LINAC-I

ILC School

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Contents of LINAC-I

- Acceleration for energy
  - Requirement on acceleration to high energy
  - Maxwell's eq. to describe microwave transmission
  - Cavity as a unit base of acceleration
  - Coupled cavity system in a SW regime
  - Pillbox cavity as a simple base to represent practical cavities
  - Circuit modeling of cavity
  - SW versus TW
  - TW linac
  - Summary of LINAC-I

Energy and luminosity

$$e\,E_c = e\,E_a\,L_{linac} = \frac{P_{linac}\;\eta_{RF->Beam}}{N\,N_b\,F_{rep}}$$

$$\frac{L}{P_{linac}} = \frac{L_0\,N_b\,F_{rep}}{e\,E_c\,N\,N_b\,F_{rep}/\eta_{RF->Beam}} = \frac{1}{e\,E_c}\,\frac{L_0}{N}\,\eta_{RF->Beam}$$

Acceleration for energy

Requirements for high energy machine

- High gradient
- High efficiency
- Emittance preservation
- Stable operation
- Low cost
  - Construction, operation,
  - Electric consumption, cooling,

Evolution of gradient and energy

Type	Beam Energy	Acceleration	scheme
Cockcroft Walton	1MeV	1MV/m	DC Rectify
Van de Graaff	10MeV	10MV/m	DC Charging
Cyclotron	100MeV	1MV/Dee	Cyclic
Synchrotron	100GeV	100MV/ring	Cyclic
Linear accelerator	1TeV	100MV/m	Periodic
Plasma accelerator	10TeV?	>10GV/m	Plasma

Early accelerator developments

after pioneering experiment with electrons or protons

- 1896 Thomson vacuum tube with thermal electrons
  - Cathode ray tube
- 1911 Rutherford Radioactive material
  - Alpha particle scattering by nucleus
- 1928 Wideröe
  - Linear accelerator idea
- 1931 Sloan & Laurence
  - Linear accelerator experiment
- 1932 Cockcroft & Walton ~1MeV
  - High voltage by rectifier
- 1933 Van de Graaff ~a few MeV
  - High voltage by carrying charge by belt

Linear accelerators

Wideröe
Driven electrodes

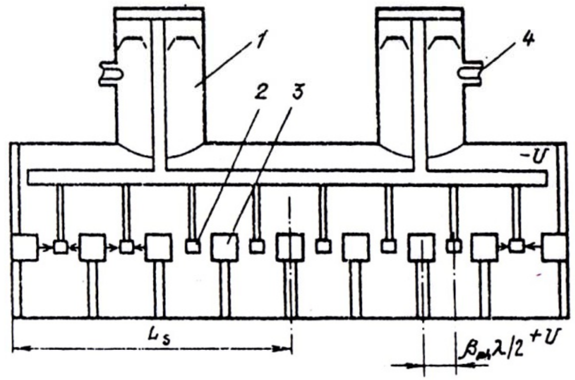


Figure 1.5. Wideröe structure: (1) stubs, (2) short drift tubes, (3) drift tubes with quadrupoles, and (4) coupling loops.

Sloan Lawrence
Independent driven electrodes

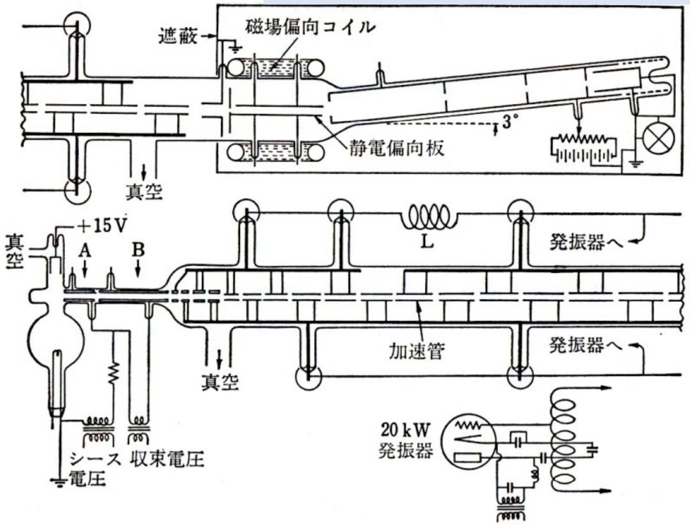
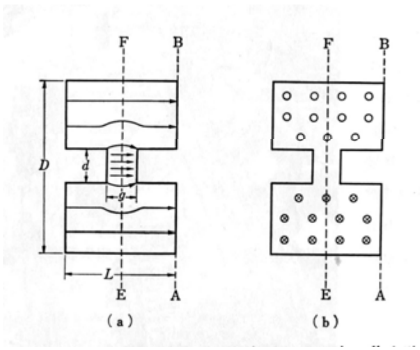


図 9·1 D. H. Sloan と E. O. Lawrence の線形加速器 (1931)[3)]



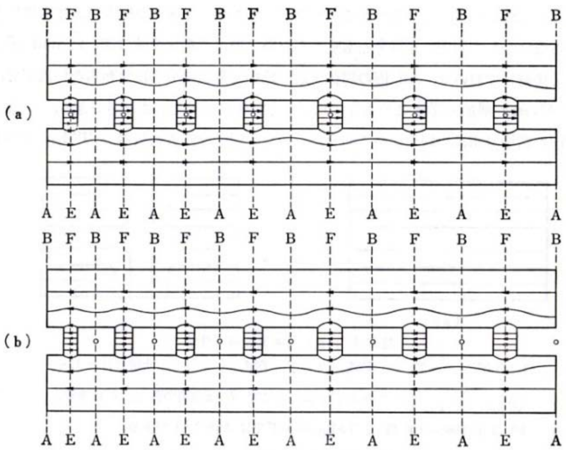


図 9·3 Alvarez 型加速空胴の加速電場の分布

Alvarez
Resonant cavity with drift tube without acceleration/deceleration

Development for higher energy

- 1931 Lawrence Several to higher than 10MeV, proton
  - Cyclotron acceleration
- 1945 McMillan and Veksler higher and higher energy
  - Synchrotron acceleration for proton
  - 1952 Courant, Snyder: Strong focus AGS (alternating gradient synchrotron)
- 1950~1960's Stanford electron linear accelerators GeV electron
  - ~1955: Ginzton, Hansen, Chodorow: Mark-II~III
    - Microwave technology the legacy of world war II
  - 1967: Panofsky: 2-mile accelerator 20GeV → ~10MV/m
- Late 1980's Richter: Stanford Linear Collider
  - Energy doubler by pulse compression technique

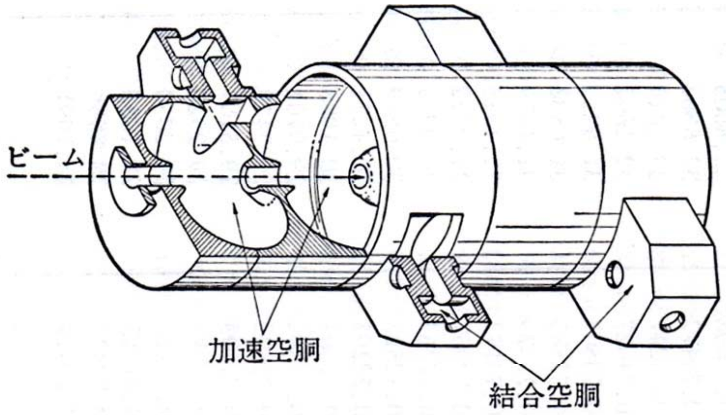
For higher energy machine

TW DLS for electron high energy machine for years

Disk Loaded Structure, 6MeV
Stanford Univ. 1947

SLAC:
Targeting highest energy with electron

SW Side-coupled cavity for proton high energy machine

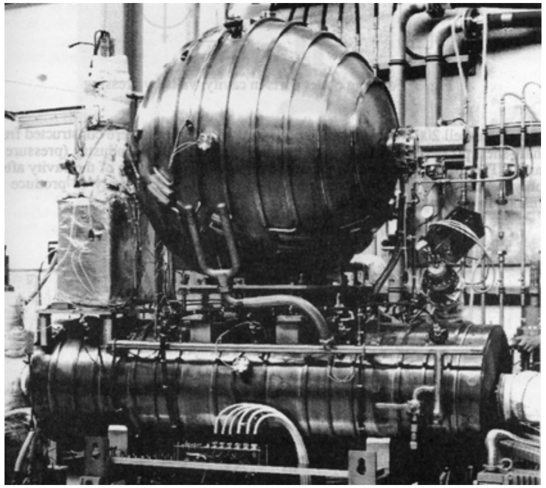


LANL SCS:
Side coupled structure

Storage ring, collider to linear collider

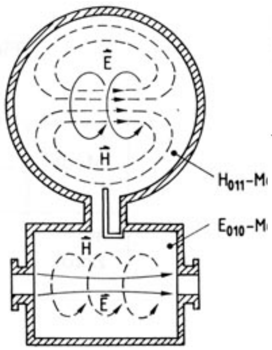
- Storage ring in e^+e^- colliding mode
  - PEP / PETRA ~17.5X2 GeV
  - TRISTAN 30X2 GeV
  - LEP 100X2 GeV
- Linear collider plans
  - 1980's VLEPP 14GHz, 100MV/m
  - 1990's TESLA 1.3GHz, 23.4MV/m → 500Gev → higher?
  - 1990's GLC/NLC 11.4GHz, 50MV/m → 1TeV
- ILC → 500GeV
- CLIC → 3TeV
- Further higher energy machine plasma, laser, etc……..

For efficiency toward high energy LEP and for stability toward high current KEKB



LEP cavity
with storage cavity
for efficiency improvement

ARES for KEKB
with storage cavity for beam stability



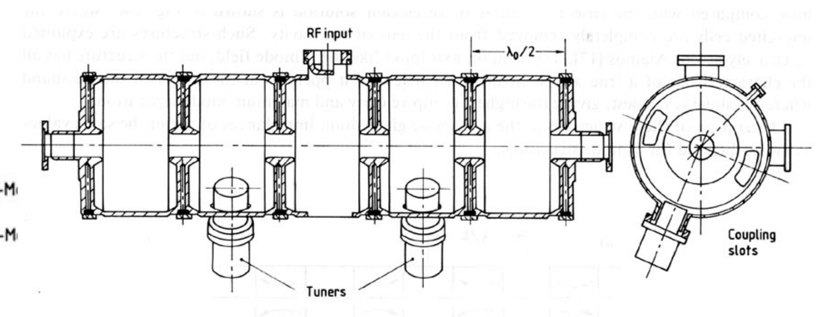


Fig. 25 The five-cell π-mode structure with magnetic coupling

Higher and higher energy for lepton linear collider

14GHz DLS   VLEPP single-bunch, high rep-rate

3GHz DLS   S-band DESY multi-bunch

1.3GHz 9-cell SCC cavity TESLA DESY

11.4GHz DLS SLAC/KEK DDS (weakly damped & detuned)

30GHz DLS CLIC (Heavily damped)

1.3GHz 9-cell SCC cavity ILC developing

12GHz DLS in study

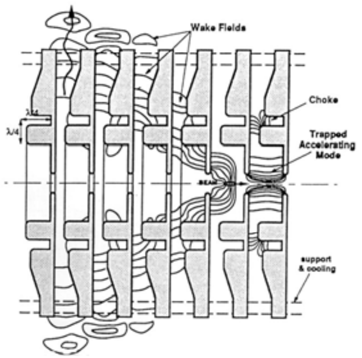
Only two types; TW DLS and SW 9-cell shaped cavity.
Simple and low cost.

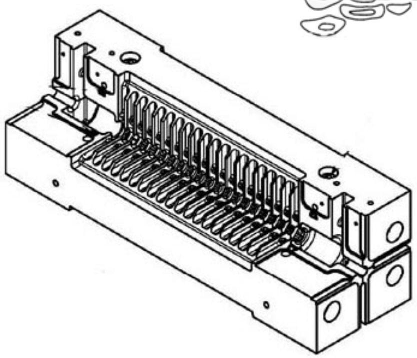
Higher electron energy being developed

With care against wake field in various manners.

Continue with TW DLS for higher-energy electron linear machine

Super-conducting cavity for higher-energy electron machine





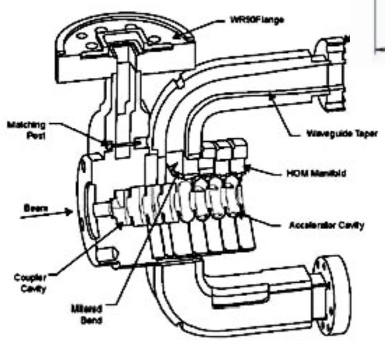
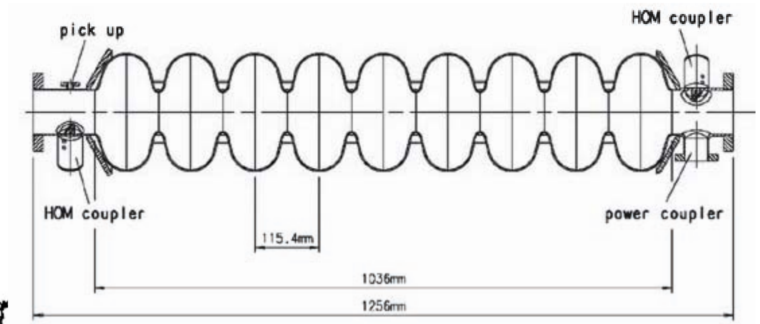


Figure 7.9: Cross-sectional view of the input end for RDDS1 structure.



SW SCC cavity
TESLA TDR 9-cell cavity
with HOM damping port

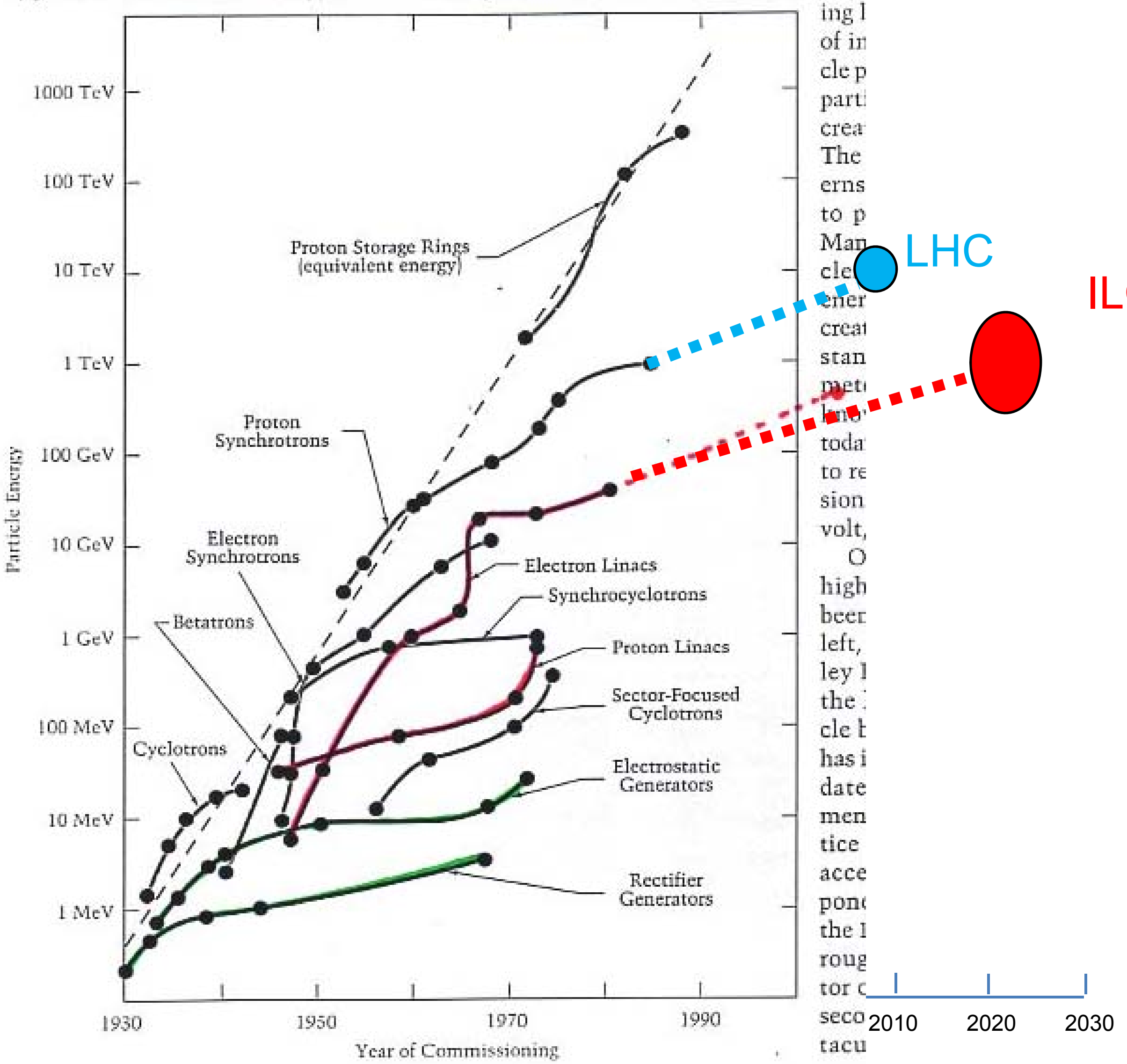
CLIC Quadrant-type DLS

DLS: Medium-damped detuned structure

Livingston plot

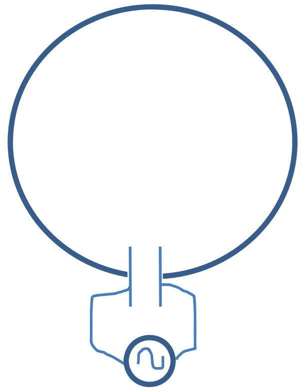
A "Livingston plot" showing accelerator energy versus time, updated to include machines that came on line during the 1980s. The filled circles indicate new or upgraded accelerators of each type.

the range of a thousand electron volts. [An electron volt is the energy unit customarily used by particle physicists; it is the energy a particle acquires when it is accelerated



Acceleration scheme

- Electric field to accelerate
- Voltage across electrodes



- DC: once
- RF in ring: many times
  - n/freq=circumference/v
- RF in line: once
  - Synchronization along a line

- It is important to focus electric field along beam axis to effectively accelerate beam.

