# School

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HOMEWORK

# 1) Ionization cooling questions

1. Assume a transverse cooling channel with, at the absorber, $\beta_\perp = 0.18$ m, $\beta_v = v/c = 0.85$, and the dimensionless constant defined by equation 40 on p 14: $C(mat, E) = 45\ 10^{-4}$. These are the values for a lattice given in Feasibility study 2. The value of $C(mat, E)$ differs somewhat from the value for pure liquid hydrogen ($C \approx 35\ 10^{-4}$) because of the effect of the hydrogen and rf windows.

   a) calculate the theoretical equilibrium emittance.

   b) What can one do to lower this value?

   c) calculate the final differential rate of cooling $d\epsilon_\perp/\epsilon_\perp$ compared with its ideal value $dp/p$.

   d) This rate would be improved if the lattice parameter $\beta_\perp$ were lowered. What is the likely problem that will be encountered if this approach were followed?

2. On p 57 we descibe a cooling ring that, with emittance exchange in wedges cools all 6 dimensions. Assume $\beta_\perp = 0.4$ m, dispersion at the hydrogen wedge $D = 7$ cm, the length of the wedge on axis $\ell = 28.6$ cm, and the height from the axis to the apex of the wedge $h = \frac{\ell}{2\ \tan(100^o/2)} = 12 \quad \mathrm{cm}$. Assume that the sum of partition functions $\Sigma J_i \approx 2.0$, $C(mat, E) = 38\ 10^{-4}$, and

assume good mixing between $x$ and $y$.

a) What are the three partition functions in this case?

b) What is the expected equilibrium transverse momentum?

c) What is the expected final rms momentum spread ?

# 2) Solenoid focusing

1. What is the wavelength $\lambda$ and lattice parameter $\beta_{\perp}$ for a particle with momentum 200 MeV in an axial field of 3.33 T ?
2. Consider a 200 MeV/c particle in this solenoidal field starting on the axis with a transverse momentum of 20 MeV/c. What is its motion and out to what distance from the axis does it get.
3. If we imagine an ideal transverse cooling system with continuous energy loss and reacceleration so that all transverse momenta are reduced to near zero, then the particle will settle at half its maximum distance from the axis and pass straight down the field lines at 200 MeV/c. What is its motion in it's Larmor frame?
4. If the particle now emerges from the end of the solenoid, it will have to cross radial field components. What transverse momentum will it have outside the field ?
5. What could one do to avoid this final momentum kick induced by exiting the solenoid (Bush's Law) ?