Beam-beam interactions with full polarization

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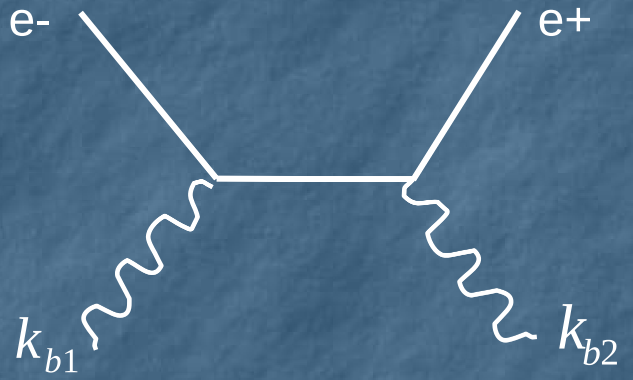
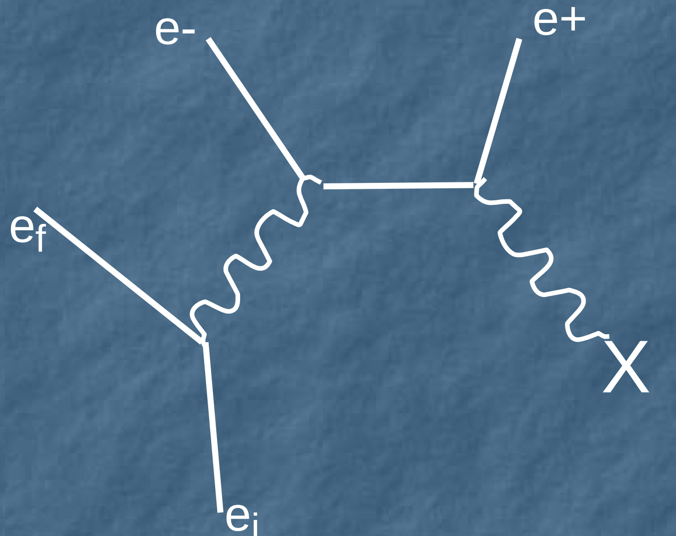
- MOTIVATION: physics requires precision treatment of polarization so implement full polarization of beam-beam processes
- review CAIN polarization treatment for beamstrahlung and incoherent processes
- Discuss implementation of Breit-Wheeler x-sect in CAIN and partial polarizations
- Implement BW x-sect with full polarizations and generate some results
- Discuss EPA dependency on virtual photon polarization
- develop expressions for virtual photon polarization

Daresbury 27.3.2008

Incoherent pair processes



- Breit-Wheeler - 2 real γ's
- no equivalent photon approximation needed
- usual helicity amplitudes
- In CAIN, by default, only circular polarisation of initial photons included – radiation and azimuthal angles neglected



- Bethe-Heitler and Landau-Lifshitz - virtual γ's
- equivalent photon approximation should be adjusted to allow for virtual photon polarization
- Need to take into account final state azimuthal angle

