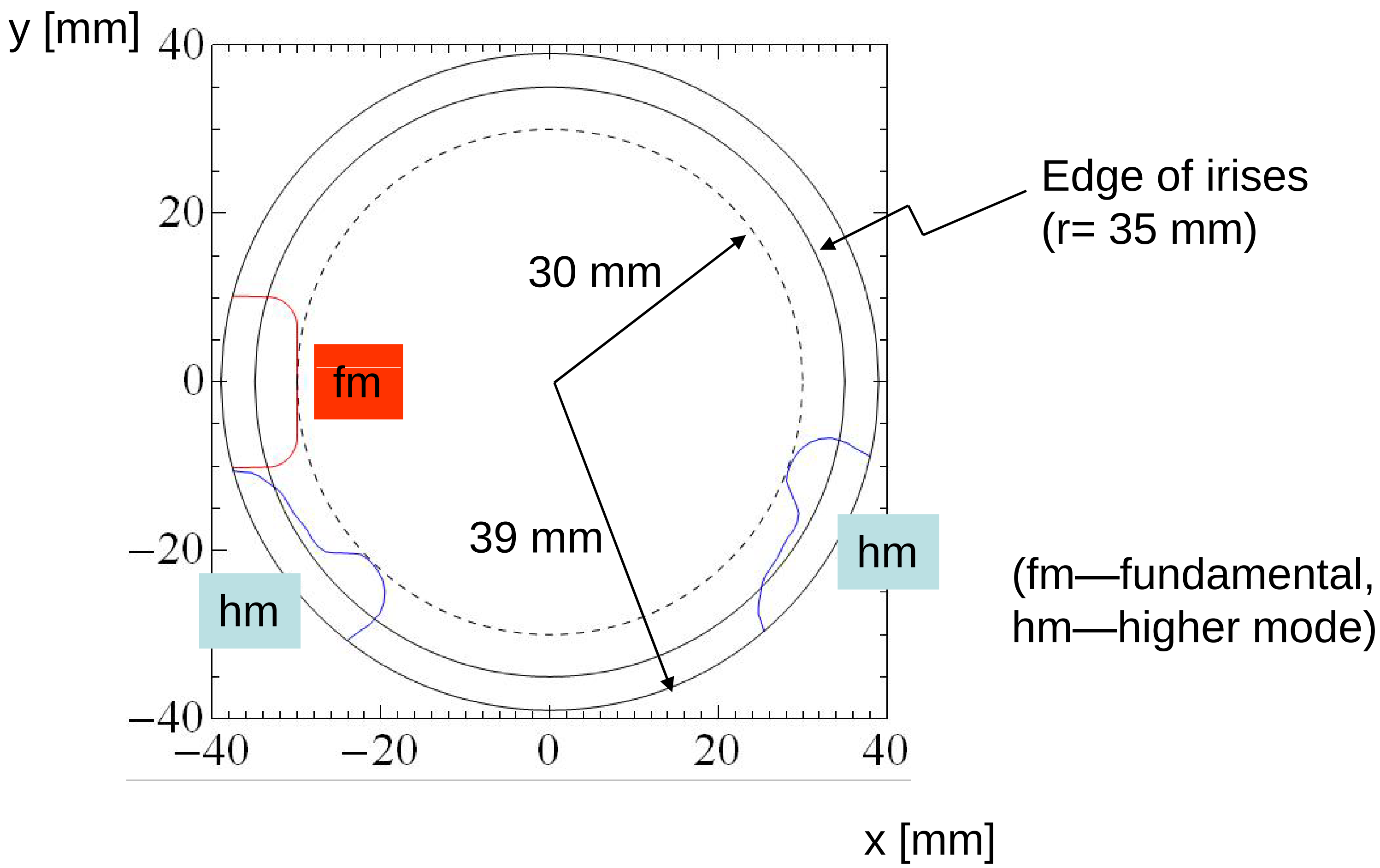
• For round beam pipe ϕ_d, ϕ_m, are simple, analytical

• for iris with $\alpha = a/b$, [a (b) minimum (maximum) radius]: $Z_{y1} = (1-\alpha^4)/\omega a^2$; for iris of angle θ: $Z_{y0} = 4(1-\alpha^2)\sin(\theta/2)/\omega\pi a$



