LINAC-II

ILC School
Chicago, Oct. 22, 2008
T. Higo, KEK

Contents of LINAC-II

- Beam quality preservation
  - Luminosity
  - Linac optics
  - Perturbations
  - Wake field
  - Wake field suppression
  - Alignment to beam
  - Breakdown rate
  - Dark current
  - Summary of LINAC-II

Energy and luminosity

$$e\,E_c = E_a\,L_{linac} = \frac{P_{linac}\;\eta_{RF->Beam}}{N\;N_b\;F_{rep}}$$

$$\frac{L}{P_{linac}} = \frac{L_0\;N_b\;F_{rep}}{e\,E_c\;N\;N_b\;F_{rep}\,/\,\eta_{RF->Beam}} = \frac{1}{e\,E_c}\,\frac{L_0}{N}\,\eta_{RF->Beam}$$

Luminosity

Luminosity related parameters

- Important parameters relevant in this lecture are

  - Beam transverse size
    - Main theme of this lecture
  - Number of bunches in a train
    - Long-range wake field
  - Number of particles in a bunch
    - Short range wake field
  - Repetition frequency
    - Wall plug power

$$L = \frac{f_{rep}\, n_b\, N^2\, H_D}{4\pi\, \sigma_x^*\, \sigma_y^*}$$

Requirements for linear collider

- Beam quality from DR and BC should be preserved.
- Phase space
  - Longitudinal
    - Energy acceptance of the final focus
  - Transverse
    - Emittance preservation to keep beam size small
- Wake field
  - Single bunch:
    - Short range (intra-bunch) wake field
    - Dispersive effect in the bunch
  - Multi-bunch:
    - Long range (inter-bunch) wake field among bunches
    - Bunch to bunch dispersive effect

Linac example parameters

ILC and CLIC

Item		units	ILC(RDR)	CLIC(500)
Injection / final linac energy	E_{Linac}	GeV	25 / 250	/ 250
Acceleration gradient	E_a	MV/m	31.5	80
Beam current	I_b	A	0.009	2.2
Peak RF power / cavity	P_{in}	MW	0.294	74
Initial / final horizontal emittance	ε_x	μm	8.4 / 9.4	2 / 3
Initial / final vertical emittance	ε_y	nm	24 / 34	10 / 40
RF pulse width	T_p	μs	1565	242
Repetition rate	F_{rep}	Hz	5	50
Number of particles in a bunch	N	10^9	20	6.8
Number of bunches / train	N_b		2625	354
Bunch spacing	T_b	ns	360	0.5
Bunch spacing per RF cycle	T_b / T_{RF}		468	6

Linac example parameters
ILC and CLIC

Item		units	ILC(RDR)	CLIC(500)
RF frequency	F	GHz	1.3	12
Beam phase w.r.t. RF	degrees		5	15
EM mode in cavity			SW	TW
Number of cells / cavity	N_c		9	19
Cavity beam aperture	a/λ		0.152	0.145
Bunch length	σ_z	mm	0.3	0.044

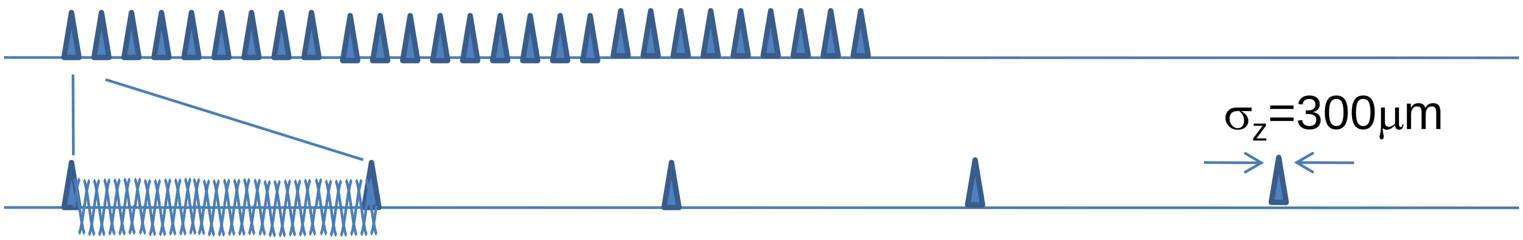
ILC parameters are taken from Reference Design Report of ILC for 500GeV.

CLIC500 parameters are taken from the talk by A. Grudief, 3rd. ACE, CLIC Advisory Committee, CERN, Sep. 2008, http://indico.cern.ch/conferenceDisplay.py?confId=30172.

Bunch pattern

ILC

2625 bunches / pulse



468 rf / separation

Many bunches / train
Many rf cycles (~10^3) till next bunch
σ_z/λ~0.0013

CLIC

354 bunches / pulse

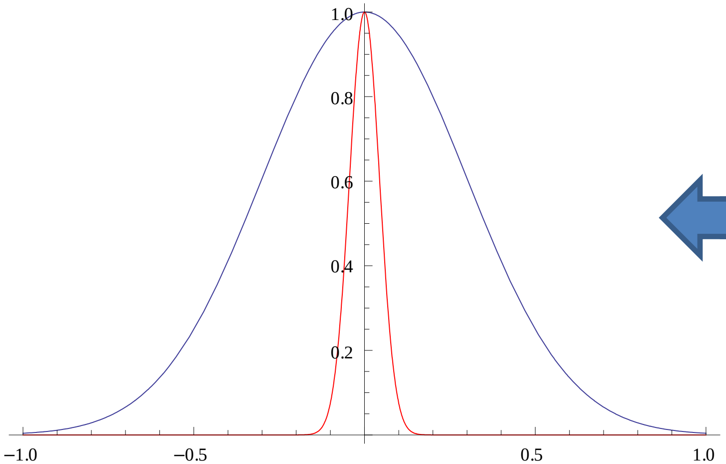
6 rf / separation

Only 6 rf cycles till next bunch
σ_z/λ~0.0018 → actual σ_z 10 times smaller

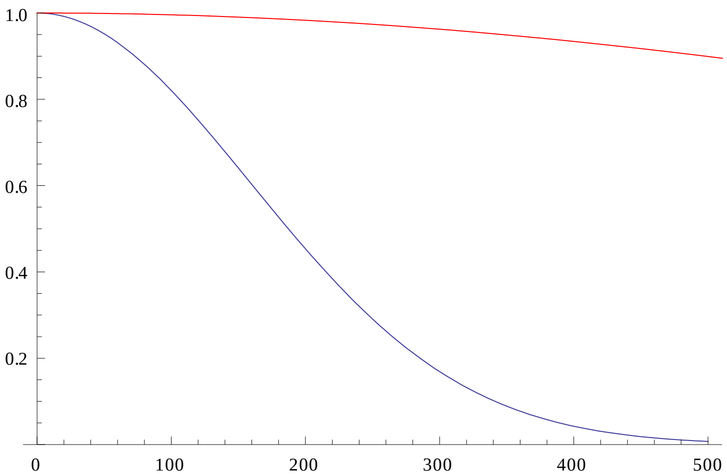
Bunch profile and power spectrum

$$f(z) \propto Exp[-\frac{z^2}{2\sigma_z^2}]$$

$$P(\omega) \propto Exp[-\frac{1}{2}(\frac{\sigma_z}{c})^2 \omega^2]$$



mm



GHz

Emittance dilution / preservation

Emittance preservation

Emittance preservation along the linac is one of
the key issues of the main linac of LC.

Emittance in 6 dimensions;
  Single bunch emittance vs multi-bunch emittance
  Longitudinal emittance vs transverse emittance in X and y

Error sources for dilution should be minimized.
  Better alignment, wake field suppression, etc.
But if diluted, we try to correct
  by realignment, corrector fast magnets, etc.

Coherent dilution can be corrected but it phase space volume is fully filled in an incoherent manner, it cannot be corrected.

Multi-bunch dilution can be corrected bunch by bunch correction.

Very basic optics of linac

Transverse motion of eq.

$$\frac{d^2}{dz^2}x+k(z)x=0 \qquad k(z)=\frac{c\,g}{E/e}[m^{-2}]$$

$$\tau=\sqrt{k}\,L_r \quad \theta=\sqrt{k}\,L_Q,\ \eta=2L_Q/L$$

Transfer matrix per period;

$$M_F=\begin{pmatrix} Cos\,\theta & \frac{1}{\sqrt{k}}Sin\theta \\ -\sqrt{k}\,Sin\theta & Cos\,\theta \end{pmatrix},\quad M_D=\begin{pmatrix} Cosh\,\theta & \frac{1}{\sqrt{k}}Sinh\theta \\ -\sqrt{k}\,Sinh\theta & Cosh\,\theta \end{pmatrix}$$

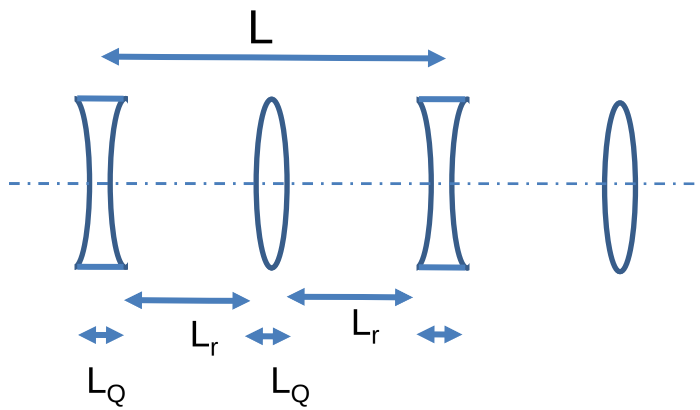
$$M_{FDF}=M_{F/2}\,M_{drift}\,M_D\,M_{drift}\,M_{F/2}$$

Twiss parameter at Q_F;

$$M_{FDF}=\begin{pmatrix} Cos\theta+\alpha\,Sin\mu & \beta\,Sin\mu \\ -\gamma\,Sin\mu & Cos\theta-\alpha\,Sin\mu \end{pmatrix}$$

In a thin lens approximation

$$Cos\mu=1-\frac{\tau^2\theta^2}{2},\ Sin\frac{\mu}{2}=\frac{\tau\,\theta}{2}=\frac{c\,g\,L^2}{8\,E/e}\eta(1-\eta)$$



Betatron oscillation along the linac is the very basic of the system.

$$x(z)\propto A\sqrt{\beta(s)}\ Sin(\psi(s)+\delta),\quad \frac{d\psi}{ds}=\frac{1}{\beta(s)}$$

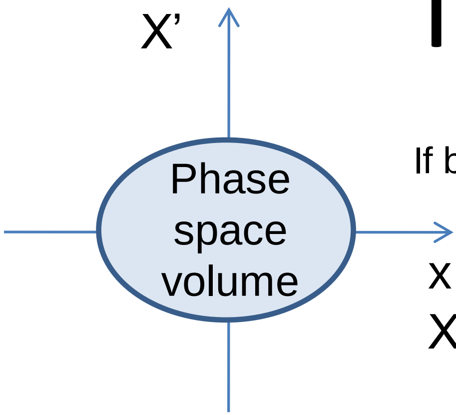
Assume $\beta(s)$ constant

$$x(z)\propto\ Sin(k_\beta\,s+\delta),\ \ k_\beta=\frac{1}{\beta(s)}=\frac{2\pi}{\lambda_\beta}$$

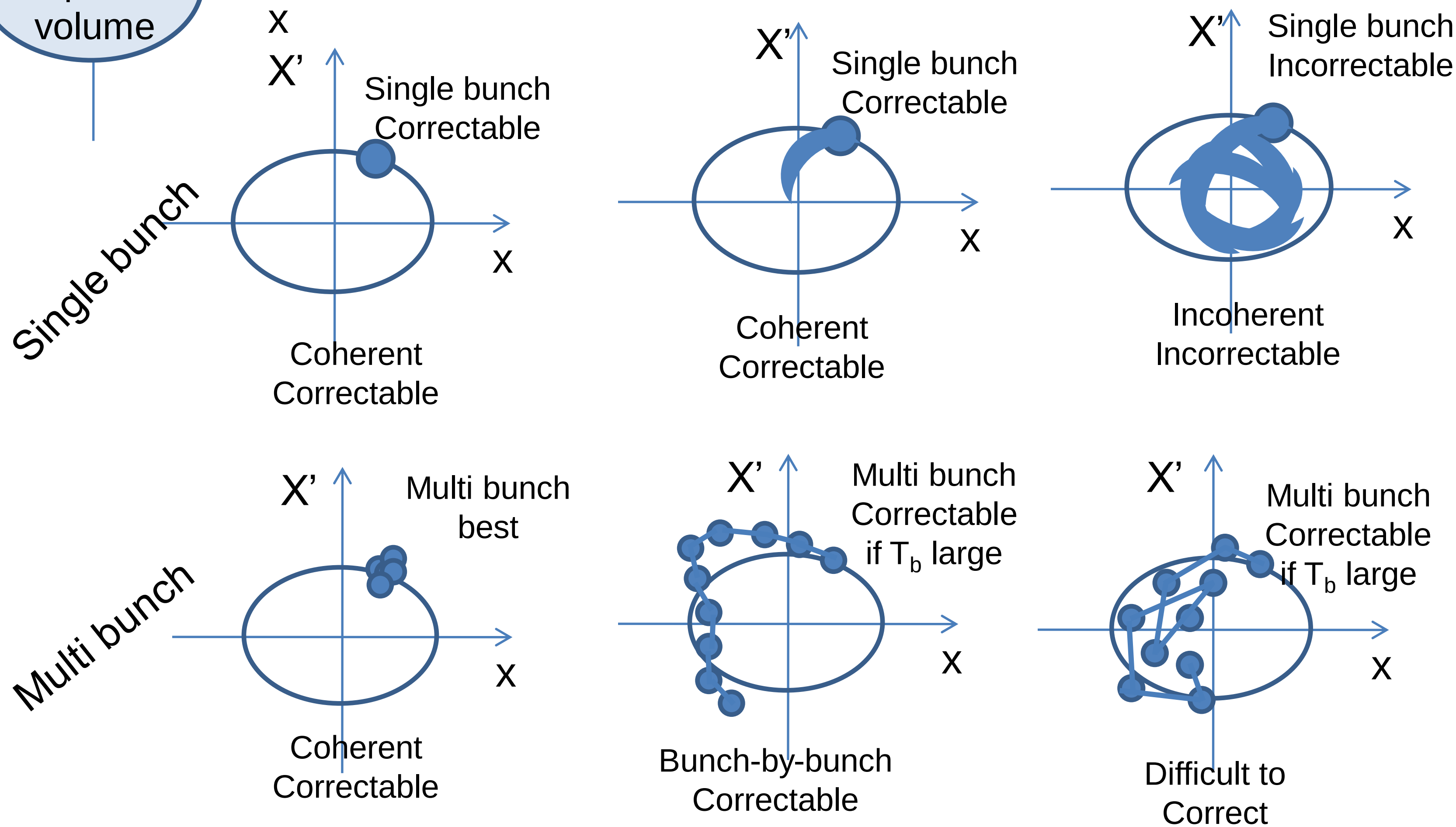
In the left FODO system,

$$k_\beta=\frac{2\pi}{\lambda_\beta}=\frac{\mu}{L}=\frac{c\,g\,L}{4\,E/e}\eta(1-\eta)$$

Transverse emittance



If before filling the whole phase space, the emittance can be controlled.



Practical errors to be considered

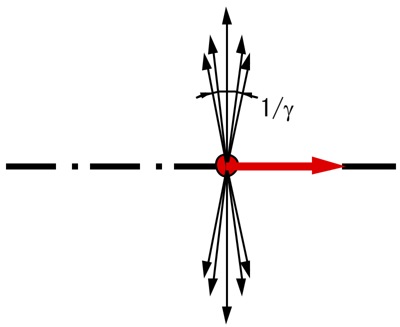
- Optical error and misalignment
  - Field stability in Q
  - Misalignment of Q, structures and BPM
- RF error
  - Phase, amplitude jitter
  - Pulse to pulse, within pulse,
  - Asymmetry in cavity to time dependent transverse kick
- Wake field
  - Long range and short range
  - Longitudinal and transverse

Possible cares and corrections

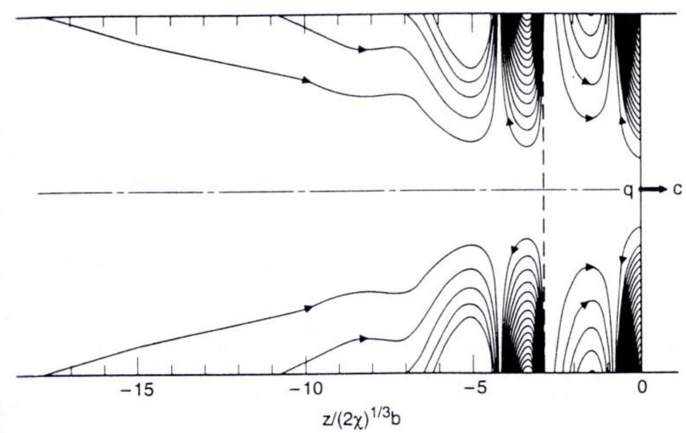
- Optical error and misalignment
  - Alignment with using beam information
  - BPM information, on- /off-energy, on- /off- Q setting
- RF error
  - Feedback with cavity field
  - Mechanical precision of cavity, offset, tilt,
  - Suppression of field asymmetry
- Wake field
  - Cavity HOM damping and cancelling
  - Multi-bunch energy compensation with injection timing, ramping pattern, etc

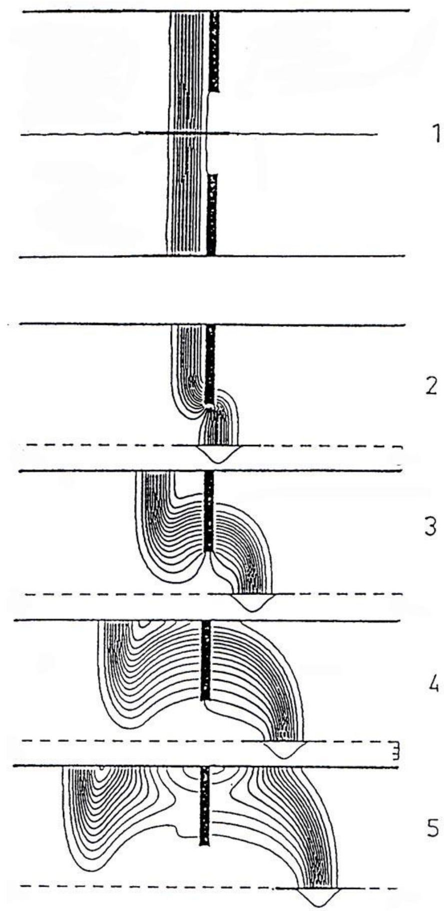
Wake field and impedance

Wake fields driven by relativistic particle



Lorentz contracted field
In free space

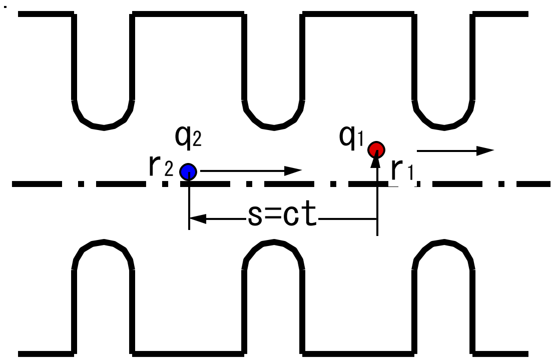




EM field associated to the particle is scattered by periphery shape.

Wake field

Driving bunch: Unit charge bunch at offset radius r
Witness bunch: trailing at z=ct behind



Wake field W(s) is the kick received by the witness bunch.

