Spin tracking and beam-beam issues

ILC-ECFA workshop Warsaw June 10 2008

Tony Hartin, Oxford

- Precision collider physics requires full treatment and tracking of polarization in the beam-beam processes
- Discuss full polarization in beam-beam background processes and implementation in CAIN
- Discuss dependency of the Equivalent Photon Approximation on virtual photon polarization
- Review calculation of the Sokolov-Ternov equation
  - Operator Method – Baier, Katkov et al
  - Volkov Solution method – 1964(Nikishov-Ritus) approximations
- T-BMT equation and anomalous magnetic moment
- Higher order beam-beam background processes

Depolarization at the IP

There is depolarization (spin flip) due to the QED process of beamsstrahlung, given by the Sokolov-Ternov equation

$$dW = -i\frac{\alpha m}{\sqrt{3}\pi\gamma}\left[\int_z^\infty K_{5/3}(z)\,dz + \frac{x^2}{1-x}K_{2/3}(z)\right]dx$$

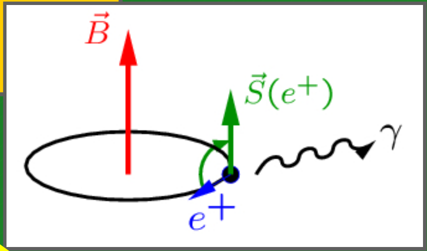
$$\text{where} \quad z = \frac{2}{3\nu\omega\epsilon_i}\,\frac{\omega_f}{\epsilon_i-\omega_f}$$

The fermion spin can also precess in the bunch fields. Equation of motion of the spin given by the T-BMT equation

$$\frac{d\vec{S}}{dt} = -\frac{e}{m\gamma}\left[(\gamma a+1)\vec{B}_T + (a+1)\vec{B}_L - \gamma\left(a+\frac{1}{\gamma+1}\right)\frac{1}{c^2}\vec{v}\times\vec{E}\right]\times\vec{S}$$

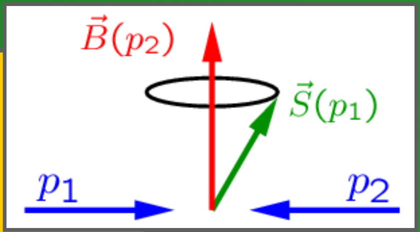
At the IP, the anomalous magnetic moment subject to radiative corrections in the presence of the bunch field

Stochastic spin diffusion from photon emission: Sokolov-Ternov effect, etc.



Parameter set	Depolarization ΔP_{lw}		
	T-BMT	S-T	total
Nominal	0.08%	0.02%	0.10%
low Q	0.04%	0.02%	0.06%
large Y	0.17%	0.02%	0.19%
low P	0.15%	0.09%	0.24%
TESLA	0.11%	0.03%	0.14%

Classical spin precession in inhomogeneous external fields: T-BMT equation.



Depol sims with CLIC parameters (I Bailey) change in polarization vector magnitude

	CLIC-G	ILC nom	ILC (80/30%)
T-BMT	0.04%	0.17%	0.14%
Beamstr.	3.20%	0.05%	0.03%
incoherent	0.06%	0.00%	0.00%
coherent	2.20%	0.00%	0.00%
total	**5.50%**	**0.22%**	**0.17%**

CAIN incoherent pair processes: Breit-Wheeler cross-section with polarizations

- Breit-Wheeler cross-section, CAIN original:

$$\sigma_{orig} \propto 2\left(1-h+\frac{2\epsilon^2-1}{2\epsilon^4}\right)\sinh^{-1}p + \frac{p}{\epsilon}\left(3h-1-\frac{1}{\epsilon^2}\right)$$

where p = electron momentum
ϵ = electron energy
$h = \xi_2\xi'_2$

full treatment due to Baier & Grozin hep-ph/0209361

$$\frac{d\sigma}{d\cos\theta\, d\phi} = \frac{\alpha^2}{4s^2x^2y^2}\sum_{ii'jj'} F^{ii'}_{jj'}\, \xi_j\xi'_{j'}\,\zeta_i\zeta'_{i'}$$