End view of iris of angle θ in round beam pipe

End on view of coupler geometry (from downstream end)



Comparison with Numerical Results

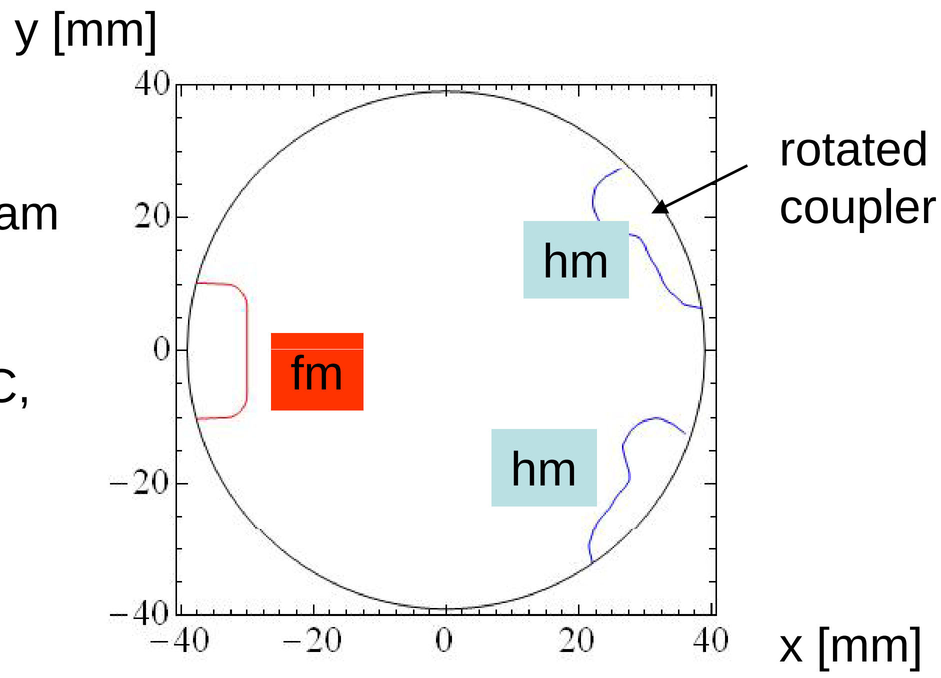
	k_x^{rms}	k_y^{rms}
Numerical	-16.5	-16.5
Analytical	-12.0	-10.0

(from plot on slide 15, Igor, May 2007)

$k_\perp^{rms}$ [V/nC] *for the three couplers in a beam pipe*

• Z. Li proposes rotating upstream coupler by 180° =>

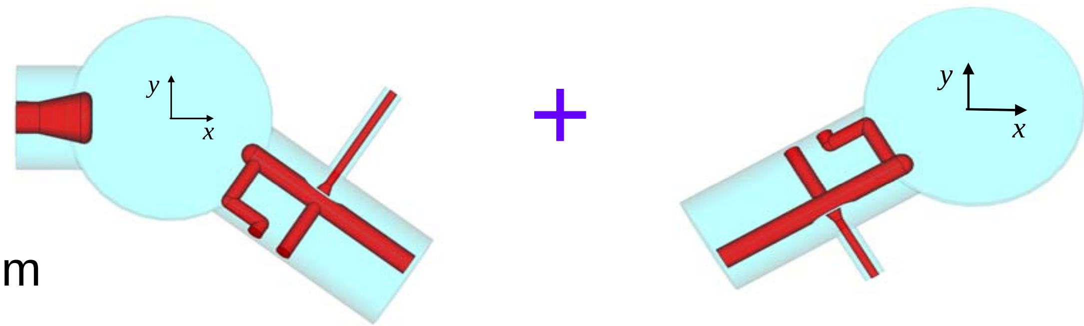
analytical result: k_x^{rms}= 3.5 V/nC, k_y^{rms}= -0.15 V/nC