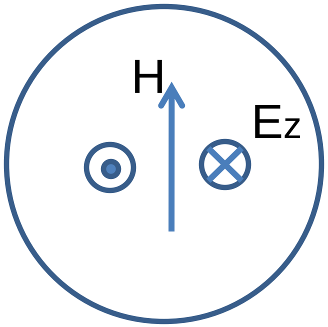
$$W(r_1,s)=\frac{1}{q_1}\int_{-\infty}^{\infty}dz\left[E(r_1,z,t)+c\,\vec{z}\times B(r_1,z,t)\right]_{t=(s+z)/c}$$

$$\Delta p=q_1\,q_2\,W(s)$$

$$W_L(r_1,s)=\frac{1}{q_1}\int_{-\infty}^{\infty}dz\left[E_z(r_1,z,t)\right]_{t=(s+z)/c}$$

$$W_T(r_1,s)=\frac{1}{q_1}\int_{-\infty}^{\infty}dz\left[E(r_1,z,t)+c\,\vec{z}\times B(r_1,z,t)\right]_{T,t=(s+z)/c}$$

Panofsky-Wenzel theorem



Think about TM11 field

$$\nabla \times E_z = -\frac{\partial}{\partial t} B_r$$

$$\vec{e_z} \times \nabla \times E = \vec{e_z} \times \left(-\frac{\partial B}{\partial t}\right)$$

$$\therefore \ \vec{e_z} \frac{\partial B}{\partial t} = \vec{e_\perp} \left(\frac{\partial}{\partial z} E_\perp - \nabla_\perp E_z\right)$$

$$\frac{d}{dz} = \frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}$$

$$\frac{\partial}{\partial z} W_T(x,y,z) = \frac{1}{q_1}\int_{-\infty}^{\infty} dz \left[\frac{\partial}{\partial z} E_T + \vec{e_z} \times B\right] = -\frac{1}{q_1}\nabla_T \int_{-\infty}^{\infty} dz\, E_z$$

$$\boxed{\begin{aligned} &\therefore \quad \frac{\partial}{\partial s} W_T = -\nabla_T W_L \\ &or \quad W_T(x,y,s) = -\nabla_T \int_{-\infty}^{s} ds' W_L(x,y,s') \end{aligned}}$$

Cylindrical symmetric system and multipole expansion

From Panofsky Wenzel theorem,

$$W_L = -\frac{\partial}{\partial s} W\,, \quad W_T = \nabla_T\, W$$

For axisymmetric environments, W can be expanded into multi-poles,

$$W(r, r', \theta, s) = \sum_{m=0}^{\infty} W_m(r, r', s)\, Cos(m\theta)$$

From Maxwell's equation, form of W_m can be found,

$$W_m(r, r', s) = F_m(s)\, r^m\, r'^m$$

Wake functions are expressed now as,

$$W_L^{(m)}(r, r', s) = \frac{\partial}{\partial s} F_m(s)\, r^m\, r'^m\, Cos(m\theta)$$

$$W_T^{(m)}(r, r', s) = m\, F_m(s)\, r^{m-1}\, r'^m\, [\vec{r}\, Cos(m\theta) - \vec{\theta}\, Sin(m\theta)]$$

$$\boxed{\text{Near axis, only m=0 for longitudinal wake and m=1 for transverse wake.}}$$

Near axis, only m=0 for longitudinal wake and m=1 for transverse wake.
Longitudinal: constant over radius
Transverse: linear over drive bunch offset but constant on witness bunch position.

Impedance

Impedance: Fourier transform of the wake function

$$Z_L(x,y,\omega) = \int_{-\infty}^{\infty} d(\frac{s}{c}) W_L(x,y,s) e^{-j\omega\frac{s}{c}}$$

$$Z_T(x,y,\omega) = -j\int_{-\infty}^{\infty} d(\frac{s}{c}) W_T(x,y,s) e^{-j\omega\frac{s}{c}}$$

Panofsky Wenzel theorem becomes,

$$\frac{\omega}{c} Z_T(x,y,\omega) = \nabla_T Z_L(x,y,\omega)$$

Wake is real → then

$$\mathrm{Re}\{Z_L(\omega)\} = \mathrm{Re}\{Z_L(-\omega)\}, \quad \mathrm{Im}\{Z_L(\omega)\} = -\mathrm{Im}\{Z_L(-\omega)\}$$

Then,

$$W_L(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega Z_L(\omega) e^{j\omega\frac{s}{c}}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega [\{\mathrm{Re}\{Z_L(\omega)\} Cos(\omega\frac{s}{c}) - \mathrm{Im}\{Z_L(\omega) Sin(\omega\frac{s}{c})\}]$$

From causality,

$$W_L(-s)\} = 0 \quad \textit{for all } s > 0$$

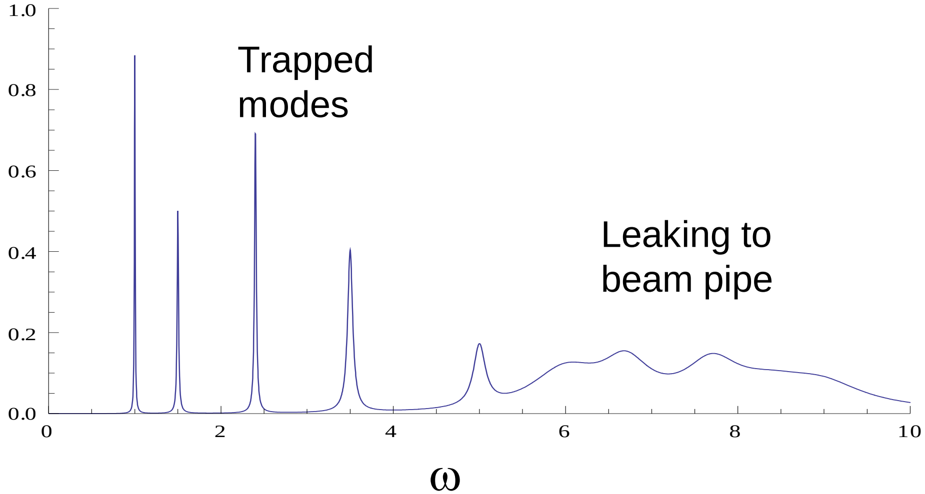
$$\therefore \int_{-\infty}^{\infty} d\omega \mathrm{Re}\{Z_L(\omega)\} Cos(\omega\frac{s}{c}) = -\int_{-\infty}^{\infty} d\omega \mathrm{Im}\{Z_L(\omega)\} Sin(\omega\frac{s}{c})$$

Actual impedance shape

Finally, wake function is calculated from only the real part of the impedance,

$$W_L(s) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega \{\mathrm{Re}\{Z_L(\omega)\} Cos(\omega \frac{s}{c})$$

Re($Z_{||}(\omega)$)



Actual real part of the impedance is illustrated as shown in the left figure.

Low frequency trapped modes and higher frequency component with escaping into the beam pipe.

Foe each resonance,

$$Z_L(\omega) = \frac{R}{1 + jQ(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})}$$

Loss parameter

Electric field is expressed as,

$$E_z(\vec{r},t) = \mathrm{Re}\{\sum_{n=0}^{\infty} E_n(\vec{r})\, e^{j\omega_n t}\}$$

$$V_n = \int_{-\infty}^{\infty} dz\, E_{z,n}(\vec{r})\, e^{j\omega_n \frac{z}{c}}$$

Define loss parameter;

$$k_n = \frac{|V_n|^2}{4U_n} \quad \text{SW}$$

$$k_n = \frac{|E_n|^2}{4u_n} \quad \text{TW}$$

For delta-function-like driving particle;

$$W_L^{\delta}(s) = \sum_{n=0}^{\infty} 2\, k_n\, Cos(\omega_n \frac{s}{c})$$

Self wake is half of the wake behind the bunch,
Energy lost in mode n is

$$\Delta U_n = k_n\, q_1^2$$

For finite bunch length,

$$\rho(\vec{r},t) = q_1\, \lambda(z - ct),$$

$$\lambda(s) = \frac{1}{\sqrt{2\pi}} \exp\{-\frac{(s-s_0)^2}{2\sigma_z^2}\}$$

Actual wake is a convolution,

$$W_L(s) = \int_0^{\infty} ds'\, \lambda(s-s')\; W_L^{\delta}(s')$$

Loss to fundamental mode;

$$k_{fund} = k_0 \exp\{-\frac{1}{2}(\frac{\omega}{c})^2\, \sigma_z^2\}$$

Loss to higher modes;

$$k_{HOM} = \sum_{n=1}^{\infty} 2\, k_n \exp\{-\frac{1}{2}(\frac{\omega_n}{c})^2\, \sigma_z^2\}$$

Longitudinal wake function in DLS

Summation of resonant modes up to a certain frequency,

$$W_L^{\delta}(s) = \sum_{n=0}^{N} 2\, k_n\, Cos(\omega_n \frac{s}{c})$$

Higher than the frequency, and in high energy limit γ>>ωa/c, optical resonator model predicts

$$\frac{dk}{d\omega} = \frac{A_0}{\omega^{3/2}}, \quad -> \; W_a(\tau) = 2\, A_0 \int_{\omega_m}^{\infty} \frac{Cos(\omega\, \tau)}{\omega^{3/2}}\, d\omega$$

Total wake field calculated for the SLAC disk loaded structure became as shown in right figure (P. Wilson Lecture)

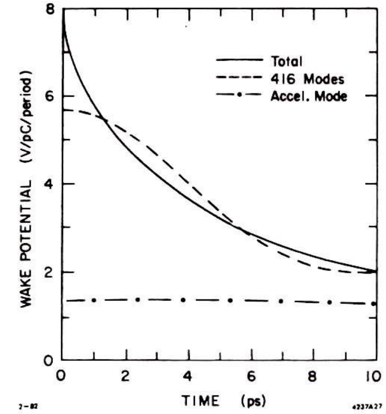


Fig. 9.3. Longitudinal wake per cell for the SLAC disk-loaded structure (0–10 ps). Cell length = 3.5 cm; beam aperture radius = 1.163 cm.

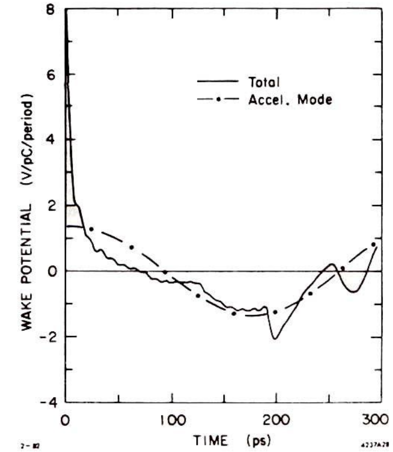


Fig. 9.4. Longitudinal wake per cell for the SLAC disk-loaded structure (0–300 ps).

Fundamental mode dominates for long-range wake, with some high Q modes superposed.

Much higher than 400th mode contributes in very short range wake field.

Transverse wake function in DLS

We follow a paper* by Zotter and Bane on transverse wake field calculation on disk-loaded structure.

Synchronous space harmonic component of the n-th TW mode, axial electric field is expressed as

$$E_{zn} = E_{0n} \left(\frac{r}{a}\right)^m Cos(m\phi)\, Cos\{\omega_n (t - z/c)\}$$

Where E_{0n} is the field at r=a, iris opening radius. Loss parameter is

$$k_n \equiv \frac{E_{0n}^2}{4u_n} \;, \quad and \quad u_n = k_n \left(\frac{r_q}{a}\right)^{2m} q^2$$

r_q is drive bunch position

$$\therefore \quad E_{0n} = -2 \left(\frac{r_q}{a}\right)^m k_n\, q$$

Therefore,

$$\therefore \quad E_{zn} = -2 k_n\, q \left(\frac{r}{a}\right)^m \left(\frac{r_q}{a}\right)^m Cos(m\phi)\, Cos\left(\omega_n \frac{z}{c}\right)$$

With Panofsky Wenzel theorem;

$$(E_T + c\, B_T)^{(cmf)} = j(c/\omega) \nabla_T\, E_z^{(cmf)}$$

* B. Zotter and K. Bane, “Transverse Resonances of Periodically widened Cylindrical Tube with Circular Crosssection”, PEP-Note-308, SLAC, 1979.

Transverse wake field (cont.)

Finally for the transverse wake field,

$$W_{Tn}(\tau)=2\left(\frac{k_n\, c}{\omega_n\, a}\right)\left(\frac{r_q}{a}\right) Sin(\omega_n\, \tau)$$

Summation gives total from resonant-like modes,

$$W_T(\tau)=2\left(\frac{r_q}{a}\right)\sum_n\left(\frac{k_n\, c}{\omega_n\, a}\right) Sin(\omega_n\, \tau)$$

Over maximum frequency ω_m, integration gives wake field using

$$\frac{d\,k}{d\omega}=\frac{A_1}{\omega^{3/2}}$$

The calculated wake field for SLAC DLS are shown in three time ranges;

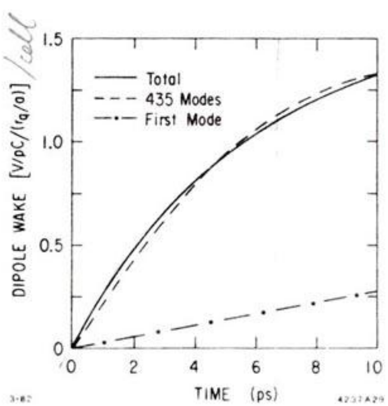


Fig. 9.5. Dipole wake per cell for the SLAC disk-loaded structure (0-10 ps).

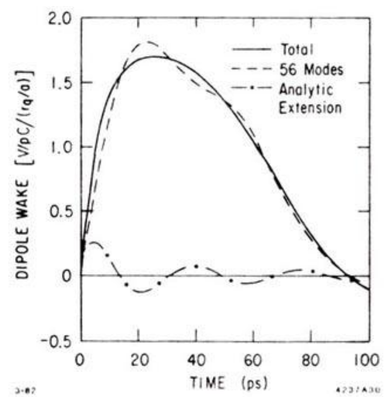


Fig. 9.6. Dipole wake per cell for the SLAC disk-loaded structure (0-100 ps).

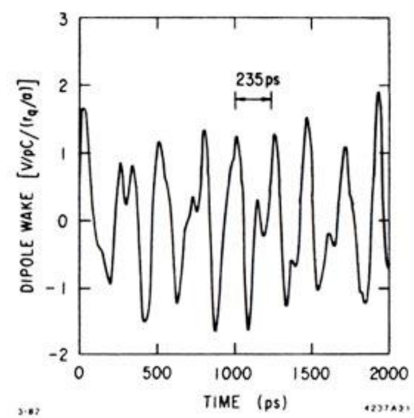


Fig. 9.7. Dipole wake per cell for the SLAC disk-loaded structure (0-2000 ps).

Dipole wake field parametrization

Transverse wake field for NLC structure;

$$W_T(s) = \frac{4Z_0\, c\, s_0}{\pi\, a^4}\{1-(1+\sqrt{s/s_0})Exp(-\sqrt{s/s_0})\}$$

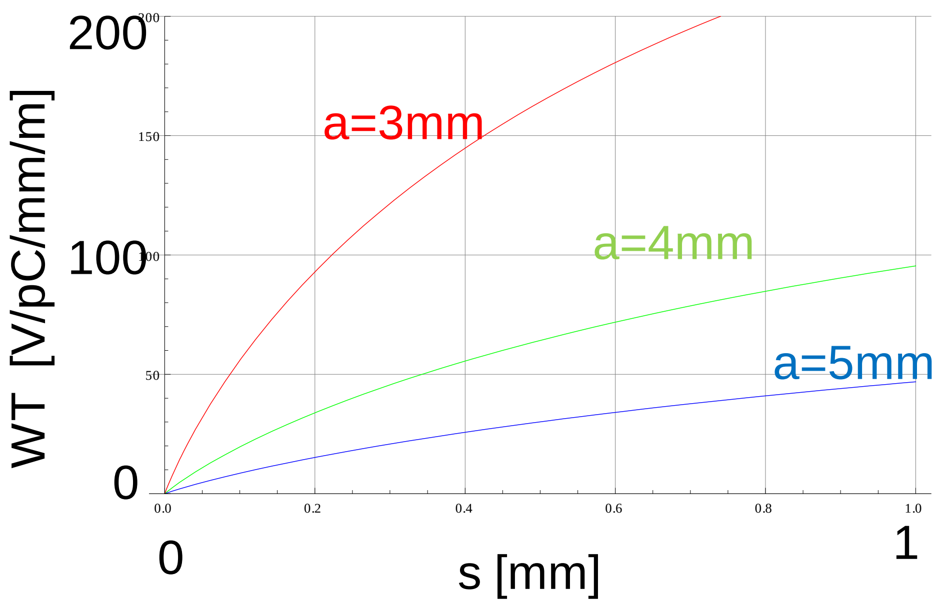
$$Z_0 = 377\,\Omega,\;\; s_0 = 0.169\,\frac{a^{1.79}\,g^{0.38}}{L^{1.17}}$$

Initial slope;

$$\frac{\partial}{\partial s}W_T(s) = \frac{2\,Z_0\,c}{\pi\,a^4}$$

Linear slope and
strong a-dependence!

Transverse wake field for NLC.



Single-bunch beam dynamics and cures

Beam dynamics under shot range transverse wake field

Force and momentum change in transverse direction;

$$F_x = \frac{dp_x}{dt} = \frac{d}{dt}\gamma\, m_0 \frac{dx}{dt} = m_0 c^2 \frac{d}{ds}\gamma \frac{dx}{ds}$$

Consider a bunch at s along the linac with the transverse position x(s,z) within the bunch with its charge distribution λ(y).

Under the force due to the transverse wake field, the equation of motion becomes

$$m_0 c^2 \frac{d}{ds}\gamma \frac{d}{ds} x(s,z) + m_0 c^2\, k_\beta^2\, x(s,z) = e\, Ne \int_z^\infty dy\, x(s,y)\, w(y-z)\, \lambda(y)$$

The second term in left hand side is the betatron oscillation term in a linac optics.
Right hand side is the force due to wake field inside the bunch at the position z due to the offset of the precedent part of the same bunch at y with offset value x(s,y).

This becomes

$$\frac{1}{\gamma}\frac{d}{ds}\gamma \frac{d}{ds} x(s,z) + k_\beta^2\, x(s,z) = \frac{N\, r_e\, 4\pi\, \varepsilon_0}{\gamma} \int_z^\infty dy\, x(s,y)\, w(y-z)\, \lambda(y)$$

$$where\ r_e = \frac{1}{4\pi\, \varepsilon_0} \frac{e^2}{m_0\, c^2}$$

Two particle model view

If no acceleration case;

$$\frac{d^2}{ds^2}x(s,z)+k_\beta^2\,x(s,z)=\frac{N\,r_e\,4\pi\,\varepsilon_0}{\gamma}\int_z^\infty dy\,x(s,y)\,w(y-z)\,\lambda(y)$$

Bunch is divided into two part, head and tail

Each charge Ne/2, separated by $2\sigma_z$ and

Head bunch behaves as

$$x_1(s)=\hat{x}_1\ Cos\,k_\beta\,s$$

The force experienced by the tail particle due to the wake field driven at the head particle

$$F_2=e\frac{N\,e}{2}\,x_1\,w(2\sigma_z)$$

Then the equation of motion of the tail particle becomes

$$\frac{d^2}{ds^2}x_2+k_\beta^2\,x_2=-\frac{N\,r_e\,4\pi\,\varepsilon_0\,w(2\sigma_z)\,x_1}{2\gamma}=-C\,\hat{x}_1\,e^{j k_\beta s}$$

This is the forced oscillation with the same oscillation frequency as the drive field.
Find a solution of the form

$$x_2=y(s)\,e^{j k_\beta s}$$

Growth estimation in two particle model

$$y''(s) + 2jk_\beta y'(s) = \hat{x}_1$$

This has a solution

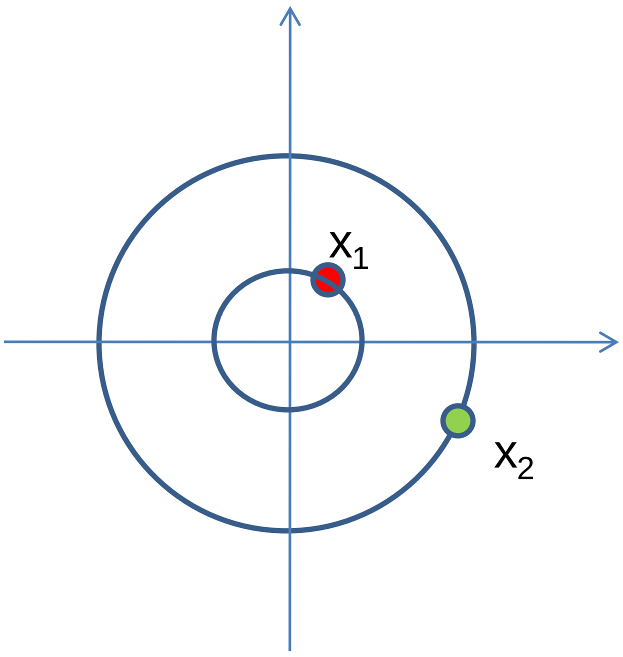
$$y(s) = -\frac{j\hat{x}_1}{2k_\beta} s$$