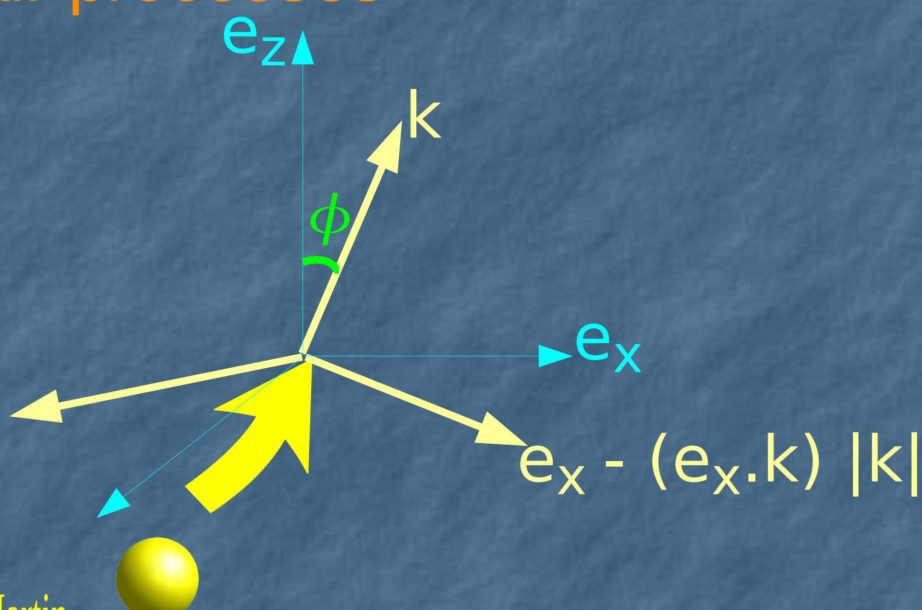
Beamstrahlung photon polarization

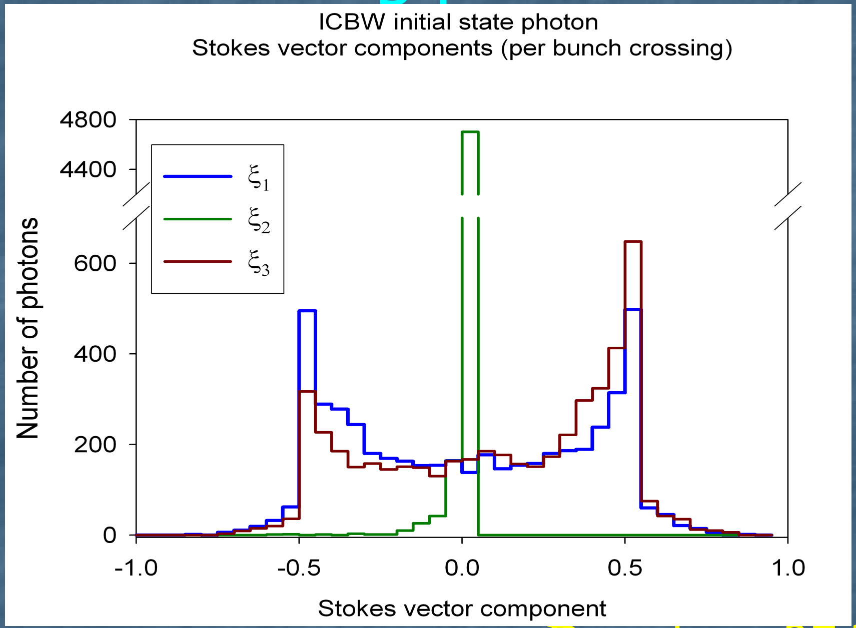
Energy spectra given by Sokolov-Ternov equation

$$dW = -i \frac{\alpha m}{\sqrt{3}\pi\gamma}\left[\int_z^\infty K_{5/3}(z)\,dz + \frac{x^2}{1-x}K_{2/3}(z)\right]dx \quad where \quad z = \frac{2B_{sch}}{3B\gamma}\frac{x}{1-x}$$

polarization first calculated with individual basis vectors then rotated to the same basis to be used as input to pair processes

A version of the Sokolov-Ternov equation with the polarizations written explicitly gives beamstrahlung polarizations





CAIN Breit Wheeler monte-carlo

$$\sigma_{BW} = \frac{m^2 r_e^2}{2\omega^2} G \quad where \quad G = \int F(\cos\theta, \phi) \; d\cos\theta \, d\phi \quad , \quad F = \frac{d\sigma_{BW}}{d\cos\theta \, d\phi}$$

1) Generate two random numbers $r_1 \epsilon (0,1)$ and $r_2 \epsilon (-1,1)$

2) Reject event if $r_1 > P$

3) If event selected then solve $F(|\cos\vartheta|) = |r2| \, G$ for $\cos\vartheta$ (sign of $\cos\vartheta$ determined from sign of r_2. Neglect φ variation)

4) Calculate event probability P in a given time interval Δt and volume. If $P > 0.1$ decrease Δt and repeat

5) Reconstruct $p_t = |p| \sin\vartheta \; (n_1 \cos\varphi + n_2 \sin\varphi)$

6) If high beamstrahlung energy then ϑ is small, but if not shouldn't neglect φ variation

BW cross-section with polarizations

- Breit-Wheeler cross-section, CAIN original:

$$\sigma_{orig} \propto 2\left(1-h+\frac{2\epsilon^2-1}{2\epsilon^4}\right)\sinh^{-1}p+\frac{p}{\epsilon}\left(3h-1-\frac{1}{\epsilon^2}\right)$$

where p = electron momentum
ϵ = electron energy
$h = \xi_2\xi_2$

full treatment due to Baier & Grozin hep-ph/0209361

$$\frac{d\sigma}{d\cos\theta\, d\phi} = \frac{\alpha^2}{4s^2x^2y^2}\sum_{ii'jj'} F^{ii'}_{jj'}\ \xi_j\xi'_{j'}\ \zeta_i\zeta'_{i'}$$