F are functions of scalar products of 4-momenta

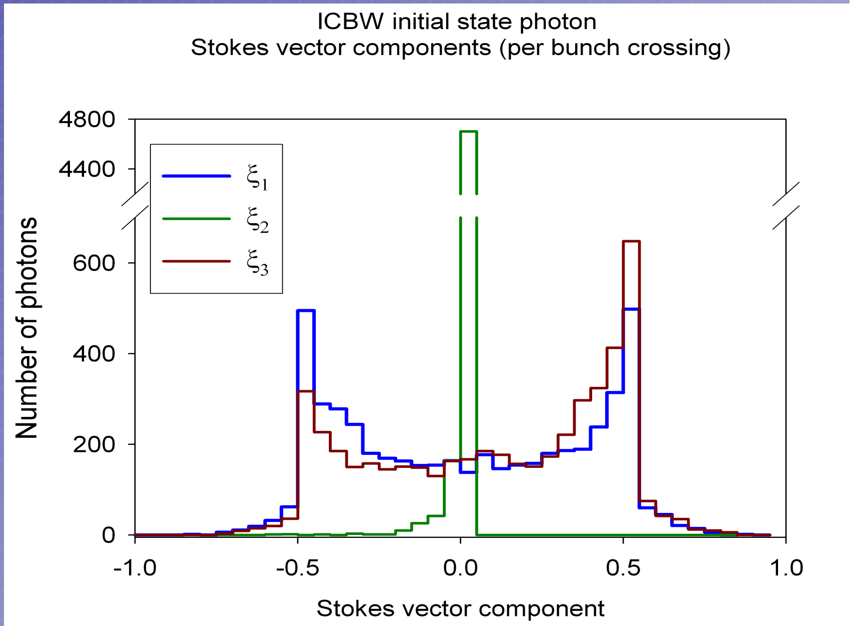
$$\sigma_{new} \propto 2\left(1-h+\frac{2}{\epsilon^2}(ha+\xi_1\xi'_1)-\frac{ha}{\epsilon^4}\right)\sinh^{-1}p + \frac{p}{\epsilon}\left(3h-1-\xi_1\xi'_1-\xi_3\xi'_3-\frac{ha}{\epsilon^2}\right)$$

where $ha = 1+\xi_3+\xi'_3+\xi_3\xi'_3$

Full expression has similar structure to original CAIN form, so can utilise existing monte-carlo methods

Final pair polarizations $\zeta^{(f)}$

$$\zeta_i^{(f)} = \frac{1}{F} \sum_{ii'jj'} F_{jj'}^{i0} \xi_j \xi_{j'}' \quad where \quad F = \sum_{jj'} F_{jj'}^{00} \xi_j \xi_{j'}'$$



Beamstrahlung photons have almost no circular polarization component – due to beam field having constant crossed field vectors

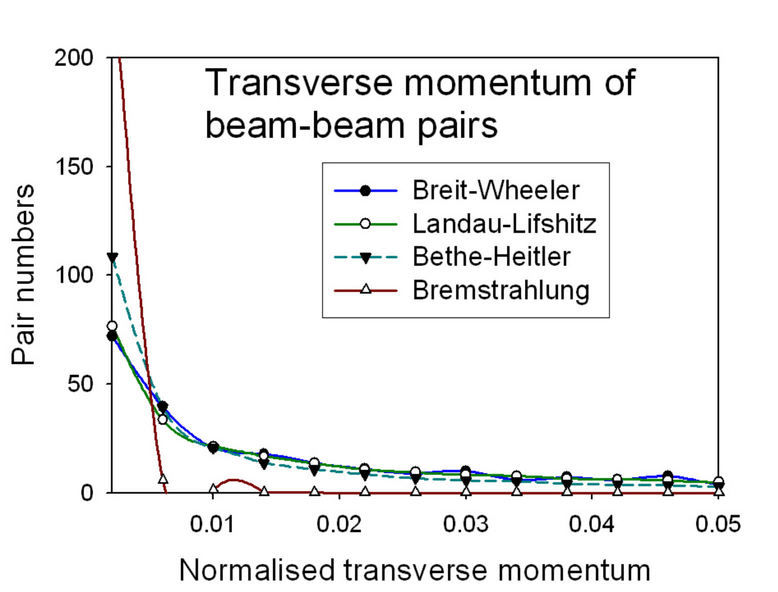
1st two components of the Breit-Wheeler pair polarization depends heavily on the photon circular polarization component, therefore ~0