Then,

$$x_2(s) = -\frac{j\hat{x}_1}{2k_\beta} s\, e^{jk_\beta s}$$

This result states that

1. Amplitude increases linearly
2. Phase is 90 degrees delayed



More general but constant energy case

Again in no acceleration case;

$$\frac{d^2}{ds^2}x(s,z)+k_\beta^2\,x(s,z)=\frac{N\,r_e\,4\pi\,\varepsilon_0}{\gamma}\int_z^\infty dy\,x(s,y)\,w(y-z)\,\lambda(y)$$

Assume initial offset without slope

$$x(0,z)=x_0 \quad , \quad \left.\frac{\partial x}{\partial s}\right|_{s=0}=0$$

And assume wake field is small

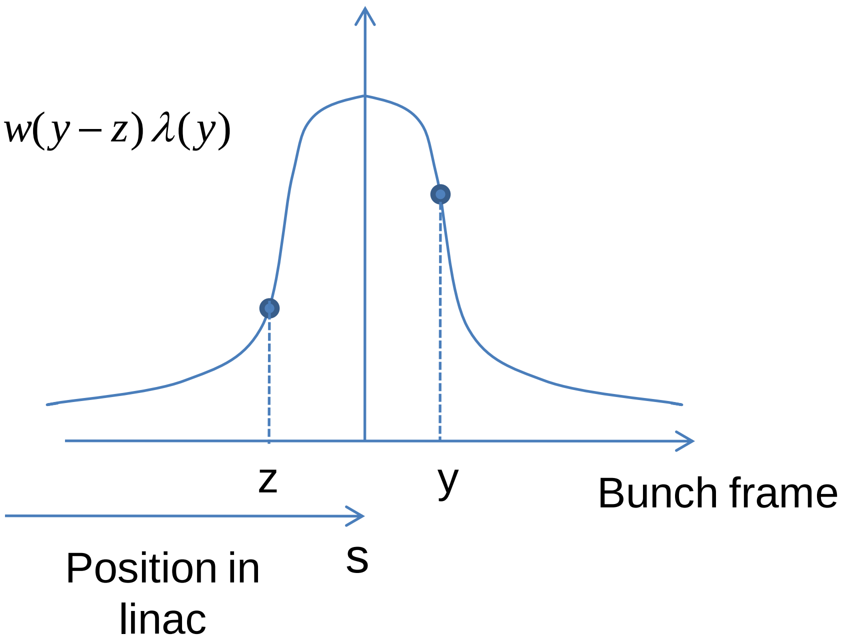
$$\frac{N\,r_e\,w}{\gamma}<<k_\beta^2$$

Let us find a solution of the form

$$x(s,z)=a(s,z)\,e^{j\,k_\beta\,s}$$

Substitute into the above equation

$$\frac{\partial a}{\partial s}=-j\frac{N\,r_e\,4\pi\,\varepsilon_0}{2\,k_\beta\,\gamma}\int_z^\infty dy\,a(s,y)\,w(y-z)\,\lambda(y)$$



Linear slope wake and square bunch

$$w = w' z$$

$$\lambda(z) = 1/2l_b$$

And define a parameter

$$r = \frac{N r_e 4\pi \varepsilon_0 w'}{2 k_\beta \gamma l_b}$$

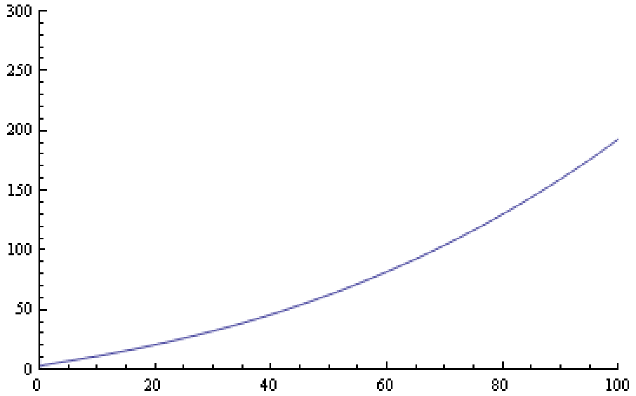
Then,

$$\frac{\partial a}{\partial s} = -j r \int_z^\infty dy (y - z) a(s, y)$$

This has an asymptotic solution,

$$a = \frac{x_0}{\sqrt{6\pi}} (2 r s z^2)^{-1/6} Exp[\frac{3^{3/2}}{4} (2 r s z^2)^{1/3}]$$

$$(2 r s z^2)^{-1/6} Exp[\frac{3^{3/2}}{4} (2 r s z^2)^{1/3}]$$



$2rsz^2$

Derived by A. W. Chao, B. Richter and C. Y. Yao, NIM, 178, p1, 1980

Take energy gain into account

Back to equation

$$\frac{1}{\gamma}\frac{d}{ds}\gamma\frac{d}{ds}x(s,z)+k_\beta^2\,x(s,z)=\frac{N\,r_e\,4\pi\,\varepsilon_0}{\gamma}\int_z^\infty dy\,x(s,y)\,w(y-z)\,\lambda(y)$$

Assume slow acceleration

$$\gamma'/\gamma \ll k_\beta$$

And assume no wake first, then

$$\frac{d^2}{ds^2}x+\frac{\gamma'}{\gamma}\frac{d}{ds}x+k_\beta^2\,x=0$$

Let us design as

$$x(s,z)=a(s,z)\,e^{jk_\beta s}$$

The equation on a becomes

$$2\,\frac{\partial a}{\partial s}+\frac{\gamma'}{\gamma}a=0$$

This has a solution,

$$a(s,z)=a_0(z)\sqrt{\gamma_0/\gamma(s)}$$

It states the adiabatic damping

Now let the wake ON,

With linear slope wake and energy gain

$$2\,j\,k_\beta\,\frac{\partial a}{\partial s}+\frac{\gamma'}{\gamma}\,j\,k_\beta\;a=\frac{N\,r_e\,4\pi\,\varepsilon_0}{\gamma}\int_z^\infty dy\,a(s,y)\,w(y-z)\,\lambda(y)$$

This time let us define

$$a(s,z)=b(s,z)/\sqrt{\gamma(s)}$$

Then the equation becomes

$$\frac{\partial b}{\partial s}=-j\,\frac{N\,r_e\,4\pi\,\varepsilon_0}{2\,k_\beta\,\gamma(s)}\int_z^\infty dy\,b(s,y)\,w(y-z)\,\lambda(y)$$

This is exactly the same as a(s,z) case but with varying γ(s)

Let us define coordinate S

$$dS=ds/\gamma(s)$$

Then

$$\frac{\partial b}{\partial S}=-j\,\frac{N\,r_e\,4\pi\,\varepsilon_0}{2\,k_\beta}\int_z^\infty dy\,b(s,y)\,w(y-z)\,\lambda(y)$$

This form is also exactly the same as that of a(s,z) before.
This has an asymptotic form as a.

Solution of growth

Assume uniform acceleration

$$\gamma(s)=\gamma_i+s(\gamma_f-\gamma_i)/L$$

Then,

$$S=\frac{L}{\gamma_f-\gamma_i}\ln\frac{\gamma(s)}{\gamma_i}\approx\frac{L}{\gamma_f}\ln\frac{\gamma(s)}{\gamma_i}$$

Therefore, as the case with a,

$$b=\frac{x_0}{\sqrt{6\pi}}(2rSz^2)^{-1/6}\,Exp[\frac{3^{3/2}}{4}(2rSz^2)^{1/3}]\quad where\ r=\frac{N\,r_e\,4\pi\,\varepsilon_0\,w'}{2k_\beta\,l_b}$$

Since S_{final} is

$$S_{final}\approx\frac{L}{\gamma_f}\ln\frac{\gamma_f}{\gamma_i}=L/\gamma_{eff}\quad where\ \gamma_{eff}=\gamma_f/\ln\frac{\gamma_f}{\gamma_i}$$

From these formula, we can estimate the growth along the bunch

$$\boxed{a=\frac{1}{\sqrt{\gamma(s)}}\frac{x_0}{\sqrt{6\pi}}(2rSz^2)^{-1/6}\,Exp[\frac{3^{3/2}}{4}(2rSz^2)^{1/3}]\quad where\quad r=\frac{N\,r_e\,4\pi\,\varepsilon_0\,w'}{2k_\beta\,l_b}}$$

BNS damping

Equation of motion in the two particle model,

$$\frac{d^2}{ds^2}x_2 + k_\beta^2 x_2 = -\frac{N r_e 4\pi\varepsilon_0 w(2\sigma_z) x_1}{2\gamma} = -C\,\hat{x}_1 e^{jk_\beta s}$$

Varying the tail particle oscillation frequency from that of the head particle,

$$\frac{d^2}{ds^2}x_2 + (k_\beta^2 + \Delta k_\beta^2) x_2 = -\frac{N r_e 4\pi\varepsilon_0 w(2\sigma_z) x_1}{2\gamma} = -C\,\hat{x}_1 e^{jk_\beta s}$$

The tail particle resonant growth is suppressed. In FODO lattice,

$$Sin\frac{\mu}{2} = \frac{c\,g\,L^2}{8E/e}\eta(1-\eta) \quad \& \quad \frac{1}{2}Cos\frac{\mu}{2}\frac{d\mu}{d\delta} = \frac{c\,g\,L^2}{8E^2/e}\eta(1-\eta) \Rightarrow \frac{d\mu}{d\delta} = -2Tan\frac{\mu}{2}$$

We make the energy variation within a bunch to introduce variation of k_β

$$\frac{dk_\beta}{d\delta} = \frac{1}{L}\frac{d\mu}{d\delta} = -\frac{2}{L}\tan\frac{\mu}{2}$$

This suppression is called BNS damping.

In practice;

Energy tapering can be produced by setting the bunch in RF slope.

The longitudinal wake function help decreased the energy toward the tail.

Autophasing

Start with the equation of motion;

$$\frac{d^2}{ds^2}x(s,z)+\{k_\beta^2\,x(s,z)+\Delta k_\beta^2\,x(s,z)\}=\frac{N\,r_e\,4\pi\,\varepsilon_0}{\gamma}\int_z^\infty dy\,x(s,y)\,w(y-z)\,\lambda(y)$$

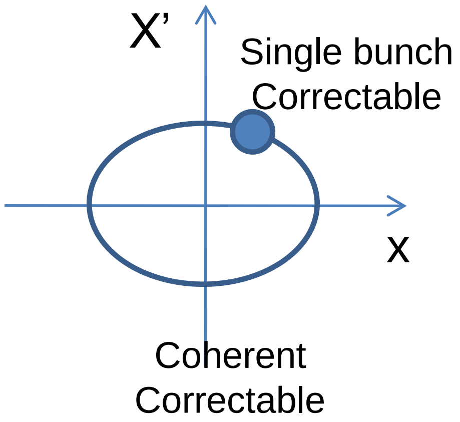
If we can vary the Δk_β as

$$\Delta k_\beta^2(s,z)\}=\frac{N\,r_e\,4\pi\,\varepsilon_0}{\gamma}\int_z^\infty dy\,w(y-z)\,\lambda(y)$$

The solution becomes simply

$$x(s,z)=x(s)=x_0\cos(k_\beta\,s)$$

This is stable and x does not depend on z, which means the both head and tail stays as in the right figure.



This suppression scheme is autophasing.
The slope on k_β is produced by energy profile inside the bunch with amplitude of the order of

$$\Delta k_\beta^2\approx\frac{N\,r_e\,4\pi\,\varepsilon_0\,\sigma_z}{\gamma}W_\perp^{'}$$

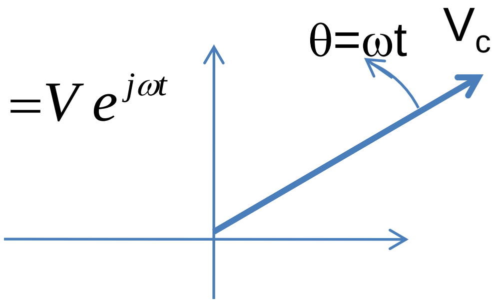
The big energy slope should be compensated at the downstream of linac.

Long range wake field

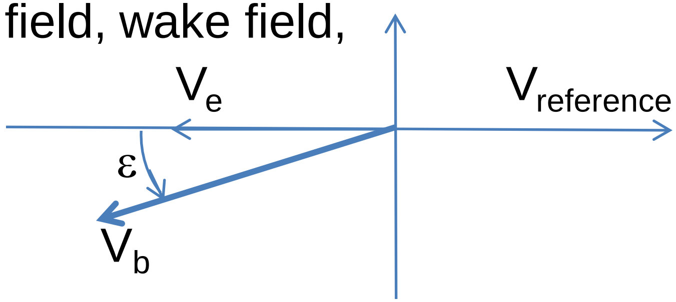
Fundamental theorem of beam loading

0: Assume a cavity field in phasor diagram with one dominant mode

$$V(t) = V\, e^{j\omega t}$$



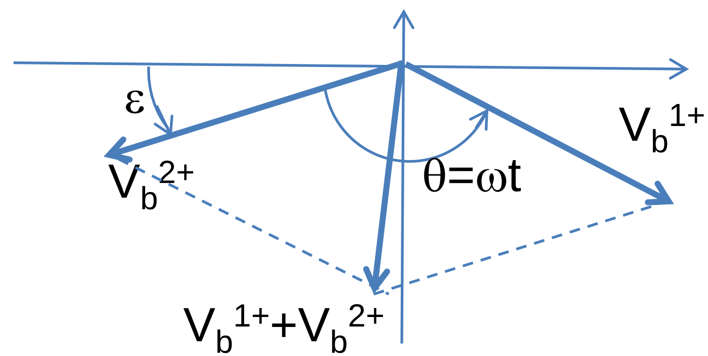
1: Point particle passed an empty cavity, leaving a field, wake field,



$$\Delta E_{1+} = qV_e = q\, f\, V_b$$ Beam energy loss

$$U_{1+} = \alpha V_b^2$$ Cavity stored energy

2: When the second bunch comes in, superposition applies;



$$V_{b1+} = V_b\, e^{j\theta}$$

$$U_{2+} = \alpha\,(V_{b1+}\, e^{j\theta} + V_{b2+})^2$$

$$= 2\,\alpha V_b^2\,(1 + Cos\theta)$$

Fundamental theorem of beam loading cont.

While loss of the second bunch;

$$\Delta E_{2+} = qV_e + qV_b Cos(\varepsilon + \theta)$$

Since particle energy loss = cavity stored energy;

$$\Delta E_{1+} + \Delta E_{2+} = U_{2+}$$

Therefore,

$$2(q\,f - \alpha V_b) + (q\,Cos\varepsilon - 2\,\alpha V_b)\,Cos\theta - q\,Sin\varepsilon\,Cos\theta = 0$$

This should always true for any θ, then

$$\therefore\ \varepsilon = 0, \qquad V_b = \frac{q}{2\alpha}, \qquad f = \frac{1}{2}$$

When a bunch passes a cavity,
it excites the cavity with the field in a decelerating direction,
or it remains a deceleration wake field in the cavity.
The bunch feels half of this excited field.

Long range wake field in a cavity

Longitudinal wake excited in a cavity is expressed as

$$W_L(s) = 2k_L e^{-\alpha \frac{s}{c}} \left(Cos(\bar{\omega}\frac{s}{c}) - \frac{\alpha}{\bar{\omega}} Sin(\bar{\omega}\frac{s}{c}) \right),$$

$$\bar{\omega} = \sqrt{\omega_0^2 - \alpha^2}\ ,\ Q = \omega_0 / 2\alpha$$

In a cavity with very high Q value,

$$W_L(s) = 2k_L e^{-\frac{\omega_L}{2Q}\frac{s}{c}} Cos(\omega_L \frac{s}{c}), \quad where\ k_L = \frac{\omega_L}{2}\left(\frac{R}{Q}\right)_L$$

$$W_T(s) = 2k_T e^{-\frac{\omega_1}{2Q}\frac{s}{c}} Sin(\omega_1 \frac{s}{c})$$

Longitudinal wake field behaves cosine-like. The bunch is decelerated and excite the field in the cavity. Point-like bunch suffer from the wake, deceleration.

Transverse wake behaves sine-like. It increases linearly in time at very short time, usually within the bunch.

Calculation of impedance in SW or TW

For longitudinal mode;

$$\left(\frac{R}{Q}\right)_L \equiv \frac{V^2}{2\omega U}\ [\Omega]\ ,\ V \equiv \int_0^L E_z(z)\, e^{jkz}\, dz\, , k = \frac{\omega}{c}, U = Stored\ energy$$

For transverse mode;

$$\left(\frac{R}{Q}\right)_T^{'} \equiv \frac{\left(\partial V / \partial r\right)^2}{2\omega U} \frac{1}{k^2} [\Omega]\ ,\ \partial V / \partial r \equiv \int_0^L \frac{\partial E_z(z)}{\partial r} e^{jkz}\, dz\, , k = \frac{\omega}{c}, U = Stored\ energy$$

Then for longitudinal wake;

$$W_L(s) \equiv -\frac{1}{Lq}\int_0^L E_z^{cmf}\, dz\ \ [V/C/m] = \sum_n 2\, k_{L,n}\, e^{-\frac{\omega_n}{2Q_n}\frac{s}{c}}\, Cos(\frac{\omega_n}{c} s)$$

Calculation of k_L by SW field solver

For SW cavity;

$$k_{L,SW} = \frac{V^2}{4U} / L = \frac{\omega}{2} (\frac{R}{Q})_L / L$$

For TW cavity;

$$k_{L,TW} = \frac{E_0^{\ 2}}{4u}$$

where only n=0 space harmonics contributes;

$$E_z = E_0 \, Cos \, \omega (t - \frac{s}{c})$$

When we consider SW field, SW = FW + BW;

$$E_z^{SW}(z,t) = E_0 \{ Cos \, \omega (t - \frac{z}{c}) + Cos \, \omega (t + \frac{z}{c}) \}$$

The field calculated by SW mode is the case with t=0 in the above equation;

$$E_z^{SW}(z,t) = 2 E_0 \, Cos \, \omega (\frac{z}{c})$$

Loss parameter formula

Therefore, E_0 can be calculated as

$$E_0 = \frac{1}{L}\int_{-L/2}^{L/2} E_z^{SW}(z)\, Cos(\frac{\omega}{c} z) = \frac{V}{L}$$

When we consider the coupling of beam to TW mode, the relevant energy is only the forward wave, which is half of U^{SW}

$$k_{L,TW} = \frac{{V_{SW}}^2}{4(U_{SW}/2)} / L = \frac{{V_{SW}}^2}{2U_{SW}}$$

$$\therefore \quad k_{L,TW} = \omega\left(\frac{R}{Q}\right)_L / L$$

Then, loss parameter is

$$k_{L,TW} = 2 \times k_{L,SW}$$

Transverse wake calculation

Transverse wake field excited by a bunch with charge q passing a cavity with transverse position offset of Δr,

$$(\vec{E}+\vec{v}\times\vec{B})_T$$

The wake field due to this field is expressed

$$W_T(s)\equiv\frac{1}{L\,q\,\Delta r}\int_0^L(\vec{E}+\vec{v}\times\vec{B})_T^{cmf}\,dz\ [\frac{V}{C\,m^2}]\ =\sum_n\frac{2k_{T,n}}{\omega/c}e^{-\frac{\omega_n}{2Q_n}\frac{s}{c}}\ Sin(\frac{\omega_n}{c}s) \qquad (\#)$$

This can be explained for both SW and TW as follows .

For SW case, from Panofsky Wenzel,

$$(\vec{E}+\vec{v}\times\vec{B})_T^{cmf}=\frac{j}{\omega/c}\nabla_T\,E_z^{cmf}$$

If we apply this to the above eq.

$$W_T^0(s)\equiv\frac{1}{L\,q\,\Delta r}\frac{j}{\omega/c}\frac{\partial V}{\partial r}$$

Transverse wake calculation (cont.)

