Finite lifetime effects in top quark pair production at threshold

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in collaboration with

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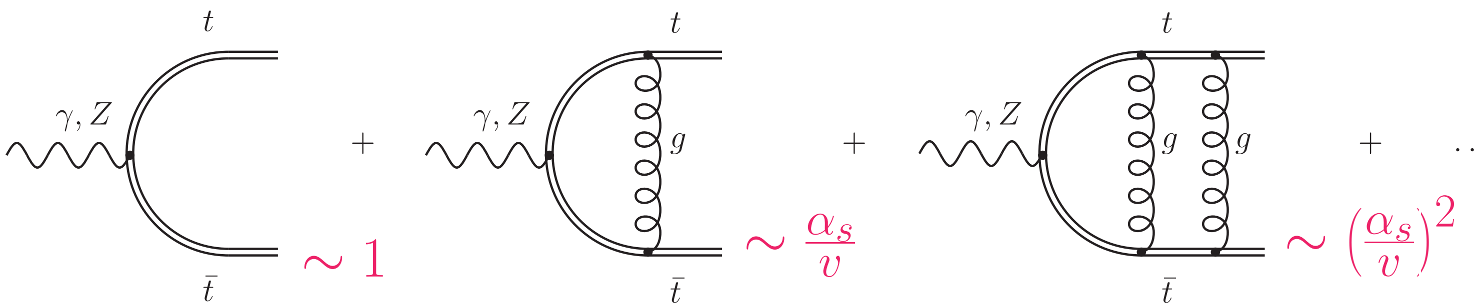
Outline

- Motivation of $t\bar{t}$ production at threshold
- Theory status
- Electroweak effects beyond leading order
- Instability of the top quark
  $\longrightarrow$ Complex Wilson coefficients
  $\longrightarrow$ Interferences
  $\longrightarrow$ Phase space divergences
  $\longrightarrow$ Phase space matching and finite imaginary renormalization
- Numerical results
- Summary and outlook

Non-relativistic top pairs

<u>e^+e^- collisions:</u> c. m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are non-relativistic $v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1$



$\Rightarrow$ Perturbation theory in α_s breaks down $v \sim \alpha_s$

$\Rightarrow$ Non-relativistic QCD (vNRQCD) $\simeq$ Schrödinger theory at LO

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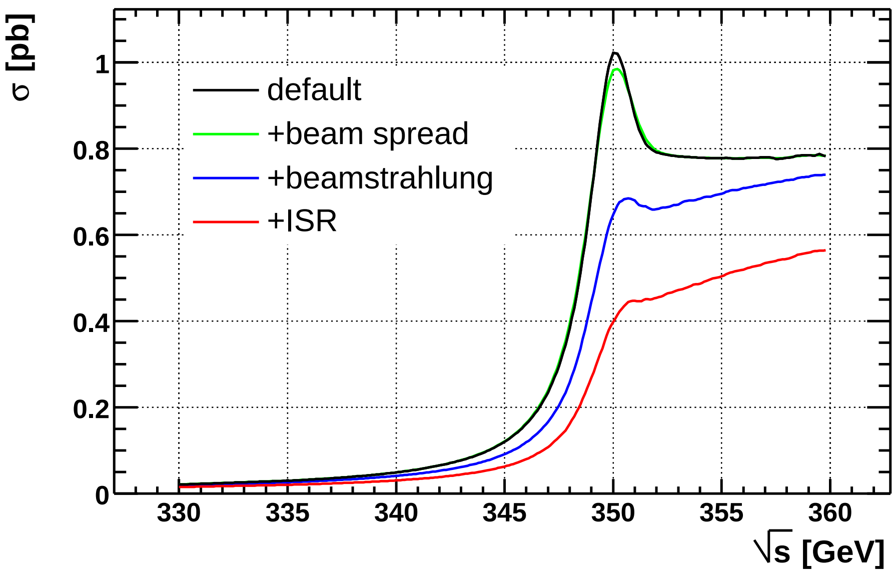
- Top quarks decay fast: $t \rightarrow Wb$ $\Gamma_t \approx 1.5\ \mathrm{GeV} > \Lambda_{\mathrm{QCD}}$

$\Rightarrow$ No bound states

$\Rightarrow$ Smooth line-shape of $\sigma_{\mathrm{thr}}(e^+e^- \rightarrow t\bar{t})$