Original beam polarization contained in the 3rd component of the produced pair

Final e- $(\zeta_1,\zeta_2,\zeta_3)=(-0.0024,-0.0024, 0.9883)$

Final e+ $(\zeta_1,\zeta_2,\zeta_3)=(0.0023,0.0079, 0.987)$

- Pairs have sufficent transverse momentum to distinguish them from outgoing beam, so...
- Could imagine a study to see how sensitive the final pair polarization is to the initial beam polarization, but...
- Breit-Wheeler pairs overlap with L-L and B-H pairs (derived using Equivalent Photon Approximation)



EPA and virtual photon polarization

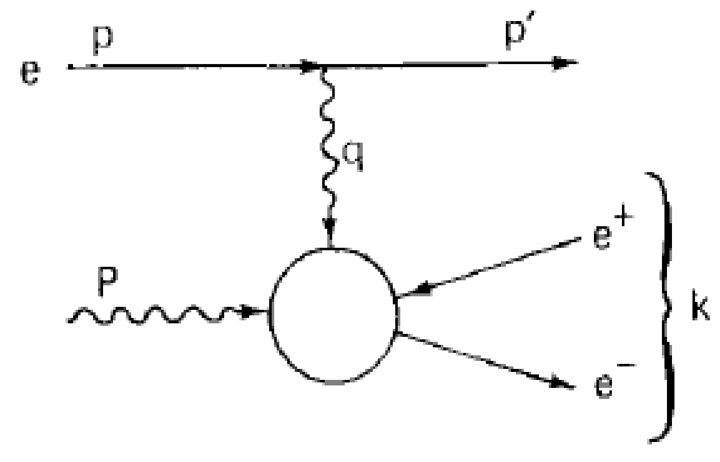
Virtual photon polarization

$$d\sigma = \frac{\alpha}{4\pi^2|q^2|}\left[\frac{(qP)^2 - q^2P^2}{(pP)^2 - p^2P^2}\right]^{1/2}(2\rho^{++}\sigma_T + \rho^{00}\sigma_S)\frac{d^3p'}{E'}$$

Full 4-f σ wriiten in terms of BW x-sects for transverse pol photons and scalar photons σ_T and σ_S – EPA neglects virtual photon polarization

If we want the final pair momenta then we need

$$d\sigma_{ep} = \left[\frac{d\sigma_\gamma}{d^3k_1} + \frac{1}{2}\xi\cos 2\varphi\,\frac{d(\sigma_\parallel - \sigma_\perp)}{d^3k_1}\right] d^3k_1\, dn(\omega, q^2)\,\frac{d\varphi}{2\pi}$$



k_1 is the 3-momenta of one of the secondaries
φ is the azimuthal angle of k_1 relative to the (p,p') plane
ξ is the virtual photon polarization

Budnev et al Phys Rep 15(4) 181-282 (1975)

Virtual photon polarization for coherent processes is the normalized spectral component of the bunch field $e_{x,y} = \hat{E}_\omega^{x,y}$

$$E_\omega^{x,y} = -\frac{ie}{\pi v}\iint \frac{q_{x,y}}{q_x^2+q_y^2}\, F(q)\, e^{i x q_x}\, e^{i y q_y}\, dq_x\, dq_y$$

$$F(q) = N\exp\left[-\frac{1}{2}(q_x\sigma_x)^2 - \frac{1}{2}(q_y\sigma_y)^2\right]$$

use flat beam approximation ($\sigma_y \ll \sigma_x$)

$$E_\omega^x = -i\,\frac{x}{\sigma_x^3}\exp\left(-\frac{x^2}{2\sigma_x^2}\right)\left[\sigma_y\exp\left(\frac{-y^2}{2\sigma_y^2}\right) + \sqrt{\frac{\pi}{2}}\,y\,Erf\left(\frac{y}{\sqrt{2}}\sigma_y\right)\right]$$