F are functions of scalar products of 4-momenta

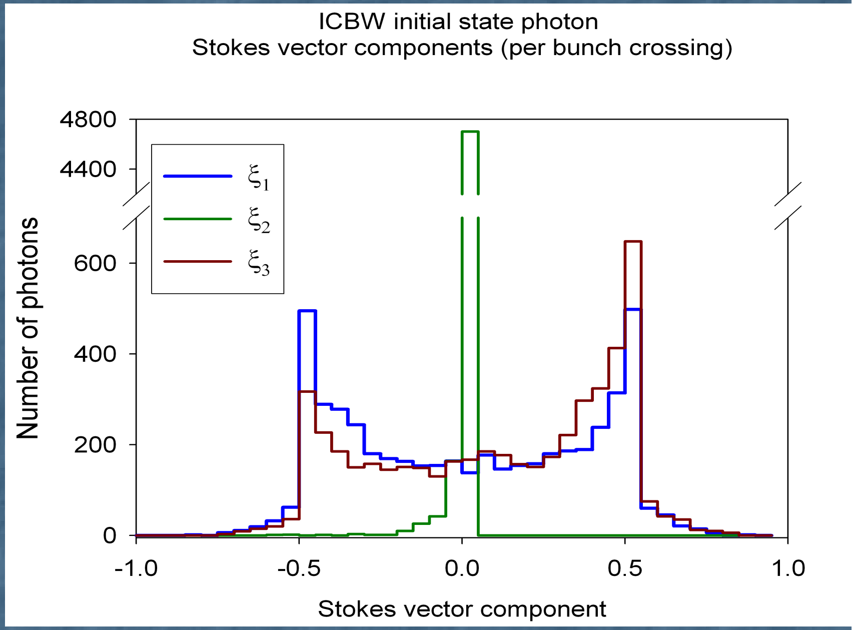
$$\sigma_{new} \propto 2\left(1-h+\frac{2}{\epsilon^2}(ha+\xi_1\xi'_1)-\frac{ha}{\epsilon^4}\right)\sinh^{-1}p+\frac{p}{\epsilon}\left(3h-1-\xi_1\xi'_1-\xi_3\xi'_3-\frac{ha}{\epsilon^2}\right)$$

where $ha = 1+\xi_3+\xi'_3+\xi_3\xi'_3$

Full expression has similar structure to original CAIN form, so can utilise existing monte-carlo methods

Final pair polarizations $\zeta^{(f)}$

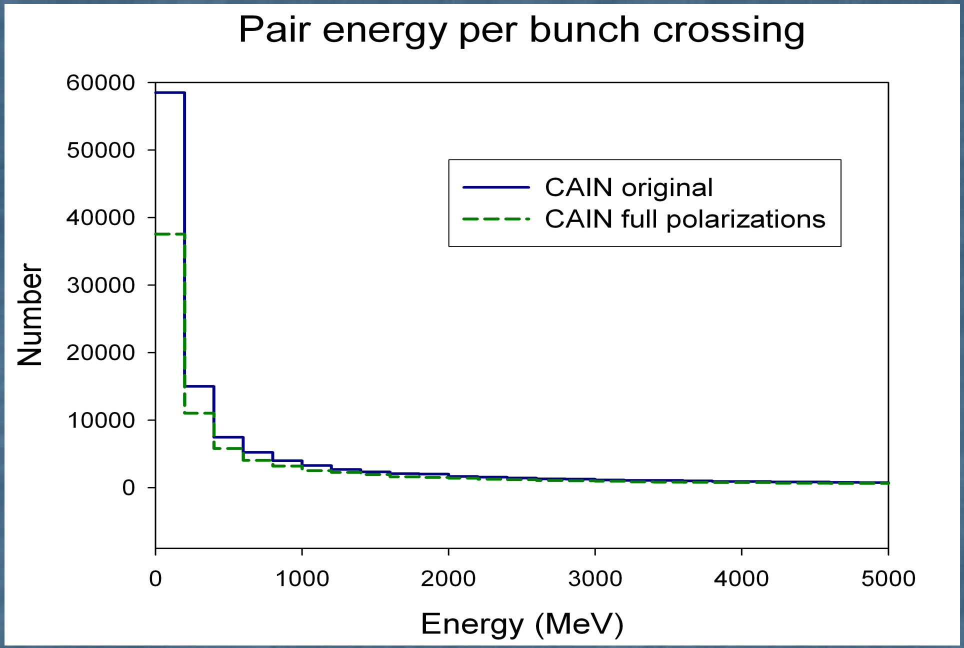
$$\zeta_i^{(f)} = \frac{1}{F} \sum_{ii'jj'} F_{jj'}^{i0} \xi_j \xi_{j'}' \quad where \quad F = \sum_{jj'} F_{jj'}^{00} \xi_j \xi_{j'}'$$