The energy of the cavity excited by charge q with offset Δr is equal to the energy loss if the bunch interacting with the field excited by itself,

$$U(\Delta r) = \frac{1}{2} q \frac{\partial V}{\partial r} \Delta r = k_T^{SW} (q \Delta r)^2 L$$

where

$$k_T^{SW} = \frac{\omega}{2} k^2 (\frac{R}{Q})'_T / L$$

and thinking

$$W_T^0 = \frac{2 k_T}{\omega / c}$$

The equation (#) is found proven. (This is for SW case)

In the TW case, as in the longitudinal wake,

$$k_{T,TW} = 2 \times k_{T,SW}$$

Therefore,

$$k_{T,TW} = \omega k^2 \left(\frac{R}{Q} \right)'_T / L$$

Frequency scaling

Per cavity;

Per unit length;

L

$$\left(\frac{R}{Q}\right)_L \equiv \frac{V^2}{2\omega U} \propto \frac{\lambda^2}{\omega\lambda^3} \propto 1$$

$$[\Omega]$$

$$\left(\frac{R}{Q}\right)_L / L \propto \omega$$

$$W_L \propto k_L \propto \omega \left(\frac{R}{Q}\right)_L / L \propto \omega^2$$

T

$$\left(\frac{R}{Q}\right)'_T \equiv \frac{(\partial V / \partial r)^2}{2\omega U} \frac{1}{k^2} \propto 1$$

$$[\Omega]$$

$$\left(\frac{R}{Q}\right)'_T / L \propto \omega$$

$$k_T \propto \omega k^2 \left(\frac{R}{Q}\right)'_T / L \propto \omega^4$$

$$W_T \propto \frac{k_T}{\omega} \propto \omega^3$$

Dipole mode field to calculate R/Q

Let us take the most typical dipole mode, TM1nl,
in a pillbox cavity

$$E_r = -\frac{\beta_z}{K_c} \quad Cos(\theta) \quad J_1^{'}(K_c\, r) \quad Sin(\beta_z\, z)$$

$$E_\theta = \frac{\beta_z}{K_c^2} \quad Sin(\theta) \quad \frac{1}{r} J_1(K_c\, r) \quad Sin(\beta_z\, z)$$

$$E_z = \qquad Cos(\theta) \quad J_1(K_c\, r) \quad Cos(\beta_z\, z)$$

$$H_r = -j\frac{\omega\varepsilon}{K_c^2} \quad Sin(\theta) \quad \frac{1}{r} J_1(K_c\, r) \quad Cos(\beta_z\, z)$$

$$H_\theta = -j\frac{\omega\varepsilon}{K_c} \quad Cos(\theta) \quad J_1^{'}(K_c\, r) \quad Cos(\beta_z\, z)$$

$$H_z = 0$$

$$where\, K_c = \rho_{1n} / a, \quad \beta_z = l\,\pi / d$$

Dipole mode field to calculate R/Q (cont.)

The slope of E_z in r-direction on θ=0 plane at the beam axis

$$\frac{\partial E_z(z)}{\partial r} = \frac{\partial}{\partial r} J_1(K_c r) = K_c [2 J_0(K_c r) - \frac{J_1(K_c r)}{K_c r}]\bigg|_{r=0} = 1.5 K_c$$

Per a cavity

$$\partial V / \partial r \equiv \int_{-L/2}^{L/2} \frac{\partial E_z(z)}{\partial r} e^{jkz}\, dz = 2\int_0^{L/2} 1.5 K_c\, Cos(kz)\, dz \;= 3 K_c \int_0^{L/2} \; Cos(kz)\, dz$$

$$\therefore \partial V / \partial r = \frac{3 K_c}{k} Sin(\frac{k L}{2})$$

Then for a cavity

$$\left(\frac{R}{Q}\right)_T^{'} \equiv \frac{\left(\partial V / \partial r\right)^2}{2\omega U} \frac{1}{k^2} \;= \frac{2 (\frac{3K_c}{k})^2\, Sin^2(\frac{k\,d}{2})}{\omega\,\mu\,\pi\, d\, a^2\, k^2\, (\frac{\omega\,\varepsilon}{K_c})^2\, J_1^{'2}(\rho_{1n})}$$

Comments on dipole mode

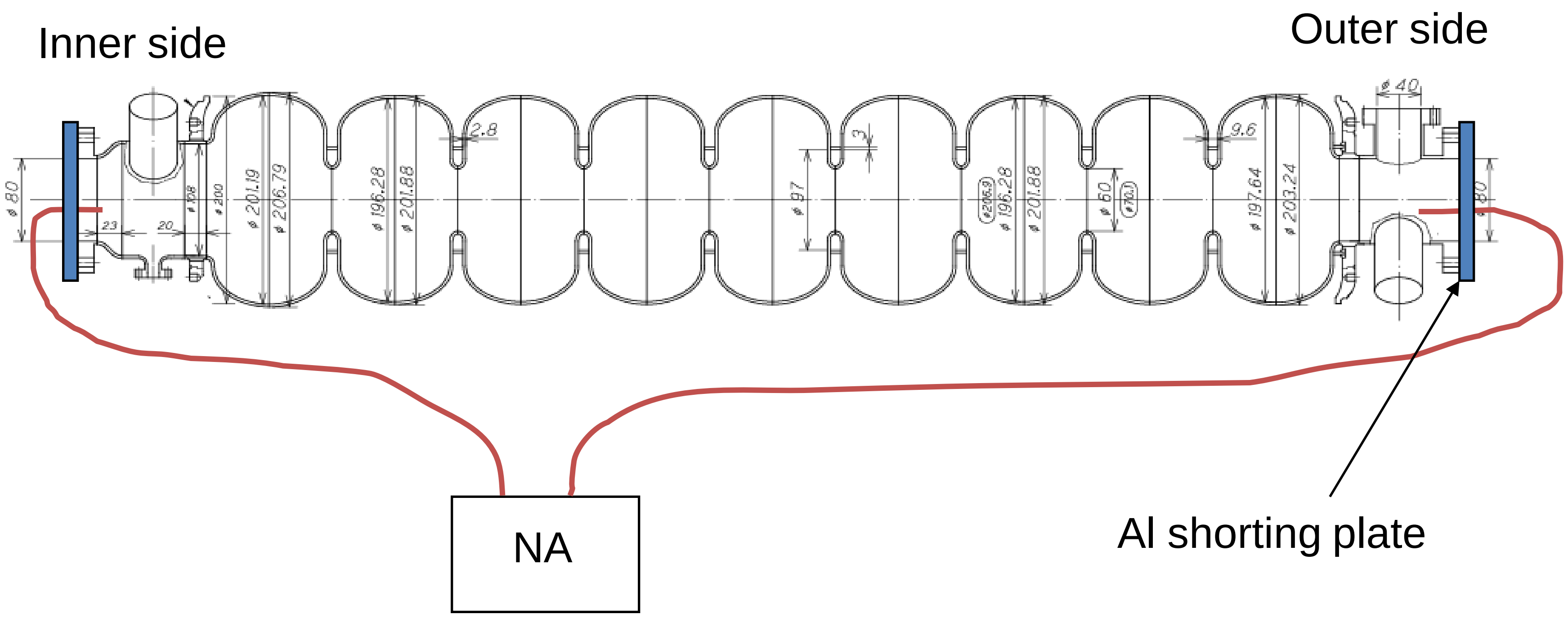
Firstly note that TE mode cannot couple to beam because of no Ez field!

Secondly, there are two polarization in dipole modes with the same field pattern.

Therefore, it is necessary to separate in frequency for these modes to be stable unless the two polarizations can be divided by geometry condition.

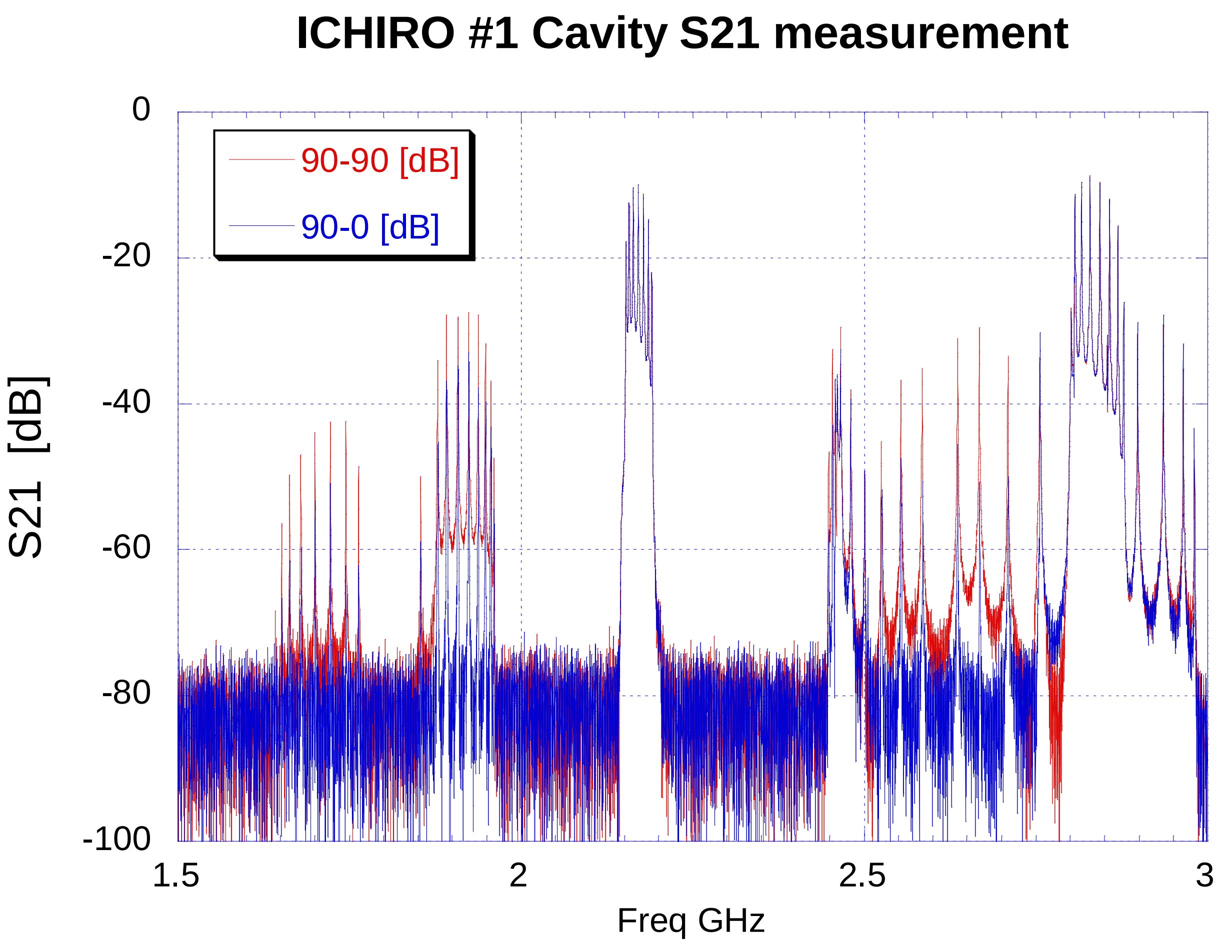
Actual higher order modes examples

Actual modes in 9-cell SCC cavity Measurement setup



Transmission measurement 1.5~3GHz

050509



Rotational symmetry identification.

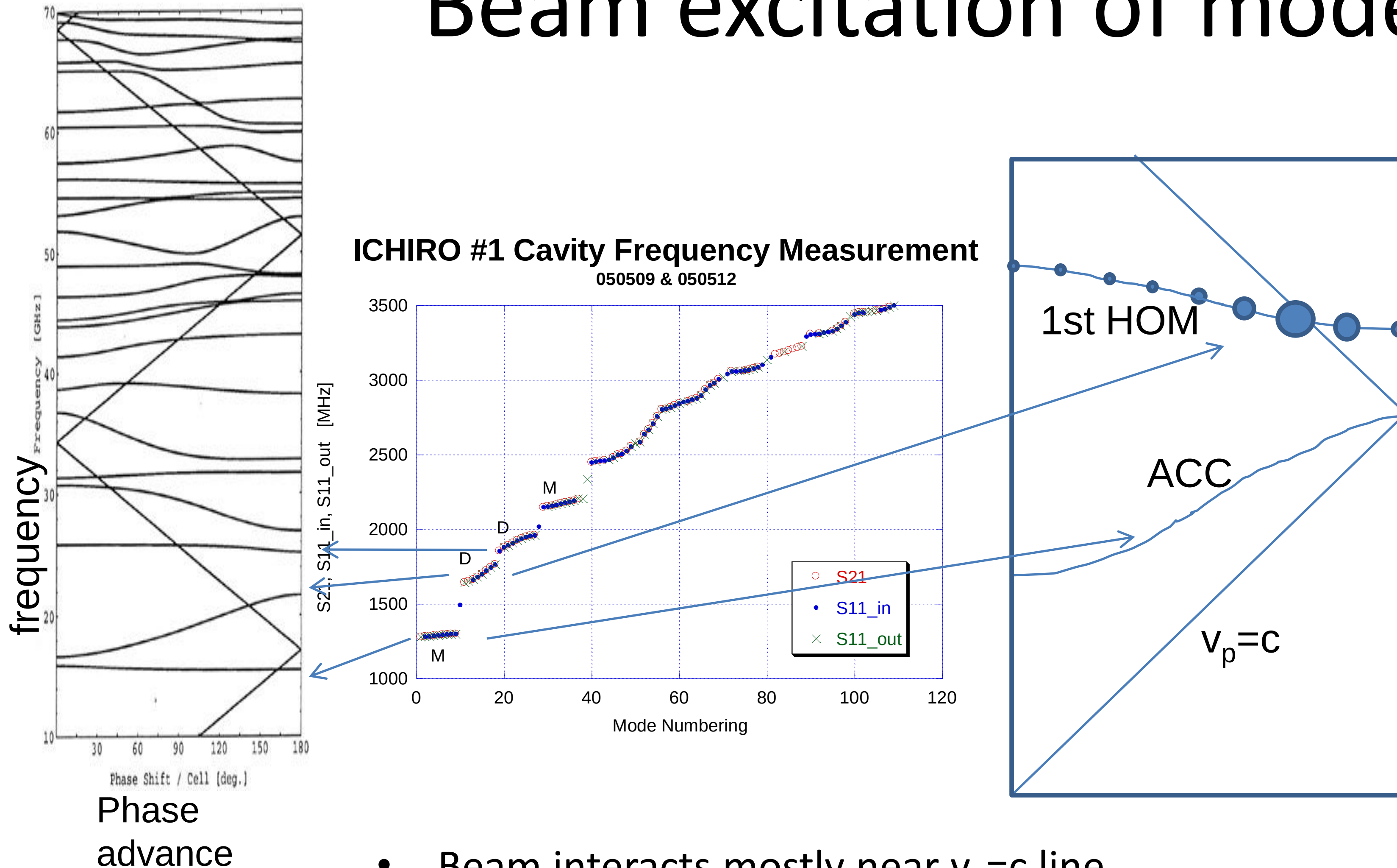
Passband nature.

Comparison with pillbox modes.

Leakage to beam pipe.

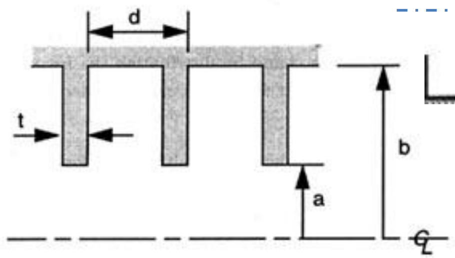
High Q modes.

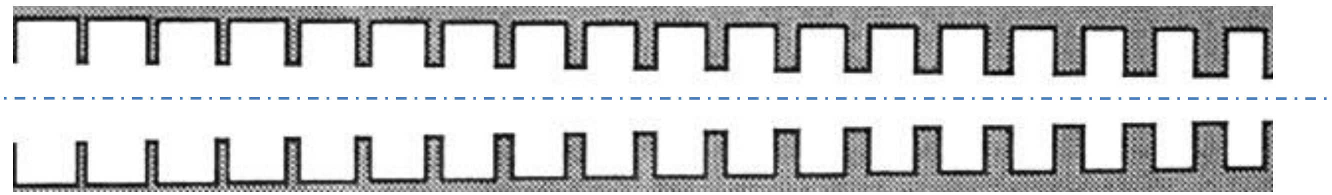
Beam excitation of modes

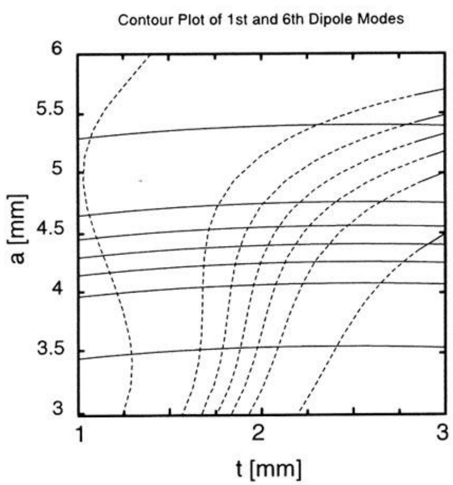


- Beam interacts mostly near v_p=c line.
- Everyband can be excited by beam.
- Most concern is the lowest dipole modes.

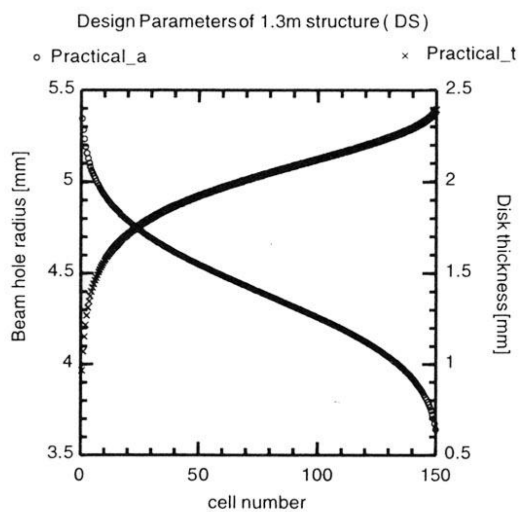
X-band detuned structure



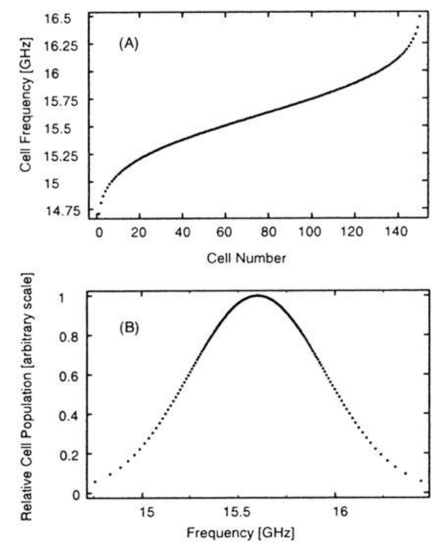




Contour of dipole mode frequency vs (a,t)