$\Rightarrow$ Non-perturbative effects suppressed

Fadin, Khoze (JETP Lett. 46, 1987)



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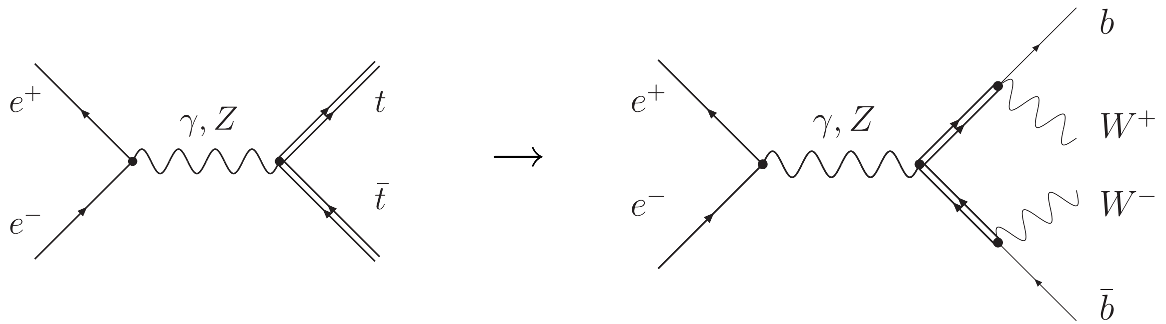
$\Rightarrow$ Instead of $e^+e^- \to t\bar{t}$ consider $e^+e^- \to bW^+\bar{b}W^-$



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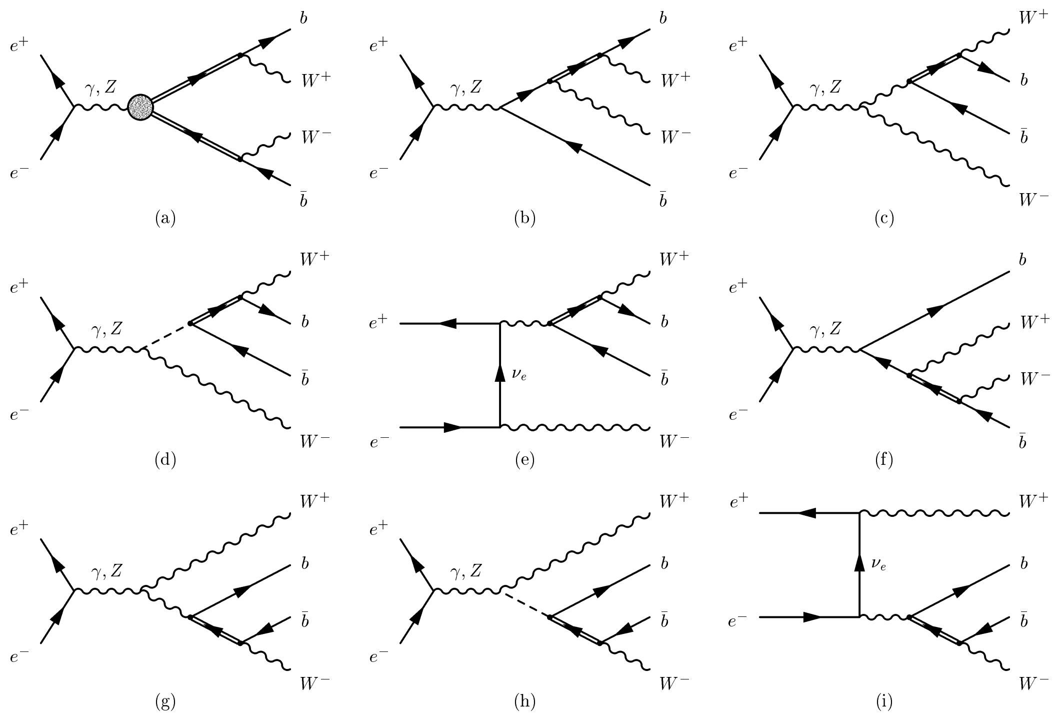
$\Rightarrow$ Interferences of double- and single-resonant diagrams