3. Asymptotic Solution
A. Numerical Study

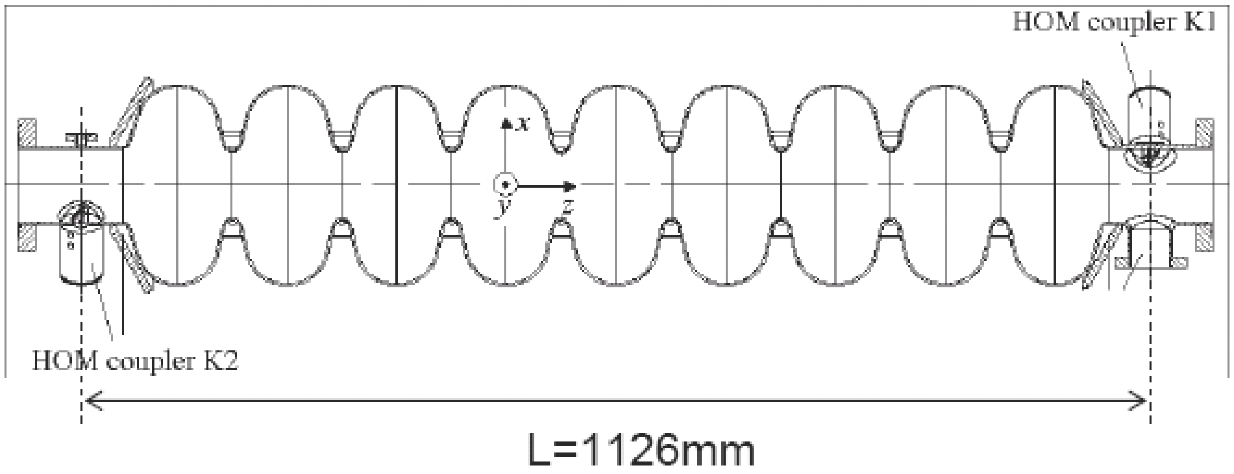
To make the problem tractable here take σ_z= 1 mm

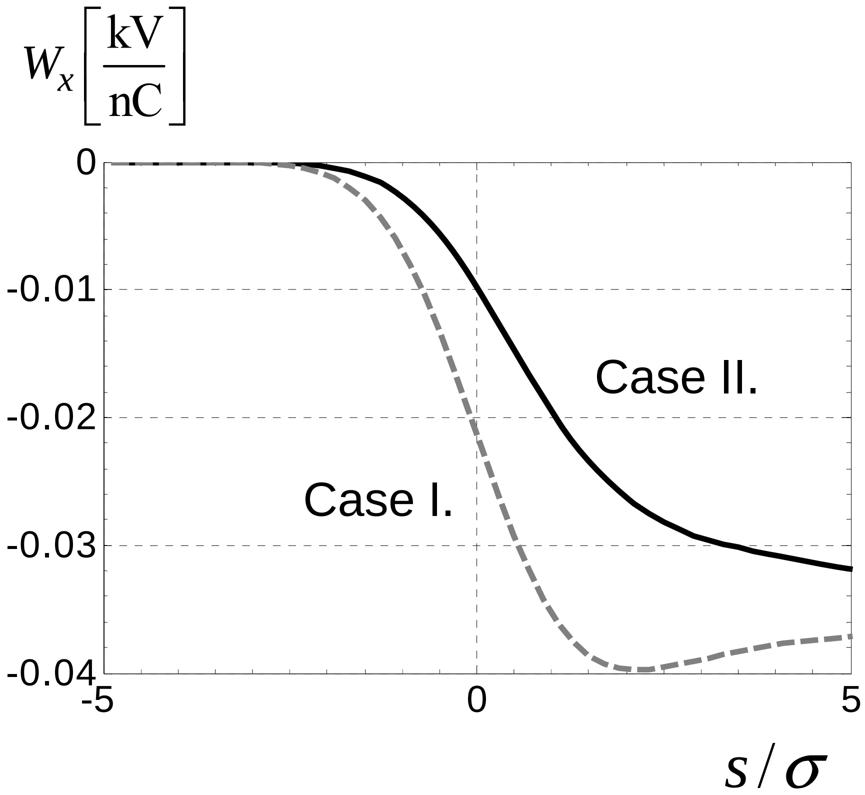
Case I. Couplers in infinite pipe of r = 39mm

