Bunch field effect on Beam-beam processes

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- MOTIVATION: Bunch field strength (ILC default parameters) is chosen to be not too large to the extent that 1st order external field processes (coherent pair production) is limited..... BUT, 2nd (and higher) order external field processes don't (necessarily) have the same dependence on the external field strength – they need to be investigated
- Discuss EPA as regards the 4-fermion process modified by bunch fields
- Derive Sokolov-Ternov equation using Dirac equation solutions in external field to determine approximations
- Describe analytic calculation of 2nd order coherent processes

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Solution of Dirac equation in beam field A^e

$$[(p-eA^e)^2-m^2-\frac{ie}{2F^e_{\mu\nu}}\sigma^{\mu\nu}]\psi_V(x,p)=0$$

- Look for a solution of the form: $\psi_V(x,p)=u_s(p)F(\phi)$
- Substitution of the general solution for ψ_V yields a first order d.e. whose solution can be expanded in powers of k,A^e

$$\psi_V(x,p)=[1+\frac{e}{2(kp)}\not{k}\not{A}^e]\exp[F(k,A^e)]e^{-ipx}u_s(p)$$

- Now look for simplifications by physical considerations

The Volkov solution in more detail

$$\psi_V(x,p)=\left[1+\frac{e}{2(kp)}\not{k}\not{A}^e\right]\exp[F(x,p,k,A^e)]e^{-ipx}u_s(p)$$

make Fourier transform to get linear term in x

$$\int dr \exp[-i(r+\nu^2/kp)kx]F_2(p,k,A^e)$$

r term interpreted as a contribution from r external field photons (r can be -ve!)

ν^2 term is a shift in electron momentum

non-external field Dirac solution

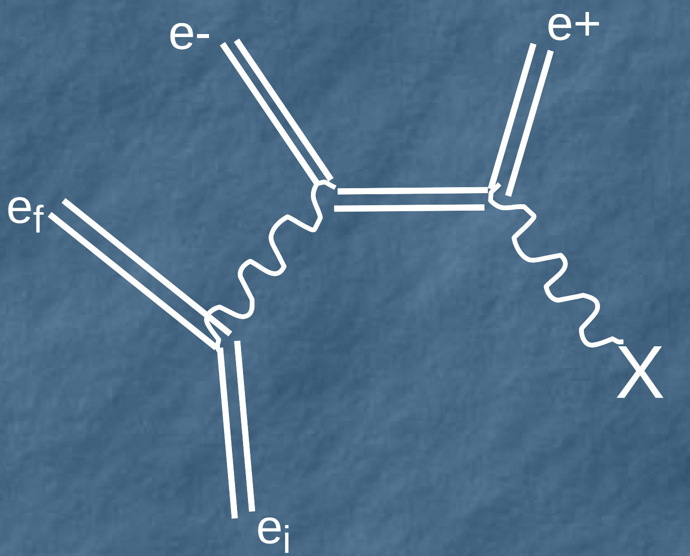
for ILC parameters

$$\frac{\omega}{m}\approx 0.06 \quad , \quad \nu^2=\frac{e|A^e|}{m}\approx 1$$

so for large E_p second term can be neglected

Volkov soln represented in Feynman diagrams by double straight lines

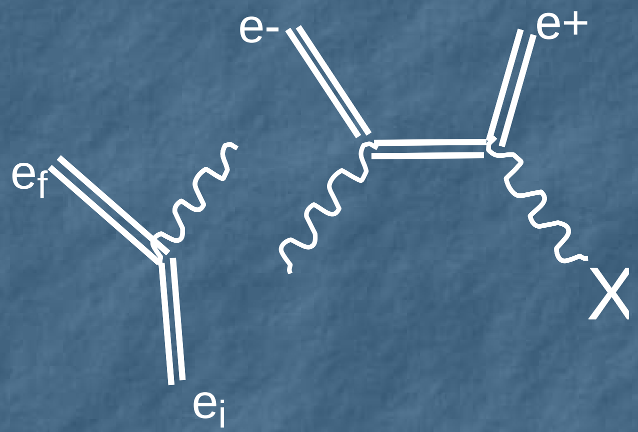
4-fermion IFQED process



- To do bunch field effect properly replace all fermion lines by Volkov solutions