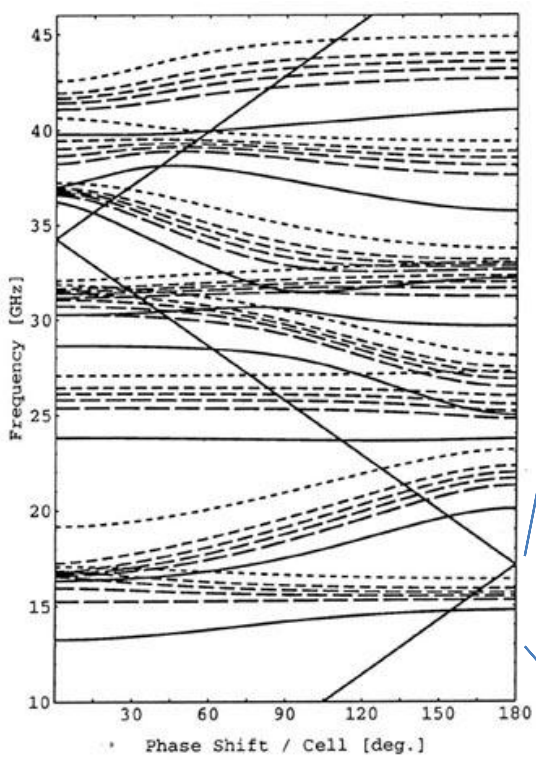


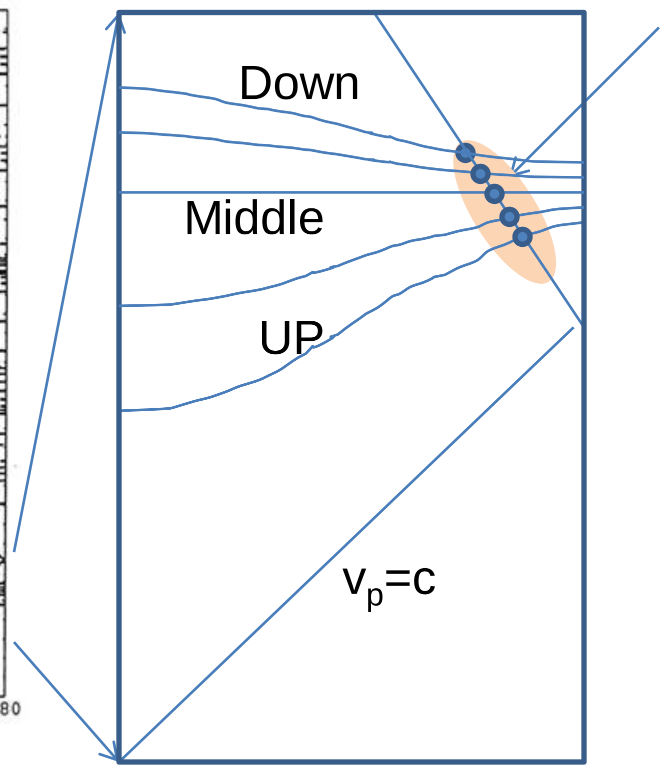
(a,t) distribution along structure



Dipole mode distribution in frequency and kick factor

Beam excited modes in detuned structure





Beam excitation

Excited modes

Trapped modes

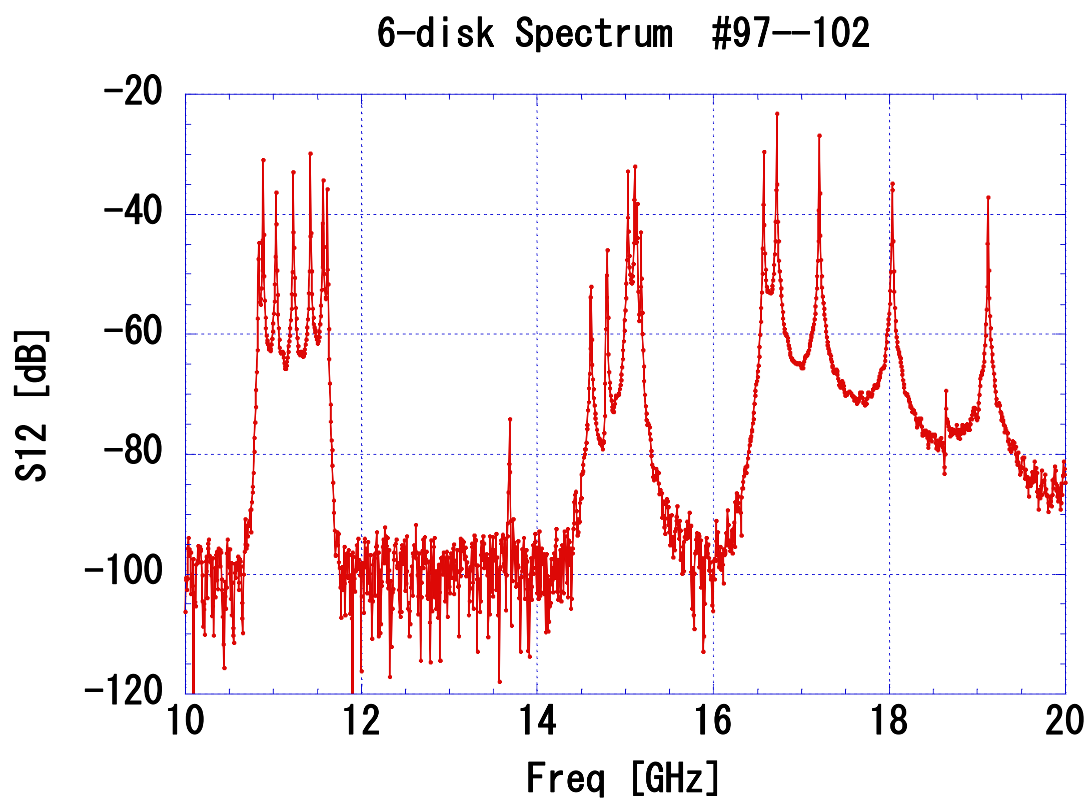
Should be damped

Position-modal frequency dependence

Can be used as SBPM

Actual modes in X-band structure

Extract a part along structure and stack 6 identical cells.
Measure dispersion characteristics to confirm the HOM.

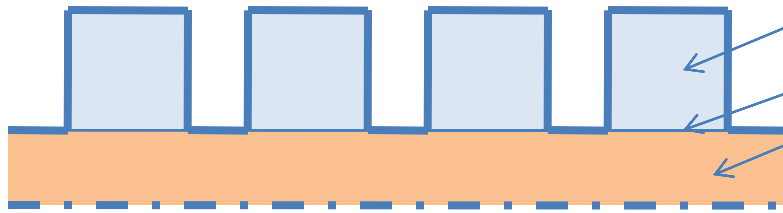


Ways to deal with coupled cavity system on HOM estimation

Various ways of treatment

- Equivalent circuit model
- Matching
  - Mode matching
  - S-parameter
  - Open mode expansion model
- Mesh based Numerical
  - Finite element model HFSS, $\Omega 2$
  - Finite difference model MAFIA, GdFidl,

Mode matching techniques



Resonant mode

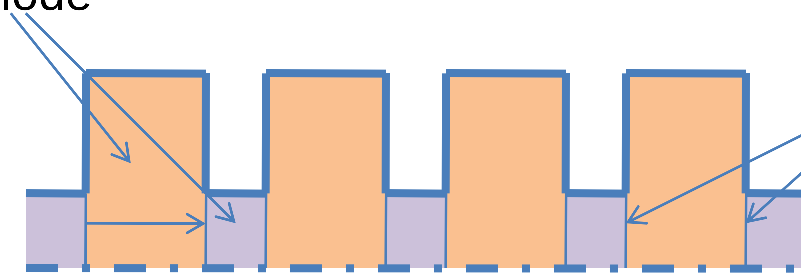
Field matching

Propagating mode

Field matching

SW & TR field matching

Propagating mode



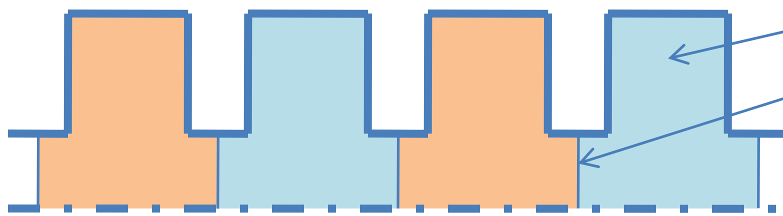
Scattering matrix

S-matrix for 3D cal can be used

Field matching

Mode matching

Expansion modes in cylindrical waveguide



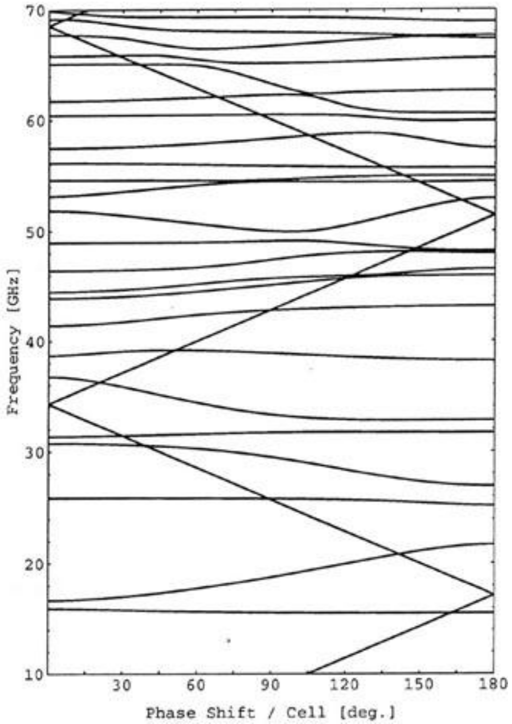
Resonant SW mode

Field matching

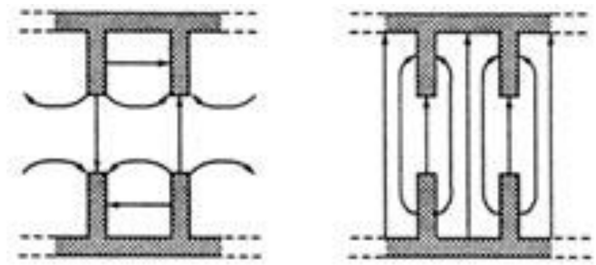
Open mode expansion

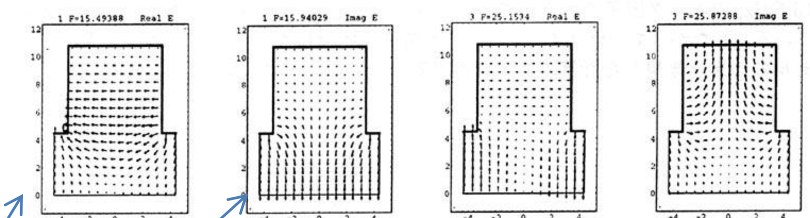
Resonators with small coupling between cells

An example:
Open mode expansion technique

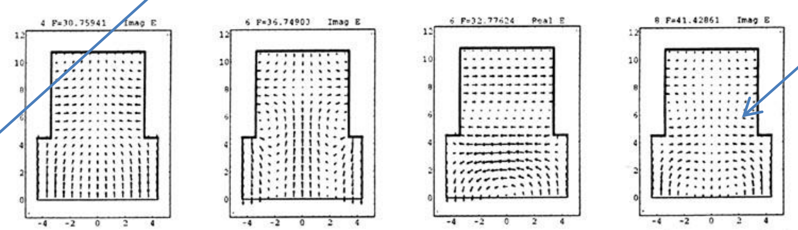


Open modes for expansion base





(a); open 1 ($\acute{e}_1$) (b); open 2 ($\acute{e}_2$) (c); open 3 ($\acute{e}_3$) (d); open 4 ($\acute{e}_4$)



(e); open 5 ($\acute{e}_5$) (f); open 6 ($\acute{e}_6$) (g); open 7 ($\acute{e}_7$) (h); open 8 ($\acute{e}_8$)

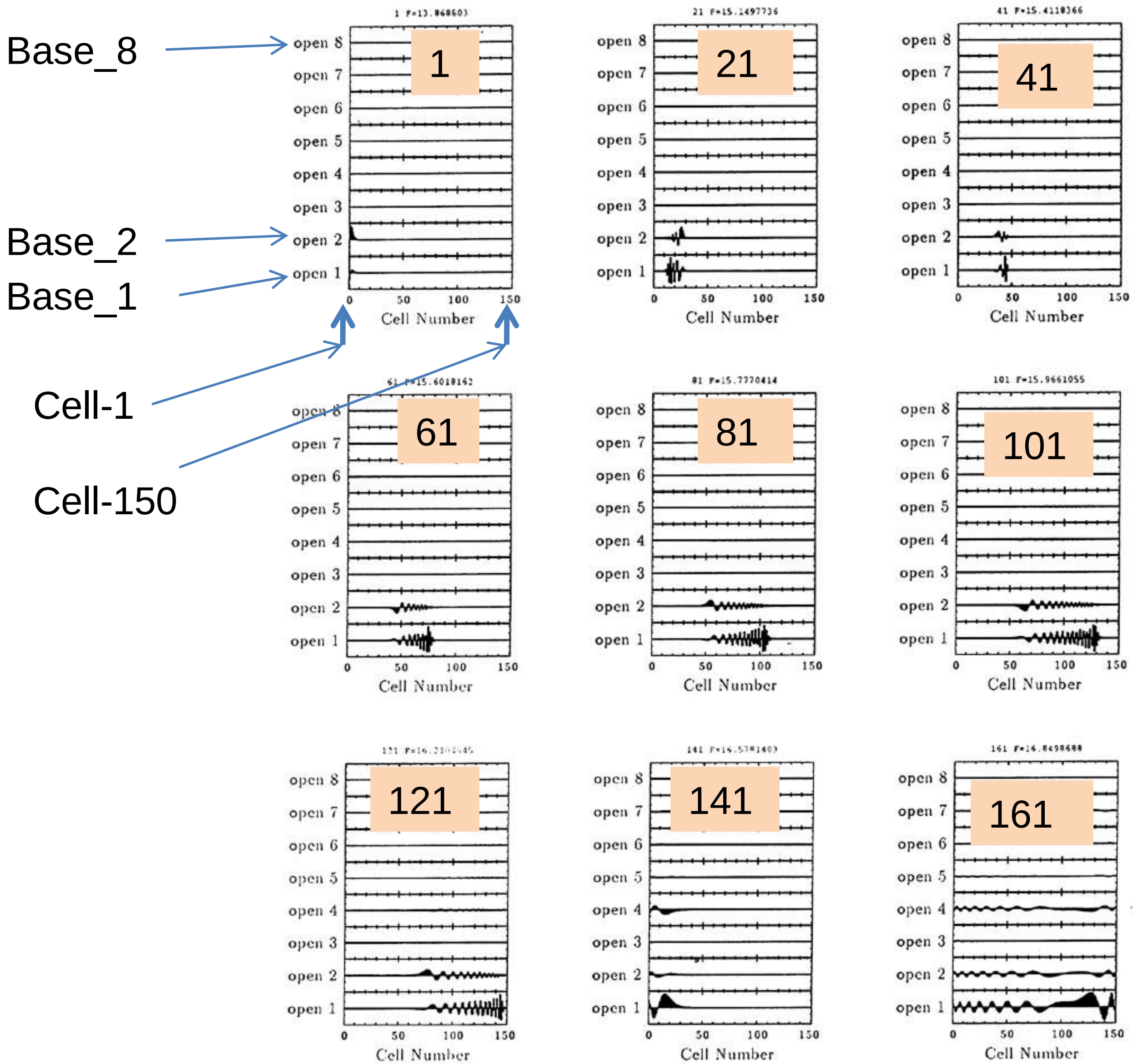
Base_1

Base_2

Base_8

Actual open modes used for calculation

Calculated field for 150-cell cavity



$$E(r,z) = \sum_{k=1}^{N_s} \sum_{n=1}^{8} a_j \, e_j^{open}$$

$$a \equiv \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{8N} \end{pmatrix}$$

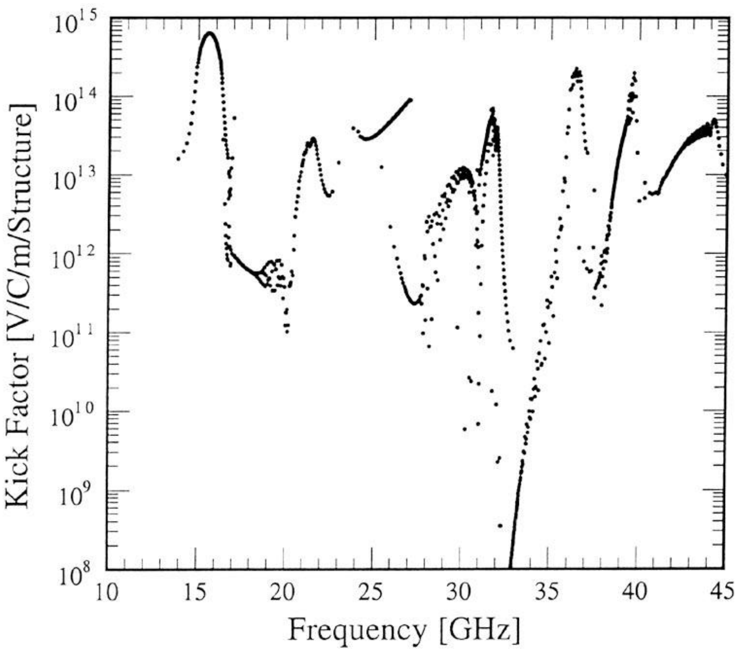
$$X\,a = \omega^2\,a$$

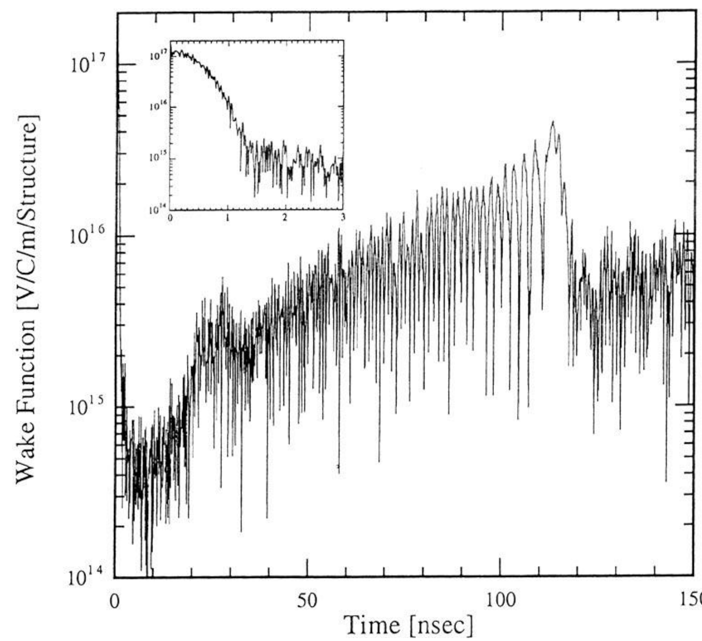
Calculation result

Kick factor

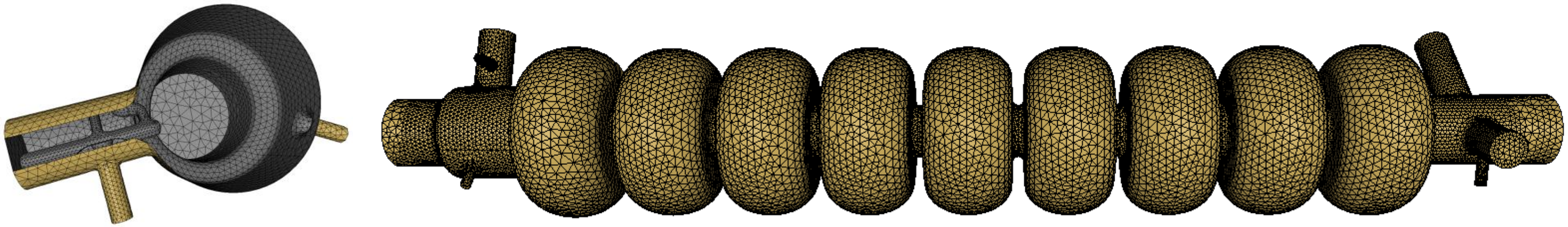
Wake field

$$W_T(t) = \sum_{n=1}^{N} 2\, k_{T,n}\, Sin(\omega_n\, t)$$





Finite element or finite difference



Example: SCC 9-cell cavity simulation with 3D FEM (Z. Li, SLAC)

- Ω3, MAFIA, GdfidL, HFSS

- Parallel PC arrays today can deal with the whole cavity in 3D as a whole.
- This is a final confirmation but these are becoming to even the tool in early design stage.

Cures against multi-bunch emittance growth by suppression of wakefield

Cures from structure design

Transverse wake field is expressed as

$$W_T(s) \propto \sum_n 2k_{T,n} e^{-\frac{\omega_n}{2Q_n}t} Sin(\omega_n t)$$

$$\frac{\omega t_b}{2Q_L} \approx \frac{f\, t_b}{Q_L/\pi} \approx \frac{468}{10^{10}} \; for\ ILC << 1$$

$$\approx \frac{6}{10^4} \; for\ CLIC << 1$$

There are two ways of suppressing wake field.
One is to align beam to axis, while the other is to suppress wake field.

Damping the mode with low Q

$$e^{-\frac{\omega t}{2Q_L}}$$

$$Q_L = \frac{\omega U}{P_{wall} + P_{ext}}$$

Intrinsic Q_0 is too large.
Both ILC and CLIC need external damping.