QED - Semi-classical methods

Derivation of Sokolov-Ternov eq^n in the literature usually done with the **'operator method'** of Baier Katkov et al

The energy levels of an ultrarelativistic electron in a magnetic field are very close together, so assume motion is classical

- Operators of the dynamical variables of the electron commute
- Retain commutator between electron and photon variables

$$\frac{\hbar\omega_0}{\epsilon_p} = \frac{B}{B_c}\left(\frac{m}{\epsilon_p}\right)^2$$

Or it can be done at Lagrangian level in the Bound Interaction (Furry) Picture

Make the external field implicit in a bound Dirac Lagrangian L_{BD}. Neglect the interaction between photons and the external field A^e.

- The interaction Lagrangian expresses interaction between the free Maxwell and the bound Dirac fields
- Requires solutions of the bound Dirac field operators

$$L = L_M + L_{BD} + L_I$$

$$L_{BD} = \bar{\psi}(x)\left(i\gamma^\mu \partial_\mu - e\gamma^\nu A^e_\nu\right) - m\psi(x)$$

$$L_I = -e\bar{\psi}(x)\gamma^\mu A^e_\mu \psi(x)$$

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Solution of Dirac equation in beam field A^e

$$[(p-eA^e)^2-m^2-\frac{ie}{2F^e_{\mu\nu}}\sigma^{\mu\nu}]\psi_V(x,p)=0$$

Look for a solution of the form:

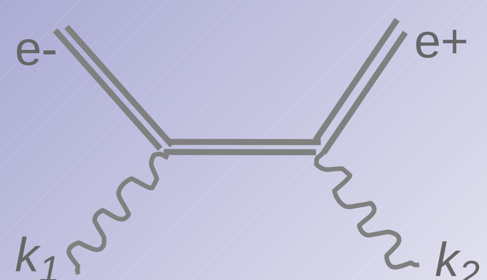
$$\psi_V(x,p)=u_s(p)F(\phi)$$

Substitution of the general solution for ψ_V yields a first order differential equation. whose solution can be expanded in powers of k, A^e

$$\psi_V(x,p)=[1+\frac{e}{2(kp)}\gamma^\mu k_\mu \gamma^\nu A^e_\nu]\ \exp[F(k,A^e)]\ e^{-ipx}u_s(p)$$

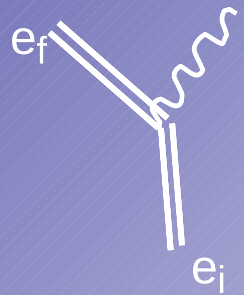
- make Fourier transform to get exponential of linear term in x
  $\int dn \exp[-i(n+v^2/kp)kx]F_2(p,k,A^e)$
- n external field photons contribute
- Fermion momentum gains $\frac{v^2}{kp}k$
- Leads to fermion mass shift m^2+v^2
- F_2 are
  - Bessel functions for circular polarized A^e
  - Airy functions for constant crossed A^e

Usual solution in the absence of A^e



fermion solutions represented by double straight lines

Beamstrahlung in an external field (Sok-Ter) – Nikishov & Ritus (1964)



Calculation first performed in a linearly polarized field

$$A_\mu = a_\mu \cos(k.x)$$

Volkov solutions introduce complicated functions B (l external field photons)

$$B_n(l,\alpha,\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos^n k.x\, e^{f(k.x)} \quad \text{where} \quad f(k.x) = i\alpha\sin(k.x) - i\beta\sin(2k.x) - il(k.x)$$