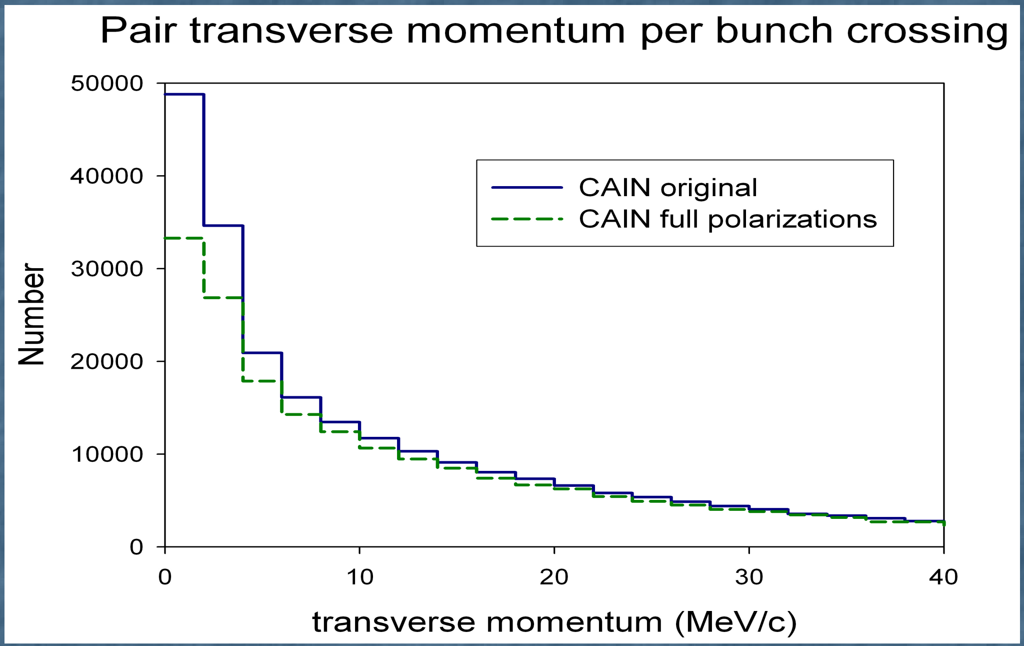


- Beamstrahlung photons have almost no circular polarization component – due to beam field having constant crossed field vectors
- 1st two components of the Breit-Wheeler pair polarization depends heavily on the photon circular polarization component, therefore ~0
- Pair polarization contained in the 3rd component

- Final **e-** ζ_1= -0.0024 ζ_2= -0.0024 ζ_3= 0.9883
- Final **e+** ζ_1= 0.0023 ζ_2= 0.0079 ζ_3= -0.987