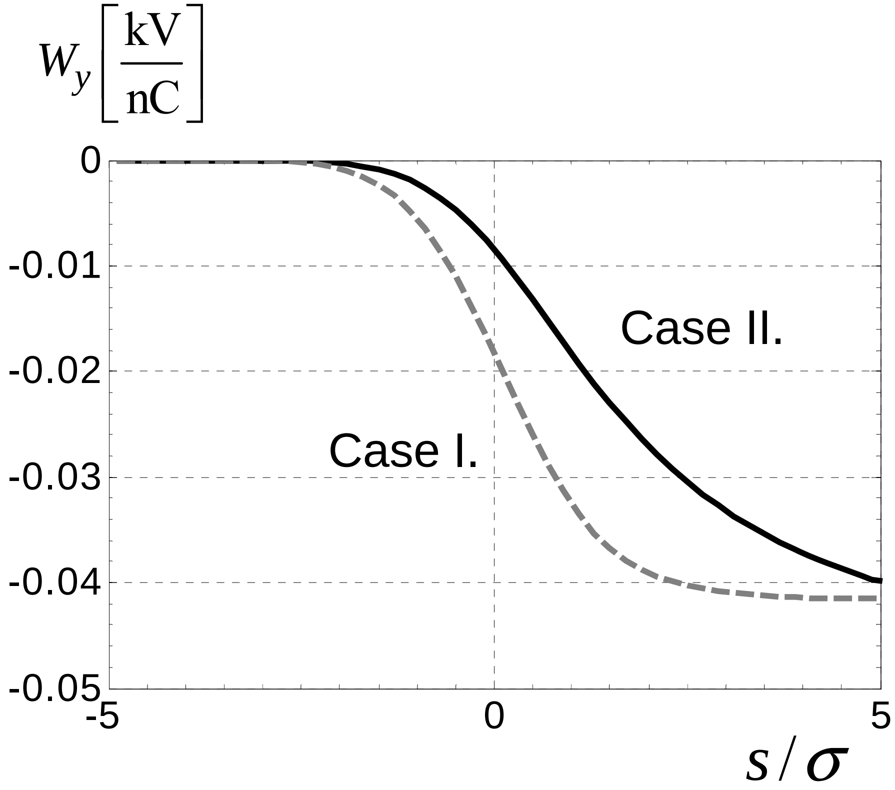


Catch-up distance:
$z \sim a^2/2\sigma_z = (35\text{ mm})^2/0.6\text{ mm} = 2\text{ m}$

Case II. Cavity with couplers in infinite pipe







Kick on the axis for Gaussian bunch with RMS width 1mm.

Case I. Couplers in infinite pipe of r=39mm

$$\mathbf{k}_\perp = \begin{pmatrix} -0.0211 \\ -0.0188 \end{pmatrix} \left[\frac{\mathbf{kV}}{\mathbf{nC}}\right]$$

Case II. Cavity with couplers in infinite pipe

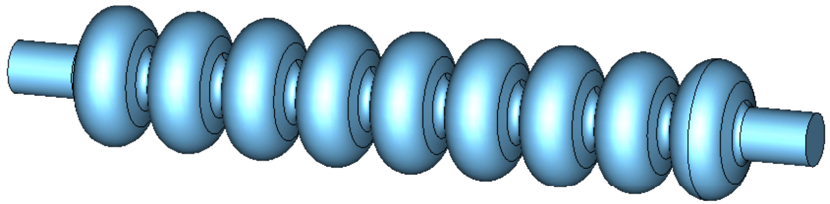
$$\mathbf{k}_\perp = \begin{pmatrix} -0.0108 \\ -0.0100 \end{pmatrix} \left[\frac{\mathbf{kV}}{\mathbf{nC}}\right]$$

