Damping Rings Lecture II Homework Problems with Answers

1. The emittance in a wiggler dominated ring can be written as:

$$\varepsilon_0 = \frac{\varepsilon_{dip}}{1+F} + \frac{\varepsilon_{wig} F}{1+F} \quad \text{where} \quad F = \frac{U_{wig}}{U_{dip}}$$

Assuming that F>>1, we have: $\varepsilon_0 \approx \varepsilon_{wig} = C_q \frac{\gamma^2}{J_x} \frac{I_{5,wig}}{I_{2,wig}}$

(a) For this problem assume that the wiggler field can be written as $B(s)=cos(ks)$ [this form will make solving the boundary conditions somewhat simpler]. Using the differential equation for the dispersion function (see Lecture 1, p. 41), write an expression for the dispersion function. Match the edge of the wiggler to a zero dispersion region at s=0 (ie, D=D'=0) and solve for the dispersion function.

Solution:

$$\frac{d^2 D(s)}{ds^2} + K(s) D(s) = \frac{1}{\rho}$$

where

$$K(s) = \frac{1}{\rho^2} \mp k(s)$$

There second term is 0 since there is no quadrupole in the wiggler and the first term is negligible for $k_w \rho_w >> 1$. Thus we must solve the integral:

$$\frac{d^2 D(s)}{ds^2} = \frac{1}{\rho_w} \cos k_w s$$

with the above boundary conditions. This yields:

$$D(s) = \frac{1}{k_w^2 \rho_w} (1 - \cos k_w s)$$

(b) Now that you have an expression for the dispersion, integrate $I_{5,wig}$ over a half period of the wiggler. You may assume that β is constant for this integration. What does this imply about the value of the α term in the integral?

Solution:

The curly-H function is given by: $\mathcal{H} = \left(\gamma D^2 + 2\alpha D D' + \beta D'^2 \right)$

If β is slowly varying, then α~0 and the second term in $\mathcal{H}$ can be neglected and γ~$1/\beta$. Thus we can write the half period contribution to I_5 as:

$$\Delta I_{5,wig} \approx \int_0^{\lambda_w/2} \frac{\left|\cos^3 k_w s\right|}{\rho_w^3} \left[\frac{(1-\cos k_w s)^2}{\beta_x k_w^4 \rho_w^2} + \frac{\beta_x \sin^2 k_w s}{k_w^2 \rho_w^2} \right] ds$$

which upon integration yields (this requires the results of several trigonometric integrations which the student can feel free to include as constants):

$$\Delta I_{5,wig} \approx \frac{36}{15\beta_x k_w^5 \rho_w^5} + \frac{4\beta_x}{15 k_w^5 \rho_w^5}$$

(c) Now determine the value of $I_{2,wig}$ and obtain an expression for the emittance. If you assume $\lambda_{wig} << \beta$, can you simplify your result?

<u>Solution:</u>

Integrating I_2 over the same region that we used for I_5 gives:

$$\Delta I_{2,wig} \approx \int_0^{\lambda_w/2} \frac{\cos^2 k_w s}{\rho_w^2} ds = \frac{\pi}{2 k_w \rho_w^2}$$

In the limit that $\beta_x >> \lambda_w$, the period of the wiggler ($=2\pi/k_w$), the first term of the result from part (b) can be dropped as negligible and we obtain:

$$\varepsilon_0 \approx C_q \frac{\gamma^2}{J_x} \frac{8\beta_x}{15\pi k_w^2 \rho_w^3}$$

(d) The CesrTA baseline lattice has B_w=1.9T, λ=0.4m, J_x=1, E=2GeV, and β_x~20m. What is the wiggler-dominated emittance?

<u>Solution:</u>

Plugging the values above yields: $\varepsilon_0 \sim 2\ nm$.