k_f

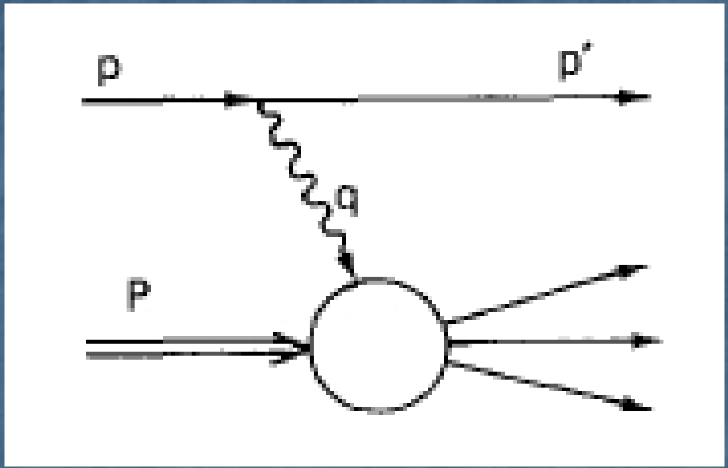
- 4^{th} order external field process is intimidating so look for an 'external field' EPA – encouraged by the fact that the photon operators are same as the 'ordinary' EPA



- So assuming EPA can still be used we are left with the 1^{st} order external field process (Sokolv-Ternov). Known to be determined by the magnitude of Υ
- The 2^{nd} order external field processes need special treatment. Propagators can reach the mass shell, the x-sections can exceed S-T and the effect does not necessarily have a simple relationship with Υ

EPA and the bunch field

$$d\sigma = \frac{\alpha}{4\pi^2 |q^2|} \left[\frac{(qP)^2 - q^2P^2}{(pP)^2 - p^2P^2} \right]^{1/2} (2\rho^{++}\sigma_T + \rho^{00}\sigma_S) \frac{d^3p'}{E'}$$



- σ_T and σ_S are now cross-sections for the coherent pair production – need to investigate validity of putting q on the mass shell and neglecting σ_S

- The dependency on fermion momenta have to be modified $P_\mu \to P_\mu + k_\mu \nu^2/2(kp)$ and $P^2 \to P^2 + \nu^2$

- If fermion energy is large and fermion and field 3-momenta are anti-parallel then 2nd term is small

    - **More theoretical work to do to understand the range of validity of the EPA approx when external field is present**

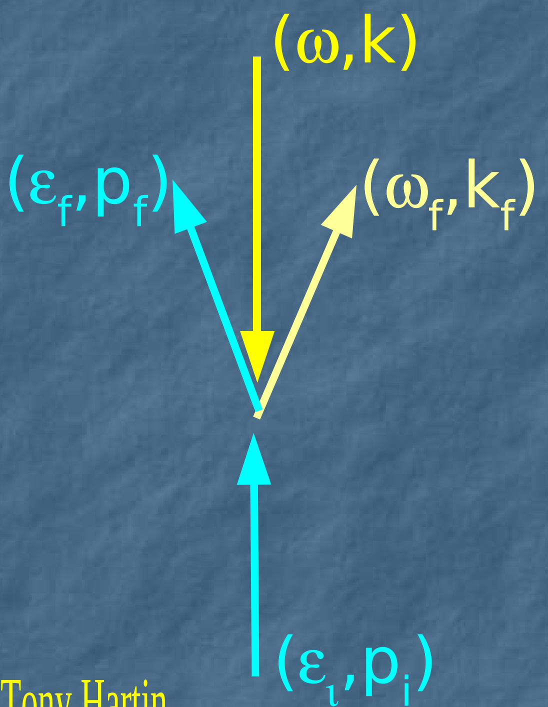
Deriving the Sokolov-Ternov equation

Sokolov-Ternov equation can be written down using the 'operator method' of Baier et al

