Phase transitions in 2HDM during Universe expansion and modern values of parameters. Problems for ILC & LHC

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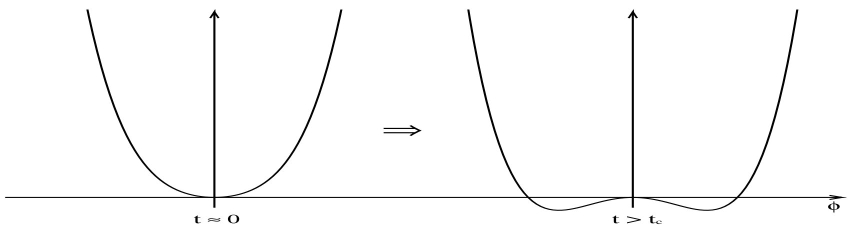
Standard rough picture: After Big Bang the temperature of the Universe T was high, in this stage vacuum expectation values of Higgs fields are given by minimum of the Gibbs potential $\Phi = V(\phi) + aT^2\phi^2$, where $V(\phi)$ is the Higgs potential, — Higgs model with mass parameters varying in time. At large T potential has EW symmetric minimum at $\langle\phi\rangle = 0$. This stage describes the phenomenon of inflation.
During the inflatory expansion, the Universe refrigerates, at some temperature the Gibbs potential transforms effectively into the well known form of the Higgs model with $\langle\phi\rangle \neq 0$ – we obtain our world with massive particles, etc.



This EWSB phase transition determines the fate of the Universe after inflation.

The potential of 2HDM and evolution of its form

The Two Higgs Doublet Model (2HDM) – the simplest extension of the minimal SM for description of EWSB – contains two scalar weak isodoublets ϕ_1 and ϕ_2 with identical hypercharge. Isoscalar combinations of the field operators

$$x_1=\phi_1^\dagger\phi_1,\quad x_2=\phi_2^\dagger\phi_2,\quad x_3=\phi_1^\dagger\phi_2\,,\quad x_{3^*}\equiv x_3^\dagger=\phi_2^\dagger\phi_1\,.$$

The most general renormalizable Higgs potential is

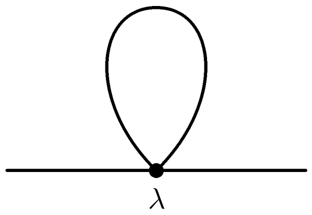
$$V=-\frac{1}{2}\left[m_{11}^2x_1+m_{22}^2x_2+\left(m_{12}^2x_3+h.c.\right)\right]+$$
$$+\frac{\lambda_1x_1^2+\lambda_2x_2^2}{2}+\lambda_3x_1x_2+\lambda_4x_3x_3^\dagger+\left[\frac{\lambda_5x_3^2}{2}+\lambda_6x_1x_3+\lambda_7x_2x_3+h.c.\right].$$

Temperature dependence

At high temperature we define instead of potential V

the Gibbs potential $V_G = Tr\left(Ve^{-\hat{H}/T}\right)/Tr\left(e^{-\hat{H}/T}\right) \equiv V + \Delta V$. .

The first correction to potential is given by tadpole diagram.



It is calculated with Matsubara diagram technic. At $T \gg m_i^2$ each loop contribute as gT^2 where g is some coefficient. In our case we have

$$\Delta m_{11}^2 = (3\lambda_1 + 2\lambda_3 + \lambda_4)gT^2\,, \quad \Delta m_{22}^2 = (3\lambda_2 + 2\lambda_3 + \lambda_4)gT^2\,,$$

$$\Delta m_{12}^2 = 2(\lambda_6 + \lambda_7)gT^2\,.$$

This variation allow to see evolution of vacuum during cooling of Universe. This evolution can influence for current state of Universe .