How to reach higher energy

- Energy ← gradient X length
- DC: Van de Graaf, CockCroft Walton
- RF based
  - Sloan Wideroe Alvarez
  - SW and TW
  - Independent RF source or Two beam acceleration
- Plasma accelerator

Limiting factors against ultimate gradient

- Peak power available
- Breakdown in structure
- Quench
- Mechanical stability
- He cooling
- Thermal / mechanical
- Dark current loading
- Phase coherency along a long line

In this lecture

- RF acceleration is the only technology with which we can reach TeV range accelerator in very near future
- We focus here, as the examples, on the RF acceleration at microwave range
  - L-band (1.3GHz) superconducting cavity
  - X-band (11-12GHz) normal conducting cavity

Wave length / Frequency / Band

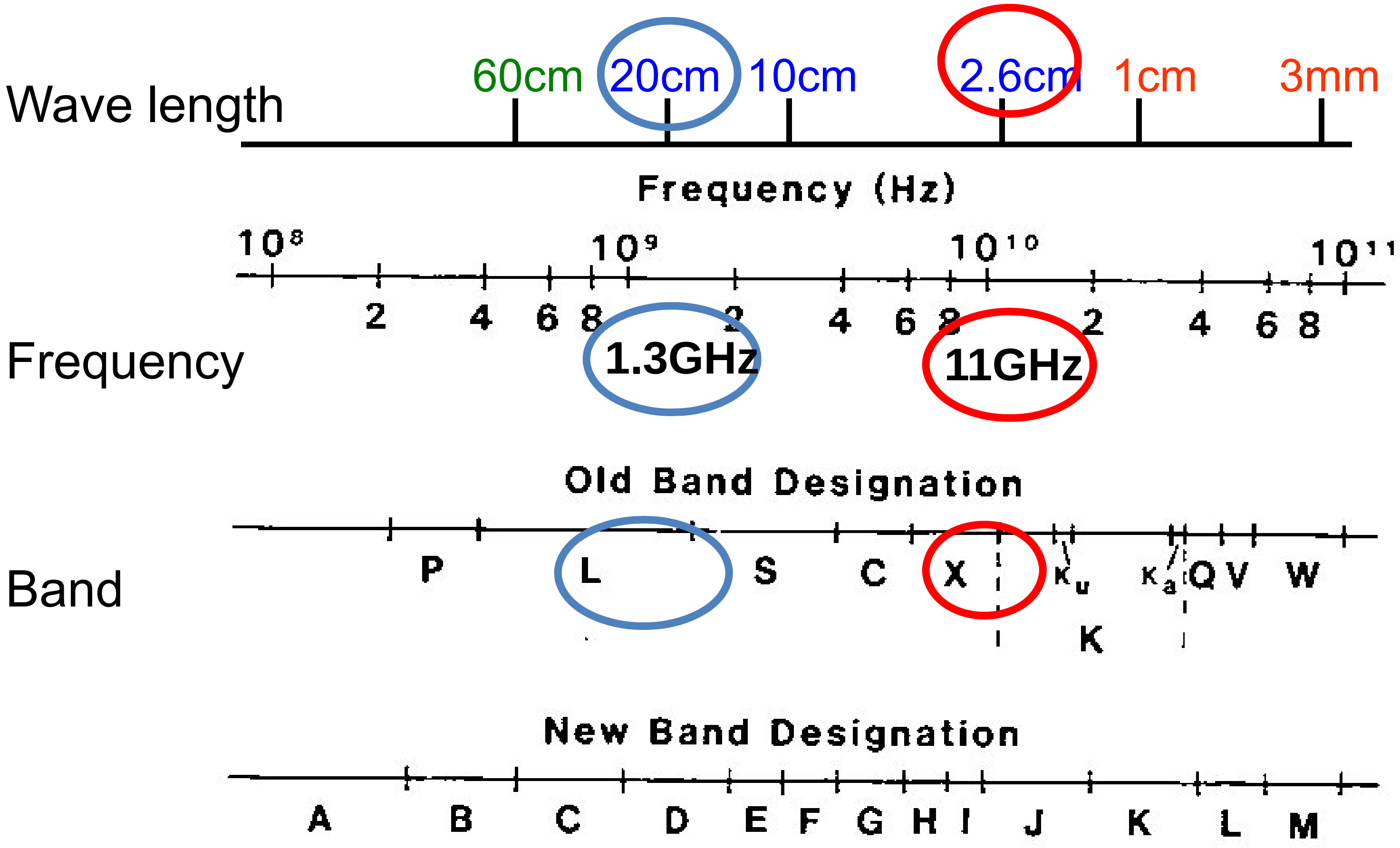


Figure 1.2 Microwave Band Designations

From Microwave Tubes by A. S. G......our, Jr.

Two types of linear accelerators

- I try to overview two types of acceleration scheme as an introduction of linear accelerator.
  - Super-conducting / Normal conducting
  - Lower frequency / Higher frequency
  - Standing wave / Travelling wave

- These happen to be two candidates for linear collider which I believe we can explore in near future.
  - ILC / CLIC

Linac example parameters

ILC and CLIC

Parameters		units	ILC(RDR)	CLIC(500)
Injection / final linac energy	E_{Linac}	GeV	25 / 250	/ 250
Acceleration gradient	E_a	MV/m	31.5	80
Beam current	I_b	A	0.009	2.2
Peak RF power / cavity	P_{in}	MW	0.294	74
Initial / final horizontal emittance	ε_x	μm	8.4 / 9.4	2 / 3
Initial / final vertical emittance	ε_y	nm	24 / 34	10 / 40
RF pulse width	T_p	μs	1565	242
Repetition rate	F_{rep}	Hz	5	50
Number of particles in a bunch	N	10^9	20	6.8
Number of bunches / train	N_b		2625	354
Bunch spacing	T_b	ns	360	0.5
Bunch spacing per RF cycle	T_b / T_{RF}		468	6

Linac example parameters

ILC and CLIC

Parameters		units	ILC(RDR)	CLIC(500)
RF frequency	F	GHz	1.3	12
Beam phase w.r.t. RF	degrees		5	15
EM mode in cavity			SW	TW
Number of cells / cavity	N_c		9	19
Cavity beam aperture	a/λ		0.152	0.145
Bunch length	σ_z	mm	0.3	0.044

ILC parameters are taken from Reference Design Report of ILC for 500GeV.

CLIC500 parameters are taken from the talk by A. Grudief, 3rd. ACE, CLIC Advisory Committee, CERN, Sep. 2008, http://indico.cern.ch/conferenceDisplay.py?confId=30172.

Maxwell’s eq. to describe microwave transmission

Maxwell's equation and wave propagation

(1) $\nabla \bullet D = \rho$

(2) $\nabla \bullet B = 0$

(3) $\nabla \times E = -\frac{\partial B}{\partial t}$

(4) $\nabla \times H = j + \frac{\partial D}{\partial t}$

If all quantities vary time harmonically;

$$E = E_0 \, e^{j\omega t}$$

$$H = H_0 \, e^{j\omega t}$$

. . . .

Then, Maxwell's eq. becomes;

$$\nabla \times E = -\, j\omega\mu \, H$$

$$\nabla \times H = (\sigma + j\omega\varepsilon)\, E$$

This has a solution of a propagation along the z-direction;

$$E = E_0 \, e^{i\omega t - \gamma_0 z}$$

$$H = H_0 \, e^{i\omega t - \gamma_0 z}$$

where

$$\gamma_0 = \alpha_0 + j\,\beta_0 = \sqrt{-\omega^2 \,(\varepsilon - j\sigma / \omega)\,\mu}$$

to wave equation

α_0 attenuation and β_0 wave number along the propagation direction

In vacuum, σ=0 → velocity

$$v = 1/\sqrt{\varepsilon\mu}$$

In a plane wave, E_x and H_y

Wave impedance becomes;

$$Z_0 = E_x / H_y = \sqrt{\frac{\mu}{\varepsilon - j\sigma/\omega}}$$

Using Z_0 and γ_0 to rewrite Maxwell's eq.;

$$\nabla \times E = -\gamma_0 (Z_0 H)$$

$$\nabla \times (Z_0 H) = \gamma_0 E$$

From these, the wave equation becomes;

$$\Delta E - \gamma_0^{\ 2} E = 0$$

$$\Delta H - \gamma_0^{\ 2} H = 0$$

Wave propagation in uniform medium

$$E = E_0\, e^{i\omega t - \gamma_0 z} \qquad H = H_0\, e^{i\omega t - \gamma_0 z}$$

$$where\ \ \gamma_0 = \alpha_0 + j\,\beta_0 = \sqrt{-\omega^2\,(\varepsilon - j\sigma/\omega)\,\mu}$$

Insulator / vacuum

$$\sigma = 0$$

$$\gamma_0 = j\omega\sqrt{\varepsilon\mu}$$

$$v = 1/\sqrt{\varepsilon\mu}$$

Velocity in free space

Poor conductor

$$\sigma/\omega << \varepsilon$$

$$\gamma_0 = \frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}} + j\,\omega\sqrt{\varepsilon\,\mu}$$

Lossy propagation

Good conductor

$$\sigma/\omega >> \varepsilon$$

$$\gamma_0 = (1+j)\sqrt{\frac{\sigma\omega\mu}{2}}$$

$$Z_0 = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$$

E and H at 45 degrees

$$\sigma(Cu) = 5.8\times10^7\ [1/(\Omega m)]$$

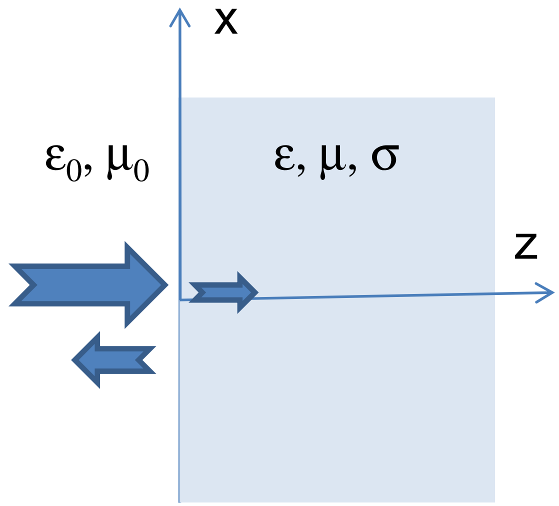
$$\varepsilon\omega/\sigma = 10^{-8} << 1\ \ at\ \ 11.4GHz$$

Reflection from good conductor and surface resistance

$$E_x / H_y \equiv Z_0 = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$$

This means phase lag between E and H.
Better conductor makes E smaller and smaller.

Assume plane wave incident in z-direction,
Wave equation for the transmitted wave into material becomes



$$\Delta E_t - \gamma_0^{\,2} E_t = \left(\frac{\partial^2}{\partial z^2} - j\omega\mu\sigma\right) E_t = 0$$

$$E_t = E_s e^{-\gamma z}$$

$$\gamma = (j\omega\mu\sigma)^{1/2} = \frac{1+j}{\delta_s}$$

Exponential decaying filed into material.

$$\delta_s = \sqrt{\frac{2}{\sigma\omega\mu}}$$

Skin depth = e-folding depth.
~ 0.6micron in copper at 12GHz.

Surface resistance

Corresponding magnetic field in medium is

$$H_t \equiv \frac{1}{-j\omega\mu} \nabla \times E_t = \frac{\gamma}{j\omega\mu} E_s \, e^{-\gamma z}$$

Then wave impedance in the medium (Cu case) is

$$Z_m \equiv \frac{j\omega\mu}{\gamma} = \frac{1+j}{\sigma\delta_s} \approx 40 m\Omega << Z_0 \equiv \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$$

From boundary condition at the surface, reflection coefficient becomes

$$\Gamma = \frac{Z_m - Z_0}{Z_m + Z_0}, \quad T = 1 + \Gamma = \frac{2 Z_m}{Z_m + Z_0} << 1$$

Almost full reflection

Surface current

$$H_s = J_s \equiv \int_0^\infty \sigma E \, dz = \frac{\sigma E_s}{\gamma} [A/m]$$

Magnetic field is terminated by the surface current J_s within the thickness δ_s. Loss occurs in the volume current with equivalent surface resistance

EM wave along a uniform guide

$$E = E_0 \, e^{i\omega t - \gamma z} \qquad H = H_0 \, e^{i\omega t - \gamma z}$$

$$\nabla_t^2 E_0 + (\gamma^2 - \gamma_0^2) E_0 = 0$$

In H field, also the same story.

This equation can be solved with proper cutoff propagation constant to satisfy boundary condition;

$$\beta_c^2 = \gamma^2 - \gamma_0^2$$

In non conducting case,

$$\gamma^2 = \beta_c^2 - \omega^2 \, \varepsilon\mu$$

Then, no propagation at low frequency;

$$\omega < \omega_c = \beta_c / \sqrt{\varepsilon\mu}$$

Propagation field along a uniform guide

$$E = (\overrightarrow{k_t}\, E_t + \overrightarrow{k_z}\, E_z)\, e^{i\omega t - \gamma z}$$

$$H = (\overrightarrow{k_t}\, H_t + \overrightarrow{k_z}\, H_z)\, e^{i\omega t - \gamma z}$$

Where E_t, H_t are transverse component vector, while E_z, H_z are both scalar and

$$\nabla^2 E_z + \beta_c^2 E_z = 0 \;\; and \;\; \nabla^2 H_z + \beta_c^2 H_z = 0$$

Them, Maxwell's equation gives

$$E_t = \frac{-\gamma}{\beta_c^2} \nabla E_z + \frac{\gamma_0}{\beta_c^2} (k_z \times \nabla Z_0 H_z)$$

$$Z_0 H_t = \frac{-\gamma}{\beta_c^2} \nabla Z_0 H_z - \frac{\gamma_0}{\beta_c^2} (k_z \times \nabla E_z)$$

A function E_z, H_z, which satisfy the wave equation, make the transverse component.

TE (H) wave / TM (E) wave

We can choose either E_z=0 or H_z=0, making

$$TE: \ H_t = \frac{-\gamma}{\beta_c^2} \nabla H_z$$

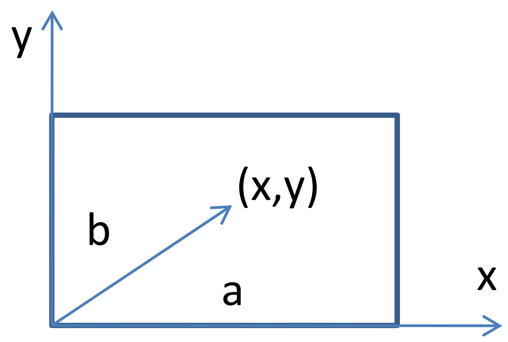
$$TM: \ E_t = \frac{-\gamma}{\beta_c^2} \nabla E_z$$

These are classified into two modes;
a pure TE (no longitudinal E field) or
a pure TM (no longitudinal H field)

If the waveguide has a modulation along z direction,
pure TE nor pure TM can exist.
This is the reality and we call it HEM, hybrid mode.

Transverse field pattern in rectangular waveguide

Solving wave equation with satisfying boundary condition



$$H_z = A_{nm} \, Cos \frac{n\pi x}{a} Cos \frac{m\pi y}{b} e^{\mp j\beta z}$$

$$H_x = \pm j \frac{\beta_{nm}}{\beta_{c,nm}^2} A_{nm} \frac{n\pi}{a} Sin \frac{n\pi x}{a} Cos \frac{m\pi y}{b} e^{\mp j\beta z}$$

$$H_y = \pm j \frac{\beta_{nm}}{\beta_{c,nm}^2} A_{nm} \frac{m\pi}{b} Cos \frac{n\pi x}{a} Sin \frac{m\pi y}{b} e^{\mp j\beta z}$$

$$E_x = \pm Z_{h,nm} \, j \frac{\beta_{nm}}{\beta_{c,nm}^2} A_{nm} \frac{m\pi}{b} Cos \frac{n\pi x}{a} Sin \frac{m\pi y}{b} e^{\mp j\beta z}$$

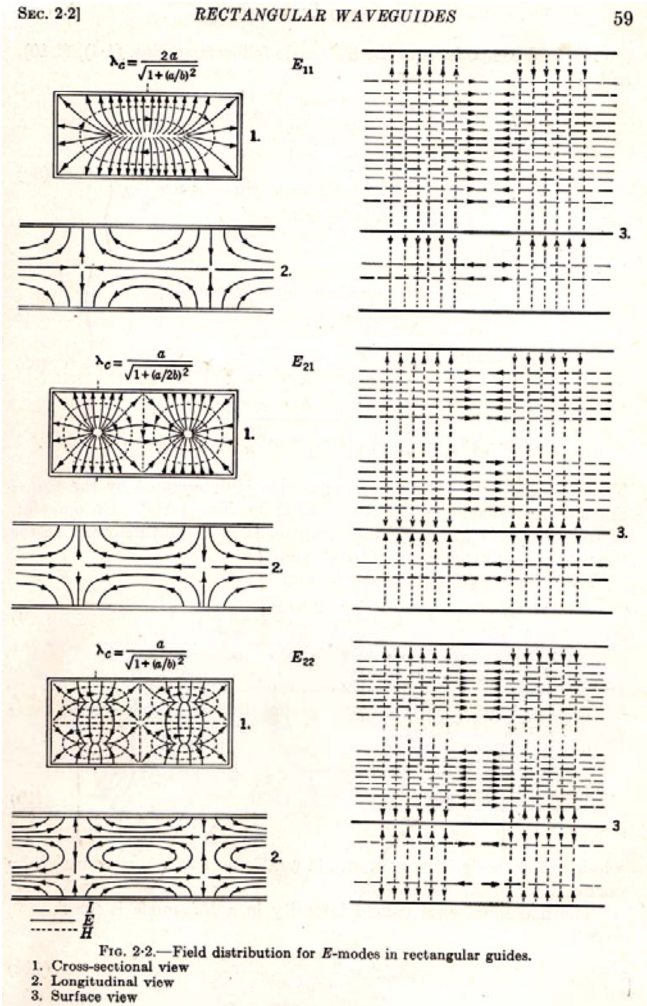
$$E_y = - Z_{h,nm} \, j \frac{\beta_{nm}}{\beta_{c,nm}^2} A_{nm} \frac{n\pi}{a} Sin \frac{n\pi x}{a} Cos \frac{m\pi y}{b} e^{\mp j\beta z}$$

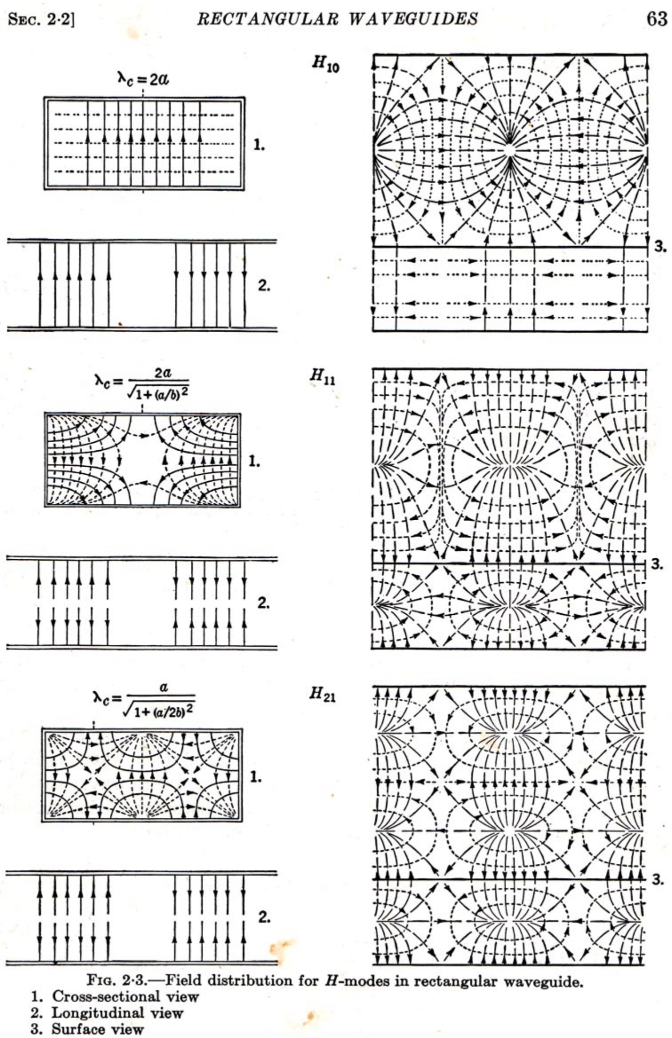
where

$$Z_{h,nm} = \frac{\beta_0}{\beta_{nm}} Z_0 = \frac{\lambda_g}{\lambda_0} Z_0$$

Typical field patterns

E. Marcuvitz ed.,
Microwave Handbook





E-mode / TM-mode

H-mode / TE-mode

Transverse field pattern in cylindrical waveguide

Solving wave equation with satisfying boundary condition

TM mode case

$$E_r = -j\frac{\beta_m}{\beta_c} \quad Cos(m\theta) \quad J'_m(\beta_c r) \quad e^{-j\beta z}$$