Pair data – Energy and P_t

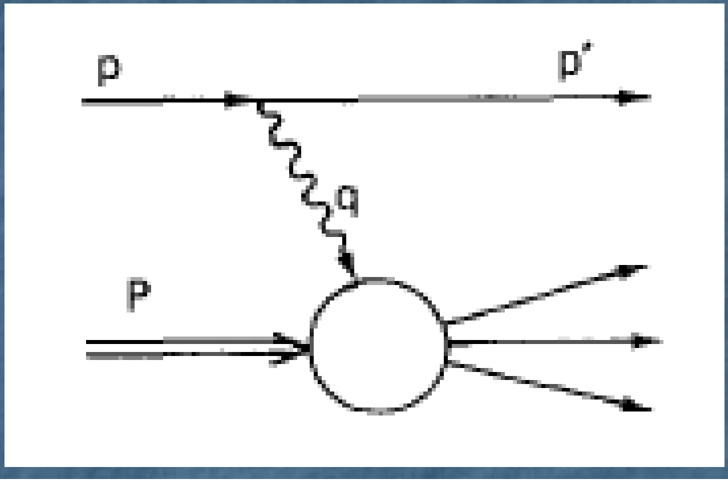




- Low energy and low Pt pairs are suppressed
- No changes at higher energies of higher P_t
- Dependency on azimuthal angle of final states not yet included in CAIN
- CAIN cross-section only terms containing products of one or two polarizations (there should be products of 3 and 4 as well)

EPA and virtual polarization

$$d\sigma = \frac{\alpha}{4\pi^2|q^2|}\left[\frac{(qP)^2 - q^2P^2}{(pP)^2 - p^2P^2}\right]^{1/2}(2\rho^{++}\sigma_T + \rho^{00}\sigma_S)\,\frac{d^3p'}{E'}$$



$$d\sigma = \sigma_\gamma(\omega)\,dn(\omega);$$

$$dn(\omega) = \int_{q^2_{min}}^{q^2_{max}} dn(\omega, q^2) = N(\omega)\,d\omega/\omega;$$

- σ_T and σ_S are the BW x-sects for transverse pol photons and scalar photons respectively
- ρ is the density matrix of the virtual photons which in general is non-diagonal and therefore the virtual photons are polarized
- If we dont detect final pair momenta then.....➔
- Otherwise... need polarised BW x-sect and the polarization state of virtual photons

$$d\sigma_{ep} = \left[\frac{d\sigma_\gamma}{d^3k_1} + \tfrac{1}{2}\,\xi\cos 2\varphi\,\frac{d(\sigma_\parallel - \sigma_\perp)}{d^3k_1}\right]d^3k_1\,dn(\omega, q^2)\,\frac{d\varphi}{2\pi}$$

Budnev et al Phys Rep 15(4) 181-282 (1975)

- **k_1 is the 3-momenta of one of the secondaries**
- **φ is the azimuthal angle of k_1 relative to the (p,p') plane**
- **ξ is the virtual photon polarization**

Virtual photon polarization I

Spectral component of bunch electric field as a function of transverse position

$$E_\omega^{x,y} = -\frac{ie}{\pi v}\iint \frac{q_{x,y}}{q_x^2+q_y^2} F(q)\, e^{i x q_x} e^{i y q_y}\, dq_x\, dq_y$$

where the form factor is

$$F(q) = N\exp\left[-\frac{1}{2}(q_x\sigma_x)^2 - \frac{1}{2}(q_y\sigma_y)^2\right]$$

and the polarization vector of virtual photons

$$e_{x,y} = \frac{E_\omega^{x,y}}{\left|E_\omega^{x,y}\right|}$$

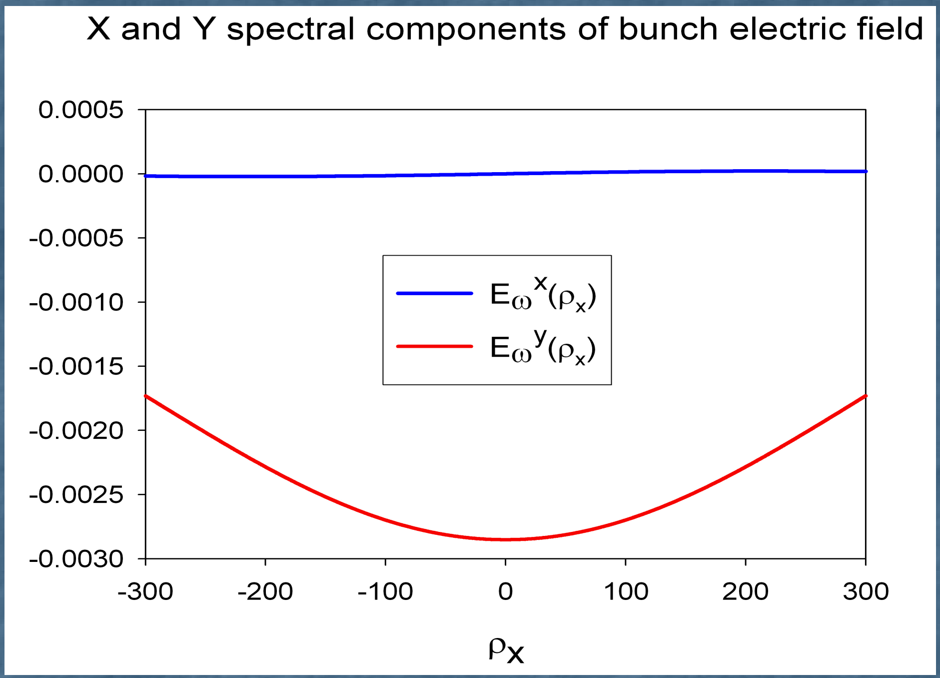
see Engel, Schiller & Serbo Z Phys C 71, 665 (1996)