$$dW = -i\,\frac{\alpha m}{\sqrt{3}\pi\gamma}\left[\int_z^{\infty} K_{5/3}(z)\,dz + \frac{x^2}{1-x}K_{2/3}(z)\right]dx \quad \text{where} \quad z = \frac{2}{3\nu\omega\epsilon_i}\,\frac{\omega_f}{\epsilon_i - \omega_f}$$

but, more generally can be obtained within limits using full Volkov solutions

lets just try to get (discovering required approximations along the way)



$$d\sigma_{fi} = \frac{1}{v(2\pi)^2}\,\frac{m^2}{4\epsilon_i\omega_i}\,\delta(P_i - P_f)\overline{\sum_i}\sum_f |T_{fi}|^2\,\frac{dp_f\,dk_f}{\epsilon_f\,\omega_f}$$

- Matrix element contains one volkov solution per Feynman diagram order, so a product of solutions for S-T
- phase integral also contains an integration over the contribution from the bunch field

(Partial) S-T derivation (continued)

constant crossed field:

$$A^e_\mu(x) = a_{1\mu}(kx)$$
$$(a_1 a_1) = -a^2$$
$$(a_1 k) = 0$$

$$\Psi^V_p(x) = E_p(x)\, u_p$$

$$E_p(x) = \left[1 + \frac{e}{2(kp)} \not{k}\not{a}_1(kx)\right] \times \exp\left(-iqx + i\frac{e^2a^2}{2(kp)}(kx) - i\frac{e(a_1p)}{2(kp)}(kx)^2 - i\frac{e^2a^2}{6(kp)}(kx)^3\right)$$

where $q = p + \frac{e^2a^2}{2(kp)}k$

Simplification 1: k || p so that $(a_1p)=0$

$$F_{1,r} = \int_{-\infty}^{\infty} t \exp\left[i(r+Q)t - iQ\frac{t^3}{3}\right] = 2\pi i Q^{-\frac{2}{3}} \mathrm{Ai}'(z)$$

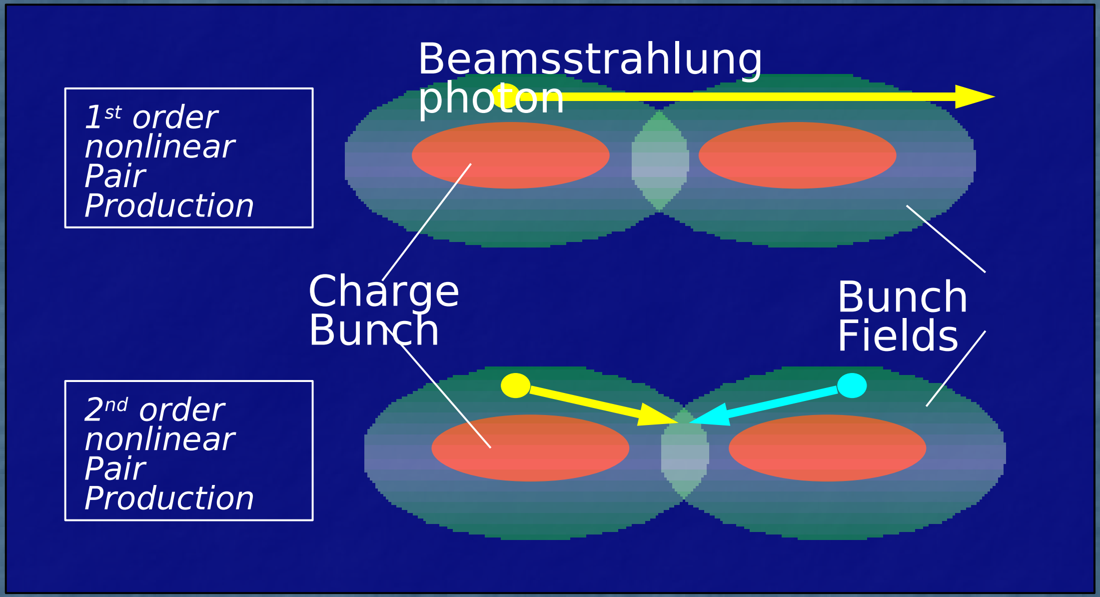
where $Q = \nu^2 \frac{(kk_f)}{(kp_i)((kp_i)-(kk_f))}$ $z = -(r+Q)Q^{-\frac{1}{3}}$