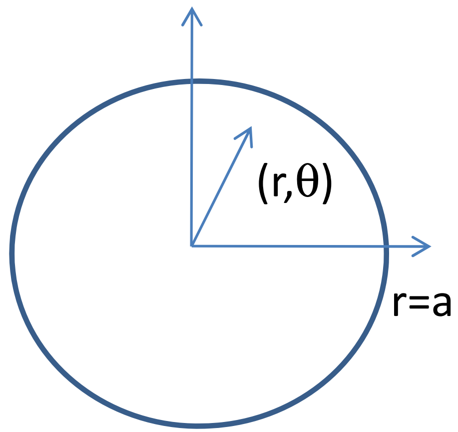
$$E_\theta = j\frac{m\beta_m}{\beta_c^2} \quad Sin(m\theta) \quad \frac{1}{r} J_m(\beta_c r) \quad e^{-j\beta z}$$

$$E_z = \quad Cos(m\theta) \quad J_m(\beta_c r) \quad e^{-j\beta z}$$

$$H_r = -j\frac{m\omega\varepsilon}{\beta_c^2} \quad Sin(m\theta) \quad \frac{1}{r} J_m(\beta_c r) \quad e^{-j\beta z}$$

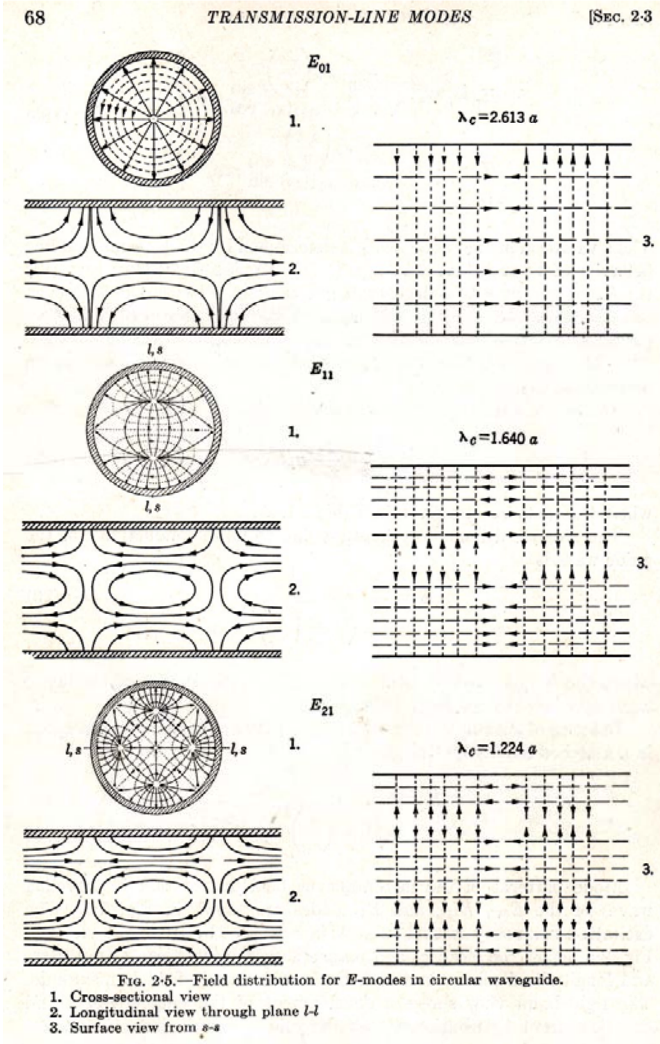
$$H_\theta = -j\frac{\omega\varepsilon}{\beta_c} \quad Cos(m\theta) \quad J'_m(\beta_c r) \quad e^{-j\beta z}$$

$$H_z = 0$$

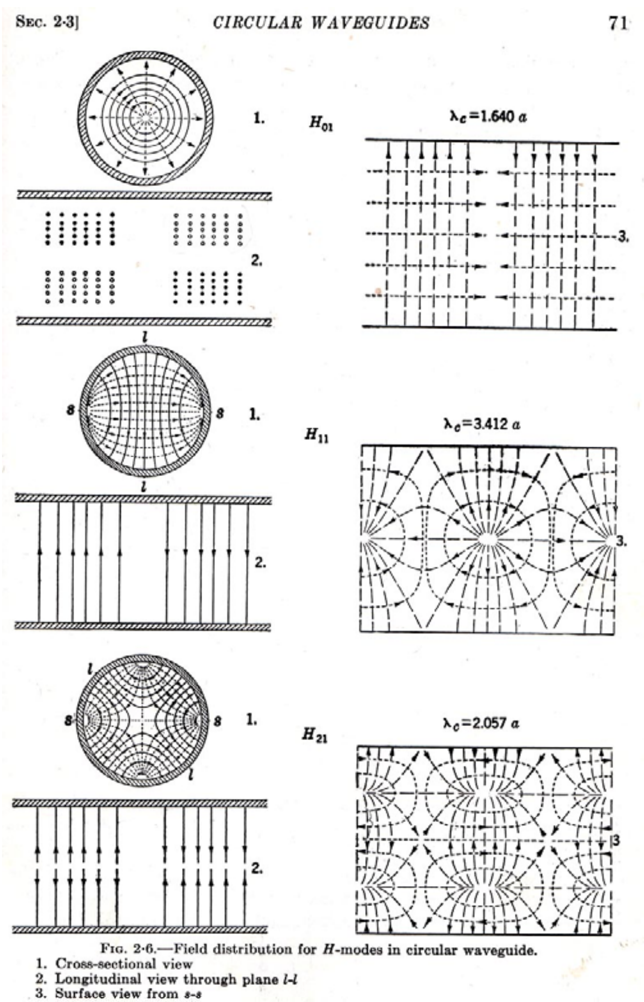


E. Marcuvitz ed.,
Microwave Handbook

Typical field patterns



E-mode / TM-mode



H-mode / TE-mode

Dispersion relation

$$\gamma^2 = \beta_c^2 - \omega^2 \varepsilon\mu = -\beta_z^2$$

Then the propagation of the form

$$e^{j(\omega t - \beta_z z)}$$

No acceleration in z-direction with TE mode because Ez=0.

No acceleration in z-direction with TM mode because phase velocity is not c, making phase slip.

Therefore, it cannot accelerate electrons in a long distance.

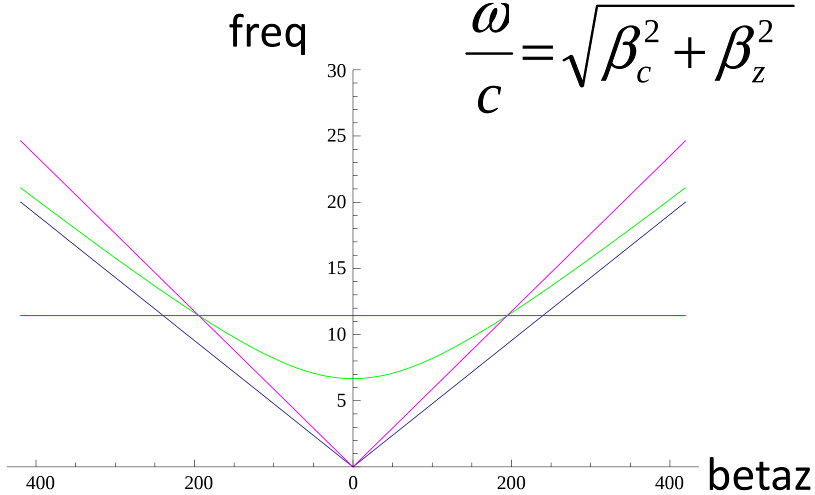
In TM case with m=0,

$$E_z = E_0 J_0(\beta_c r) \; e^{j(\omega t - \beta z)}$$

$$E_r = j E_0 Z_0 (1 - (\omega_c / \omega)^2) \; J_1(\beta_c r) \; e^{j(\omega t - \beta z)}$$

$$H_\theta = j E_0 \; J_1(\beta_c r) \, e^{j(\omega t - \beta z)}$$

$$\frac{\omega}{c} = \sqrt{\beta_c^2 + \beta_z^2}$$

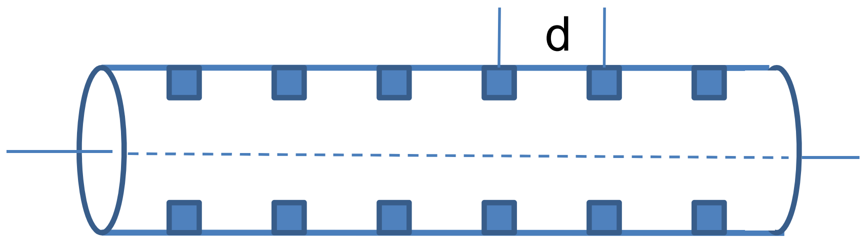


Reducing phase velocity to meet with beam velocity

Add periodical perturbation with its period=d.

If d=half wavelength, then reflection from each obstacle add coherently, making large reflection, resulting in a stop band.

Then wave component with harmonics $\beta_z=2\pi/d$ suffer from significant reflections, making a stop band.



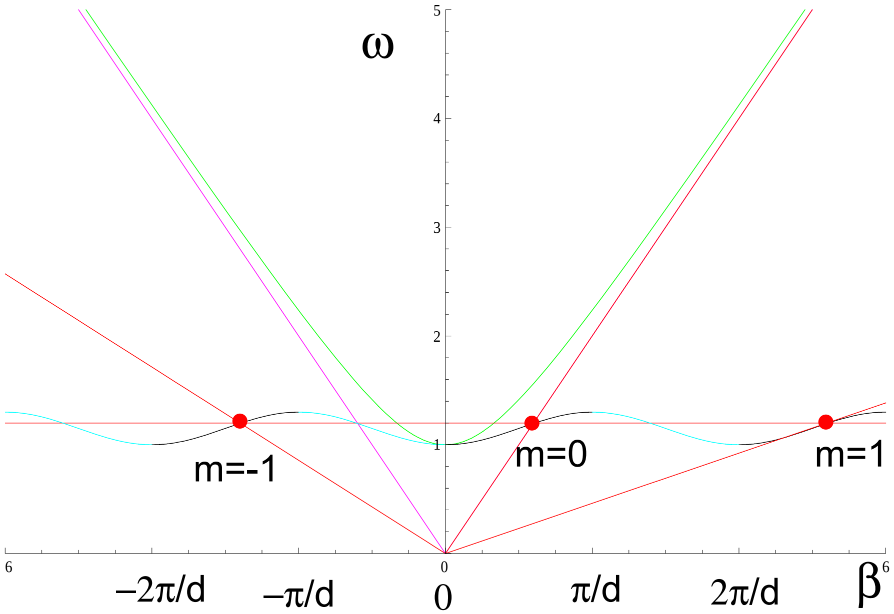
Expansion to space harmonics

$$E_z = \sum_{n=-\infty}^{n=\infty} a_n \, J_0(k_{rn}\, r)\; e^{j(\omega t - \beta_n z)}$$

where

$$\beta_n = \beta_0 + 2\pi n / d$$

$$k_{rn}^2 = k^2 - \beta_n^2$$



This is equivalent to the Floquet's theorem.

Now it can be tuned to have a phase velocity of light.
This is required for high energy linac structure.

The accelerating field contains infinite number of space harmonics, driven at frequency ω.

There are stop bands. No propagation mode exists.

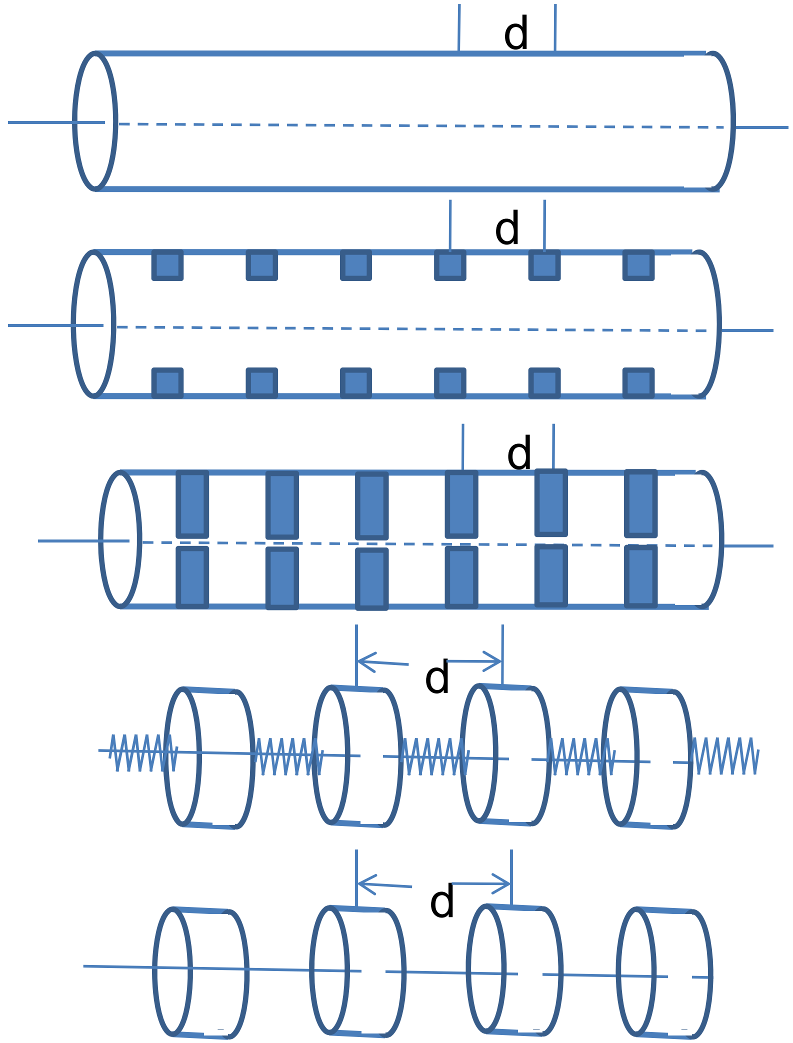
Uniform waveguide to isolated cavity

If the perturbation becomes large, reflection from the obstacle is so large that each cell becomes almost isolated cavity.

Power propagation only through a very small aperture.

In this extreme, the system can better be analyzed by a weakly coupled cavity chain model.

Now let us start from isolated cavities.



Cavity as a unit for acceleration

The extreme of isolated cavities

- Isolated cavities
  - Should be synchronized with beam
    - With external reference
    - With phasing among cavities
- External control
  - Need many input circuits along linac
  - Space factor is not high.
- Coupling between cavities
  - The only way to practically apply for very long linac as linear collider
  - Two ways, TW and SW.

Inside a cavity

- Frequency
  - Field phase should be synchronous to beam
- Acceleration field on axis
  - Focusing Ez to beam axis
  - $R/Q=V^2/2\omega U$ focusing the field within stored energy
- R shunt impedance
  - $R=V^2/P_{wall}$ keep field by feeding power
- Loss factor
  - beam cavity interaction
  - $k_L= V^2/4\omega U$ beam loading energy → kq^2
  - beam loading voltage $-kq\text{Cos}\theta$

Most basic parameter: Frequency

Resonant frequency in electric circuit

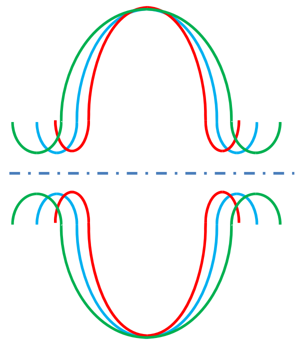
$$Freq = \frac{1}{2\pi\sqrt{LC}}$$

Cavity frequency can be tuned by changing L and/or C by perturbing magnetic field and/or electric field.

Slater's perturbation theory states;

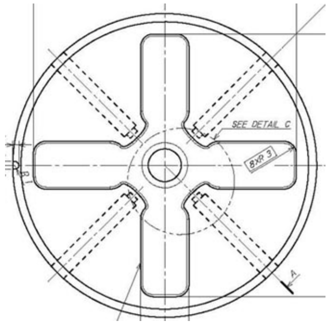
$$\frac{\omega^2 - \omega_0^2}{\omega_0^2} = \int_{\delta V} (H^2 - E^2)\, dV$$

$$\int_{Cavity} H^2\, dV \equiv 1, \quad \int_{Cavity} E^2\, dV \equiv 1$$



SCC cavity tuning
Blue nominal freq
Freq up green
Freq down red

Actual cavity tuning can be done by deforming cell shape, local dimple tuning, inserting rod, etc.



Four dimple tuning per cell in NCC.

Acceleration related parameters

Basic acceleration-related parameters. In a cavity or in a unit length.

$$V = \int E_z(z,t)\,dz$$

$$R \equiv \frac{V^2}{P_c}$$

$$R/Q \equiv \frac{V^2}{2\omega U}$$

$$Q \equiv \frac{\omega U}{P_c} = \frac{G}{R_s}$$

$$E_{acc} = V/L$$

$$R/L = \frac{E_{acc}^2}{(P_c/L)}$$

$$(R/L)/Q = \frac{E_{acc}^2}{(P_c/L)}$$

$$Q = \frac{\omega(U/L)}{(P_c/L)}$$

Wall loss by surface integral
Stored energy by volume integral

$$P_c = \frac{R_s}{2}\int |H^2|\,dS$$

$$U = \frac{\mu}{2}\int |H^2|\,dV = \frac{\varepsilon}{2}\int |E^2|\,dV$$

$$G \equiv \omega\mu \frac{\int |H^2|\,dV}{\int |H^2|\,dS}$$

Geometrical factor due to geometry.

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

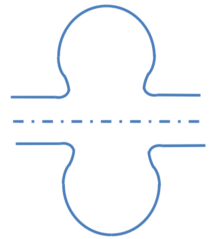
Surface resistance due to surface loss mechanism.

Efficient acceleration R/Q

How to concentrate the E_z field on axis to make an efficient acceleration? → Increase R/Q.

$$R/Q \equiv \frac{V^2}{2\omega U}$$

For higher R/Q
→ Smaller beam aperture → smaller cell-to-cell coupling.
→ Nose cone → same as above → need other coupling mechanism



ILC super-conducting cavity
→ smooth, polish with liquid, high pressure rinse, etc.
→ with circle-ellipsoid smooth connection,
→nose cone is difficult
→ less effort on higher R/Q, simply decreasing beam hole aperture because storing large energy with longer period is possible

Choke mode cavity needs field at choke area to establish imaginary short
→ sacrifice several % loss in R/Q

Shaped disk-loaded structure
→ only change R/Q by beam hole aperture

Loss factor

Loss factor K_L described later

$$k_L = \frac{\omega R}{4Q}$$

The energy left after a bunch, with change
q, passes a cavity is

$$U_m = k_{L,m} q^2$$

Larger R/Q makes bigger energy left in the cavity.
It may cause various problems;
- Phase rotation of accelerating mode
- Transverse kick field
- Heating beam pipe

In a ring application, such as storage ring and DR, sometimes R/Q should be reduced.

In the linac application, it usually tuned to be maximized to get a better acceleration efficiency.

Acceleration: Transit time factor

Assume TM010 mode in a pillbox of length L

$$E_z(z,t) = E_0\, e^{j\omega t}$$

Maximum acceleration occurs if the electric field is maximum when the beam passes the center of the cavity.

In case of thin cavity, where L<< c / f,

$$R_{un} = \frac{V_0^{\,2}}{P}\ ,\ \ V_0 = E_0\, L$$

The acceleration felt by the beam decays as time,

$$E_z(z,t) = E_0\, Cos\ (\omega\, t), \qquad z = c\, t$$

Voltage acquired by beam is then

$$V(L) = \int_{-L/2}^{L/2} (E_0\, Cos\ \ \omega\, t)\, d(c\, t) = \frac{2\, c\, E_0}{\omega}\, Sin\ (\frac{\omega\, L}{2\, c})$$

Transit time factor:

$$T \equiv V(L)/V_0 = \frac{2\, c}{\omega\, L}\, Sin\ (\frac{\omega\, L}{2\, c}) = \frac{Sin\ (x)}{x},\ \ where \quad x \equiv \frac{\omega\, L}{2\, c}$$

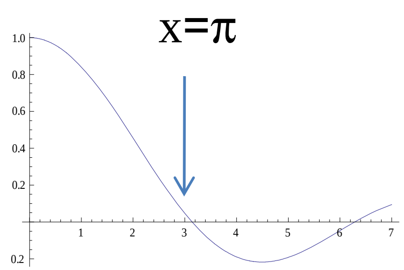
In π mode cavity

$$c\, \frac{1}{2\, f} = L$$

Then transit time factor becomes

$$T = \frac{Sin\ (\pi\, /\, 2)}{\pi\, /\, 2} = \frac{2}{\pi} \approx 0.64$$

$$R = R_{un}\, T^{\,2} = 0.4\, R_{un}$$



Surface loss and Q_0

$$Q = \frac{G}{R_s}$$

Super conductor, Nb case:

$$R_{BCS}(\Omega) = 2\times10^{-4}\frac{1}{T}\left(\frac{f}{1.5}\right)^2 e^{-\frac{17.67}{T}}$$

$$f(GHz),\ T(°K)$$

at 1.3GHz, T=2K << 9K → ρ_{BCS}=11nΩ

Higher freq → larger BCS loss.