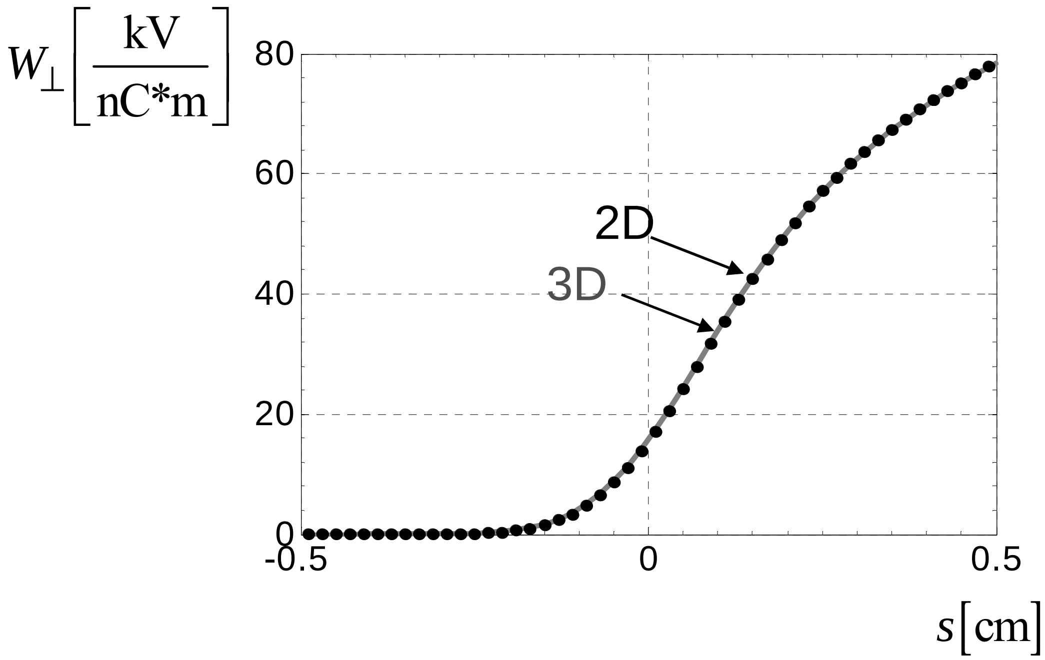
The on-axis kick is reduced by factor 2

The calculations are done with serial code ECHO.
The CPU time for the cavity with couplers is about 1 hour.
The estimated accuracy is about 5%.

Case III. Cavity alone in infinite pipe

To check accuracy we have done 3D calculations for TESLA cavity alone with the same mesh resolution and compared the result with accurate 2D simulations. The difference in dipole kick factor is ~1%.