Effectively lower Q by frequency spread

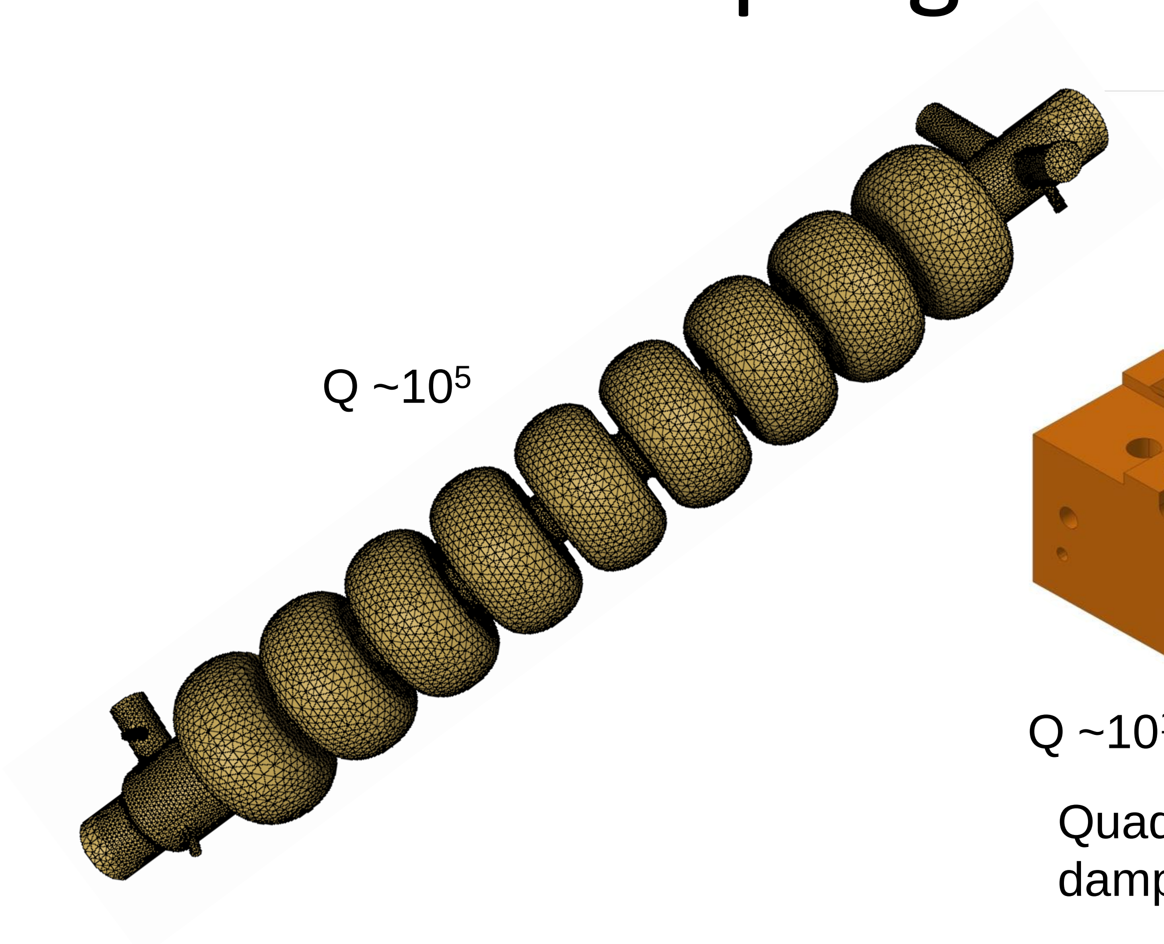
$$W(t) \propto \int f(\omega) e^{j\omega t}\, d\omega$$

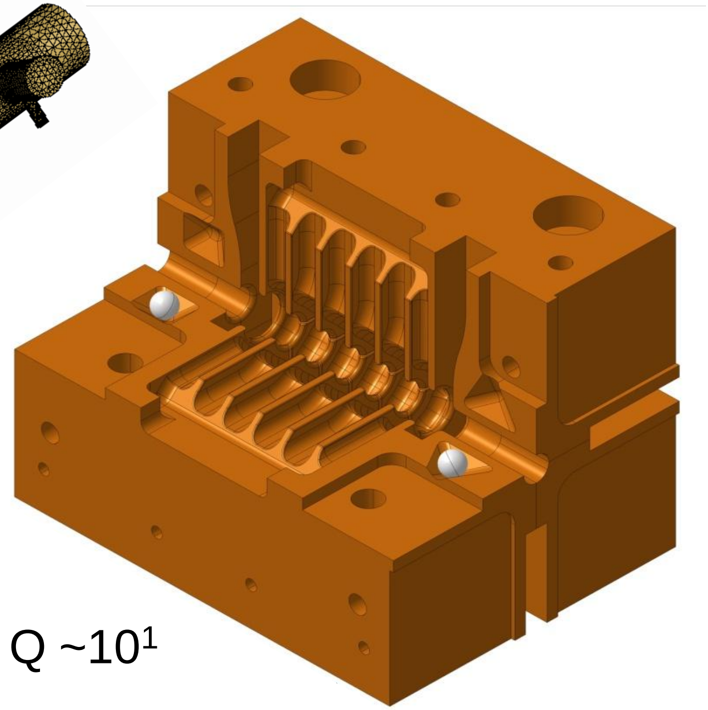
X-band approach utilizes this.

The relative ratio depends on design.

External coupling   ILC and CLIC

Q ~10^5





Q ~10^1

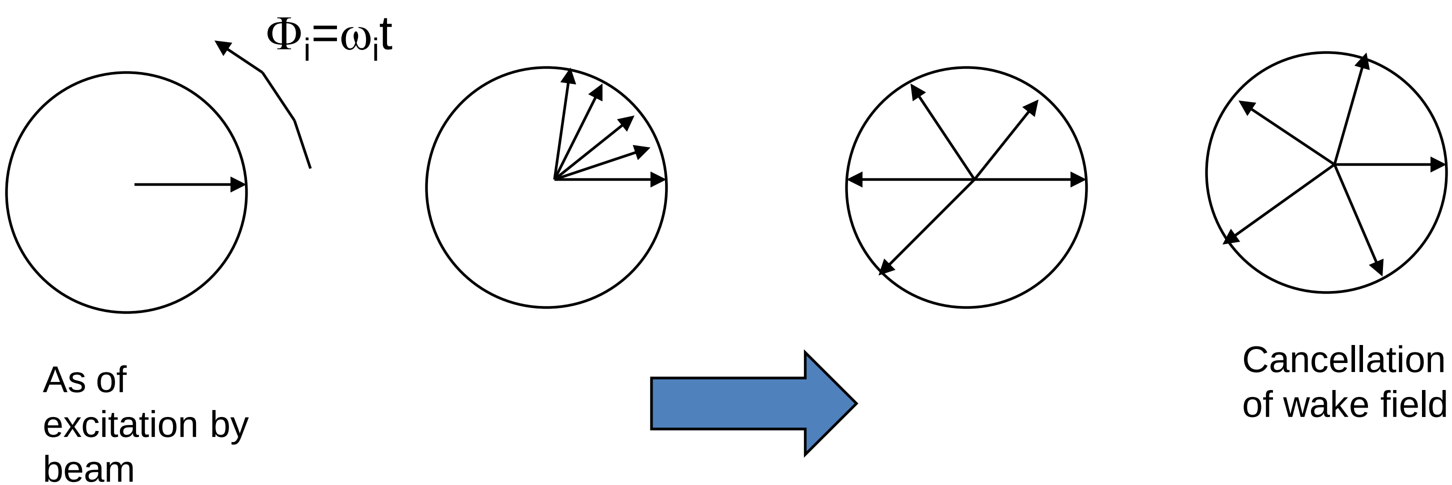
Quadrant-type heavily damped structure

It is not easy to make Q very low by external coupling.
This results in the application of frequency spread for effective damping (cancellation) of wake field in addition to low Q.

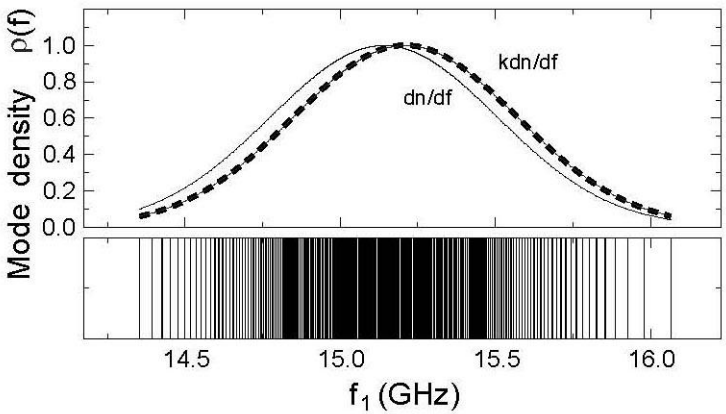
Detuning to make frequency spread

Wake field $\Leftrightarrow$ Fourier transform $\Leftrightarrow$ Frequency distribution

$$\boxed{W(t) = \sum_i K(\omega_i) Sin(\omega_i t)}$$



Calculation with equivalent circuit



Geometry parameters along a structure is distributed to make the coupling to beam (kick factor) as gaussian like.

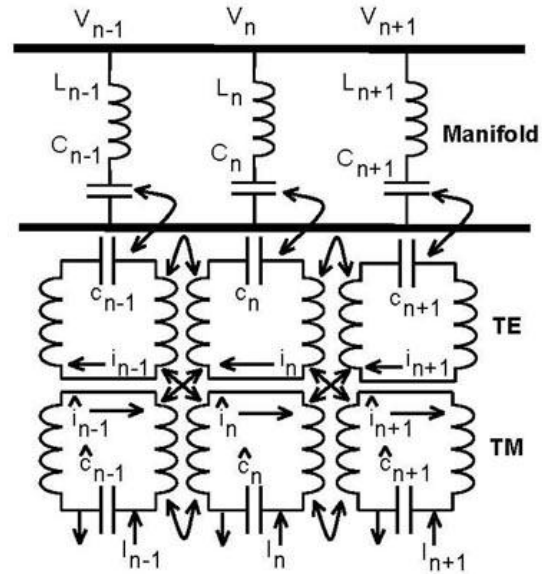
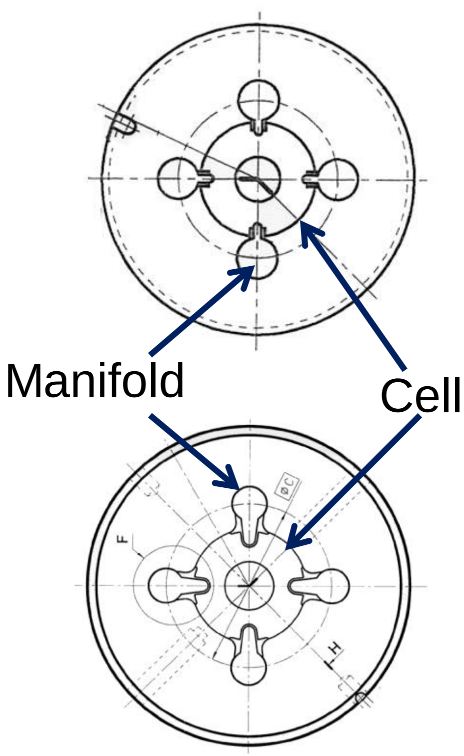


Figure 7.4: Circuit model for DDS structures.

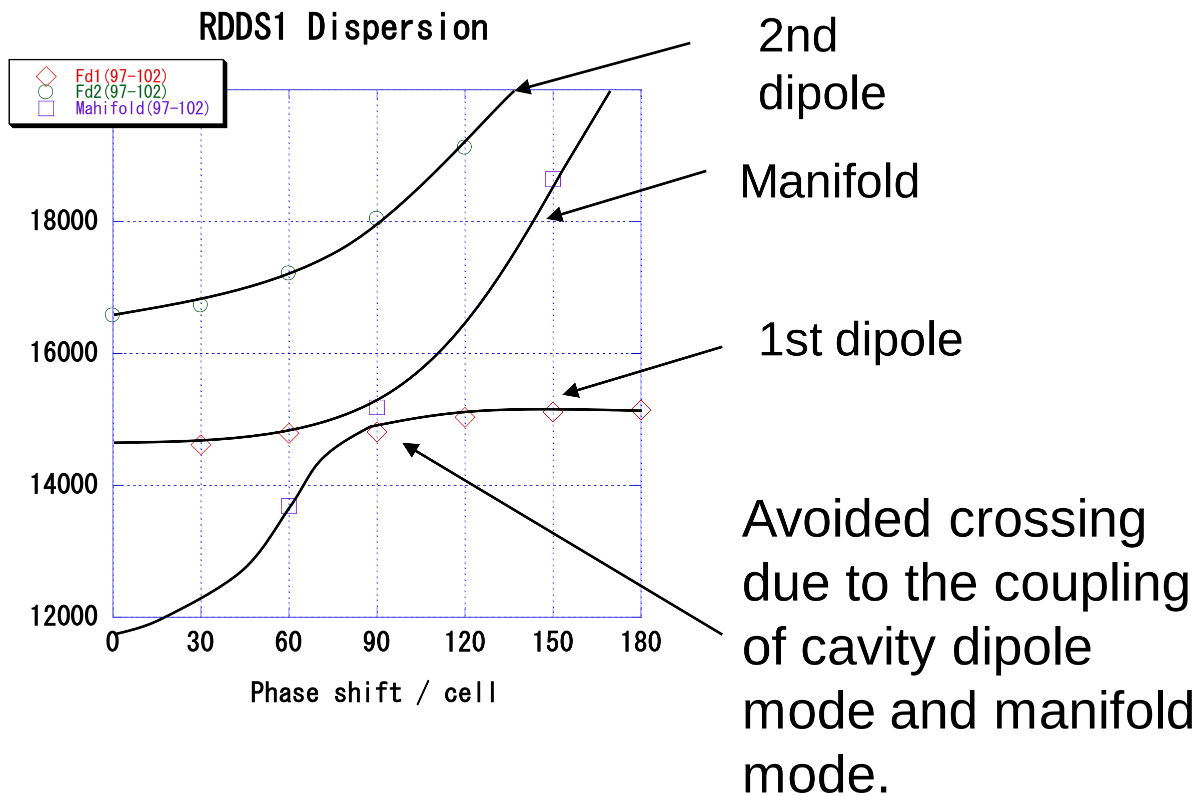
Analyse the whole system with coupled resonator equivalent circuit model. pioneered by K. Band and R. Gluckstern and explored by R. Jones.

Moderate damping by extraction of dipole modes into manifold

Example of middle cells of RDDS1

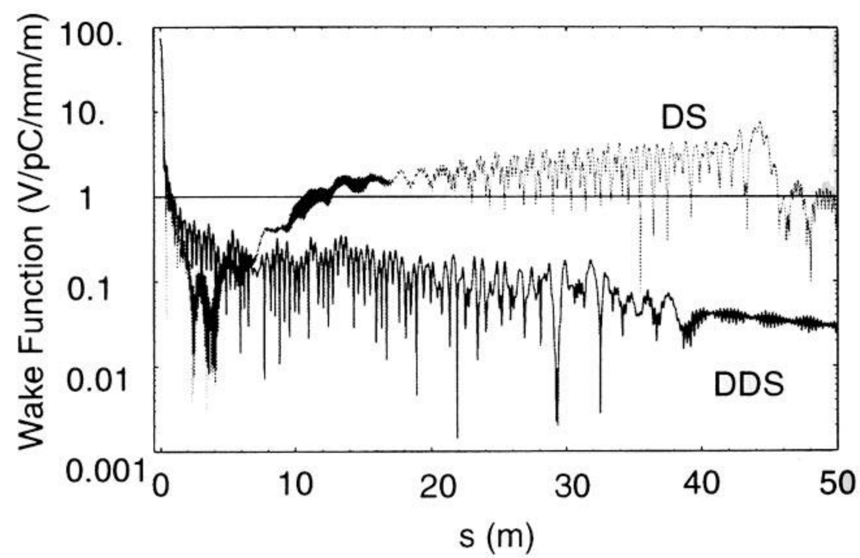


Example cell shape

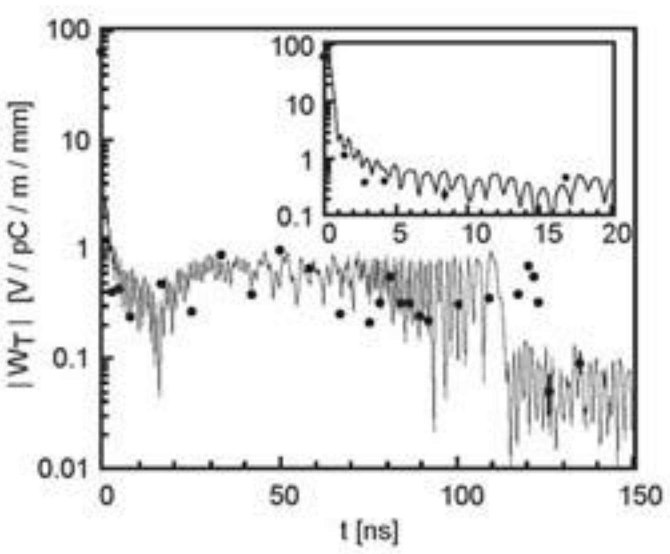


Distribution of (a, t) and introduction of damping

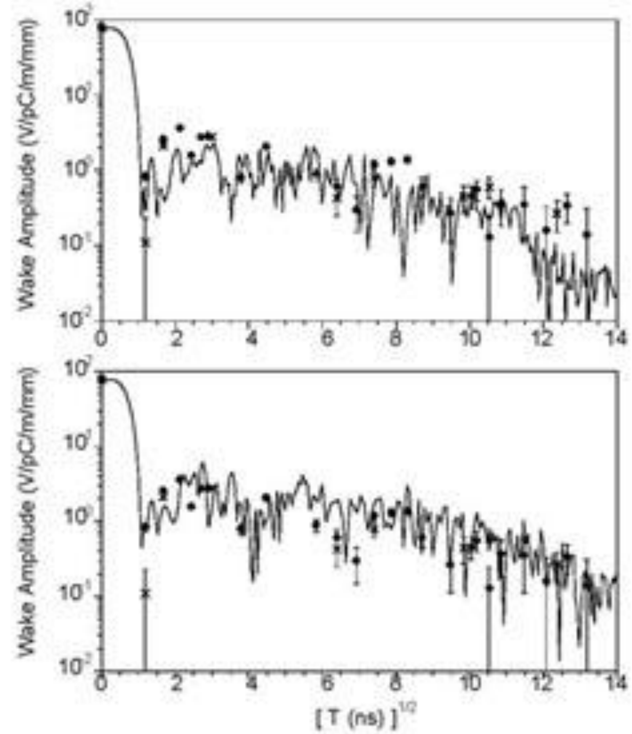
- Faster damping need larger width
- Truncation makes tail up in wake field
- DS detuned only → DDS damped detuned
- Recurrent due to finite number of distribution points
- Interleaving makes longer recurrent



Result of equivalent circuit model calculation and estimate of tolerances

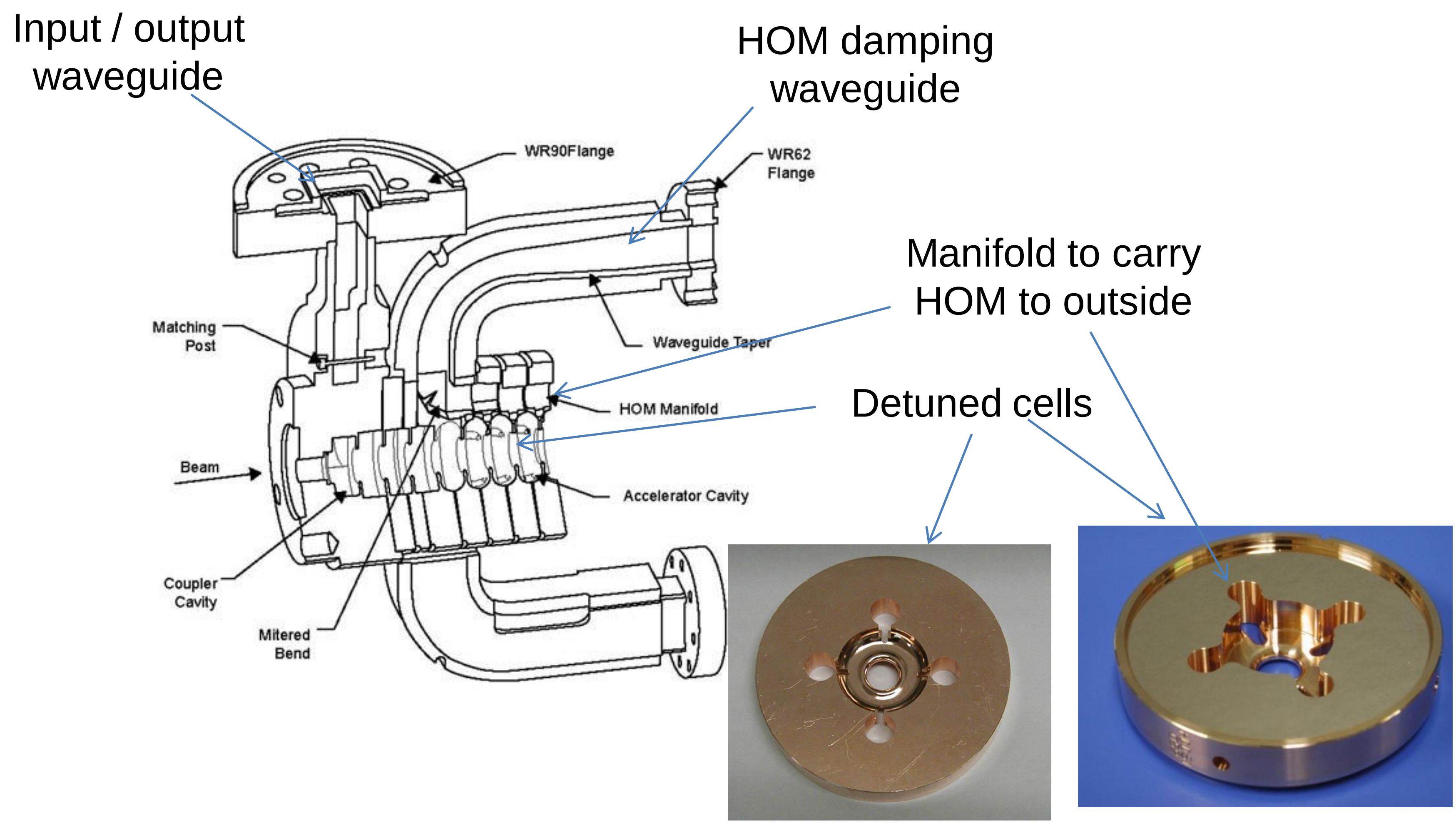


Wake field calculation based on frequency error info from fabrication.