Possible to increase geometrical factor, G by shaping. It reduces cryogenic power consumption.

Actually, $R_s = R_{BCS} + R_{residual}$

Need to keep smaller R_s by making proper material surface.

Suppressing multipacting and field emission loading.

Normal conductor:

Equivalent surface current in thin skin depth δ_s with surface resistance R_s.

R_s depend on mostly choice of material.

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

$\sigma_{Cu}=5.8\times10^{7}(1/\Omega)$ → R_s~28mΩ

Higher Rs makes larger pulse surface heating during short pulse.

How to increase Q_0

ILC SCC
→ TESLA to LL shape
→ expanding cell outer area → reduce H field → against quench
→ eventually increase Q_0 by larger G

Heavily damped cavity
→ Loss in choke mode cavity,
to establish imaginary short by storing power at choke

Loss in heavily damped cavity with damping waveguide
→ opening toward damping waveguide, magnetic field gets higher

For disk-loaded structure
→ good to have higher Q0 to reduce wall power, higher transfer efficiency
→ near round cell shape, make it close to sphere

Suppression of local field enhancement

Electric field; Ep / Eacc

Peak surface electric field

→ Field emission source

→ Breakdown in NCC

Magnetic field; Hp / Eacc

→ Surface temperature rise within a pulse in NCC

→ Quenching of superconductor above magnetic field threshold

How to decrease these ratios?

→ Shaping global cell shape

→ Make it smooth locally

Need care on SCC EWB welding quality

NCC remove burrs and sharp corners

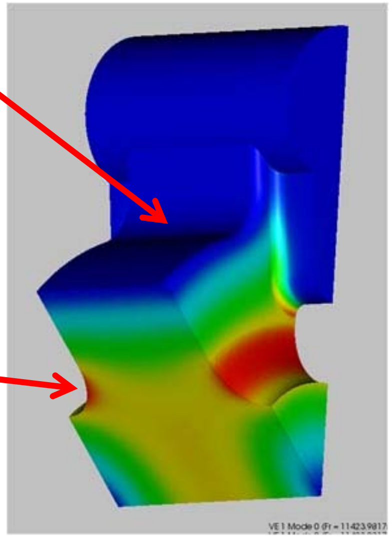
EBD: Electron beam welding

Cares on local field enhancement

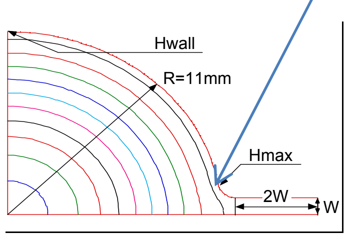
Care on the opening edge to damping waveguide.

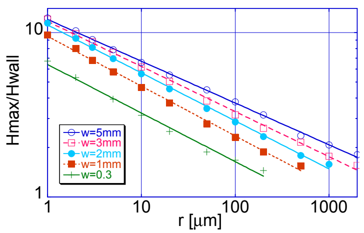
Smooth opening to reduce Hp

Smooth shape to suppress Ep



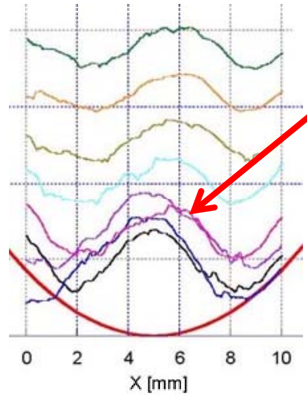
Enhancement at small edge on opening





2D calculation of cylinder with radial opening channel

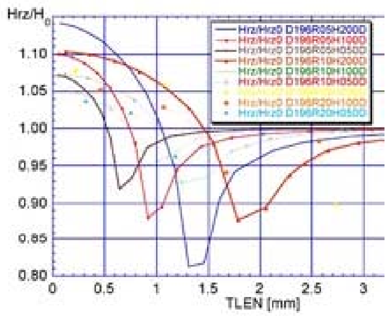
Care on EBW bead shape in SCC cavity.



Bump above design contour

0.2mm

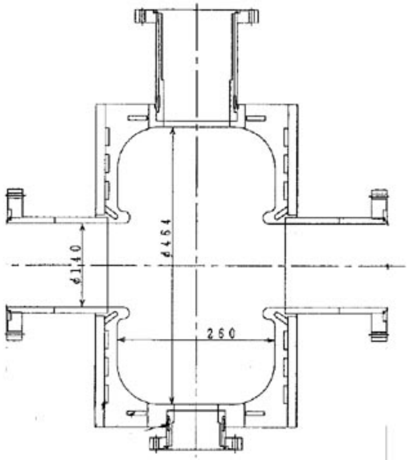
Bump over EBW line



Hs enhancement over a small sphere sitting on a spherical cavity.

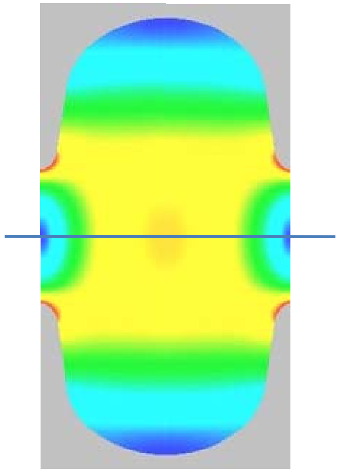
Height / radius < 0.001 for δHs/Hs<a few %

Shaping of accelerator cell profile examples

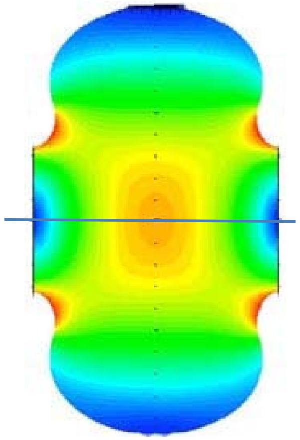


Damped cavity for storage ring
With nose cone.

Single cell
Damped cavity

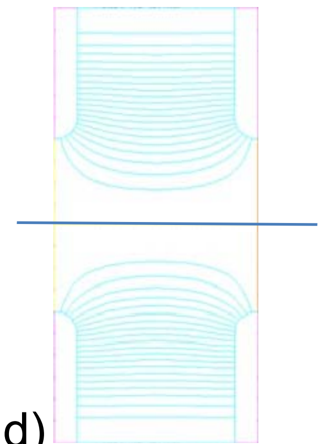


SCC
Typical cavity



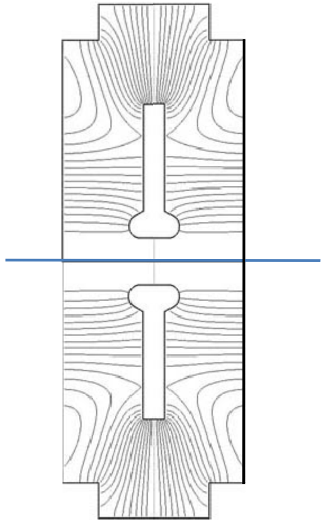
SCC
Reentrant cavity

SW TM010
SCC π mode
Smooth
More E_z on axis
Less H_s at outer



HDDS for GLC/NLC

TW DLS
TM010
$5\pi/6$-mode
(figure shows π field)

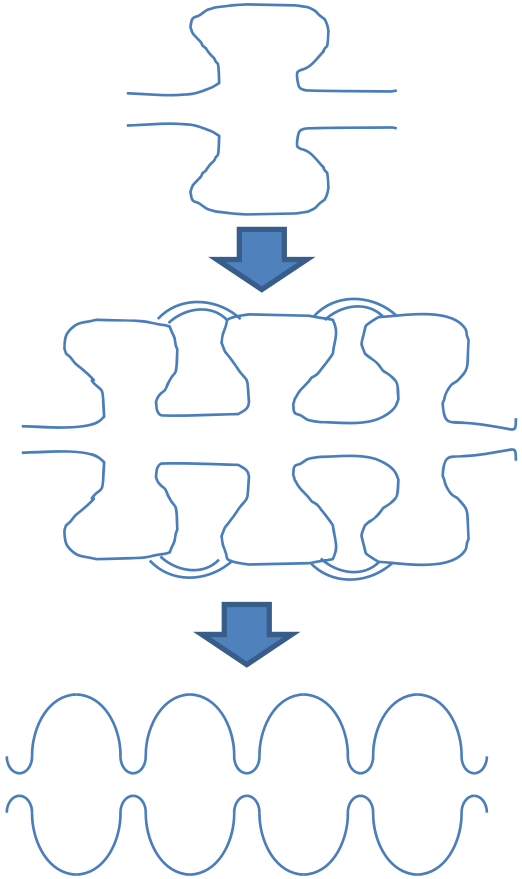


DAW cavity

TM020-like $\pi/2$
Floating washer
Coupling mode in
addition to
accelerating mode.
High Q, high R

Coupled cavity system in a SW regime

Cell-to-cell coupling



Ep/Eacc increases, but easy to confine field to increase *T* if normalized in the field-existing area.

Coupled cavity needs coupling between cells through some mechanism other than beam aperture.

Weakly coupled-cell through beam aperture
SCC cavity for ILC
Transit time factor cannot be improved, similar to that of pillbox or less.

Weakly coupled resonators

- Each resonator has
  - Internal freedom
  - Eigen modes in the cavity in an almost closed surface
  - Excited resonant modes couple to beam
  - Acceleration, deceleration, transverse kick, etc.

- Total system
  - Weak coupling usually to adjacent cavity through some apertures
  - Total system is described as coupled resonator system

- Mathematically equivalent to
  - Mechanically coupled oscillator model
  - Electrically coupled resonant circuit model

Coupled resonator model to describe the total system

Assume each cavity is represented by a resonant circuit. (described later)

$$I_0 = X_0 \left(1 + \frac{\omega_0}{j\omega Q} - \frac{\omega_0^2}{\omega^2}\right) + k X_1$$

$$I_n = X_n \left(1 + \frac{\omega_0}{j\omega Q} - \frac{\omega_0^2}{\omega^2}\right) + \frac{k}{2}(X_{n-1} + X_{n+1})$$

$$I_N = X_N \left(1 + \frac{\omega_0}{j\omega Q} - \frac{\omega_0^2}{\omega^2}\right) + k X_{N-1}$$

where $\omega_0^{-2} = 2LC$

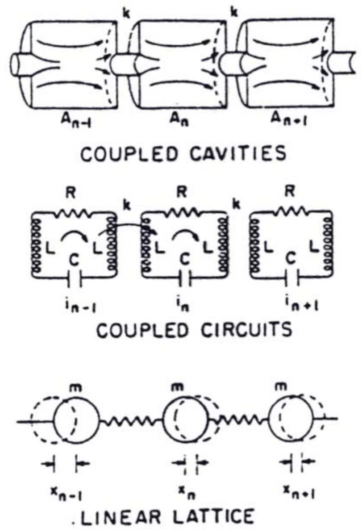
$$X_n = \sqrt{2L}\, i_n$$

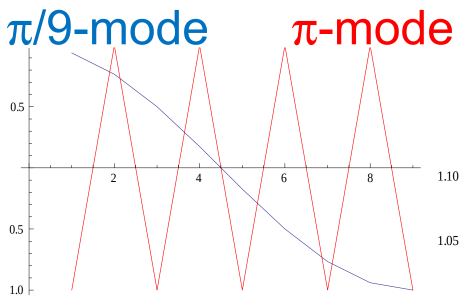
$$Q = 2\omega_0 L / R$$

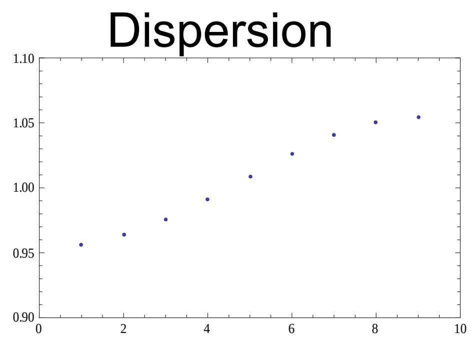
If Q>>1,

$$X_n^q = const \quad Cos\left(\frac{\pi q n}{N}\right) e^{j\omega_q t}$$

$$with \qquad \omega_q^2 = \frac{\omega_0^2}{1 + k\ Cos\left(\frac{\pi q}{N}\right)}$$







Perturbation analysis

$$X \equiv \begin{bmatrix} X_0 \\ X_1 \\ \\ X_N \end{bmatrix}, \quad M \equiv \begin{bmatrix} 1 & k & 0 & & & 0 \\ k/2 & 1 & k/2 & 0 & & 0 \\ & k/2 & 1 & k/2 & 0 & \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & & 0 & k/2 & 1 & k/2 \\ 0 & 0 & & 0 & k & 1 \end{bmatrix}$$

Equation:

$$M\ X - (\frac{\omega_0}{\omega})^2\ X = 0$$

Solution:

$$X^q = \begin{bmatrix} 1 \\ Cos(\pi q / N) \\ \\ Cos(\pi q n / N) \\ \\ Cos \pi q \end{bmatrix}$$

Frequency errors

$$\omega_{0n}^{2'} = \omega_0^2 + \delta\omega_{0n}^2$$

First order equation

$$M\ \delta X^q = \frac{\omega_0^2}{\omega_q^2}\delta X^q + \begin{bmatrix} \delta(\frac{\omega_{00}^2}{\omega_q^2}) & 0 & 0 & 0 \\ 0 & \delta(\frac{\omega_{01}^2}{\omega_q^2}) & 0 & 0 \\ \bullet & \bullet & \bullet & 0 \\ 0 & 0 & 0 & \delta(\frac{\omega_{0N}^2}{\omega_q^2}) \end{bmatrix} X^q$$

Perturbation analysis (cont.)

For π mode

$$\delta X_n^{\pi} / X_n^{\pi} = \sum_{P=1}^{N} \tilde{\varepsilon}_p \, (1-k)\, Cos\pi \frac{pn}{N} / k\,(1 - Cos\frac{\pi p}{N})$$

For π/2 mode

$$\delta X_n^{\pi/2} / X_n^{\pi/2} = \frac{1}{k} \sum_{P=1}^{N} \tilde{\varepsilon}_p \, (Sin\frac{\pi pn}{N} Sin\frac{\pi n}{2} / Sin\frac{\pi p}{N})$$

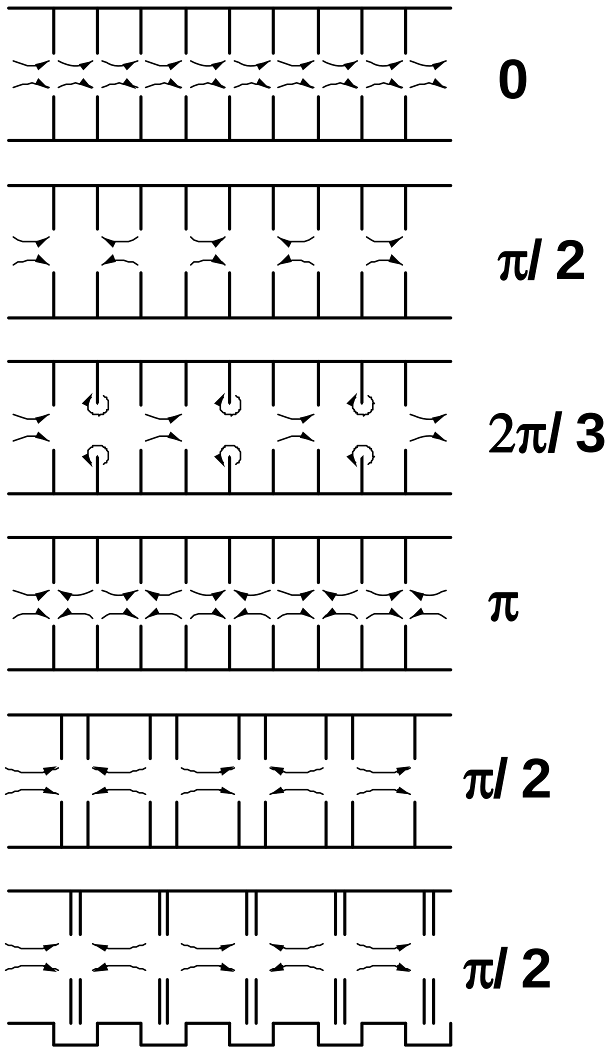
Where $\tilde{\varepsilon}_p$ is Fourier component of frequency perturbation of $Cos\frac{\pi pn}{N}$

Both perturbation scales as 1/k.
As for number of cells, π mode scales as N^2, while π /2 mode linearly on N.

For longer structure, π mode becomes difficult.
This is related to no energy exchange in π mode because of zero group velocity.

For energy transfer, we need other mode than π mode, which destroys π mode itself.

π-mode and π/2 mode



Most basic but no net acceleration

Stable cavity system but half acceleratin

Good compromize in TW linac

Most efficient but weak against perturbation

Electrically π/2 but acceleration efficiency ~ π-mode

Actual π-mode acceleration with π/2 coupling element outside accelerating cell.

SCC 9-cell cavity example

- Cell to cell coupling $\kappa \sim 2\%$
- Dispersion curve $f_0/(1+\kappa \, \text{Cos}\phi)^{1/2}$
- Band width BW ~ κf_0
- Small mode separation $f_\pi - f_{8\pi/9} \sim 0.06 * BW$
- Tuning of SCC 9-cell cavity see next page
- Shunt Impedance of Total system
  - $R_{total} = R_{single}$ X 9 if flat field and right frequency

Practical issues in SCC being analyzed with coupled resonator model

- SCC 9-cell cavity is basically expressed as
  - a single chain of coupled resonators.
- Field flatness consideration and tuning
  - Frequency of cells
  - Lorentz force detuning
  - EP deformation of cell shape
- Coupling between cells
  - It makes coupling coefficient between resonators.
  - It represent robustness of field flatness against perturbations
  - It gives spacing to the nearest resonance, $8\pi/9$ mode.
- Some other system such as super-structure
  - Also can be described by a weekly coupled two 9-cell systems.