First goal: To see possible scenarios of evolution of Universe

For representative discussion –

explicitly CP-conserving potential with softly broken Z_2 symmetry,

Useful notations (for definiteness we take $k > 1$)

$$\lambda_1 = \lambda\,,\;\; \lambda_2 = k^4\lambda\,,\;\; \lambda_3 = \lambda\rho_3\,,\;\; \lambda_4 = \rho_4\lambda\,,\;\; \lambda_5 = \rho_5\lambda\,,$$
$$m_{11}^2 = m^2(1+\delta)\,,\quad m_{22}^2 = k^2 m^2(1-\delta)\,,\quad m_{12}^2 = \mu m^2,\quad \lambda_{6,7} = 0.$$

At $\delta = 0$ our potential has an extra symmetry which we denote here as $\mathcal{ZK}$ symmetry

$$\phi_1 \leftrightarrow k\,\phi_2\,.$$

During evolution, system can pass through this symmetry point possibly providing new types of phase transitions

We present many equations for the case of weak violation of $\mathcal{ZK}$ symmetry – $\boxed{\delta \ll 1}$.

In the forthcoming discussion we assume that λ_i are not large, so that perturbative approach can be used.

Useful quantities

The scales of field and energy values at the extremum points, similar to SM

$$Y = m^2/(2\lambda)\,, \qquad \varepsilon = m^4/(8\lambda)\,.$$

Y_0, ε_0 – modern values of the same quantities (at $T = 0$)

We use

$$\rho_{345} = \rho_3 + \rho_4 + \rho_5, \quad \tilde{\rho}_{345} = \rho_3 + \rho_4 - \rho_5.$$

Types of possible extremes of potential

The extrema of the potential define the values $\langle\phi_{1,2}\rangle$ of the fields $\phi_{1,2}$ via equations:

$$\partial V/\partial\phi_i|_{\phi_i=\langle\phi_i\rangle} = 0\,, \qquad \partial V/\partial\phi_i^\dagger|_{\phi_i=\langle\phi_i\rangle} = 0\,.$$

These equations have the electroweak symmetry conserving (EWc) solution $\langle\phi_i\rangle = 0$ and the electroweak symmetry breaking (EWSB) solutions.

The extremum energy is

$$\mathcal{E}_N^{ext} = V(\langle\phi_i\rangle_N)\,.$$

For each EWSB extremum one can choose the z axis in the weak isospin space so that $\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}$ with real $v_1 > 0$ ("neutral direction"). The residuary $\langle\phi_2\rangle$ has generally an arbitrary form $\Rightarrow$ After this choice the most general electroweak symmetry violating solution of extremum condition can be written in a form with real v_1 and complex v_2

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 \end{pmatrix} \;\; with \;\; v_1 = |v_1|,\, v_2 = |v_2|e^{i\xi}\,.$$

It is natural to distinguish two types of extrema, with $Z \neq 0$ (charged extrema, $u \neq 0$) and with $Z = 0$ (neutral extrema, $u = 0$).

I. Charged extremum, $u \neq 0$

If this extremum realizes the vacuum, it is not possible to split the gauge boson mass matrix into the neutral and charged sectors, the interaction of gauge bosons with fermions will not preserve electric charge, photon becomes massive, etc. Certainly, this case is not realized in our World. But in the past?

This extremum is defined by parameters of potential unambiguously only in some limited region of parameters of potential, for our potential

$$\mathcal{E}_{ch}^{ext} = -2\varepsilon \left(\frac{k^2}{k^2 + \rho_3} + \frac{\mu^2}{\rho_4 + \rho_5} + \delta^2 \frac{k^2}{k^2 - \rho_3} \right) .$$

If charged extremum is minimum, it is global one – vacuum

Neutral extrema, $u = 0$. General

Other extrema obey a condition for $U(1)$ symmetry of electromagnetism:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \; \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 = |v_2| e^{i\xi} \end{pmatrix},$$

another parameterization: $\quad v_1 = v\, cos\beta\,, \quad v_2 = v\, sin\beta\,.$

The physical Higgs bosons in one of extrema can have definite CP parity if only potential can be written in explicitly CP conserving form (with all real λ_i, m_{ij}^2) (Haber, Gunion; IFG, Krawczyk). We consider here this very case.

Model contains two fields with identical quantum numbers
$\Rightarrow$ pure Higgs sector can be described both in terms of fields ϕ_k, and in terms of fields ϕ'_k obtained from ϕ_k by a generalized rotation
The correspondent transformations form $SO(3,1)$ group, the same as rotation group of Minkowski space. It allows to use in general analysis geometrical approach. (I.P. Ivanov. *Phys. Lett* **B632** (2006) 360; *Phys. Rev.* **D75** (2007) 035001).

The geometrical description of the most general case shows that the results obtained for presented representative model are practically exhaustive.