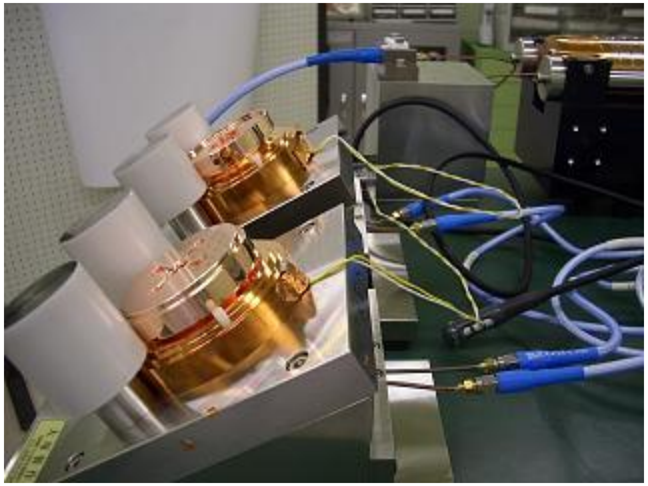


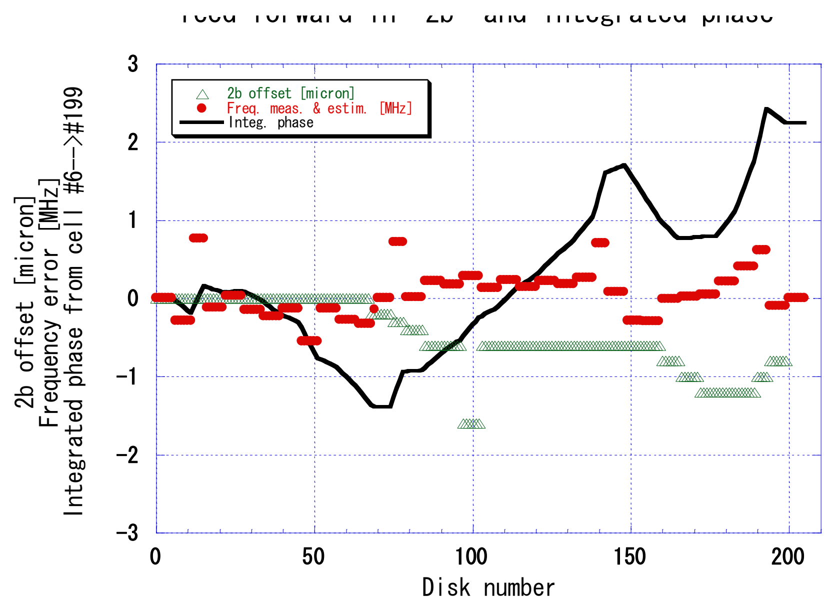
Wake field simulation with more frequency errors to investigate frequency tolerances.

Actual design (RDDS1) and typical cells



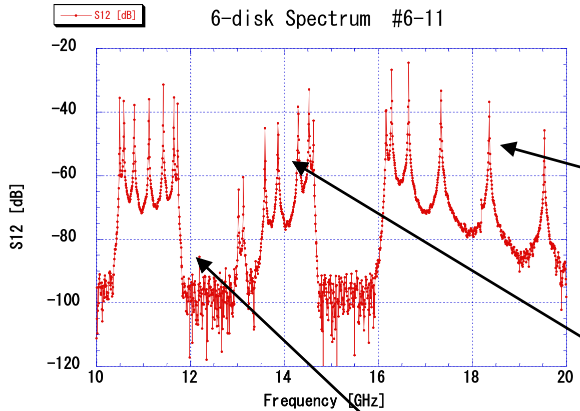
Frequency control of actual structure

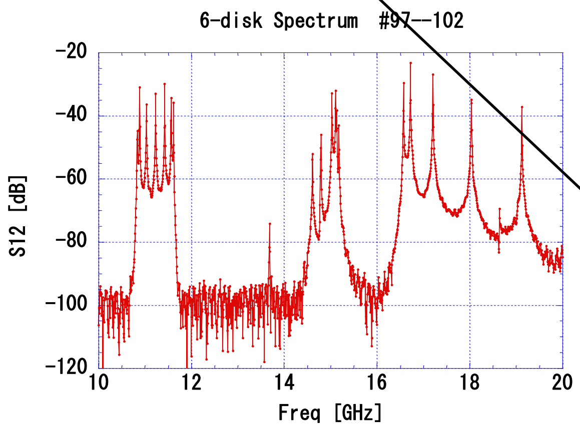


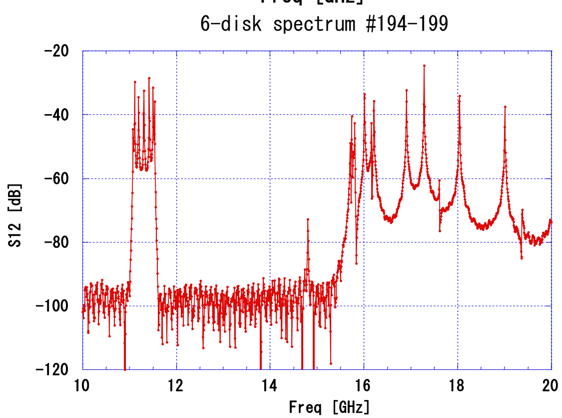


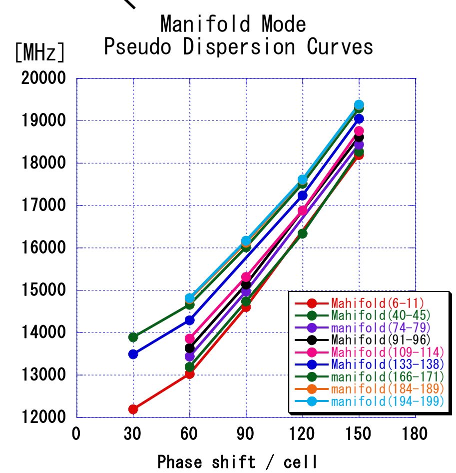
Check each cell frequency and feedforward to the later cell fabrication.

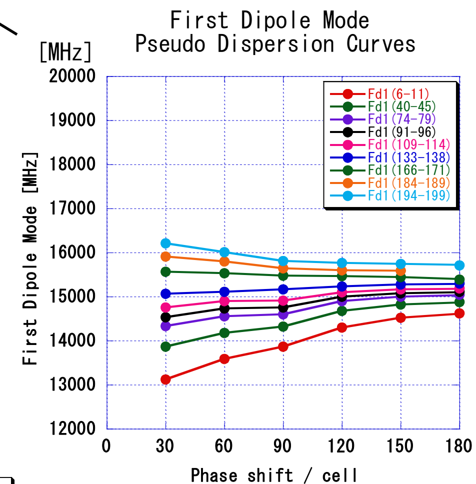
RDDS1 dispersion

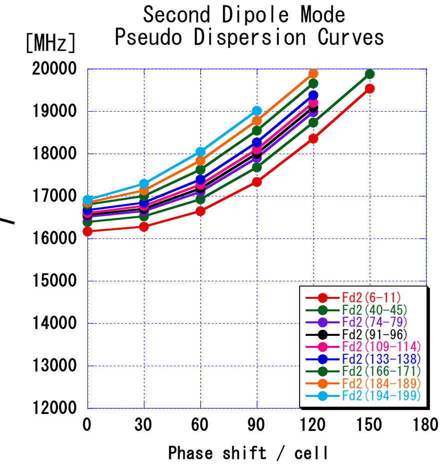




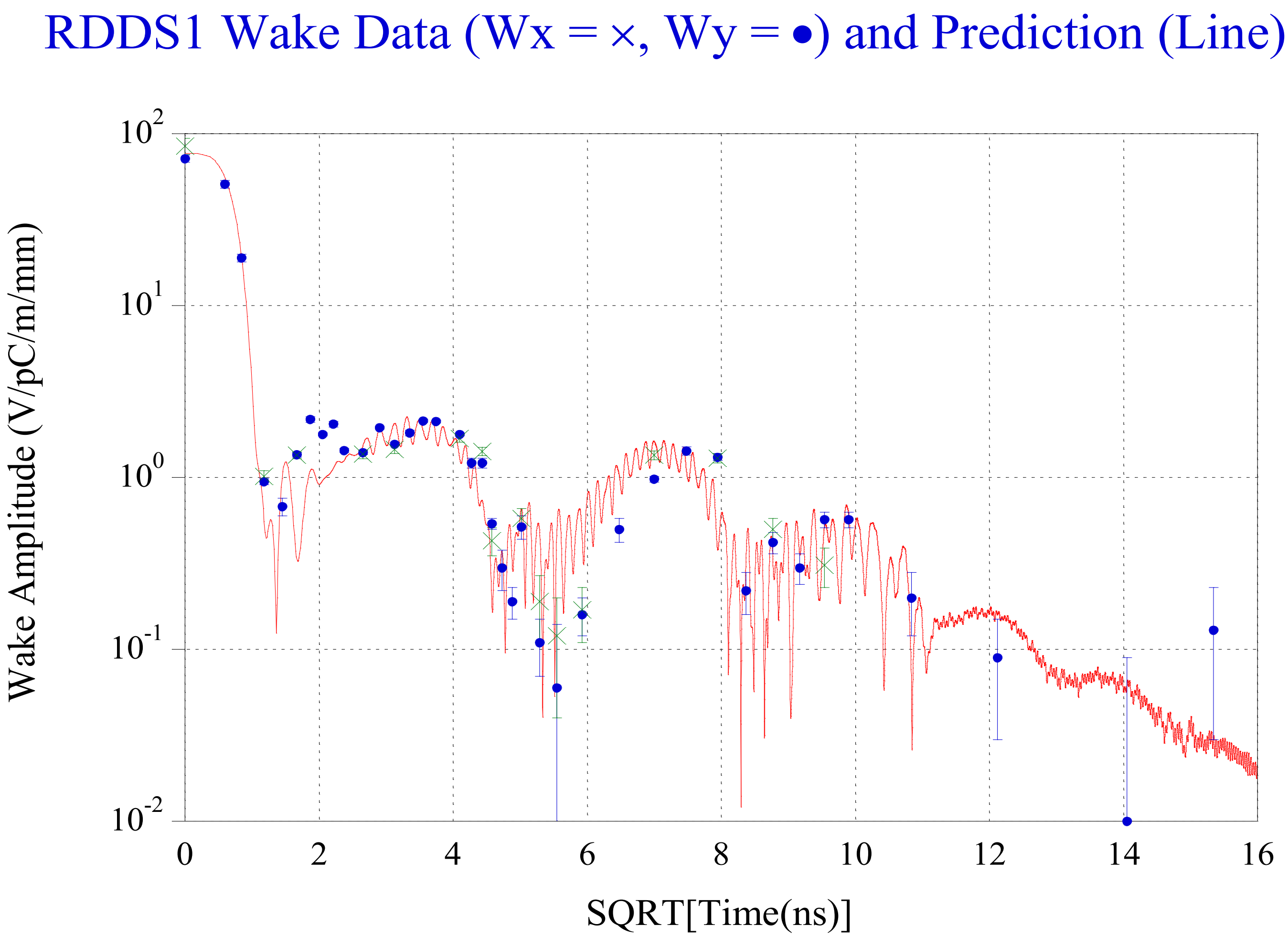








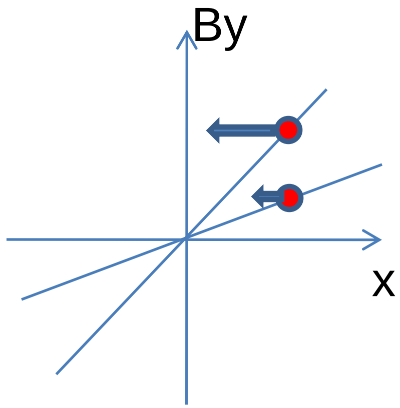
Proof of wake field in RDDS1



Cures by improving alignment

One-to-one steering

If every BPM is aligned perfectly to the magnetic center of each quadrupole magnet, it is easy to adjust the beam with respect to those quad's. Just align the beam to zero the BPM reading.



Changing the Q strength → transverse kick measured at downstream BPM's to know the beam position w.r.t. Q center and BPM calibration.

It is straightforward way but suffers from errors in BPM reading, BPM misalignment w.r.t. Q magnet, etc.

This is the local correction, but there is better way of correcting more globally, DF or WF correction scheme.

DF and WF correction

The equation of motion in transverse plane in high energy linac;

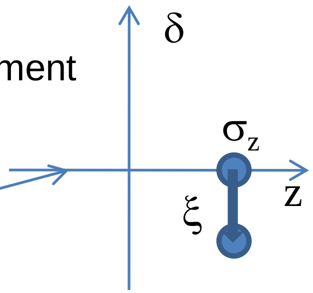
$$\frac{1}{\gamma(s)}\frac{d}{ds}\gamma(s)\frac{d}{ds}x(s;z,\delta)+(1-\delta)K[x(s;z,\delta)-x_q]$$
$$=(1-\delta)G+\frac{1-\delta}{\gamma_0(s)}N\,r_e\int_z^{\infty}dz'\int_{-\infty}^{\infty}d\delta'\,\rho(z',\delta')W_{\perp}(s;z-z')[x(s;z',\delta')-x_a]$$

where
δ = δE/E,
G(s) steering field, K(s) = Q magnetic field,
W_t = transverse wake field, N = number of particles in a bunch,
r_e = classical electron radius,
ρ(z,d) = charge distribution,
x_q = Quad misalignment, x_a structure misaligment

$$K(s)=\frac{e}{p_0\,c}\frac{dB_y}{dx},\quad G(s)=\frac{e}{p_0\,c}B_y$$

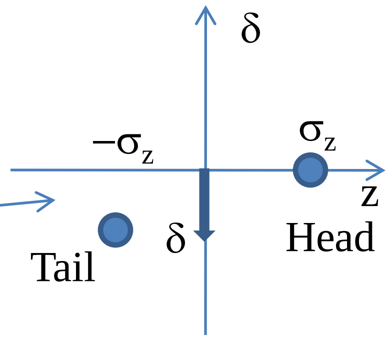
Correlated energy spread δ

$$\Delta x_d=x(\sigma_z,0)-x(\sigma_z,\xi)$$



Uncorrelated energy spread ξ

$$\Delta x_w=x(\sigma_z,0)-x(-\sigma_z,\delta)$$



Equation of motion and force term

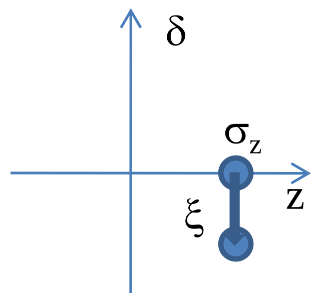
The equation for the difference are in the first order approximation

No wake field

Oscillation

$$\frac{1}{\gamma(s)}\frac{d}{ds}\gamma(s)\frac{d}{ds}\Delta x_d(s;\sigma_z,\xi)+K(s)\Delta x_d(s;\sigma_z,\xi)$$
$$=\xi(G(s)\ +\ K(s)[x_q(s)-x(s;\sigma_z,0)])$$

Steering Q mag



With wake field

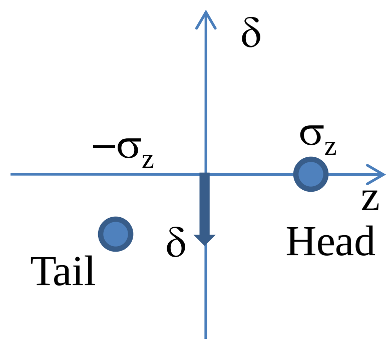
Oscillation

$$\frac{1}{\gamma(s)}\frac{d}{ds}\gamma(s)\frac{d}{ds}\Delta x_w(s;\sigma_z,\delta)+K(s)\Delta x_w(s;\sigma_z,\delta)$$
$$=\ \delta\, G(s)\ +\ \delta\, K(s)\ \{x_q(s)\ -x(s;\sigma_z,0)\}$$
$$-\frac{N\, r_e}{2\gamma_0(s)}W_\perp(s,2\sigma_z)\ \{(x(s;\sigma_z,0)-x_a(s)\}$$

Q mag

Steering

Wake field



Dispersion free (DF) correction

In the first order approximation, the solutions for these are written as the transferred position originated from the kick at s' upstream

No wake field

$$\Delta x_d(s;\sigma_z,\xi) = \int_0^s ds' R_{12}(s,s')\,\xi\{G(s') \; + \; K(s')[x_q(s') - x(s';\sigma_z,0)]\}$$

Minimize Δx_d is equivalent to locally minimize the following value small

$$\begin{aligned} &G(s') \; + \; K(s')[x_q(s') - x(s';\sigma_z,0)] \\ &= \underbrace{[G(s') \; + \; K(s')\, x_q(s')]}_{\text{Q misalignment}} - \underbrace{K(s')\, x(s';\sigma_z,0)]}_{\text{Local correction}} \end{aligned}$$

In reality, from the i'th BPM reading m_i and its difference Δm_i with their predicted values, x_i and $\Delta x_{i,}$ minimization does dispersion free correction;

$$\min\left(\sum_{i=1}^{N_{BPM}} \frac{x_i^{\;2}}{\sigma_{prec}^2 + \sigma_{BPM}^2} + \frac{\Delta x_i^{\;2}}{\sigma_{prec}^2} \right)$$

Wakefield free (WF) correction

In the first order approximation, the solutions for these are written as the transferred position originated from the kick at s' upstream

With wake field

$$\Delta x_w(s;\sigma_z,\delta) = \int_0^s ds' R_{12}(s,s')\,\delta\,[G(s') \;+\; K(s')\{x_q(s') - x(s';\sigma_z,0)\;]$$
$$- \frac{N r_e}{2\gamma_0(s')} W_\perp(s',2\sigma_z)\,\{x(s';\sigma_z,0) - x_a(s')\}$$

Minimize Δx_w is equivalent to locally minimize the following value small

$$\delta[G(s') + K(s')\{x_q(s') - x(s';\sigma_z,0)\;] - \frac{N r_e}{2\gamma_0(s')} W_\perp(s',2\sigma_z)\,\{x(s';\sigma_z,0) - x_a(s')\}$$
$$= \delta[G(s') + K(s')\;x_q(s')] - \{\delta K(s') - \frac{N r_e}{2\gamma_0(s')} W_\perp(s',2\sigma_z)\}\,x(s';\sigma_z,0) + \frac{N r_e}{2\gamma_0(s')} W_\perp(s',2\sigma_z)\,x_a(s')$$

Q misalignment

Wake field correction

Structure misalignment

Wake field term cannot be cancelled out by δ term because of constant W_t while alternating in K(s'). Taking QF only or QD only makes the correction of W_t.