External field strength expressed by dimensionless parameter ν which has a direct relationship to field potential or strength and an inverse relationship to the field frequency ω

$$\nu = \frac{ea}{m} \propto \frac{B}{\omega}$$

Constant field calculation performed for $\nu \to \infty$ $(\omega \to 0)$

Saddle point approximation used to write B_n as a function of Airy functions and the phase ψ of the slowly alternating external field

$$B_n \propto \frac{1}{\nu \sin\psi}\frac{Ai(y)}{\sqrt{y}} \quad \text{where} \quad y = \left(\frac{\nu}{\sin\psi}\right)^{2/3}$$

- other approximations made

Transformation to constant crossed field using solutions of a Schlömilch eqn

$$\text{if} \quad W(B) = \frac{2}{\pi}\int_0^{\pi/2} F(B\sin\psi)\,d\psi \quad \text{then} \quad F(B) = W(0) + B\int_0^{\pi/2} W'(B\sin\psi)\,d\psi$$

Clearly it would be better to do the calculation directly in the constant field, for arbitrary n and without approximations – work in progress

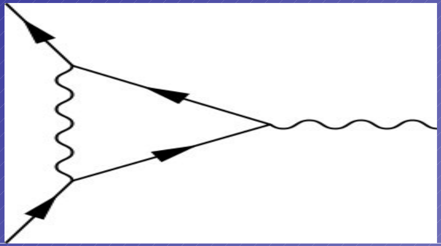
Anomalous magnetic moment in a strong field
Moortgat-Pick/Hesselbach

Needed in T-BMT equation to calculate the rate of depolarization due to beam-beam effect

$$\vec{\Omega} = -\frac{e}{m\gamma}\left[(\gamma a+1)\vec{B}_T + (a+1)\vec{B}_L - \gamma\left(a+\frac{1}{\gamma+1}\right)\frac{\beta}{c}\vec{e}_v \times \vec{E}\right]$$

Main contrib^{n} from vertex diagram

$$a = \frac{\alpha}{2\pi} + O(\alpha^2)$$



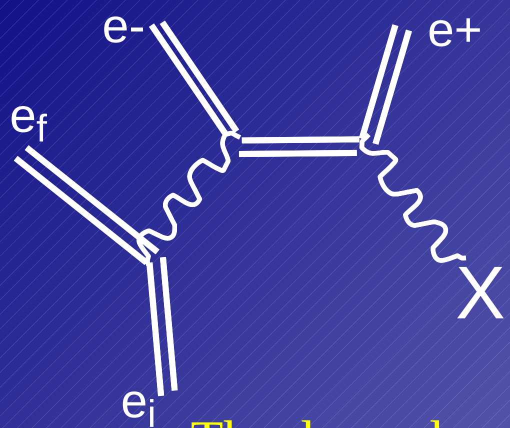
when fermion is embedded in a strong external field characterised by $\Upsilon = \nu^2 \frac{(k.p)}{m^2}$ the anomalous magnetic moment develops a dependence on Υ and is given by (Baier-Katk)

$$a(\Upsilon) = -\frac{\alpha}{\pi\Upsilon}\int_0^\infty \frac{x}{(1+x)^3}dx \int_0^\infty \sin\left[\frac{x}{\Upsilon}\left(t+\frac{1}{3}t^3\right)\right]dt$$

However...we can envisage

- recalculating the vertex diagram in BP with Volkov solutions replacing all fermion lines
- Making mass correction (including self-energies)

4-fermion IFQED process

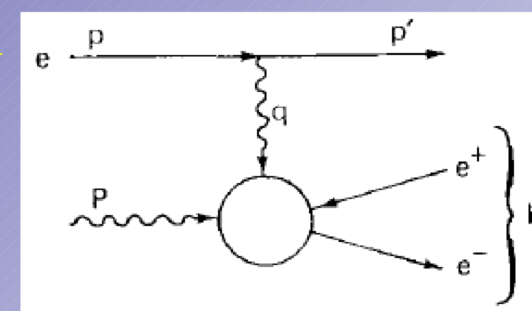


- To do bunch field effect properly replace all fermion lines by Volkov solutions
- 4^{th} order external field process is intimidating so look for an 'external field' EPA