Frequency error and field distribution

$$M_0 \cdot x_0 = \lambda_0 \, x_0$$

$$(M_0 + \delta) \cdot (x_0 + \delta x) = (\lambda_0 + \delta\lambda)\,(x_0 + \delta x)$$

$$\therefore \quad \delta \cdot x_0 + M_0 \cdot \delta x = \delta\lambda \cdot x_0 + \lambda_0 \cdot \delta x$$

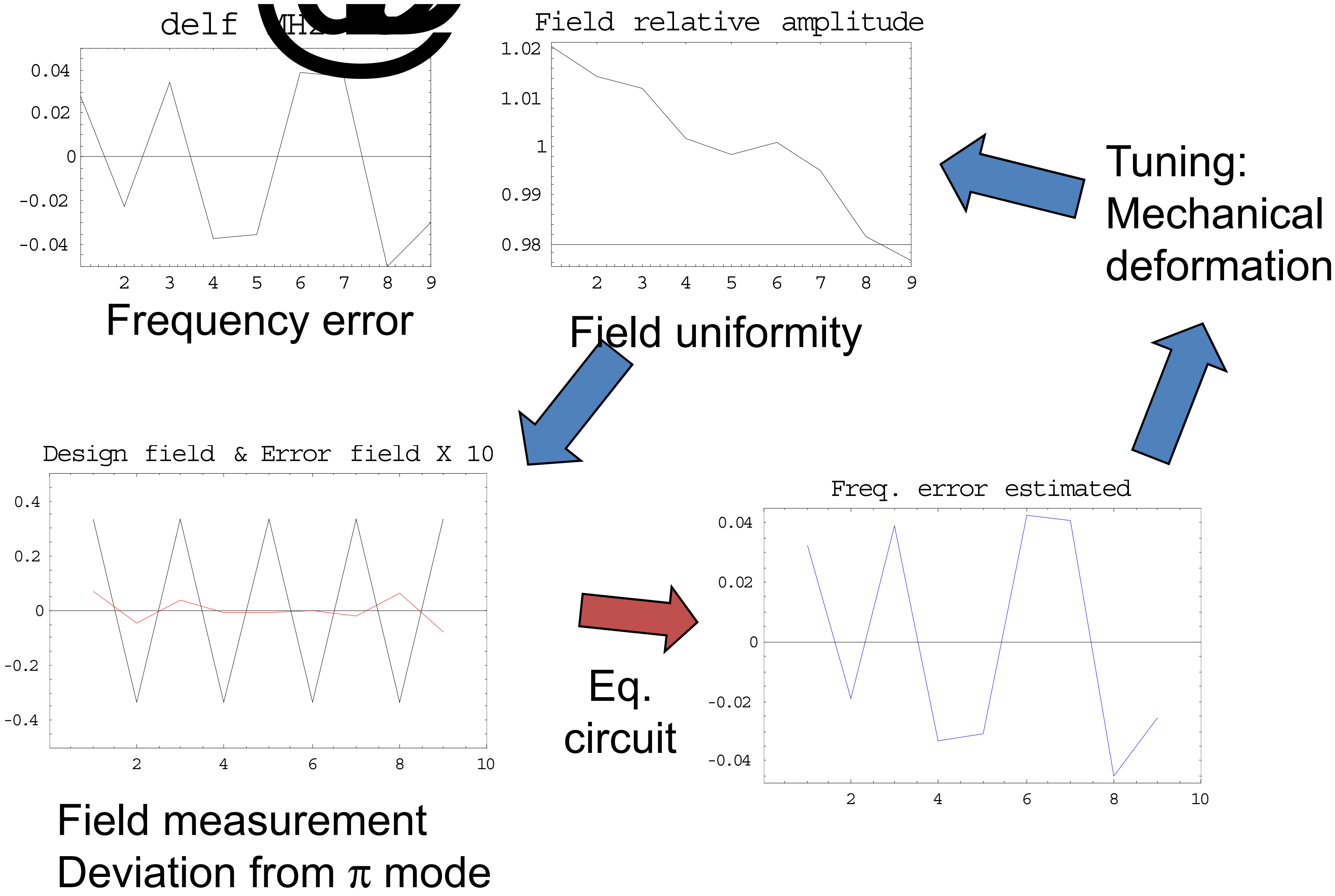
$$because \quad \delta\lambda / \lambda_0 << \left|\delta x\right| / x_0$$

$$then \quad \delta \cdot x_0 = (\lambda_0 - M_0) \cdot \delta x$$

$$we\, know\, design\, values: \; x_0, \lambda_0, M_0 \; and\; measured\; \delta x$$

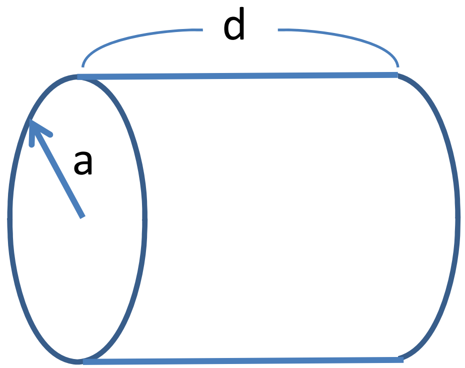
$$so\, that\, we\, get \quad \delta = 2 \cdot (\delta\omega / \omega_0)$$

Frequency error estimation from measured field



Pillbox cavity as a simple base to represent practical cavities

SW Cavity example: pillbox



In a cylindrical waveguide, two propagation modes exist;

$$e^{i(\omega t-\beta)z} \quad and \quad e^{i(\omega t+\beta)z}$$

Forward wave and Backward wave

For satisfying the boundary condition at both end plates, the solution with the superposition of these two counter-propagating modes in proper phase and amplitude becomes SW in a pillbox cavity.

$$E_r = -\frac{\beta_z}{K_c} \; Cos(m\theta) \; J'_m(K_c r) \; Sin(\beta_z z)$$

$$E_\theta = \frac{m\beta_z}{K_c^2} \; Sin(m\theta) \; \frac{1}{r} J_m(K_c r) \; Sin(\beta_z z)$$

$$E_z = \; Cos(m\theta) \; J_m(K_c r) \; Cos(\beta_z z)$$

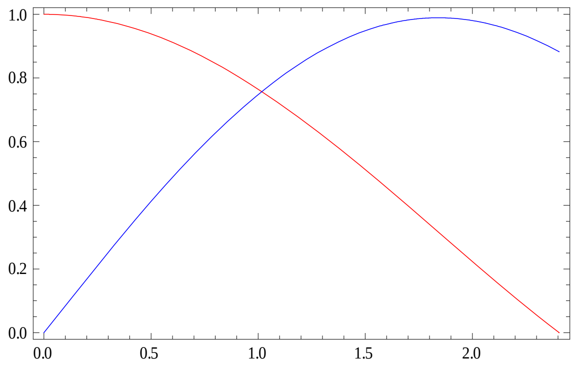
$$H_r = -j\frac{m\omega\varepsilon}{K_c^2} \; Sin(m\theta) \; \frac{1}{r} J_m(K_c r) \; Cos(\beta_z z)$$

$$H_\theta = -j\frac{\omega\varepsilon}{K_c} \; Cos(m\theta) \; J'_m(K_c r) \; Cos(\beta_z z)$$

$$H_z = 0$$

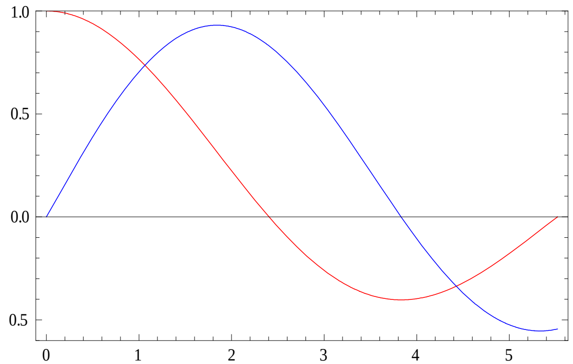
$$where \quad K_c = \rho_{mn}/a, \quad \beta_z = l\pi/d$$

Bessel's functional form representing pillbox field to satisfy boundary condition



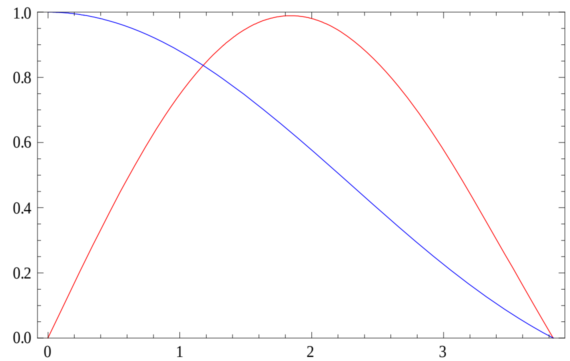
TM01
Ez and Hr

Acceleration
Max at center and 0 at r=a



TM02
Ez and Hr

Acceleration
More energy storage for a given acceleration



TM11
Ez and Hr

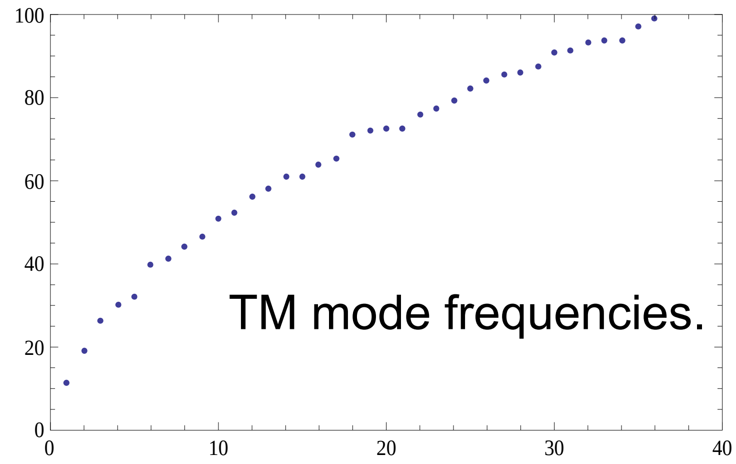
Transverse kick
Two polarizations
Zero at center and linearly increase as r increases.
Big kick at center H field.

Mode frequency in a pillbox cavity

Modes are classified as TM and TE mode.
Frequencies are determined to satisfy boundary condition at two end surface,

$$\left(\frac{\omega_{mnl}}{c}\right)^2 = \left(\frac{\rho_{mn}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2$$

where ρ_{mnl} is Bessel's functions zero for TM or derivative of Bessel's function becomes zero for TE.
Modal density increases as higher frequency region.



Accelerating mode is usually TM010 mode.
If we want more stored energy with the same acceleration, TM011, TM020 or others can be used.

Q value of modes in pillbox cavity

$$U = \frac{\mu}{2}\int |H|^2 dV = \frac{\varepsilon}{2}\int |E|^2 dV , \quad P = \frac{R_s}{2}\int |H|^2 dS , \quad Q = \frac{\omega U}{P}$$

For TM modes;

Volume integral;

$$\int |H|^2 dV = \int (H_r^2 + H_\theta^2)\, r\, dr\, d\theta\, dz$$

$$= A\, d \int [H_r^2(z=0) + H_\theta^2(z=0)]\, r\, dr\, d\theta$$

$$where \quad A = 1 \ for \ \ell = 0 \ and \quad A = 1/2 \ for \ \ell \neq 0$$

$$for \ \ m \neq 0$$

$$\int |H|^2 dV = A\pi d \left(\frac{\omega\varepsilon}{K_c}\right)^2 \int [(\frac{m}{K_c r})^2 J_m^2(K_c r) + J_m'^2(K_c r)]\, r\, dr$$

$$= A\pi d \left(\frac{\omega\varepsilon}{K_c}\right)^2 I(r = a)$$

$$= A\pi d \left(\frac{\omega\varepsilon}{K_c}\right)^2 \frac{a^2}{2} J_m'^2(\rho_{mn})$$

$$for \ \ m = 0 \ similar \quad result$$

$$I \equiv \int [(\frac{m}{K_c r})^2 J_m^2(K_c r) + J_m'^2(K_c r)]\, r\, dr = \frac{r^2}{2}[J_m'^2(K_c r) + \frac{2}{K_c r} J_m'(K_c r) J_m(K_c r) + \{1 - (\frac{m}{K_c r})^2\} J_m^2(K_c r)]$$

Q (cont.)

Finally we get,

$$\int |H|^2 \, dV \;=\; A\,B\,\pi\,d\,(\frac{\omega\varepsilon}{K_c})^2 \frac{a^2}{2} J_m^{'2}(\rho_{mn})$$

$$where \quad B = 2 \;for\; m = 0 \;and\; B = 1 \;for\; m \neq 0$$

Surface integral;

$$\int |H|^2 \, dS = 2\int [H_r^2(z=0) + H_\theta^2(z=0)]\, r\, dr\, d\theta + \int [H_\theta^2(r=a)]\, a\, d\theta\, dz$$

$$= 2B\pi(\frac{\omega\varepsilon}{K_c})^2 \frac{a^2}{2} J_m^{'2}(\rho_{nm}) + A\,B\,\pi\,d\,(\frac{\omega\varepsilon}{K_c})^2 a\, J_m^{'2}(\rho_{nm})$$

$$= B\pi(\frac{\omega\varepsilon}{K_c})^2 a\,(a + Ad)\, J_m^{'2}(\rho_{nm})$$

Therefore, for TM modes,

$$Q = \frac{1}{\delta_s} \frac{a}{1 + \frac{a}{Ad}}$$

Where δ_s is skin depth,

$$\delta_s = \sqrt{\frac{2}{\sigma\omega\mu}}$$

Similarly we obtain for TE modes;

$$Q = \frac{1}{\delta_s} \frac{a[\rho_{mn}^{'2} + (\frac{a}{d})^2 (\ell\pi)^2](\rho_{mn}^{'2} - m^2)}{\rho_{mn}^{'4} + 2(\frac{a}{d})^3(\ell\pi\rho_{mn}^{'})^2 + (\frac{a}{d})^2(1 - \frac{2a}{d})(\ell\pi m)^2}$$

Acceleration in a single pillbox cavity

$$E_{acc} = \frac{1}{d}\int_{-d/2c}^{d/2c} \mathrm{Re}\,[E_z(z,t)|]\,dt$$

$$= \frac{1}{d}\int_{-d/2c}^{d/2c} \mathrm{Re}\,[E_z(z)|_{beam}\; e^{j(\omega t+\phi)}]\,dt$$

$$= \frac{1}{d}\int_{-d/2}^{d/2} \mathrm{Re}\,[E_z(z)|\{Cos\,(\beta_z z+\phi) + j\,Sin\,(\beta_z z+\phi)\}]\,dz$$

$$(\,for\;\; synchronous\;\; beam\quad \omega t - \beta_z z = 0)$$

$$= \frac{1}{d}\int_{-d/2}^{d/2} E_z(z)\,Cos\,(\beta_z z+\phi)\;dz$$

For an even function, such as the case of TM010, the ϕ=0 to maximize acceleration. This is the case of on-crest acceleration.

R/Q value of modes in pillbox cavity

Normalization: $\int |E|^2 \, dV = 1 \; -> \; U = \frac{\varepsilon}{2}$

Definition of field integral:

$$E_{acc} = \frac{1}{d} \int_{-d/2}^{d/2} E(z,t) \, Cos(\omega t) \, dz$$

$$E_0 = \frac{1}{d} \int_{-d/2}^{d/2} E(z,t) \, dz$$

Transit time:

$$T = E_{acc} / E_0$$

Impedance:

$$\frac{R}{Q} = \frac{1}{d} \frac{(E_{acc} d)^2}{2\omega U} = \frac{E_{acc}{}^2}{2\omega (U/d)} = \frac{2 d Z_0}{2\omega / c} E_{acc}{}^2$$

R/Q for TM_{0nl} field

In a previously obtained field expression of pillbox mode,

$$\int |E|^2 \, dV = A_{nl}$$

$$A_{n0}{}^2 = \pi \, d \, a^2 \, J_1^2(\rho_{n0}) \quad for \quad l = 0$$

$$A_{nl}{}^2 = \pi \, d \, a^2 \, J_1^2(\rho_{nl}) \, \frac{1}{2} \{1 + (\frac{l \pi a}{d \, \rho_{nl}})^2\} \quad for \quad l \neq 0$$

Then, to make the normalization, $\int |E|^2 \, dV = 1$

$$E_z(z) = \frac{1}{A_{nl}} Cos\,(m\theta)\, J_m(K_c r)\, Cos\,(\beta_z z)$$

For TM010,

$$E_{010,z}(z) = \frac{1}{A_{10}} \; J_0(r = 0)$$

Finally,

$$E_{acc} = \frac{J_0(r=0)}{A_{10} \, d} \int_{-d/2}^{d/2} Cos\,(\frac{\omega}{c} z + \phi_s)\, dz = \frac{J_0}{A_{10} \, d} \frac{2c}{\omega} Sin\,(\frac{\omega d}{2c})\, Cos\ \phi_s$$

$$T = \frac{2c}{\omega \, d} Sin\,(\frac{\omega d}{2c})\, Cos\ \phi_s$$

$$\frac{R}{Q} = \frac{2 \, d \, Z_0}{\omega / c} \, (\frac{2 \, c \, J_0}{A_{10} \, d \, \omega})^2 \, Sin^2(\frac{\omega d}{2c})\, Cos^2 \phi_s$$

For TM_{0nl} mode

For TM_{0nl}, with $\ell \neq 0$

$$E_{acc} = \frac{J_0}{A_{nl}\ d} \int_{-d/2}^{d/2} Cos\ (\frac{\ell\pi}{d} z)\ Cos\ (\frac{\omega}{c} z + \phi_s)\ dz$$

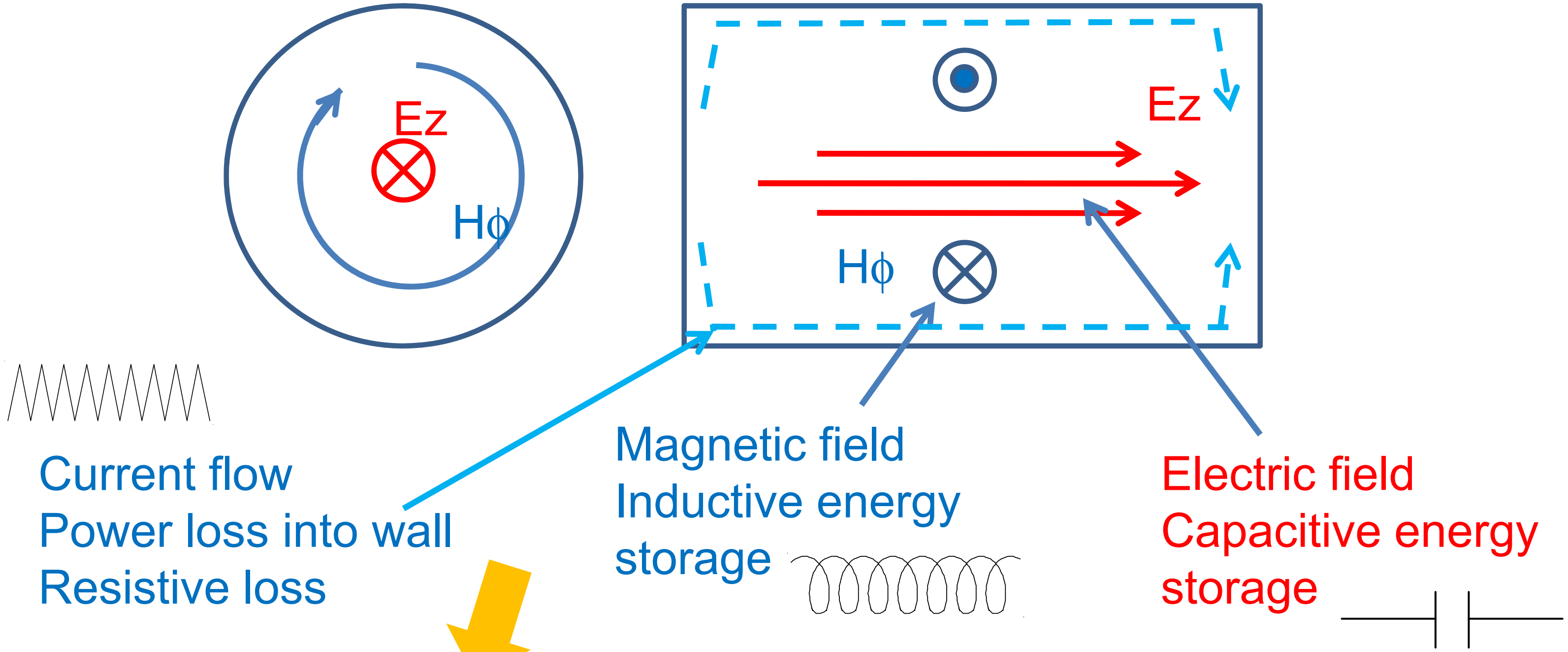
$$= \frac{J_0\ Cos\ \phi_s}{A_{nl}\ d} [\frac{Sin\ (\frac{\omega}{c} + \frac{\ell\pi}{d})\frac{d}{2}}{\frac{\omega}{c} + \frac{\ell\pi}{d}} + \frac{Sin\ (\frac{\omega}{c} - \frac{\ell\pi}{d})\frac{d}{2}}{\frac{\omega}{c} - \frac{\ell\pi}{d}}]$$

$$= \frac{2\ \omega\ J_0\ Cos\ \phi_s}{A_{nl}\ d\ c} \frac{1}{(\frac{\omega}{c})^2 - (\frac{\ell\pi}{d})^2} B_l$$

$$where \quad B_l = \begin{cases} Cos\ (\frac{\ell}{2}\pi)\ Sin\ (\frac{\omega\ d}{2\ c}) & for \quad \ell = even \\ Sin\ (\frac{\ell}{2}\pi)\ Cos\ (\frac{\omega\ d}{2\ c}) & for \quad \ell = odd \end{cases}$$

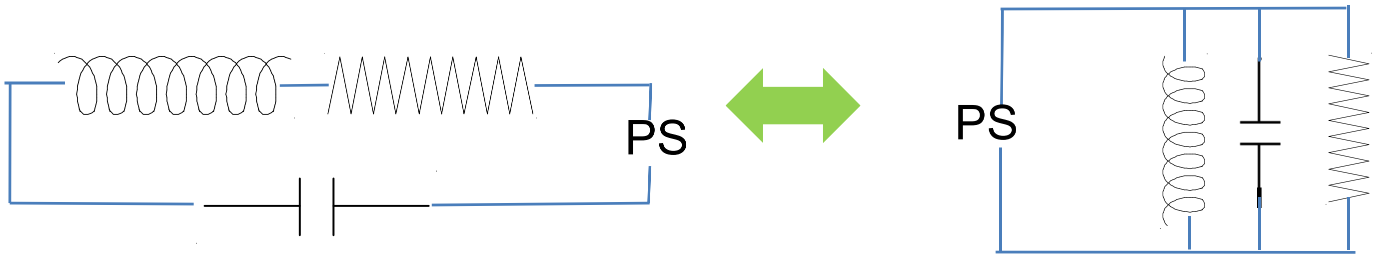
Circuit modeling of cavity

Cavity to equivalent circuit



This system can intuitively be expressed with series resonant circuit in electric circuit

The differential equiation is mathematically equivalent and the system can also be presented by parallel resonant circuit.



