Electron Source

(a) Calculate the space charge limit according to Child-Langmuir law with parameters

- V=120kV,
- S:1cm diameter
- d:5cm

(b) In this limit, what is the bunch length, to extract 3.2nC charge for ILC?

(c) Compare this bunch length to 1.3 Ghz RF period. Calculate energy spread due to RF curvature with 1/10, 1/100, 1/300, and 1/1000 of this bunch length.

Hint: Energy spread due to RF curvature is

$$\frac{\Delta E}{E} = 1 - \cos\left(\omega \frac{\delta t}{2}\right), \tag{1}$$

where δt is bunch length.

Positron Source

(d) How much beam energy, E, is neccesary to obtain 10 MeV photons from undulator? Undulator strength parameter, K, is given as

$$K = 93.4 B \lambda_p, \tag{2}$$

where B is the peak magnetic flux density (T) and λ_p is undulator period (m). Please assume following parameters

- λ_p, 0.01m
- n: harmonic number,1
- B: Peak magnetic flux desity, 1.0 T

(e) How much beam energy, E, is neccesary to obtain 10 MeV photons from Laser Compton? Assume $1\mu\mathrm{m}$ wave length for laser.

- Planck constant: 6.63E-34 Js
- Speed of light : 3.00E+8 m/s

Bunch Compressor

(f) Calculate the expected final bunch length after BC section assuming $\sigma_0 = 0.15(\%)$ and $R_{56} = -0.2(m)$.

(g) How much voltage (V_{RF}) is required to compose this BC section?

Spin Rotator

(h) Derive Precession angle by solenoid rotator from Thomas-BMT equation.

(i) Calculate the bending magnetic filed integral (T.m) to rotate the spin vector with 90 degree.

Physics constatns

- Electronic charge e : $1.60 \times 10^{-19} C$.
- Planck constant h : 6.63×10^{-34}Js.
- Speed of light c : $3.00 \times 10^{8}\mathrm{m/s}$.
- electron mass m : $9.1 \times 10^{-31} kg$

Answer

(a) $\mathrm{S} = 7.85 \times 10^{-5}\mathrm{m}^2$, d=0.05m give perviance $P = 7.32 \times 10^{-8}$. Beam current at the space charge limit is given by

$$J = PV^{1.5}, \tag{3}$$

substituting $V = 1.2 \times 10^5$ (V), the answer is 3.05A.

(b) The total charge is 3.2nC ($3.2\times10^{-9}C$), then it takes $3.2\times10^{-9}/3.05 = 1.05 \times 10^{-9}$. The answer is 1.05 ns ($1.05 \times 10^{-9}s$).

(c) For $\delta t = 1.05 \times 10^{-10}$, 1.05×10^{-11}, and 3.5×10^{-12}, the relative energy spread is calculated as 9.0×10^{-2}, 8.8×10^{-4}, and 9.8×10^{-5} respectively.

(d) K is calculated to be 0.93 with given parameters. Photon energy from undulator is given by

$$E_{ph}(eV) = 9.50\frac{nE^2(GeV^2)}{\lambda_p(1 + K/2)}, \tag{4}$$

where the emission angle θ is assumed to be 0. Substituting $E_{ph} = 1.0 \times 10^7$, $n = 1$, $\lambda_p = 0.01$, and $K = 0.93$, the beam energy is extracted as $E = 124GeV$.

(e) Photon energy from Laser compton scattering is

$$E_{ph} \sim 4\gamma^2 E_L, \tag{5}$$

where E_L is laser photon energy. Energy of $1\mu\mathrm{m}$ wave length photon in electron volt is

$$E_L = \frac{h}{e}\frac{c}{\lambda} = 1.24. \tag{6}$$

Then, γ^2 is calculated as

$$\gamma^2 = \frac{10^7}{4 \times 1.24} = 2.02 \times 10^6, \tag{7}$$

γ is 1.42×10^3 and the electron eneryg is $\gamma mc^2 = 0.72GeV$.

(f) The final bunch length is given by

$$z_2 = \sigma_0 \times R_{56}. \tag{8}$$

Substituting $\sigma = 0.0015$ and $R_{56} = -0.2$, the final bunch length is $z_2 = 0.0003\ m = 300\ \mu m$.

(g) To satisfy the matching condition for bunch compression, $R_{65} = -1/R_{56} = 5.0$. R_{65} is given as

$$R_{65} = \frac{eV_0}{E}\frac{\omega}{\beta c}, \tag{9}$$

where V_0 is RF field, E is beam energy, ω is angular frequency, βc is speed of the bunch. Substrituting $R_{65} = 5.0$, $E_0 = 5GeV$,$\omega = 2\pi \times 1.3 \times 10^9$, $\beta c = 3.0 \times 10^8$, the required RF field is $eV_0 = 0.92\,GeV$.

(h) Skip

(i) The rotation angle by bending magnet is given by

$$\phi = \frac{eG}{m\beta c}\int Bds, \tag{10}$$

where G is anomalous magnetic moment 0.00116 for electron. Substituting physical constans, it gives

$$\phi = 0.68\int Bds, \tag{11}$$

where β is 1. Then, 90 degree rotation requires 2.3 T.m.