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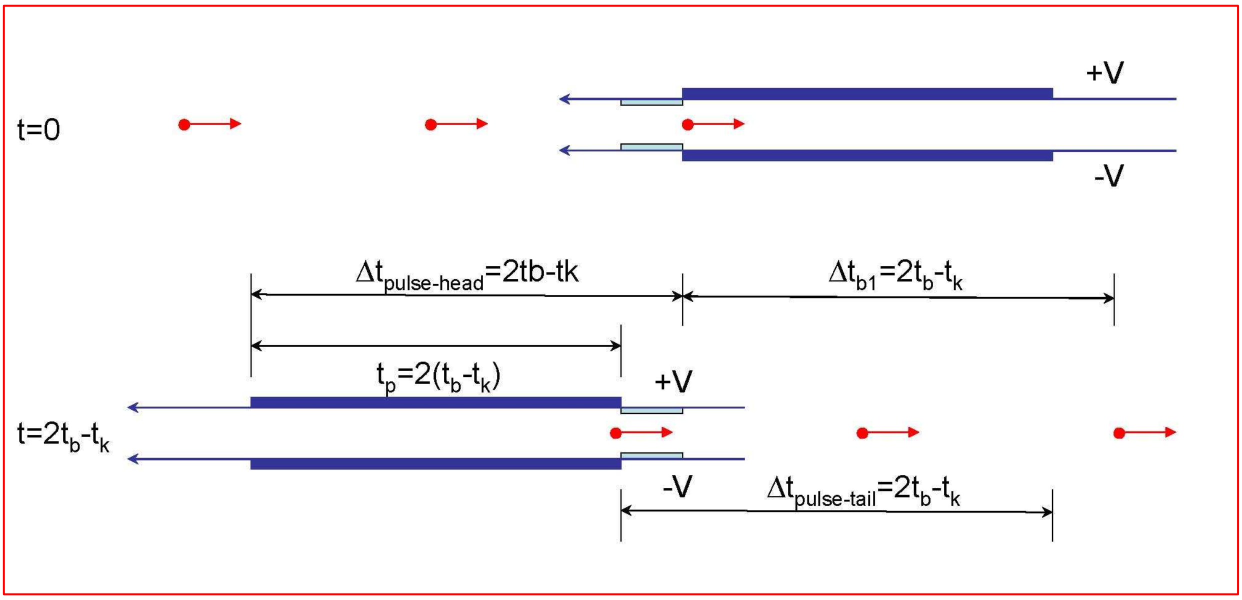
2. On page 26 of today's lecture, we stated that the relationship between the fast kicker pulse, the stripline length, and the bunch spacing to ensure that a bunch could be kicked without affecting its neighbors (before or after) is:

$$t_p \le 2\left(t_b - t_k\right)$$

(a) Please confirm this relationship



As shown in the sketch, the pulse must enter the kicker structure after bunch 1 exits the structure. Then the pulse must exit the structure before bunch 3 enters the structure. Using the relationships shown in the sketch above, the time between the leading and trailing edge of the pulse must satisfy the specified relationship.

(b) Assuming that the pulser produces a perfect square wave and that there are no limitations on its width, derive the condition that maximizes the kick to the middle bunch (while still not kicking the preceding or following bunches).

<u>Solution:</u>

In order for the bunch of interest to receive the maximum amplitude kick and in order that no energy is wasted by having the pulse persist in the stripline kicker structure when the bunch is not present, we want the pulse to have filled the kicker upon the entrance of the bunch and to leave the kicker immediately upon the bunch's exit. This gives rise to the drawing on the following page:

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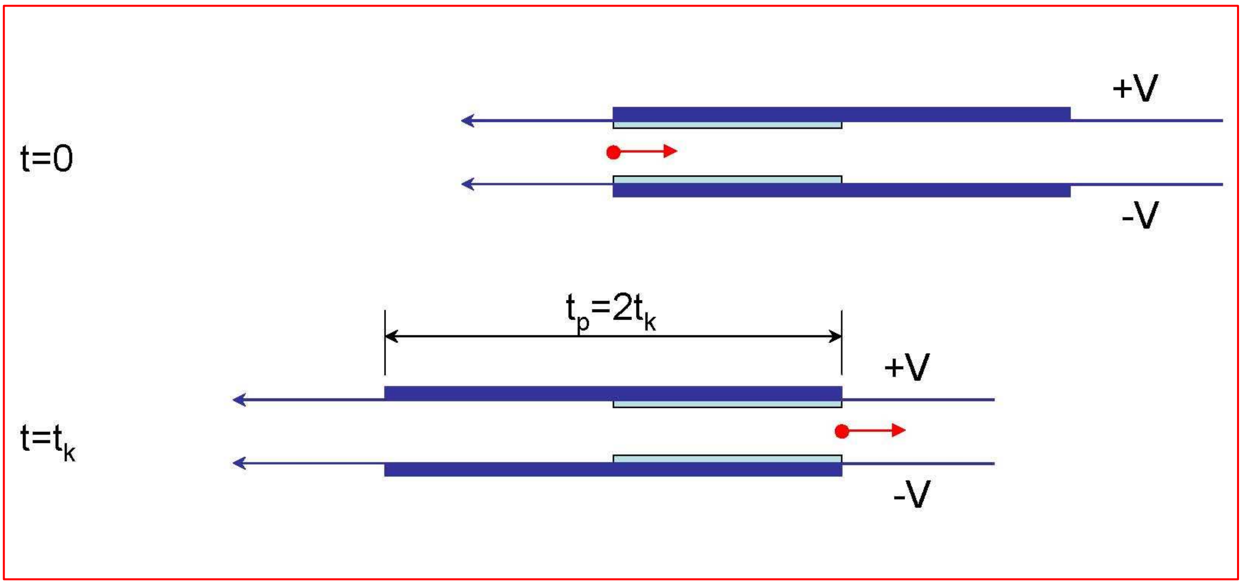

Thus the condition that maximizes the kick to the bunch of interest for a given stripline length is: $t_p = 2t_k$

If we have no constraints on our ability to make a high voltage square wave pulse, we can combine this result with the result from part (a) to obtain:

$$t_p \le 2\left(t_b - t_k\right)$$

$$2t_k \le t_b$$

and the maximum kick will occur for $t_k=0.5t_b$. For 3.1 ns spaced bunches, this corresponds to a 47 cm stripline.

2.(c) Using the result from part B, and assuming that bipolar pulsers (ie, one electrode receives a positive voltage and the other receives a corresponding negative voltage), how many stripline structures are required to achieve the total kick of 43 kV-m as described in the lecture.

Solution:

Let's assume that the pulsers deliver +/- 10kV to the stripline electrodes (the target discussed in the lecture). Then each stripline provides 9.4 kV m of kick and a minimum of 5 kickers are required.

(d) The required fractional repeatability of the kick angle for the ILC DR is 7×10^{-4}. Assuming that the amplitude jitter between pulsers in part (d) is uncorrelated, what is the required pulse amplitude stability for each individual pulser?

<u>Solution:</u>

The fractional stability of the pulsers required is given by:

$$\frac{\sigma_\theta}{\theta} = \left.\frac{\sigma_V}{V}\right|_{\text{all modules}}$$

If we have N modules producing the total kick voltage, then:

$$V_{\text{total}} = NV_{\text{single}} \quad \text{and} \quad \sigma_{V_{\text{total}}} = \sqrt{N}\sigma_{V\text{single}}$$

and the fractional stability specification for a single module is:

$$\frac{\sigma_{V_{\text{single}}}}{V_{\text{single}}} = \sqrt{N}\,\frac{\sigma_{V_{\text{total}}}}{V_{\text{total}}} = \sqrt{N}\left(7.5\times10^{-4}\right) \approx 0.16\%$$

for a set of 5 pulsers whose amplitude noise is uncorrelated.

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3. On page 48 of today's lecture we wrote down the stability criteria for trapping ions with molecular mass A in the potential well of the electron beam.

(a) Calculate the minimum molecular mass ion that can be trapped immediately after injection. The injection and extraction parameters are given in the table below.

Solution:

Using the injection values in the table below with the stability criterion from the lecture:

$$A \geq \frac{r_p N_0 s_b}{2\sigma_y\left(\sigma_x + \sigma_y\right)}$$

we obtain:

A>0.05

where r_p=1.5×10^{-18} m. Thus all gas species can be effectively trapped given the injection conditions.

(b) Repeat the calculation for the time immediately before extraction.

Solution:

A>9

Thus the lightest gas species will no longer be trapped as the beam damps down.

	N_0	s_b	σ_x	σ_y
Injection	2×10^{10}	0.9 m	600 µm	300 µm
Extraction	2×10^{10}	0.9 m	250 µm	6 µm