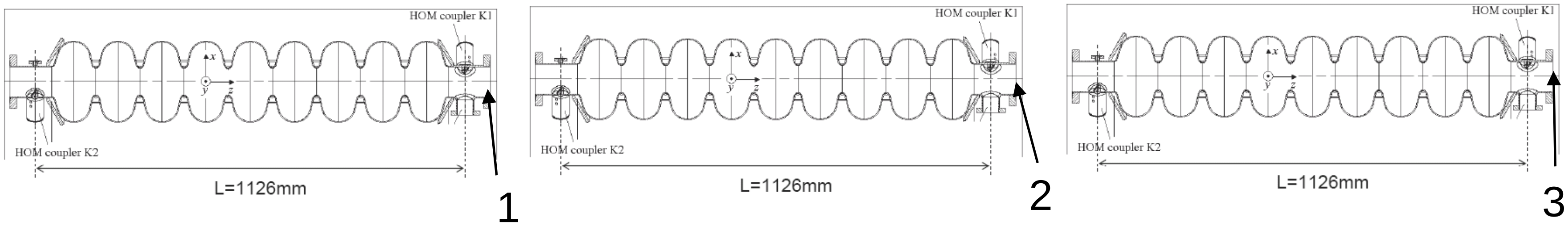


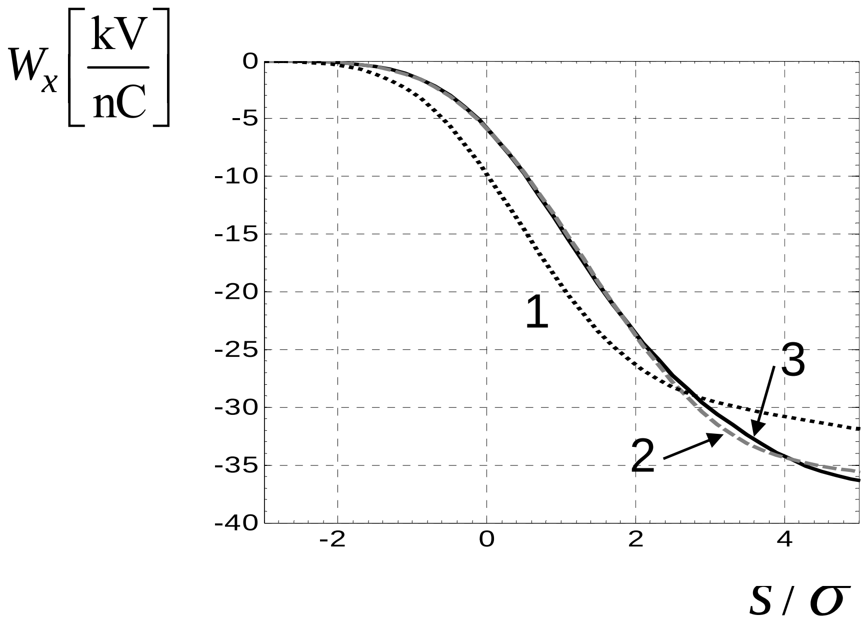
$$k_{\perp}^{2D} = 18.31\left[\frac{\text{kV}}{\text{nC*m}}\right]$$

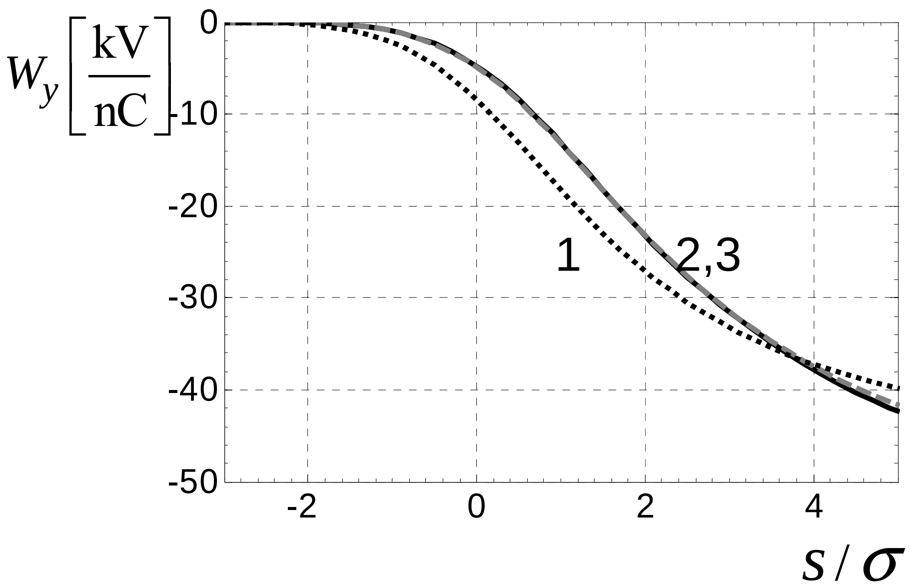
$$k_{\perp}^{3D} = 18.55\left[\frac{\text{kV}}{\text{nC*m}}\right]$$