II. Spontaneously CP violating extremum, $\xi \neq 0$.

In this case v_1, v_2 and $\cos\xi$ are described by parameters of the potential unambiguously.

The physical neutral Higgs states have no definite CP parity. This extremum is doubly degenerated in the "direction" of CP violation.

At small δ

$$v_1^2 = \frac{2Y\,k^2}{k^2+\tilde{\rho}_{345}}, \quad v_2^2 = \frac{2Y}{k^2+\tilde{\rho}_{345}}, \quad \tan\beta = \frac{1}{k}\left(1-\delta\frac{k^2+\tilde{\rho}_{345}}{k^2-\tilde{\rho}_{345}}\right),$$

$$\cos\xi = \frac{\mu\left(k^2+\tilde{\rho}_{345}\right)}{2k\rho_5};$$

$$\mathcal{E}^{ext}_{sCPv} = -\varepsilon\left(2\frac{k^2}{k^2+\tilde{\rho}_{345}}+\frac{\mu^2}{\rho_5}+2\delta^2\frac{k^2}{k^2-\tilde{\rho}_{345}}\right).$$

This extremum can be minimum of potential if only $\rho_5 > 0$.

With radiative (loop) corrections (RC) main qualitative features of obtained picture are changed weakly. These corrections are essential if they violate some artificial symmetry of the potential. In our case that is its explicitly CP conserving form. Radiative corrections contain contributions e.g. of light quarks, having imaginary parts for the considered mass interval.
The simplest example — correction to λ_5, obliged by b-quark. Rough estimate gives additional $Im\lambda_5 \lesssim (m_b/v)^4(m_b/M_h)^2 \sim 10^{-10}$, where factors m_b/v are from Yukawa coupling and factor $(m_b/M_h)^2$ – from loop integral itself.
These imaginary parts eliminate degeneracy of the sCPv extrema in accordance with the arrow of time, and it is natural to expect that the energy difference between these two states is small – we deal with *almost* degenerate states. In simple words, one can write that the phase with left violation of CP is real vacuum.
The corresponding corrections to other extrema are negligible.

III. CP conserving (CPc) extrema

In this case equation for extremum written for $t = \tan\beta$ and v_i^2 have form

$$\mu(k^4t^4 - 1) + t\left[(k^2t^2 - 1)(k^2 - \rho_{345}) + \delta\,(k^2t^2 + 1)(k^2 + \rho_{345})\right] = 0\,,$$

$$v_1^2 = 2Y\,\frac{1+\delta+t\mu}{1+\rho_{345}t^2}\,, \qquad v_2^2 = t^2v_1^2\,.$$

Generally this equation has 4 solutions. We classify them by the case of precise $\mathcal{ZK}$ symmetry ($\delta = 0$).

Solutions $A\pm$

$$At \;\; \delta=0: \quad t=t_{A0\pm}=\pm\frac{1}{k}, \qquad v_1^2=2Y\frac{k(k\pm\mu)}{k^2+\rho_{345}}, \quad v_2^2=\frac{v_1^2}{k^2};$$

$$at \;\; \delta\sim 0: \quad t=t_{A\pm}=\pm\frac{1}{k}\left[1-\delta\,\frac{k^2+\rho_{345}}{k^2\pm 2k\mu-\rho_{345}}\right].$$

Necessary condition for realization of extremum $A\pm$ is $k\pm\mu+\delta>0$.

One can see that at $\mu>0$ the extremum $A+$ is more deep than $A-$ and at $\mu<0$ the extremum $A-$ is more deep than $A+$:

$$\mathcal{E}_{CPcA\pm}=-2\varepsilon\,\frac{(k\pm\mu)^2+k\delta\cdot(k\pm\mu)}{k^2+\rho_{345}}.$$

Solutions $B\pm$

At $\delta = 0$ we have

$$t = t_{B0\pm} = \frac{\rho_{345} - k^2 \pm \sqrt{(\rho_{345} - k^2)^2 - 4\mu^2 k^2}}{2\mu k^2}\,;$$

$$v_{1\pm}^2 = \frac{Y}{2}\left(1 \mp \sqrt{1 - \frac{4\mu^2 k^2}{(\rho_{345} - k^2)^2}}\right), \quad v_{2\pm}^2 = \frac{v_{1\mp}^2}{k^2}\,.$$

The states B_+ and B_- are degenerated in energy

$$\mathcal{E}_{CPcB\pm} = -\varepsilon\left[1 + \frac{2\mu^2}{\rho_{345} - k^2}\right].$$

This degeneracy is broken at $\delta \neq 0$.