Wakefield free (WF) correction

In reality, from the i'th BPM reading m_i and its difference Δm_i with their predicted values, x_i and Δx_i, minimization does wake field free correction;

$$\min\left(\sum_{i=1}^{N_{BPM}} \frac{x_i^{\,2}}{\sigma_{prec}^2+\sigma_{BPM}^2}+\frac{\Delta x_i^{QF\,2}}{2\sigma_{prec}^2}+\frac{\Delta x_i^{QD\,2}}{2\sigma_{prec}^2}\right)$$

Where the Δx_i^{QF} and Δx_i^{QD} are those difference orbit due to the variation of only QF and QD.

Correction example; NLC case from T. Raubenheimer, NIM A306, p63,1991.

Method	$\boldsymbol{\varepsilon_y}$	**Trajectory rms**
1-to-1	23 ε_{y0}	72 μm
DF	9 ε_{y0}	55 μm
WF	1 ε_{y0}	44 μm

Alignment with using excited field in the actual structure

If we measure dipole mode in a structure, we can estimate the position of beam which excites the mode. The coupling is linear as offset.

If the modal frequency depends on the position of the mode, it can distinguish the position there by frequency filtering.

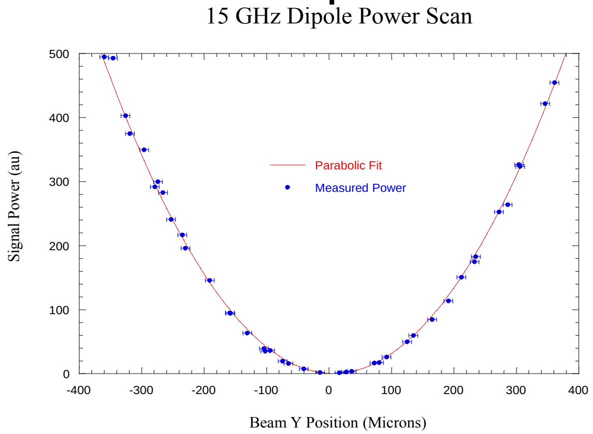
In such cavity as ILC, it can be done with using power from HOM couplers.

In such cavity CLIC, it can be done with extracted power from manifold or damping waveguide.

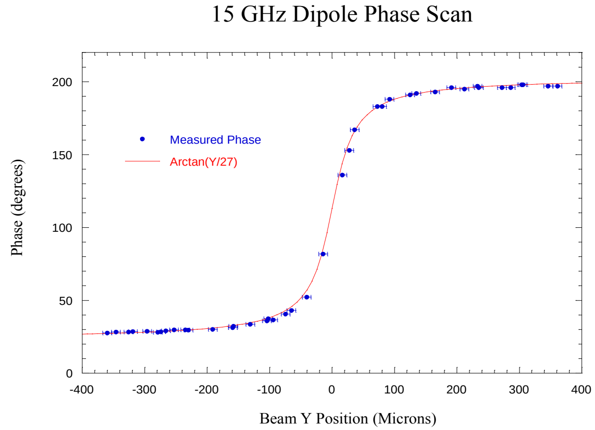
Both directions x and y are measured with distinguishing two modes in different polarizations, almost degenerate but with some frequency difference.

Alignment measurement in situ as SBPM

Excited power



Phase of excited field



Power from manifold

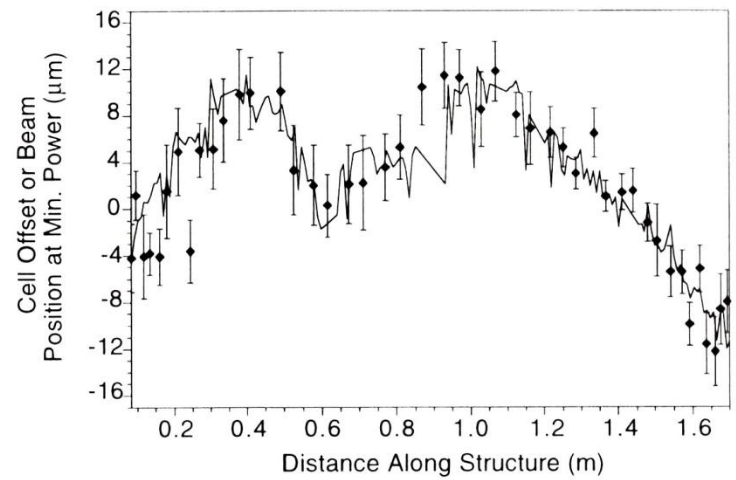
Frequency filtered

Phase and amplitude

Frequency-to-position

Straightness measure

Position measured from frequency filtered signal compared to mechanical measurement



Dark current issue

Dark current

What is dark current?

Dark current is a stable emission of electrons under high field.

DC field emission is the tunneling feature of electron migration near surface.

It is studied by Fowler-Northeim.

Here work function and field enhancement factor play important roles.

Actually the field E_0 is replaced by the local field βE_0.

$$\boxed{i_{FE,\,DC} = F\, E_0^2\, Exp(-\frac{G}{E_0}) \quad [Am^{-2}], \quad F = 1.54\times10^{-6}\times10^{4.52\phi^{-0.5}}\,\phi^{-1},\; G = 6.53\times10^{9}\,\phi^{1.5}}$$

RF field emission is estimated to be the superposition of DC field emission.

It is calculated by J. Wang and G. Loew.

It should exist in ant RF field, whether or not in SW and TW or in NCC and SCC .

$$\boxed{i_{FE,\,RF} = F\, E_0^{2.5}\, Exp(-\frac{G}{E_0}) \quad [Am^{-2}], \quad F = 5.7\times10^{-12}\times10^{4.52\phi^{-0.5}}\,\phi^{-1.75},\; G = 6.53\times10^{9}\,\phi^{1.5}}$$

Tracking of FE electrons are studied by various authors.

Nowadays, numerical tracking is usual one.

Simple analytical estimate gives minimum threshold field for capture.

Acceleration in linear accelerator

Acceleration field is expressed as the summation of all space harmonics in the periodic structure.

$$E_z = \exp\, j(\omega t - k_z z) \sum_{n=-\infty}^{n=\infty} - j\, E_n\, J_0(k_{rn} r) \exp^{-2 j \pi n z / d}$$

$$k_{rn} = k^2 - (k_z + 2\pi n / d)^2$$

$$k = \omega / c = 2\pi / \lambda$$

$$\frac{d\gamma}{dz} = - \sum_{n=-\infty}^{n=\infty} \varepsilon_n\, J_0(k_{rn})\, Sin(\theta + \frac{2\pi n z}{d})$$

$$\varepsilon_n = e\, E_n \,/\, m_0 c^2$$

$$\theta = \int (\frac{1}{\beta_p} - \frac{1}{\beta})\, dz$$

If, β_p~β, then θ is slowly varying function. Therefore, only n=0 is dominant.

$$\frac{d\gamma}{dz} = -\varepsilon_0\, Sin(\theta) \quad and \quad \frac{d\theta}{dz} = \frac{k}{\varepsilon_0}(\frac{\sqrt{p^2+1}}{\beta_p} - p) + A$$

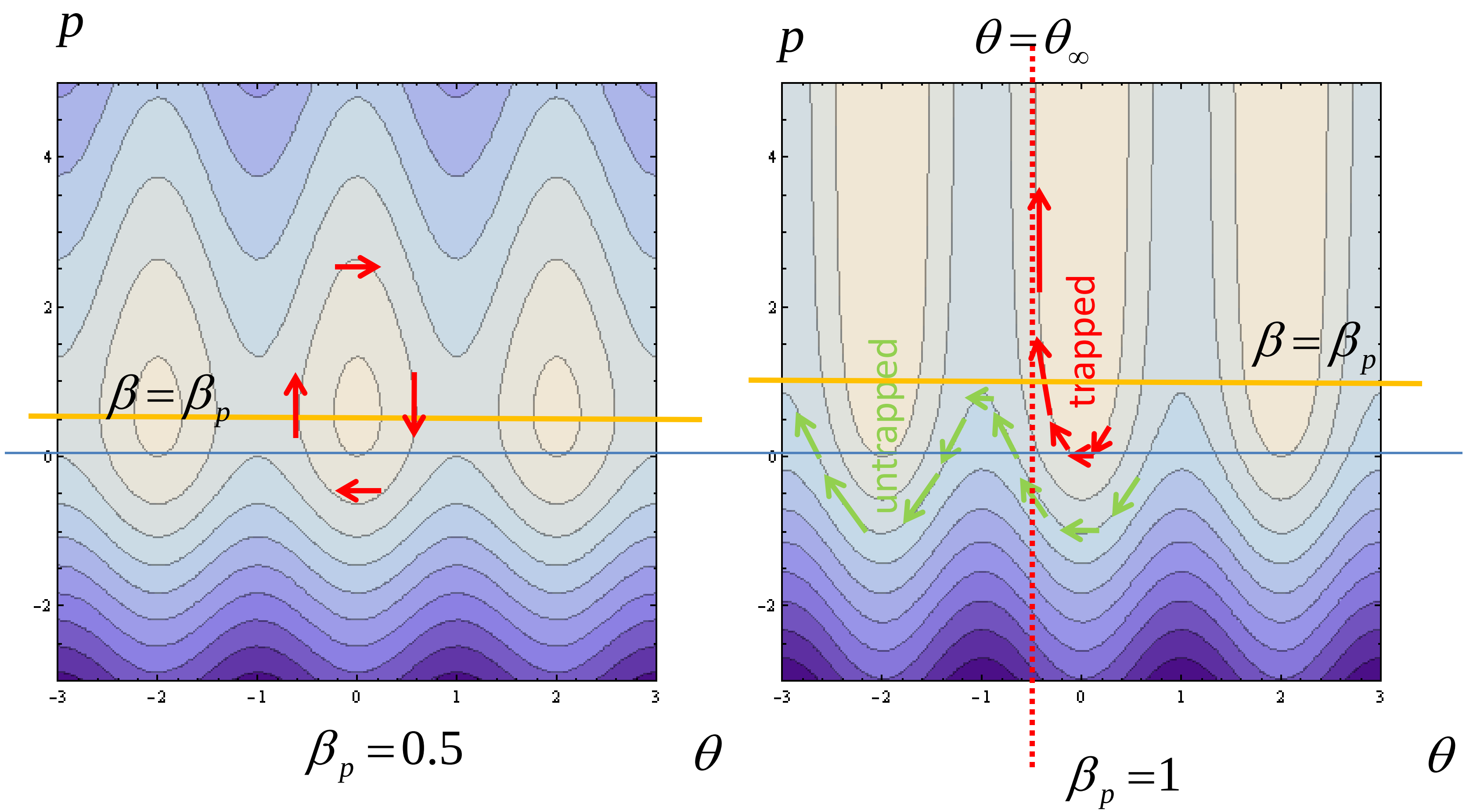
Finally, by combining these equations,

$$Cos(\theta) = \frac{k}{\varepsilon_0}(\frac{\sqrt{p^2+1}}{\beta_p} - p) + A$$

Plot the contour of this equation with A in the next page.

R. Helm and R. Miller, in Linear Accelerators, ed. by P. M. Lapostolle and A. L. Septier, North-Holland Publishing Co., 1970

Separatrics describing longitudinal motion



$\frac{p}{m_0 c} \equiv \sqrt{\gamma^2 - 1}$ *Normalized momentum*

θ RF phase seen by particle

Capture threshold

$$Cos(\theta) = \frac{k}{\varepsilon_0}\left(\frac{\sqrt{p^2+1}}{\beta_p} - p\right) + A$$

For β_p=1, A=Cos(θ_m) when approaching at p→infinity

$$Cos(\theta) - Cos(\theta_m) = \frac{k}{\varepsilon_0}\left(\sqrt{p^2+1} - p\right)$$

Minimum field ε_0 for being tramped, eq. to being max in left-hand side,

$$Cos(\theta) - Cos(\theta_m) = 2$$

For zero-energy electron, p=0 to be captured,
the minimum field becomes

λ = 26.242mm at 11.424GHz
λ = 230.6 mm at 1.3GHz

$$\boxed{E_{acc}^{threshold} = \frac{\pi m_0 c^2}{e\lambda}} \Rightarrow$$

61 MV/m at 11.424GHz
7 MV/m at 1.3GHz

Tolerable breakdowns or quenches

To keep the beam energy under RF failure

It is important to make the integrated luminosity high by keeping the instantaneous luminosity high.

Once some failure happens in some cavity, the power feeding the cavity or the bunch of cavities is shut off.

Then, the power or pulse width will be recovered taking some pulses starting from a little lower power level. During this period, the cavities in recovering mode are off in timing from acceleration for the linac if powered by independent power supply.

In such a system as CLIC two beam scheme, the off-timing operation cannot be applied and gradual power recovery is needed.

During this recovery period, other cavities than nominal should be used to keep the beam energy. Therefore, linac needs extra acceleration capability than the nominal one.

Extra spare cavities or extra power/gradient capability is required.

Simple estimation of tolerable failure rate

Compensation with spare cavitires:

$N_{unit}\ N_{str}$ $\alpha\ N_{unit}\ N_{str}$

Nominal Spare

R_{fail} failure rate for a cavity (1 trip / N pulses)
T_{rec} recovery time in the unit of pulses

Number of failures during the recovery time for a cavity

$$N_{unit}\, N_{str}\, R_{fail}\, T_{rec}$$

It should be less than the number of spare units

$$N_{unit}\, N_{str}\, R_{fail}\, T_{rec} < \alpha\, N_{unit}$$

Then the tolerable failure rate is

$$R_{fail} < \frac{\alpha}{T_{recov}\, N_{str}}$$

An example;
Order estimation

$$N_{unit} = 3000$$

$$N_{str} = 8$$

$$T_{rec} = 60\, s \times 50\, Hz$$

$$\alpha = 0.01$$

In this example,

$$R_{fail} < 4 \times 10^{-7}$$

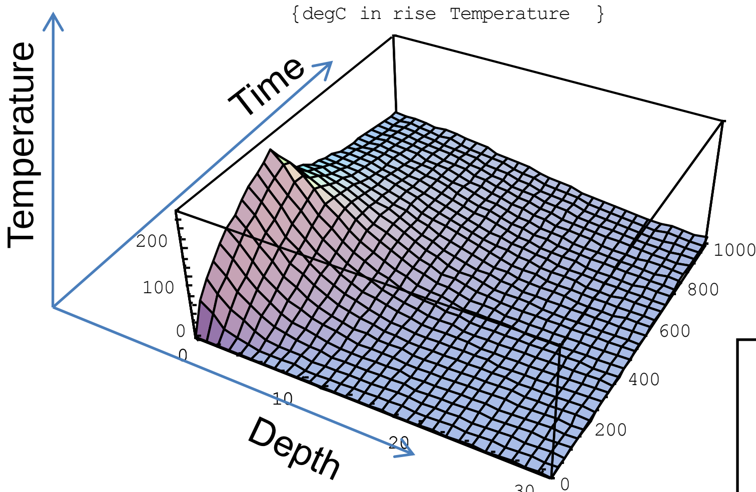
Reduce failure rate or more margin in accelerator gradient

- Cares on SCC system
  - Margin of accelerator gradient
    - Increase quench field due to H_s
    - Suppress FE
    - Increase Q_0
    - Variable feeding system
  - Mechanical long life for tuner
- Reduce breakdown rate in NCC
  - Possible trigger source of breakdown
    - Surface quality chemically and physically
    - Reduce micro protrusions
    - Reduce pulse temperature rise?

Pulse temperature rise

Surface heating and heat diffusion into body.

$$\frac{\partial}{\partial t}u(x,t)-\kappa^2 u(x,t)=\frac{P_w(x,t)}{c_p\,\rho}$$



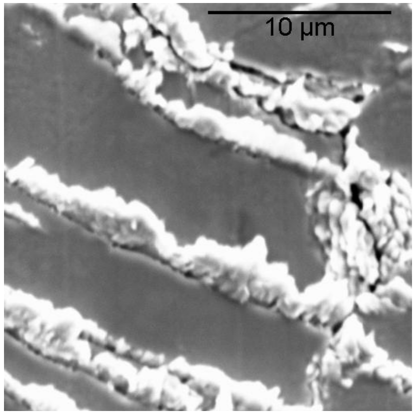
$$\Delta T=\frac{1}{2}R_s\left|H_s\right|^2\frac{2\sqrt{T_p}}{\rho\,c_\varepsilon\sqrt{\pi\,\alpha_d}}$$

Pritzkaw Thesis, p99, SLAC-Report 577.

Cu
Rs=27.85mΩ
Hs=1MA/m
Tp=400ns=4*10^-7
ρ=8.93 10^3 kg/m^3
Cε=380 J/kg/K
αd=k/ρcε= m^2/s
k=401 W/m/K

ΔT=270degC

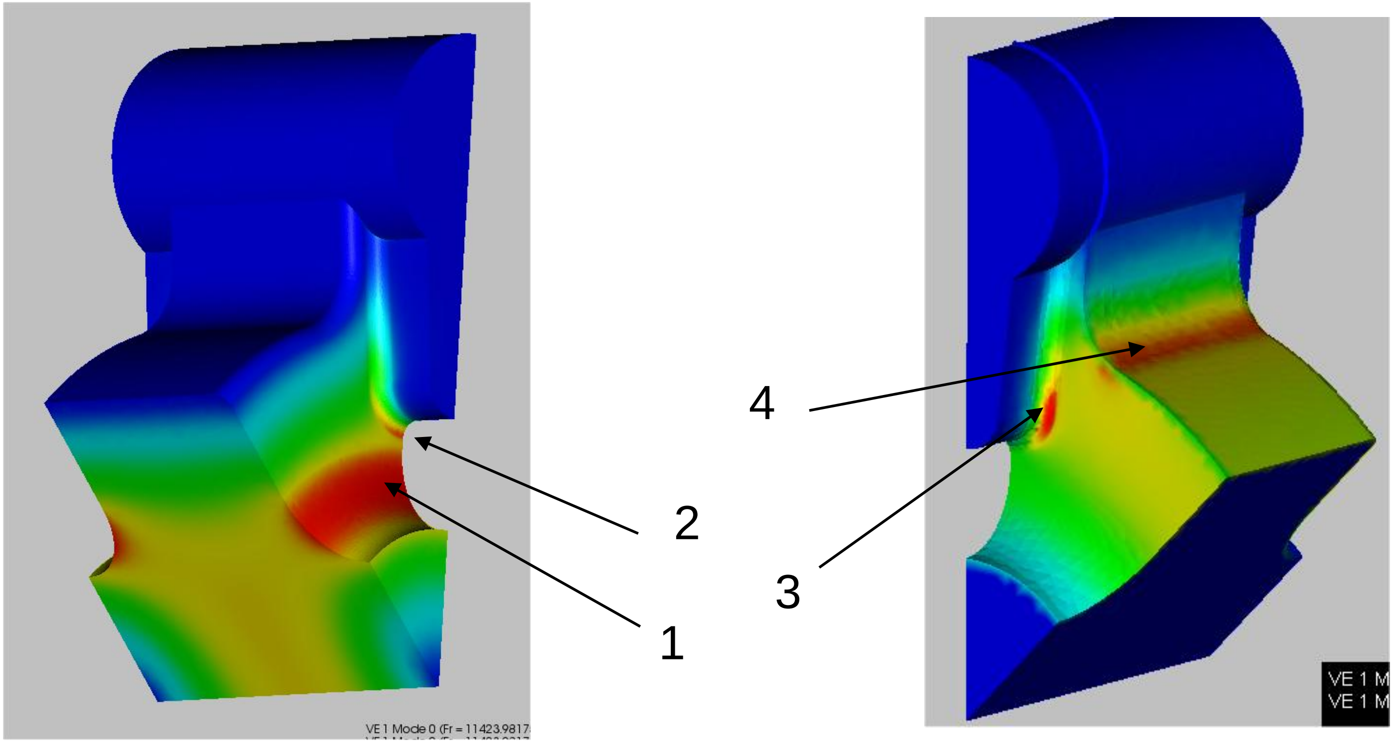
Pulse heated surface



V. Dolgashev and L. Laurent, AAS08

There may trigger breakdowns.

Care in complicated shape formation



$E_{surface}$ $H_{surface} \rightarrow \Delta T_{pulse}$

We need to avoid additional local field enhancement
due to non-smoothness especially at red areas.
Special care is taken at points (2,3,4) where smooth junction is difficult
due to the junction between milling and turning

Summary of LINAC-II

- By keeping the emittance growth within a tolerable level, the luminosity will be kept.

- Various sources, especially wake-field origin, were discussed.
- Various cures on HOM origin are discussed.
- Cares on alignment to suppress single-bunch wake field was discussed, using structure BPM and BBA.

- These perturbations and cures are almost similar to both warm and cold linac.