Circuit model of cavity

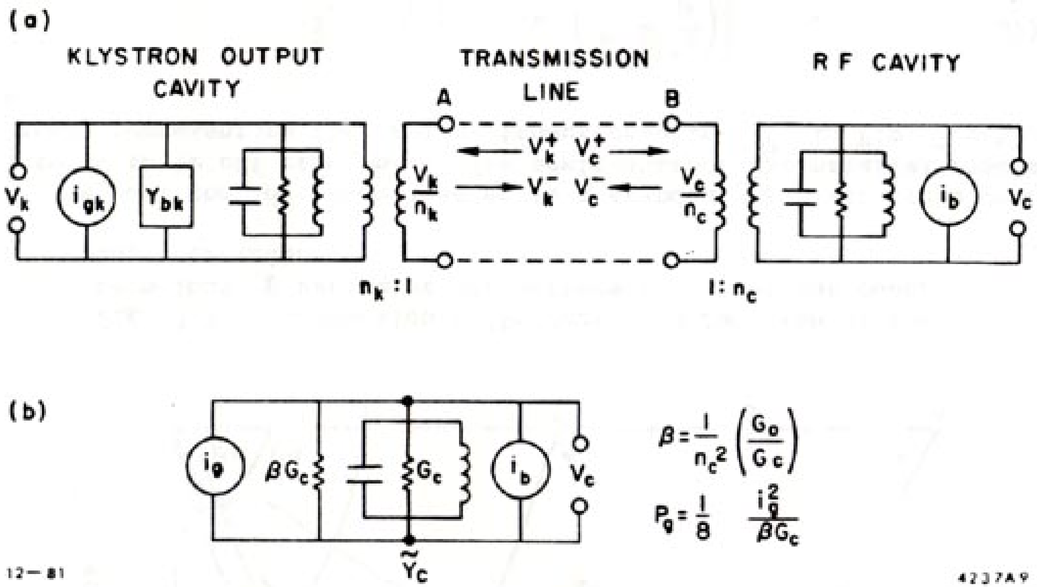
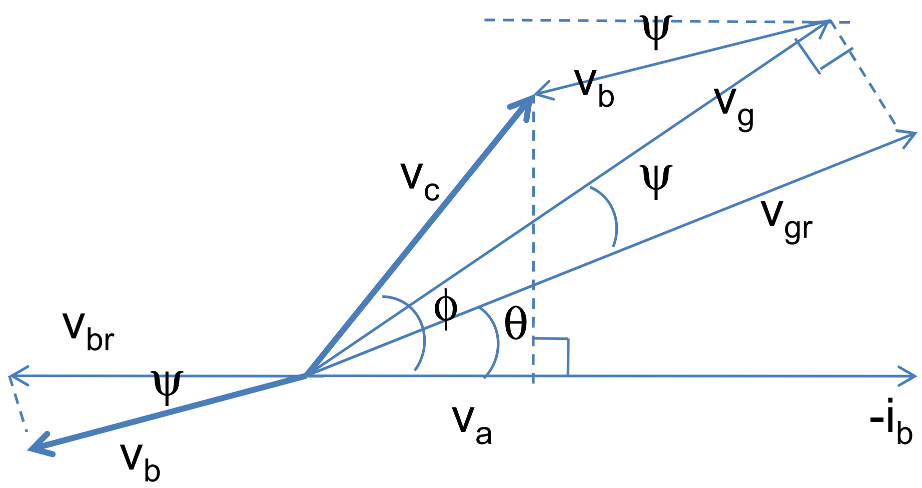


Fig. 3.9. (a) Equivalent circuit for a beam-loaded cavity coupled to a klystron; (b) simplified circuit assuming a matched RF source.

From P. Wilson Lecture Note.

RF power source, transmission line and cavity can be described as an equivalent circuitry.

Cavity response and beam



Phasor diagram

ψ: Cavity tuning angle

$$\tan\psi = -2 Q_L \delta$$

$$\delta = (\omega - \omega_0)/\omega_0$$

If frequency changes to the order of Q band width, ψ becomes as big as 45deg! v_g/v_{gr} = 0.7 (reduced much)

To save power, need smaller $\delta \ll 1/Q_L$

Need v_{gr} to keep $\phi > 0$.

Transient field around cavity

$$E_{out} = E_e + \Gamma E_{in}$$

$$P_{in} = P_{out} + P_c + \frac{dU_c}{dt}$$

$$\Gamma \approx -1 \qquad \beta_c \equiv \frac{P_{out}}{P_c}$$

$$t_c \equiv \frac{2Q_L}{\omega} = \frac{2Q_c}{(1+\beta_c)\,\omega}$$

$$t_c \frac{dE_e}{dt} + E_e = \frac{2\beta_c}{1+\beta_c} E_{in}$$

Emitted field from cavity stored field

$$E_e = \frac{2\beta_c}{1+\beta_c} E_{in} \, (1 - e^{-t/t_c})$$

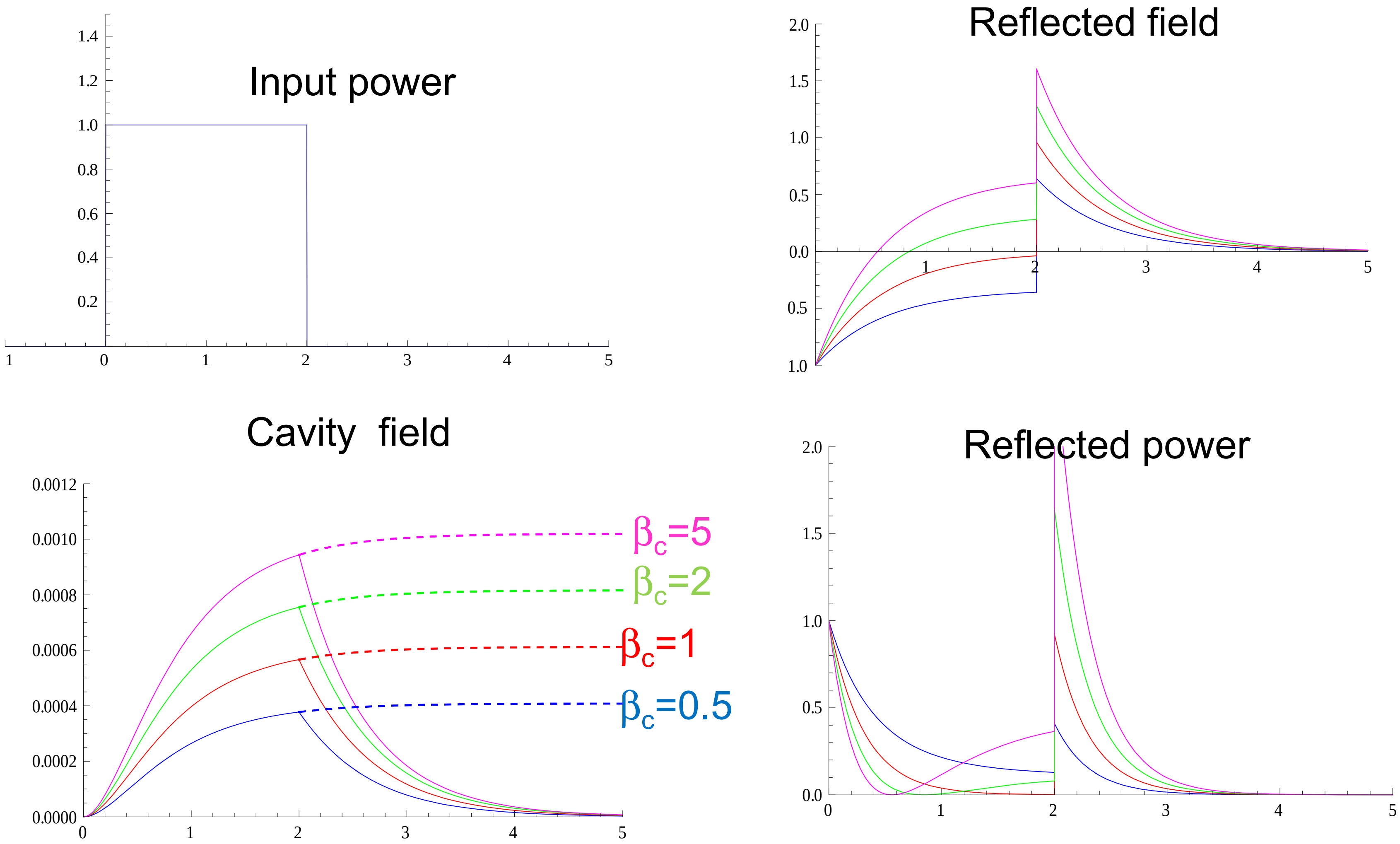
Emitted (out-going, reflected) field from cavity

$$E_{out} = E_e - E_{in} = \frac{2\beta_c}{1+\beta_c} E_{in} \, (1 - e^{-t/t_c}) - E_{in}$$

Cavity filled voltage

$$V(t) = (1 - e^{-t/t_c}) \sqrt{(\frac{R}{Q})\, \omega t_c \, P_{in} \, \frac{2\beta_c}{1+\beta_c}}$$

Cavity transient response



SW cavity beam loading

From energy conservation,

$$P_{in} = P_{out} + P_c + \frac{dU_c}{dt} + I_b V_a$$

With proper timing $t=t_r$, we can make

$$\frac{dU_c}{dt} + I_b V_a = 0$$

Then, we can make the beam loading compensation to keep the voltage gain the same within the bunch train.

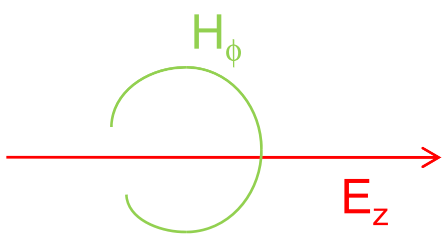
This is realized in a proper timing, feeding power and beam current to adjust to the transient behavior of the cavity.

SW versus TW

SW and TW

$$SW \;:\; e^{j\omega t} Sin(kz) \;,\; e^{j\omega t} Cos(kz)$$

$$\nabla \times H = \frac{\partial D}{\partial t}$$

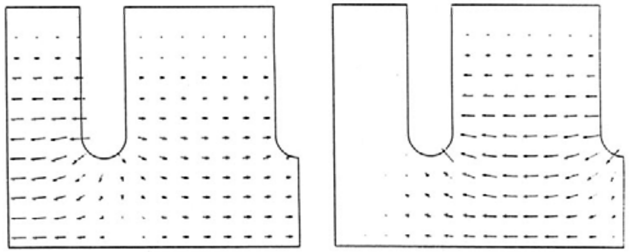


Superposition of Cos + j Sin
Example pillbox TM010 mode
E_z and H_ϕ is 90 degrees out of phase

Forward or backward wave F+B or F-B
E_r and H_ϕ in phase to make Poynting vector

$$TW \;:\; e^{j(\omega t - kz)} \;,\; e^{j(\omega t + kz)}$$

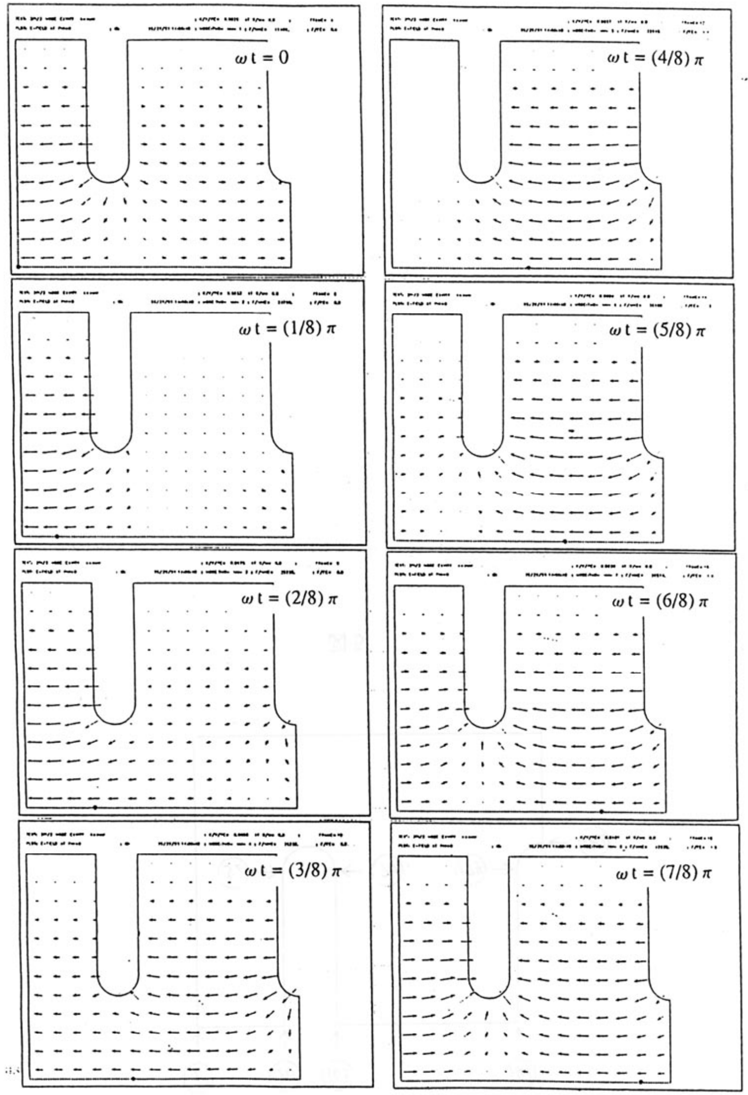
SW field ←→ TW real field



Short b.c.
Cosine

Open b.c.
Sine

$$E^{TW} \propto E^{SW}_{Short} + j\,E^{SW}_{Open}$$



SW ← → TW

- $R/Q_{TW} = R/Q_{SW}$ X 2
  - The space harmonics of SW propagating against beam cannot contribute to the net acceleration, while the reverse-direction power is needed to establish the SW field.
- SCC cavity operated in TW mode
  - Resonant ring like: return the outgoing power from cavity to feed again into the upstream of the cavity. But need sophisticated system outside.
- Many NCC at high gradient use TW
  - High field need high impedance.
  - Power not used for beam acceleration nor wall loss need be absorbed by outside RF load.
- High gradient in SW cavity
  - Probably resistive against damage from arcing due to easy detuning.
  - Stability requires short cavity→ need more feeding points

TW linac

TW basic idea different than standing wave

- TW
  - Travelling wave = microwave power flow in one direction
  - Beam to be coupled to the field associated to this power flow
- Power flow
  - Acceleration field decreases along the structure
    - Due to wall loss and energy transfer to beam
  - Attenuation parameter is a key parameter
  - Extracted power is absorbed by load or re-inserted into the structure
- Shaping of attenuation along a structure
  - Accelerating mode → CZ, CG
  - Higher mode consideration → Detuned, …
- Synchronization to beam
  - Control of frequencies of cells to make the phase velocity right
- Input matching with mode conversion
  - Matching to TM01-type mode in the periodic chain of cavities
  - From TE10 in waveguide

Acceleration field in TW linac

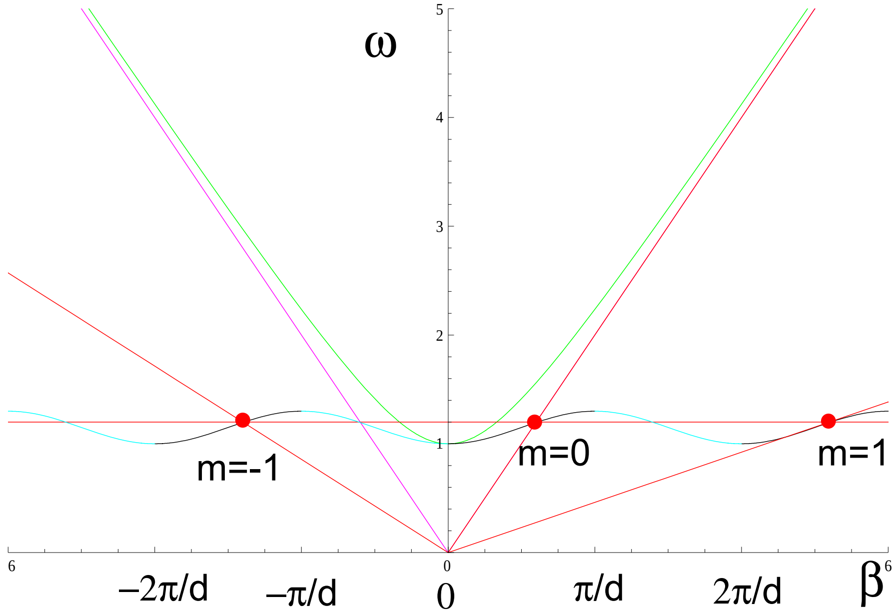
$$E_z(r,\theta,z,t)=\sum_{n=-\infty}^{n=\infty} E_p(r,\theta,z)\ e^{j\omega t-\gamma z}$$

$$=\sum_{n=-\infty}^{n=\infty} E_n(r,\theta)\ e^{j(\omega t-\beta_n z)}\quad e^{-\alpha z}$$

where

$$\beta_n=\beta_0+2\pi n/d$$

$$E_n(r,\theta)=\frac{1}{d}\int_z^{z+d} \quad E_d(r,\theta,z)\ e^{j(2\pi n/d)z}$$



$$E_z(r,\theta,z,t)=\sum_{n=-\infty}^{n=\infty} a_n\ J_0(k_{rn}\,r)\ e^{j(\omega t-\beta_n z)}$$

$$where\quad \beta_n=\beta_0+2\pi n/d \quad and \quad k_{rn}^2=k^2-\beta_n^2$$

Structure parameters in a uniform structure (CZ)

SW cavity → TW linac

Acceleration voltage in a cavity → Acceleration gradient

Stored energy in a cavity → Stored energy in a unit length

Power loss in a cavity → Power attenuation in a unit length

$$r \equiv \frac{E_a^2}{\left|dP/dz\right|} \qquad Q \equiv \frac{\omega u}{\left|dP/dz\right|} \qquad r/Q \equiv \frac{E_a^2}{\omega u}$$