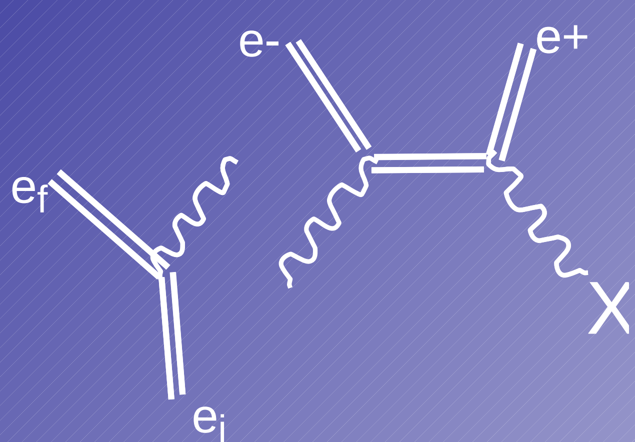
The dependency on fermion momenta have to be modified

$P_\mu \rightarrow P_\mu + k_\mu \, \nu^2/2(kp)$ and $P^2 \rightarrow P^2 + \nu^2$

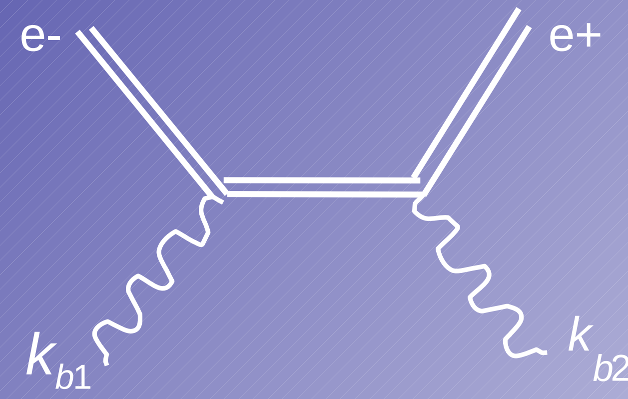
$$d\sigma = \frac{\alpha}{4\pi^2 |q^2|} \left[\frac{(qP)^2 - q^2P^2}{(pP)^2 - p^2P^2} \right]^{1/2} (2\rho^{++}\sigma_T + \rho^{00}\sigma_S) \frac{d^3p'}{E'}$$