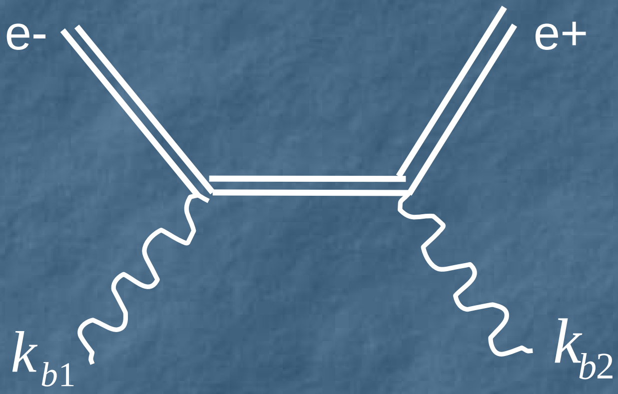
Simplification 2: k || $-k_f$ and $\epsilon_i >> m_e$ then $Q = \frac{\nu^2}{\omega\epsilon_i}\frac{\omega_f}{\epsilon_i - \omega_f}$

- $\int$(P.I.)dr yields r→Q
- $Q^{-2/3}\,\mathrm{Ai}'(Q^{-2/3}) = K_{2/3}(2Q/3)$

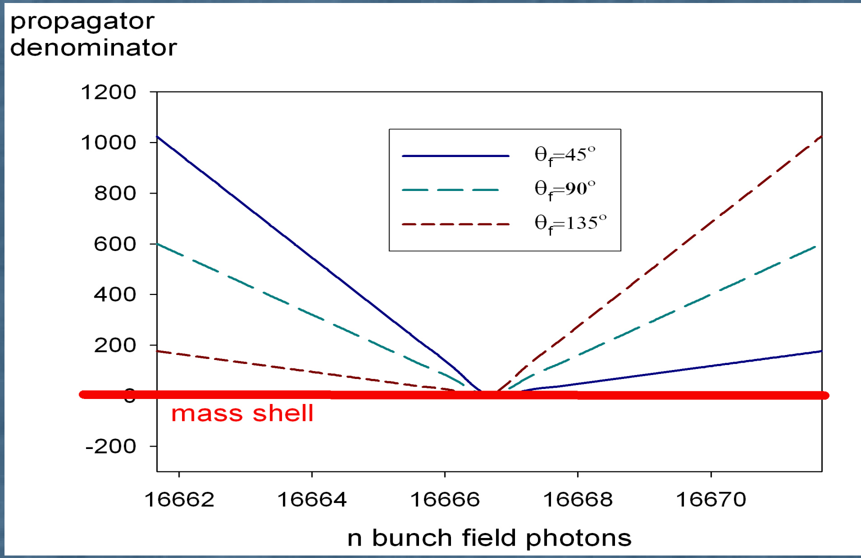
$$(S.T.)\, z = \frac{2}{3\nu\omega\epsilon_i}\frac{\omega_f}{\epsilon_i - \omega_f} \qquad (volkov)\, z = \frac{2\nu^2}{3\omega\epsilon_i}\frac{\omega_f}{\epsilon_i - \omega_f}$$

2^{nd} order external field process: Coherent Breit-Wheeler (CBW) process





- 2^{nd} order process contains twice as many Volkov E_p
- Double integrals over products of 4 Airy functions – mathematical challenge!
- spin structure same as ordinary Breit-Wheeler