Group velocity and attenuation parameter

$$P \equiv v_g \cdot u \quad with \quad v_g = d\omega / d\beta$$

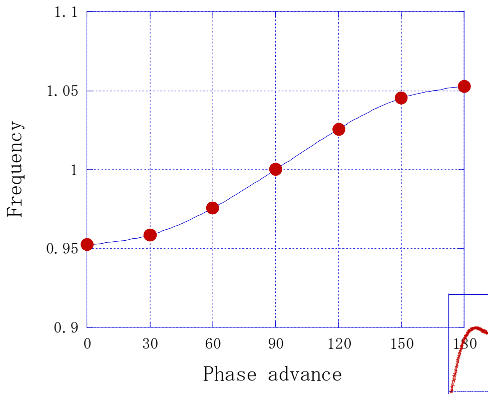
$$\frac{dP}{dz} = -2\alpha P \qquad \frac{dE_a}{dz} = -\alpha E_a \qquad \alpha \equiv \frac{\omega}{2 v_g Q}$$

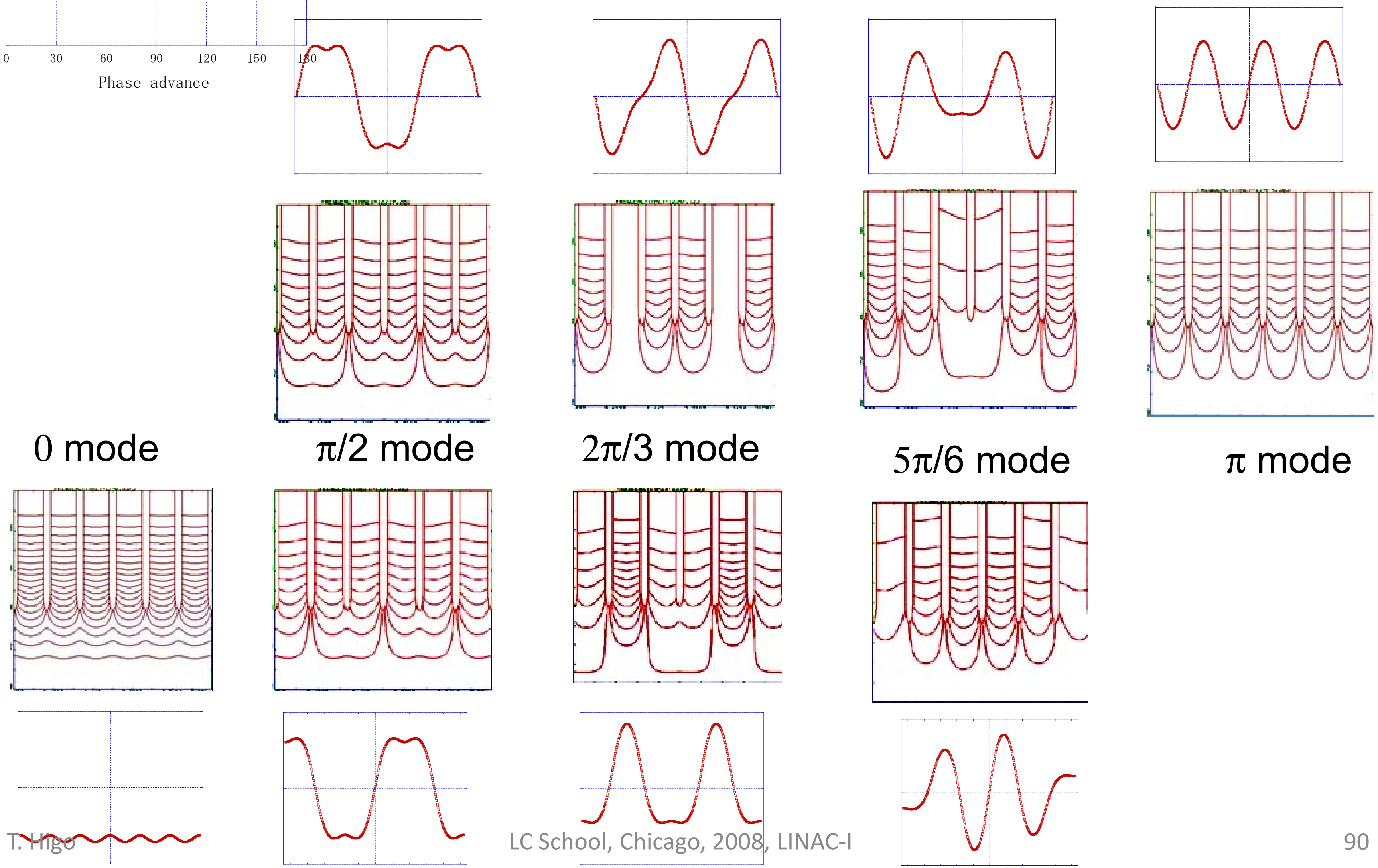
$$E_a = E_0 e^{-\alpha z} \qquad P_a = P_0 e^{-2\alpha z}$$

At the end of a structure, of length L

$$E_L = E_0 e^{-\alpha L} \qquad P_L = P_0 e^{-2\alpha L} \qquad \tau \equiv \alpha L = \frac{\omega L}{2 v_g Q}$$

DLS mode
Cal. With SW field patterns





Shunt impedance vs phase advance

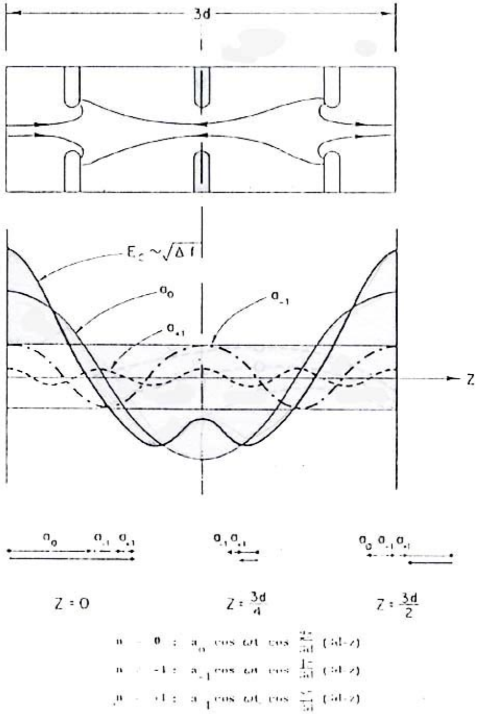


Fig. 26. Example of three-cavity stack resonating in $2\pi/3$ mode with illustration of how first three space harmonics add and subtract to give cavity field observed by bead measurement. The perturbation Δf is proportional to E_z^2.

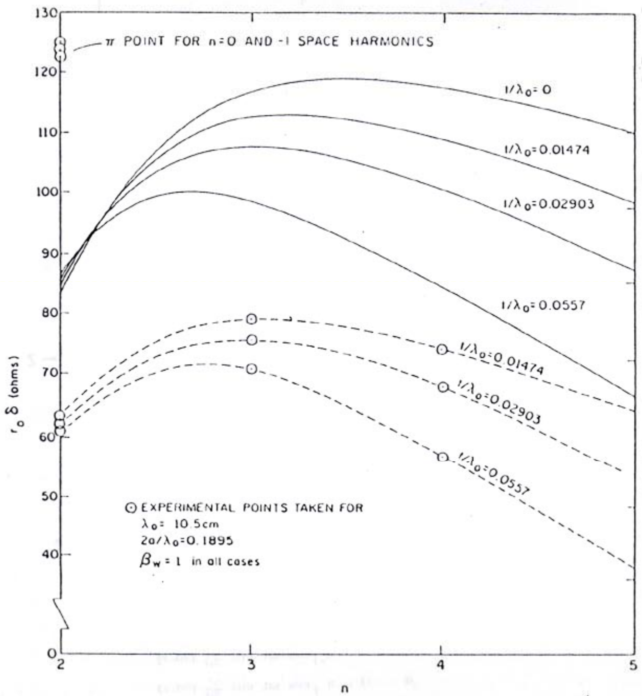


Fig. 27. Theoretical and experimental curves of normalized shunt impedance ($r_0\delta$) vs. number of disks per wavelength (n) for various values of t/λ_0 at $\beta_w = 1$.

Actual acceleration field on axis can be decomposed into space harmonics.

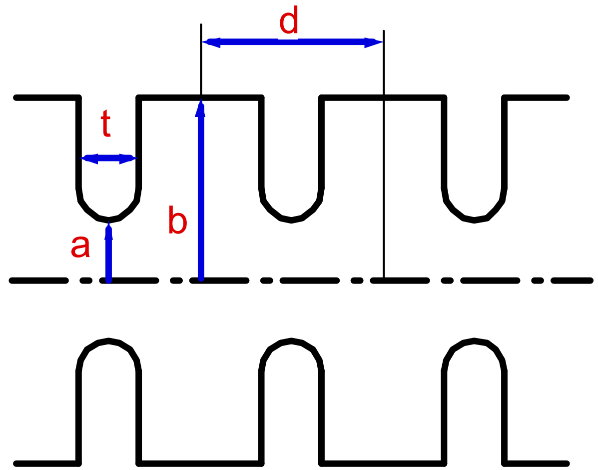
Net acceleration for long distance is realized by only synchronous one.

n=number of disks per wavelength.

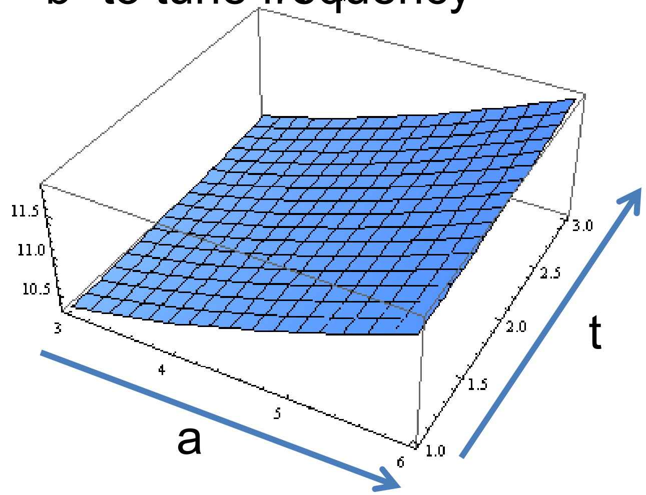
Thinner disk makes larger Gross peak at $2\pi/3$ mode

DLS basic design with changing (a,t)

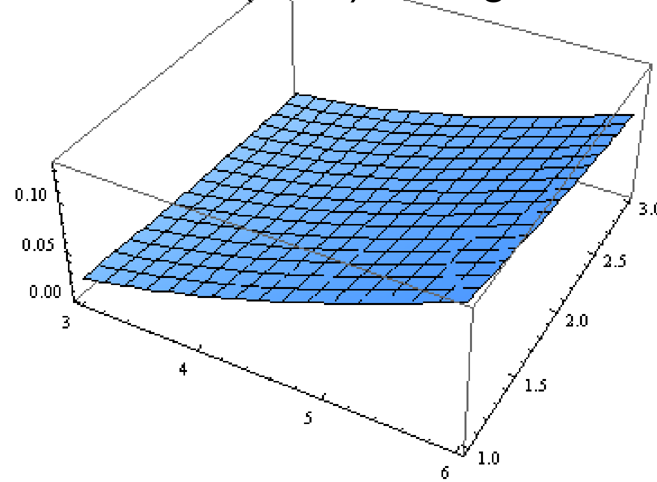
Simple geometry for DLS design



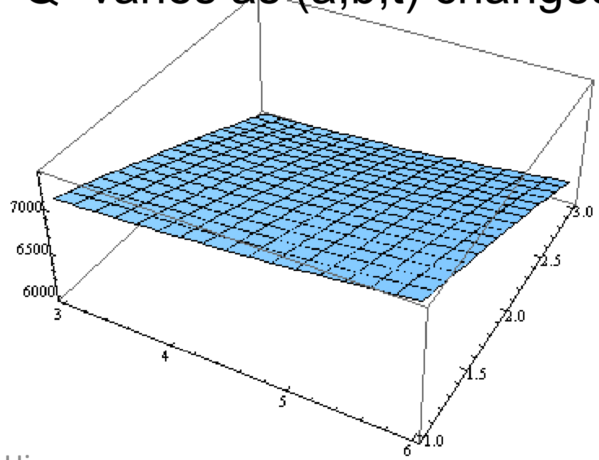
“b” to tune frequency



“vg” varies as (a,b,t) changes

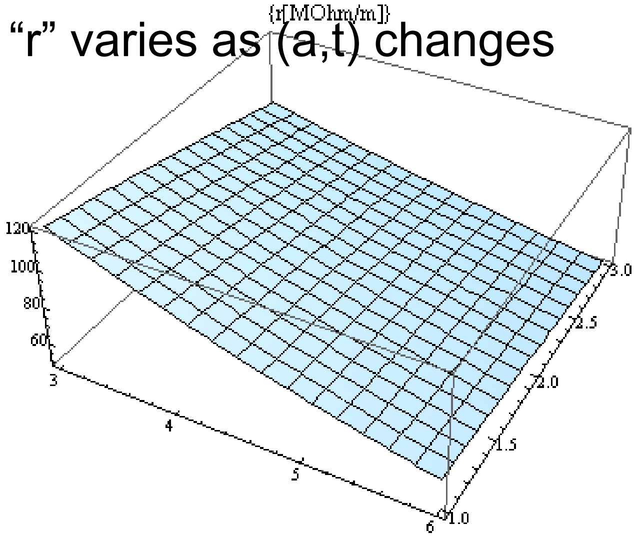


“Q” varies as (a,b,t) changes

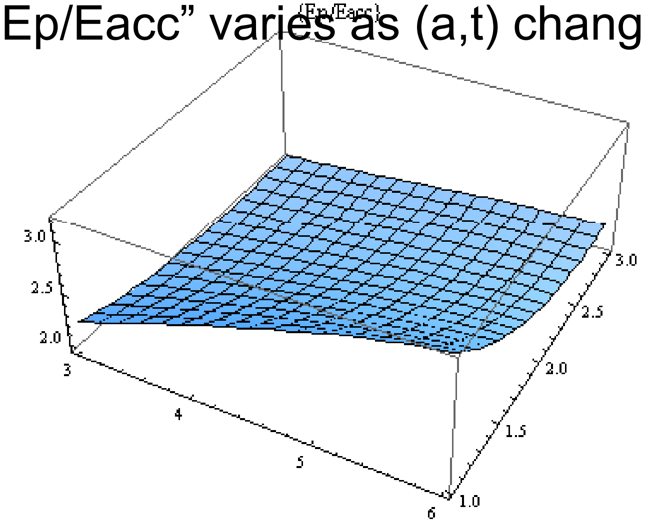


DLS Parameter variation with (a,t)

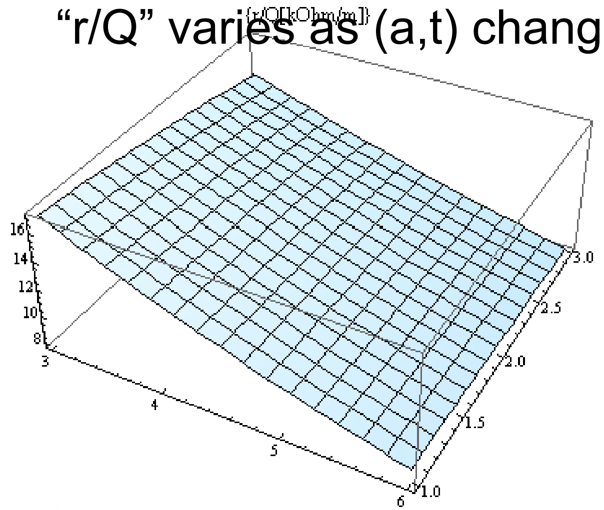
“r” varies as (a,t) changes



“Ep/Eacc” varies as (a,t) changes



“r/Q” varies as (a,t) changes



Hp/Eacc

CG: Constant gradient structure

Assume CG case is realized.
By neglecting weak variation of r,

$$|dP/dz| = const$$

$$\Rightarrow \quad \frac{P(z)}{P_0} = 1-(z/L)(1-e^{-2\tau})$$

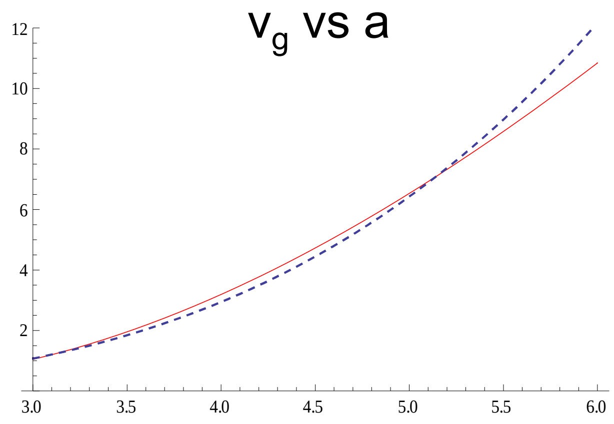
Group velocity should be varied linearly as

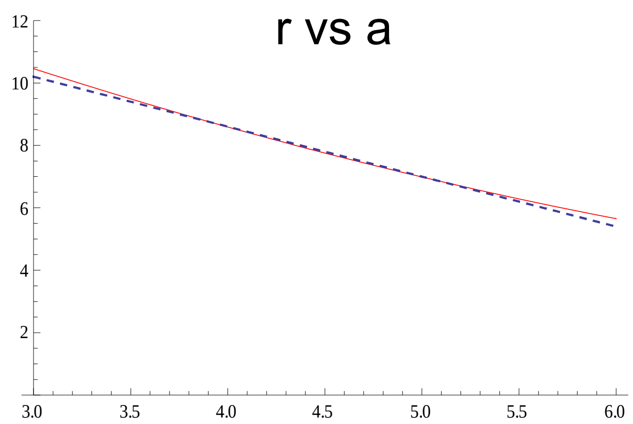
$$v_g(z) = \frac{\omega L}{Q}\,\frac{1-(z/L)(1-e^{-2\tau})}{1-e^{-2\tau}}$$

Filling time becomes

$$T_f = \int_0^L \frac{dz}{v_g(z)} = \tau\frac{2Q}{\omega}$$

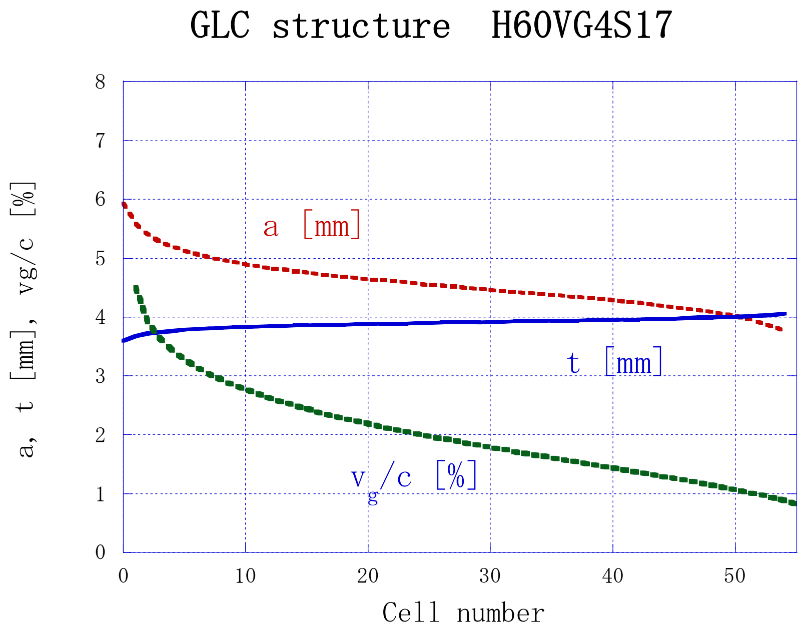
This is the same as CZ case





2π/3 mode in X-band disk-loaded structure

E_{NL} and E_{LD} actual and CG characteristics



Basically changing beam aperture “a” to make it roughly CG

Roughly constant gradient by linearly tapered vg

Reduce field near input coupler with initial taper

Acceleration with/without beam

Steady state case

Field with beam is
superposition of externally-driven field + beam-induced field

Acceleration along a structure without beam

$$CZ : V_0 = (r\,L\,P_0)^{1/2}\,[(2/\tau)^{1/2}\,(1-e^{-\tau})]$$

$$CG : V_0 = (r\,L\,P_0)^{1/2}\;(1-e^{-2\tau})^{1/2}$$

Beam induced field (beam loading) in an empty structure

$$\frac{dP}{dz} = I_0\,E_b - 2\,\alpha\,P$$

E_b : Beam induced field
I_0 : DC current of beam
P : Power flow of beam induced field