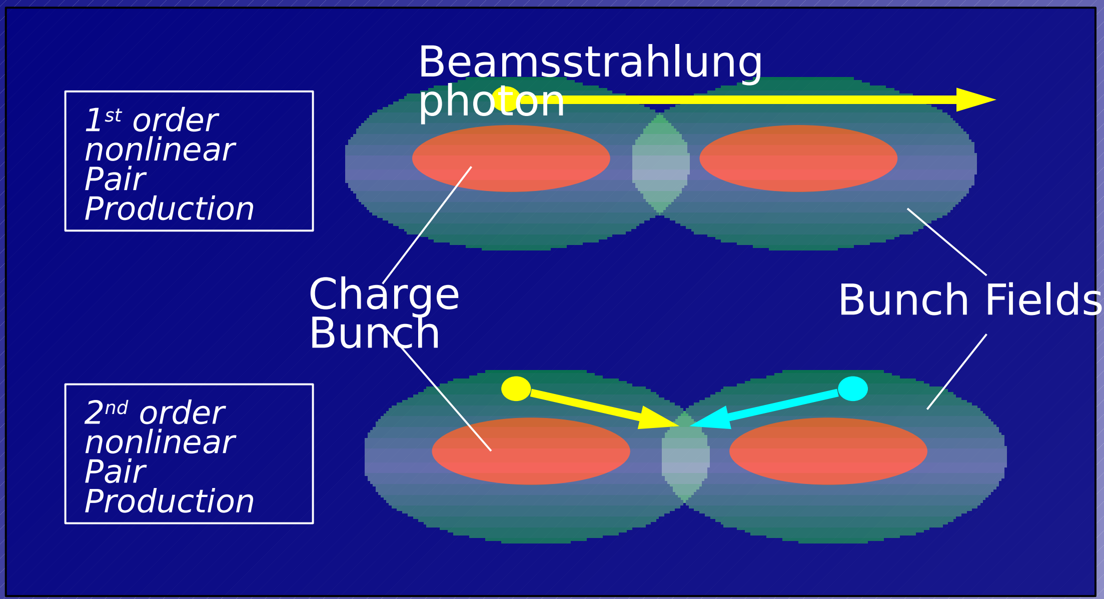


- So assuming EPA can still be used we are left with the 1st order external field process (Sokovlov-Ternov). Known to be determined by the intensity of the bunch field Υ
- The 2^{nd} order external field processes need special treatment. Propagators can reach the mass shell, the x-sections can exceed S-T and the effect does not necessarily have a simple relationship with Υ

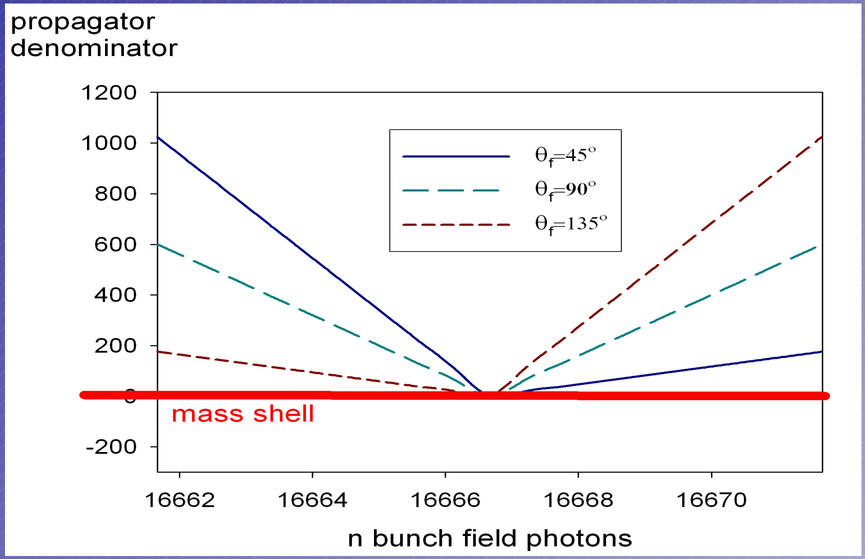


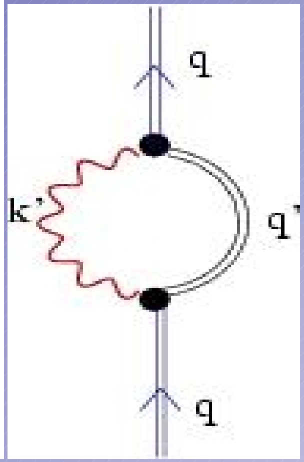
2^{nd} order external field process: Coherent Breit-Wheeler (CBW) process





- 2^{nd} order process contains twice as many Volkov E_p
- Double integrals over products of 4 Airy functions – mathematical challenge!
- spin structure same as ordinary Breit-Wheeler





fermions recieve a mass shift due to bunch field and the propagator can reach mass shell whenever $r\,\omega \sim \omega_b$

Discussion & Future work

(1) Full polarization of Coherent pairs involving real photons (B-W) implemented – original beam polarization carried by final particles

(2) Depolarization for CLIC parameters **significant** and needs further study

(3) Present Sokolov-Ternov equation assumes small Upsilon, but larger values (CLIC) require more exact calculation using Volkov solutions

(4) Previous Volkov solution calculations (1964) use several approximations – calculation with no approximations in progress

(5) T-BMT equation sensitive to anomalous magnetic moment calculation – use Volkov solutions

(6) EPA calculation has to be altered to take into account polarization of virtual photons – analytic forms have been developed and need to be implemented

(7) 4-fermion processes with Volkov solutions also requires modification of the EPA

(8) Higher order coherent processes can progress using exact calculation of the beamstrahlung in external field