At $k \neq 1$ solutions B_+ and B_- describe quite different physics.

- $v^2 = v_1^2 + v_2^2$ are different $\Rightarrow$ M_W and M_Z are different.
- Yukawa couplings $\Rightarrow$ fermion masses are different.

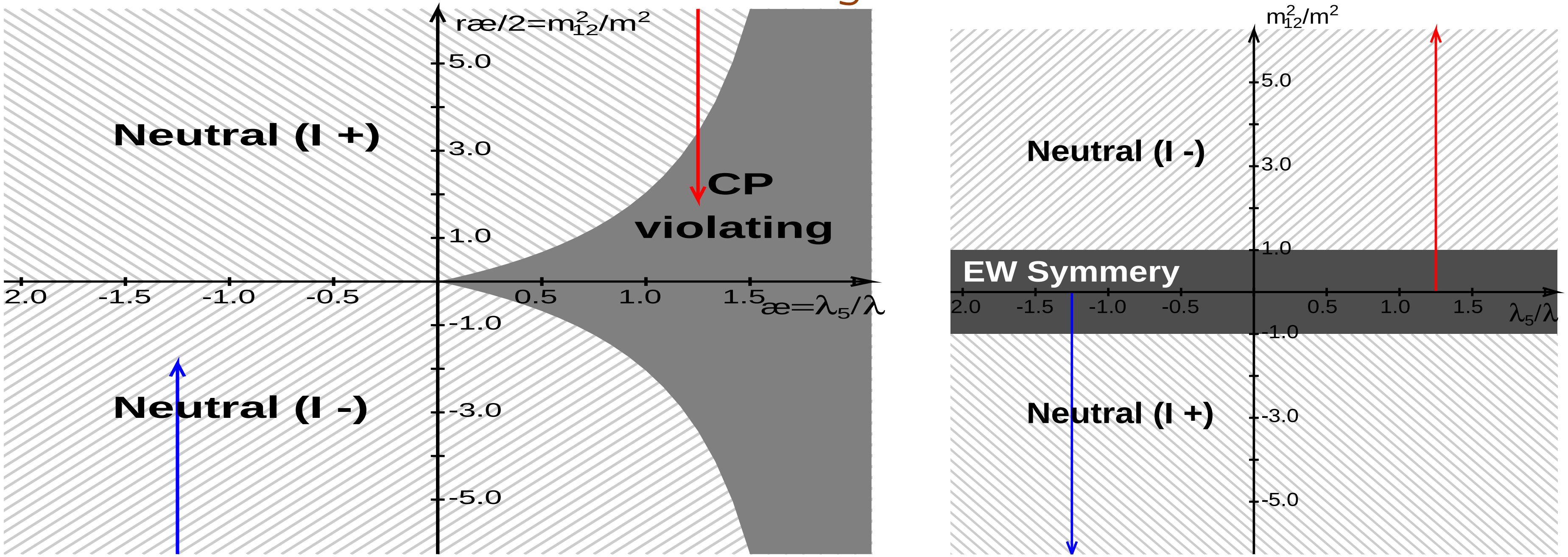
These phenomena shift degeneracy point in δ.

At the temperature variation near the point of $\mathcal{ZK}$ symmetry the system exhibits first order phase transition with stepwise variation of order parameter (v_1, v_2) and particle masses.

Certainly, the transition point is shifted due to EW and Yukawa corrections in potential but the phenomenon of the first order transition will take place. We expect that the transition latent heat will appear in this approximation.