Since $E_b^2 = 2\,\alpha\,r\,P$, then

$$\frac{dE_b}{dz} = I_0\,\alpha\,r - \alpha\,E_b + \frac{E_b}{2\alpha}\frac{d\alpha}{dz}$$

This becomes simple when we consider CZ or CG structure,

$$CZ\;case : \;\frac{dE_b}{dz} = I_0\,\alpha\,r - \alpha\,E_b$$

$$CG\;case : \;\frac{dE_b}{dz} = I_0\,\alpha\,r$$

Beam loading voltage

Beam loaded field along a structure is described as,

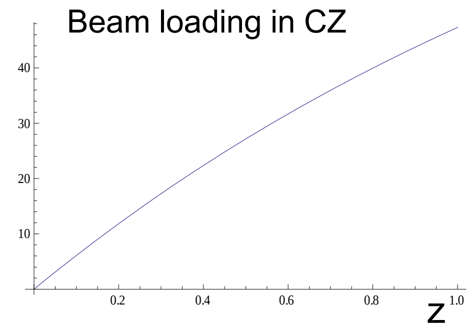
$$CZ: \quad E_b(z) = I_0\, r\, (1 - e^{-\alpha z})$$

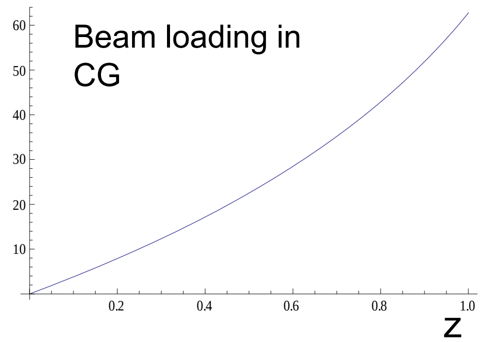
$$CG: E_b(z) = -\frac{I_0\, r}{2} \ln(1 - \frac{1 - e^{-2\tau}}{L} z)$$

By integrating along a structure

$$CZ: \quad V_b = I_0\, r\, L [1 - \frac{(1 - e^{-\tau})}{\tau}]$$

$$CG: \quad V_b = I_0\, r\, L [\frac{1}{2} - \frac{\tau e^{-2\tau}}{1 - e^{-2\tau}}]$$



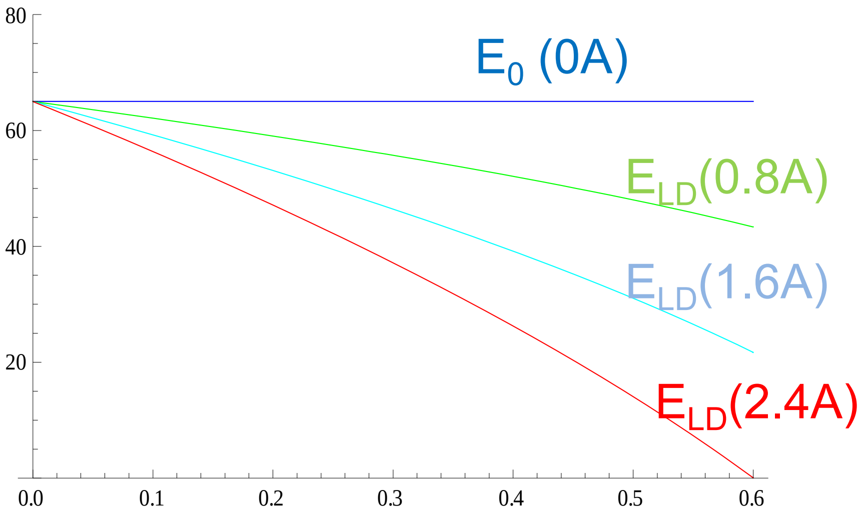


The net acceleration voltage then becomes,

$$CZ, CG: \quad V_{LD} = V_0\, Cos\, \theta - V_b$$

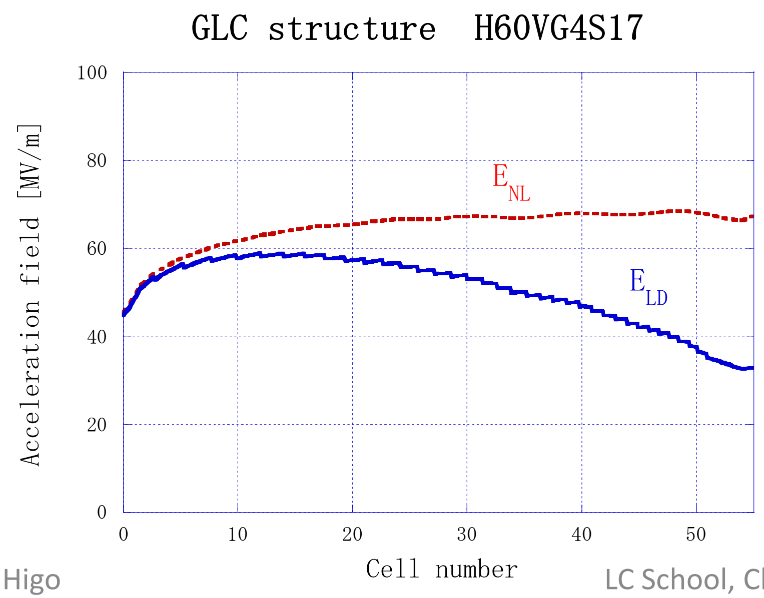
Where θ is the off crest angle w.r.t. RF on-crest phase.

Beam loaded field with beam



Estimation with CG case

r=60 MΩ/m
τ=0.6
L=0.6 m
E0=65 MV/m
I=0 - 0.8 - 1.6 - 2.4 A



Recursive calculation at I_b=0.9A with actual parameters (a,b,t) → r, Q along the structure → Field

Beam loading voltage build up toward downstream end

Full loading in CTF3

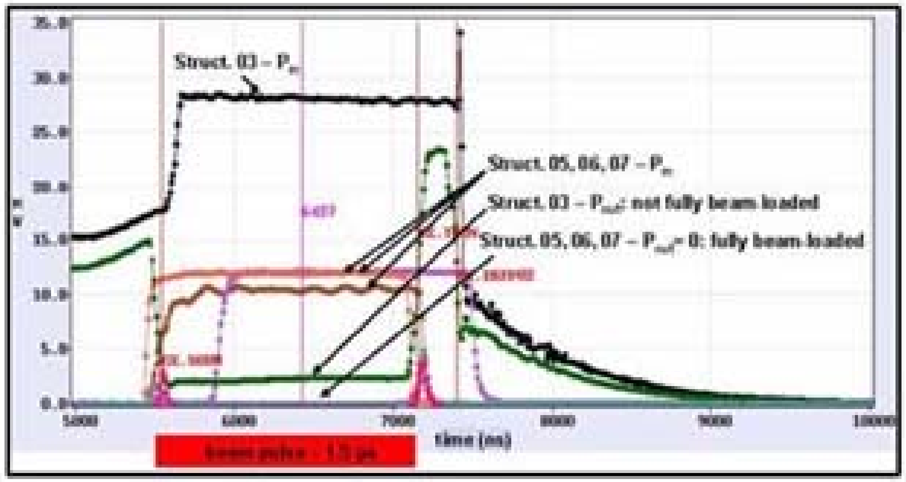


Figure 5: RF power levels at the structure input and output for always one accelerating structure per module as a function of time.

P. Urschütz et al., “Efficient Long-Pulse Full y Loaded CTF3 Linac Operation”, LINAC06.

In a steady state regime, beam can fully absorb stored energy of the structure. This is fully loaded condition.

Transient property of travelling wave propagation

- Wave propagation suffers from the dispersive effect of the periodic structure.

$$E_0(t) = \mathrm{Re}\int_0^t e^{j\omega t}\, h(t)$$

$$E_q(t) = \mathrm{Re}\int_0^t e^{j\omega(t-\tau)}\, G_q(\tau)\, d\tau$$

$$= Z_q(t)\, Cos(\omega t - \pi q - \varphi)$$

- $\pi/2$ mode → $\phi=0$, only amplitude modulation
- Accelerated particle sees as function of time.

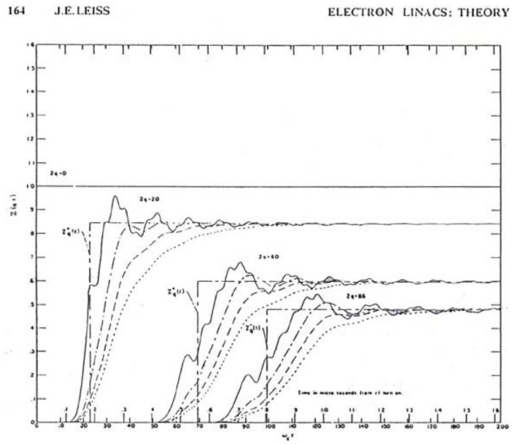


Fig. 5. Pulse transmission (Leiss *et al.* [1962]) in a specific $2\pi/3$ mode S-band accelerator of 86 disks for different values of input risetime, plotted against time in units of $\omega_c t$. The amplitude $Z(q, t)$ is shown for the signal at different points in the waveguide. The solid curve corresponds to the solutions found in eqs. (4.16) to (4.20). $Z_q{}^0(t)$ is calculated using eq. (4.21). Dispersive calculations similar to these but for a constant gradient waveguide are given in Helm [1966].

Rising front of pulse transmission at a certain position q down the structure. (Linear Accelerators)

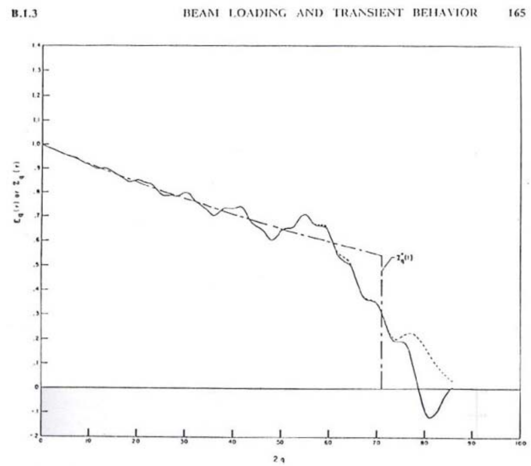


Fig. 6. Similar to fig. 5 but indicating the variation in rf amplitude with distance down the waveguide for a specific time $\omega_c t = 82$. The dotted curve is the amplitude $Z_q(t)$. The solid curve is the field $E_q(t)$ which synchronous particles would see. The difference between the two is caused by phase oscillations.

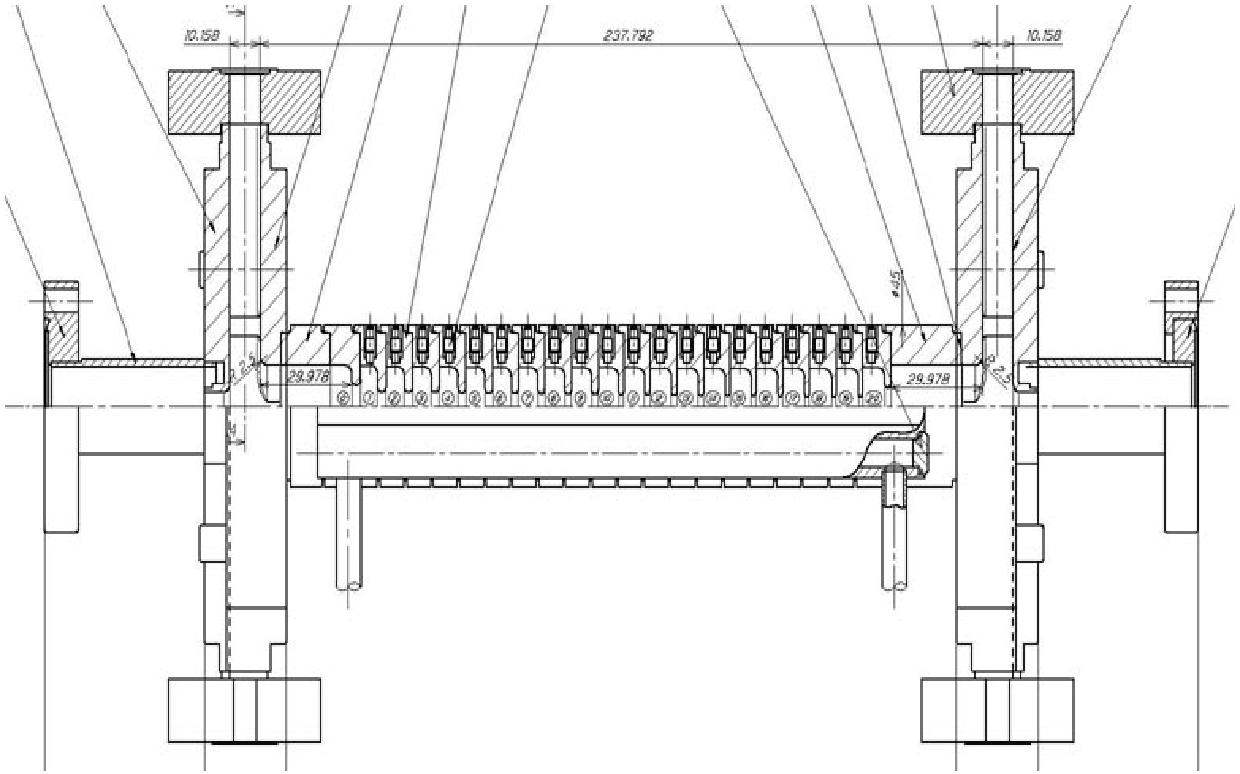
Solid line: Field amplitude along the structure. Dotted line: Field a particle sees. (Linear Accelerators)

The same property is calculated straightforwardly based on the equivalent circuit model by T. Shintake. (Frontiers in Accelerator Technology, 1996)

TW accelerator structure in practice

- Design
- Cell fabrication
  – Frequency control
- Bonding
- Tuning
  – Matching
  – Phase tuning
  – Minimize small reflection
  – HOM

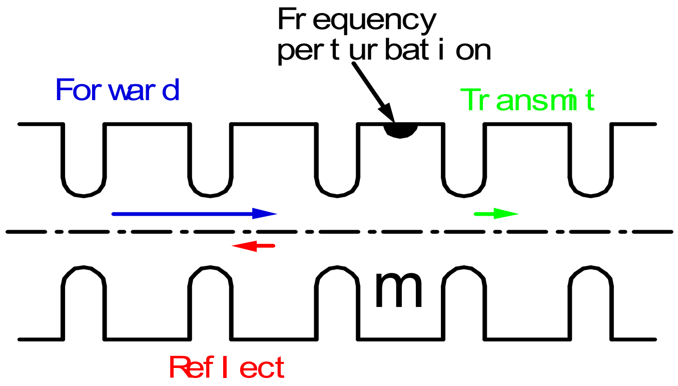
Typical TW structure made of stacked disks



Small reflection and tuning

Express wave propagation

$$V_k = \begin{cases} 1e^{-k(j\theta+\Gamma)} + R\,e^{-2m(j\theta+\Gamma)}\,e^{k(j\theta+\Gamma)} & \text{for } \; k \le m \\ T\,e^{-k(j\theta+\Gamma)} & \text{for } \; k > m \end{cases}$$



From continuity at m-th cell,

$$T = 1 + R$$

From coupled resonator model,

$$(\delta\omega_m^2 + \omega_m^2 - \omega^2 + j\frac{\omega\omega_m}{Q_m})V_m = \frac{1}{2}\omega_m^2\,(k_{m-1/2}\,V_{m-1} + k_{m+1/2}\,V_{m+1})$$

Explicitly written as,

$$(\delta\omega_m^2 + \omega_m^2 - \omega^2 + j\frac{\omega\omega_m}{Q_m})(T\,e^{-m(j\theta+\Gamma)}) = \frac{1}{2}\omega_m^2\,k_{m-1/2}\,(e^{-(m-1)(j\theta+\Gamma)} + e^{-2m(j\theta+\Gamma)}\,R\,e^{(m-1)(j\theta+\Gamma)})$$

$$+\frac{1}{2}\omega_m^2\,k_{m+1/2}\,T\,e^{-(m+1)(j\theta+\Gamma)}$$

Tuning condition,

$$\delta\omega_m^2 + \omega_m^2 - \omega^2 + j\frac{\omega\omega_m}{Q_m} = \frac{1}{2}\omega_m^2\,(k_{m-1/2}\,e^{-(m-1)(j\theta+\Gamma)} + k_{m+1/2}\;e^{-(m+1)(j\theta+\Gamma)})$$

Reflection (cont.)

Finally reflection from cell m becomes,

$$R = \frac{\delta\omega_m^2 / \omega_m^2}{\delta\omega_m^2 / \omega_m^2 + k_{m-1/2}(\Gamma Cos\theta + j\, Sin\theta)}$$

Assume; loss-less TW with $2\pi/3$ mode, k~0.02, $\delta\omega^2/\omega^2$ ~ 10^{-4}

$$R \approx \frac{\delta\omega_m^2 / \omega_m^2}{k_{m-1/2}\, Sin\theta} \approx \frac{10^{-4}}{0.02(\sqrt{3}/2)} \approx 10^{-2}$$

If coherent error from 10 cells, those cohere so that R becomes as much as 0.1.
Such systematic error is not allowed.
In contrast, random error makes much smaller and such amount of error is allowed.

Phase error

Dispersion relation;

$$\omega = \frac{\omega_0}{\sqrt{1+\kappa Cos(\phi)}}$$

Group velocity formula;

$$\frac{\partial\omega}{\partial\phi} = v_g = -\frac{1}{2}\omega_0\,\kappa Sin(\phi)$$

Frequency error to phase advance error

$$\delta\Phi = \delta\phi\, d = \frac{\delta\omega}{v_g} d = \frac{d/c}{v_g/c}\delta\omega = \frac{\lambda\,(\Phi/2\pi)/c}{v_g/c}\delta\omega = \frac{\Phi}{v_g/c}\frac{\delta\omega}{\omega}$$

At X-band, 1MHz gives phase advance error of 0.6degree/cell.

In 20 cell structure such as CLIC,
Systematic error of this order is not good
but random error should be OK, as long as acceleration mode.

Summary of LINAC-I in comparison of SCC and NCC

- How the linac design can be determined from gaining energy point of view?
- Energy mode
  - SW or TW
- Confinement mechanism
  - SCC or NCC
- Frequency choice
  - 1GHz-10cm or 10GHz-1cm?

Choice of material for EM field confinement

- SCC
- Nb
- Γ Geometrical factor
- Intrinsic limit Hs
- Mechanical strength
- Quench
- Cryogenic power

- NCC
- Cu
- R Shunt impedance

- Thermal
- Breakdown
- RF generation

Choice of frequency

1GHz	10GHz
10cm	1cm
Drawing or hydroforming	High precision turning / milling
Longer pulse ~1ms	Shorter pulse ~100ns
Longer power	High peak power

Choice of EM mode & Efficiency

- SW
- SCC
- Power loss
  - Cavity wall → cryogenic
  - Reflection from cavity

- TW
- NCC
- Power loss
  - Cavity wall
  - Transmitted to RF load

Potential for higher energy

- Es/Ea
- Hs/Ea
- Multi pactor
- Field emission
- Lorentz detuning
- Quench
- Q_0 → cryogenic power

- Es/Ea
- Hs/Ea
- Surface temperature rise in a pulse
- Fatigue
- Breakdown
- R/Q * Q → efficiency