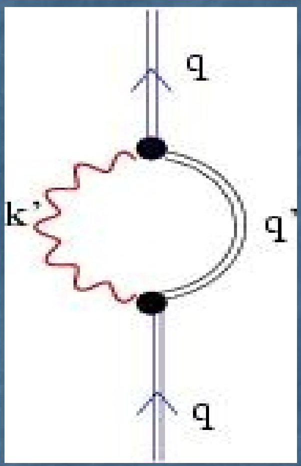
fermions recieve a mass shift due to bunch field and the propagator can reach mass shell whenever $r\omega \sim \omega_b$

CBW cross-section with simplifications

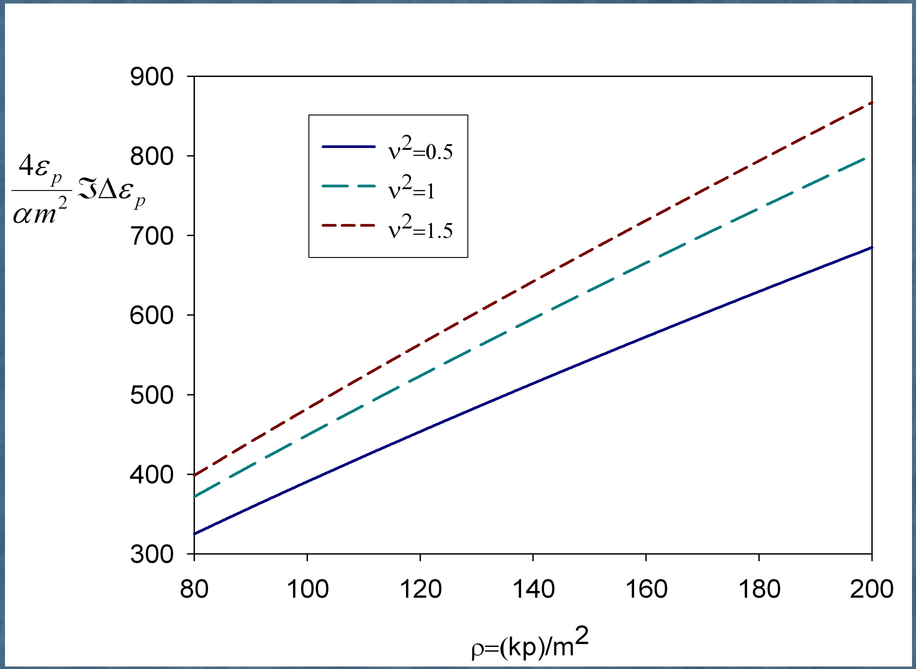
$$\frac{d\sigma_{CBW}}{d\Omega} \approx \frac{d\sigma_{BW}}{d\Omega} \int_{-\omega_1/\omega}^{\infty} \frac{dn}{\left[(n\omega \pm \omega_1)^2 + \Gamma^2\right]^2} F$$

- Can write CBW diff x-section as the ordinary BW diff x-section times a function F and a resonance
- lower bound of integration is determined physically – c of m energy must be at least 2x0.511 MeV
- F is an integration of products of Airy functions for crossed beam field – numerically difficult
- Γ is a resonance width determined from a self energy calculation

Calculation of Resonance widths



- The Electron Self Energy must be included in the Coherent Breit-Wheeler process
- This is a 2[nd] order IFQED process in its own right.
- Renormalization/Regularization reduces to that of the non-external field case



- The Electron Self Energy in external CIRCULARLY POLARISED e-m field originally due to Becker & Mitter 1975 for low field intensity parameter $(ea/m)^2$. Has been recalculated for general field intensity parameter
- ESE in external CONSTANT CROSSED field is due to Ritus, 1972
- Optical theorem: the imaginary part of the ESE has the same form as the Sokolov-Ternov equations

Summary and things to do

- 4-fermion processes should be modified to include the bunch fields
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- Volkov solutions of Dirac field with external field replace **all** fermion lines
- EPA has to be studied for validity when external fields are included
- 1st order IFQED beamstrahlung process calculated using Volkov solutions and compared to Sokolov-Ternov equation. Small difference in the argument of McDonalds function discovered. Needs more investigation
- 2nd order IFQED Coherent Breit-Wheeler discussed. Calculation has some mathematical challenges