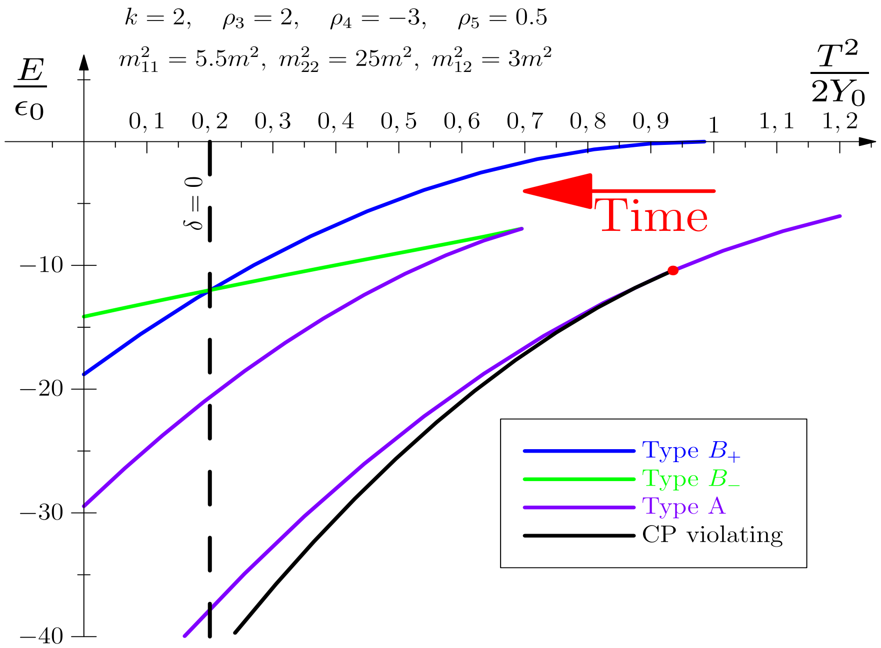
1. Toy potential with $k = 1$, $\delta = 0$, $\rho_3 = 1$, $\rho_4 = 0$

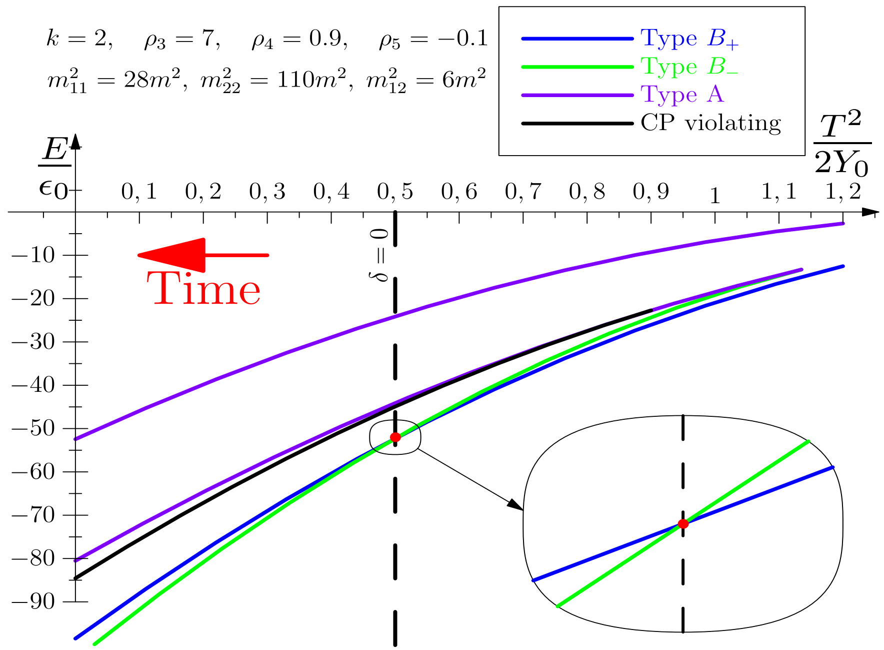
Phase diagram



Vacuum states in the plane $\varkappa r = \mu$ (vertical axis) — $\varkappa = \rho_5$ (horizontal axis). Left plot: $m^2 > 0$, right plot: $m^2 < 0$.