Case IV. Several cavities with couplers

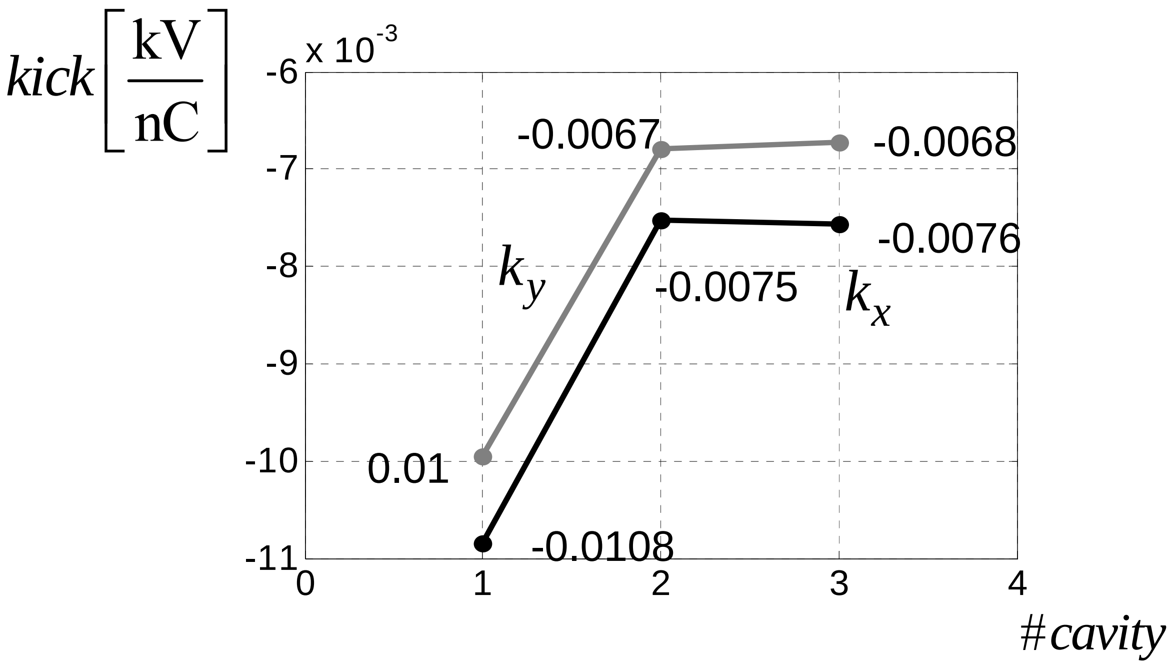
To reach the “periodic” solution we have calculated the wakes in chain from 3 cavities.







Summary



Case I. Couplers in infinite pipe of r=39mm

$$\mathbf{k}_\perp = \begin{pmatrix} -0.0211 \\ -0.0188 \end{pmatrix} \left[\frac{\mathbf{kV}}{\mathbf{nC}}\right]$$

Case II. Cavity with couplers in infinite pipe

$$\mathbf{k}_\perp = \begin{pmatrix} -0.0108 \\ -0.0100 \end{pmatrix} \left[\frac{\mathbf{kV}}{\mathbf{nC}}\right]$$

Case IV. "Periodic" solution

$$\mathbf{k}_\perp = \begin{pmatrix} -0.0076 \\ -0.0068 \end{pmatrix} \left[\frac{\mathbf{kV}}{\mathbf{nC}}\right]$$

From a simple analytical calculation, obtain k_x= -0.009 kV/nC, k_y= -0.006 kV/nC

About a factor 3 reduction compared to Case 1

4. Effect on Beam

Wake has two components: offset and slope

Slope:

from numerical results W_{av}~ 2.4 V/nC/mm/m; for periodic case should reduce a factor 2~3 to ~1 V/nC/mm/m, which is a factor 20 smaller than the cavity wake

Offset:

a constant driving term to the equation of motion generates a kind of dispersion => the closed orbit depends on longitudinal position;

model: $y'' + y/\beta^2 = e^2 N\, W(s)/E$ has solution

$$y = \beta^2 e^2 N\, W(s)/E + \text{betatron oscillation(s)}$$

Particles will perform free betatron oscillation about different centers, depending on *s*; no real wake effect

Assume eN= 3 nC, β= 100 m, $\gamma\epsilon$= 2*10^{-8} m, head-to-tail wake W= 20 V/nC => at 15 GeV beam is spread out in y by 5 σ_y, at 250 GeV by 1.6 σ_y

• Z. Li's modified coupler configuration will have ~0 coupler wake; smaller iris cavity, that shadows couplers, will also have ~0 coupler wake

5. Conclusions

Coupler wake of ILC structure will spread out the beam size by ~70%

Slope of wake is negligible compared to cavity wake

Z. Li's modified geometry will make the effects become negligible; smaller iris cavity that shadows couplers will also have ~0 coupler wake effect

Need to do:

Numerically verify that slope of wake of couplers asymptotically reduces by a factor ~2—3

Numerically calculate 3 cavity wake for σ_z= 300 μm

Simulate coupler wake effect for discrete focusing lattice