integration is performed by expanding in a taylor series and using the limit $\sigma_x >> \sigma_y$ for flat beams

$$E_\omega^x = -i\,\frac{x}{\sigma_x^3}\exp\left(-x^2/2\sigma_x^2\right)\left[\sigma_y\exp\left(-y^2/2\sigma_y^2\right) + \sqrt{\frac{\pi}{2}}\,y\,Erf\left(y/\sqrt{2}\sigma_y\right)\right]$$

and similar for E_ω^y

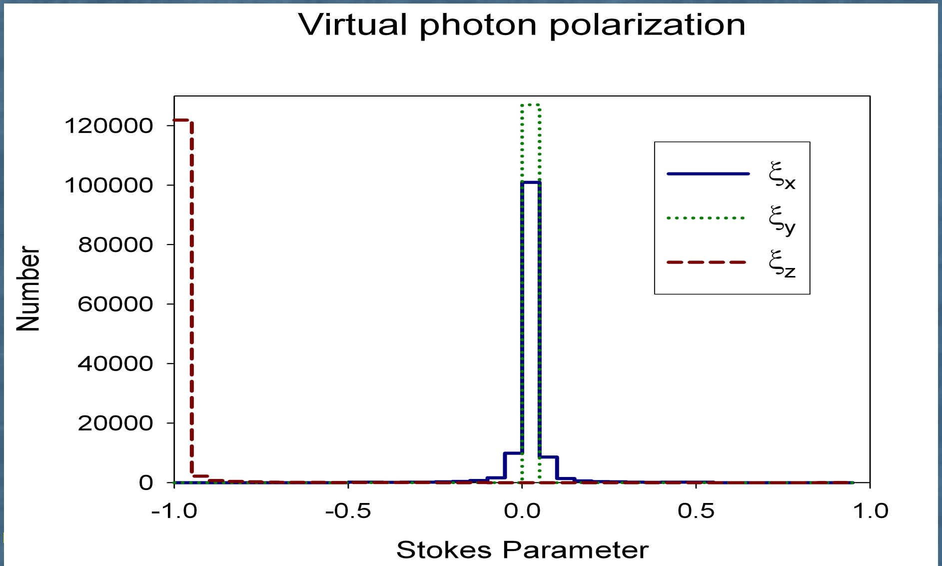
Virtual photon polarization II



Magnitude of y component of spectral electric field is much greater than x component. Has consequences for stokes parameters since

$$\xi_1 = \hat{E}_\omega^x \hat{E}_\omega^y * + \hat{E}_\omega^y \hat{E}_\omega^x * \quad ; \quad \xi_3 = \hat{E}_\omega^x \hat{E}_\omega^x * - \hat{E}_\omega^y \hat{E}_\omega^y *$$

$$\xi_2 = \Im\left[\hat{E}_\omega^y \hat{E}_\omega^x * - \hat{E}_\omega^x \hat{E}_\omega^y *\right] = 0$$



No circular polarization **BUT** processes occur in bunches undergoing pinch effect and other disruption, could use a more realistic form factor for the bunch field

Summary and things to do

- Investigated present treatment of polarization in CAIN and developed expressions for BW x-sect with full polarizations
- CAIN modified, using present monte-carlo scheme to include Breit-Wheeler x-sect with full polarization
- Compared to CAIN default (BW with circular polarization only) there is a suppression of low energy pairs when we account for full polarizations
- Circular polarization of initial states very low (beam field is a constant crossed field) consequently final states almost completely depolarised
- Expressions for virtual polarization developed
- CAIN needs proper treatment of azimuthal angle in order to implement virtual photon polarization