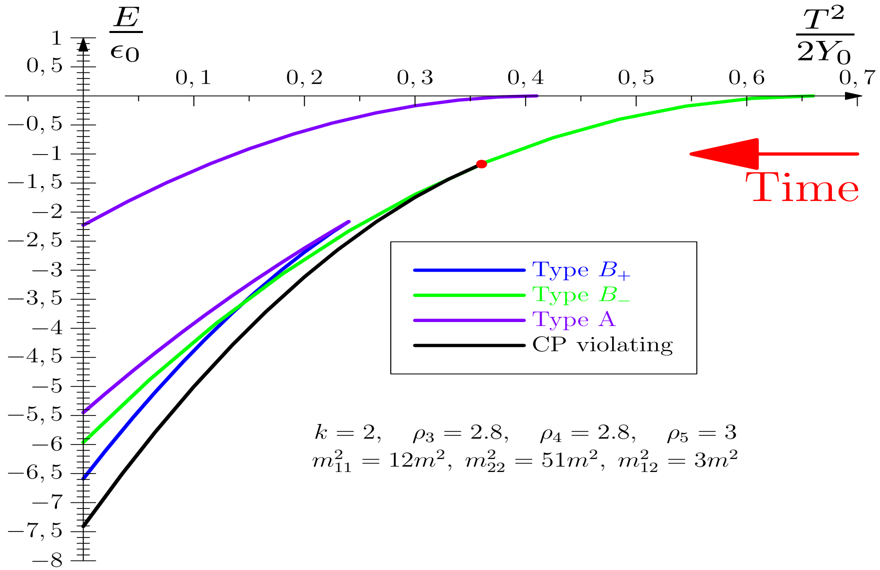
Arrows – evolution of vacuum states during cooling of Universe

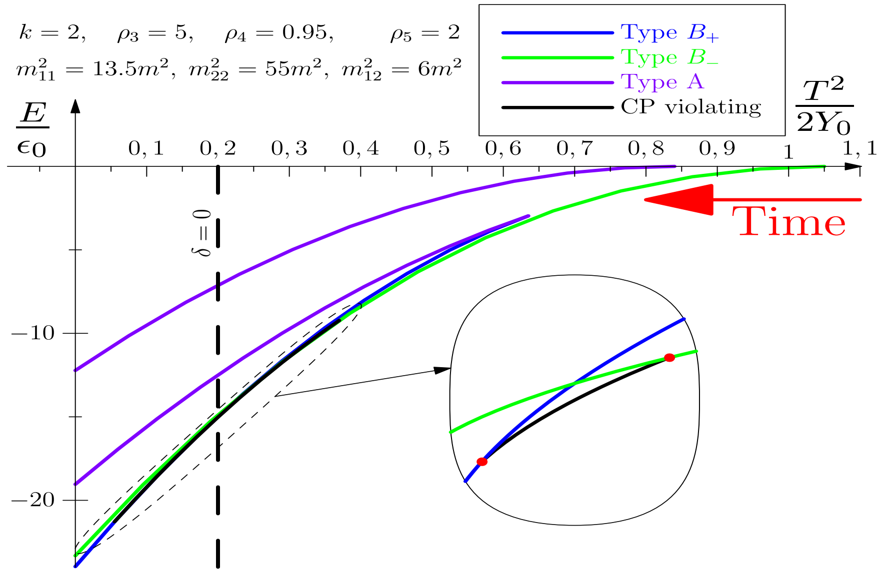




Left – 2 phase transition: EWc phase → CPc phase B (2 order) → sCPv phase (2 order)

Right – 2 phase transition: EWc phase → CPc phase B (2 order) → CPc phase B (1 order)





Left – 2 phase transition: EWc phase → CPc phase A (2 order) → sCPv phase (2 order)

Right – 3phase transition: EWc phase → CPc phase B (2 order) → sCPv phase (2 order) → CPc phase B (2 order);

the CPC phases B at high and low temperatures are different like in the case with 1-st order transition, physical properties changes quickly.

GOALS FOR FUTURE WORK

• To determine possible values of parameters of 2HDM, measurable at LHC, ILC, corresponding to different scenarios.

• Which quantities must be measured at LHC, ILC and with what precision for choice of one scenario?

For example

The $\mathcal{ZK}$ symmetry reaches at the temperature T_{ZK}, obtained from modern values of parameters via equation, describing $\delta = 0$:

$$m^2_{11,0} - m^2_{22,0}\sqrt{\lambda_1/\lambda_2} = gT^2_{ZK}(1 - \sqrt{\lambda_1/\lambda_2})\left[3\lambda_1 - (2\lambda_3 + \lambda_4)\sqrt{\lambda_1/\lambda_2}\right].$$

Condition $T^2_{ZK} > 0$ limits range of parameters allowing 1-st order phase transition in the earlier history of an Univerese.