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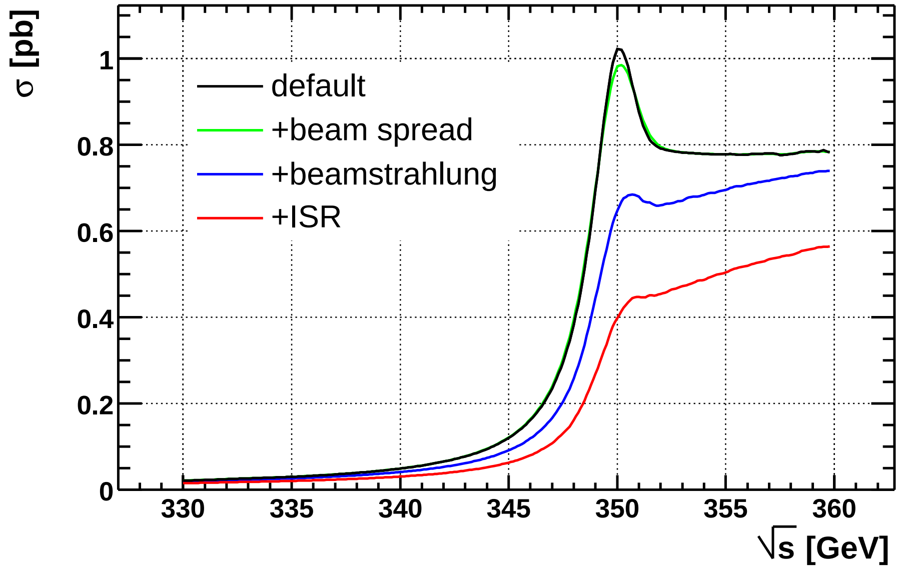
- Measured cross section contains luminosity spectrum $$\sigma^{\mathrm{obs}}(s) = \int_0^1 dx\, \mathcal{L}(x) \sigma^{\mathrm{theo}}(x^2 s)$$

  $\to$ pure QED effects not considered here



Measurements

Simulations of threshold scan at ILC ($\int \mathcal{L}\, \mathrm{dt} \sim 300\, \mathrm{fb}^{-1}$):

Martinez, Miquel (Eur. Phys. J. C 27, 2003)

- Top quark mass

  $(\delta \mathrm{m_t})^{\mathrm{exp}} \sim 50$ MeV

- Top Yukawa coupling

  $(\delta \mathrm{y_t}/\mathrm{y_t})^{\mathrm{exp}} \sim 0.35$ (if $\mathrm{M_H} < 140$ GeV)

- Strong coupling

  $(\delta \alpha_s(\mathrm{M_Z}))^{\mathrm{exp}} \sim 0.001$

- Top decay width

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$\Rightarrow$ Required theoretical precision

$$(\delta\sigma_{\mathrm{thr}}/\sigma_{\mathrm{thr}}) \leq 3\,\%$$

Effective theory (stable quarks)

- Relevant scales

$$m_t \text{ (hard)} \gg m_t v \text{ (soft)} \gg E \sim m_t v^2 \text{ (ultrasoft)}$$

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- Power counting $\boxed{\mathsf{v} \sim \alpha_s \ll 1}$

$$\boxed{\left(\frac{\alpha_s}{\mathsf{v}}\right) \sim 1} \qquad \boxed{(\alpha_s \ln \mathsf{v}) \sim 1}$$

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$$\text{LL} \quad \sim \sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m$$

$$\text{NLL} \quad \sim \{\alpha_s, v\} \sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m$$

$$\text{NNLL} \sim \{\alpha_s^2, \alpha_s v, v^2\} \sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \ln v)^m$$

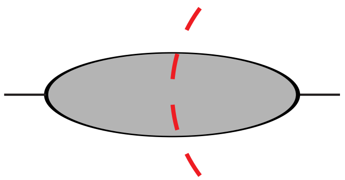
Effective theory (stable quarks)

Total cross section from $e^+e^- \to e^+e^-$ using the Optical Theorem

Strassler, Peskin (Phys. Rev. D 43, 1991)

$$\sigma_{\rm tot} \quad \propto \quad \mathrm{Im}\left[i \sum_{\mathbf{p},\mathbf{p}'} \int d^4x\, e^{-i\hat{q}\cdot x} \left\langle 0 \left| T \left(C(\mu) \mathbf{O}^\dagger_{\mathbf{p}}(0) \right) \left(C(\mu) \mathbf{O}_{\mathbf{p}'}(x) \right) \right| 0 \right\rangle \right]$$

$$\propto \quad \mathrm{Im}\left[C(\mu)^2\, G(0,0,E) \right]$$



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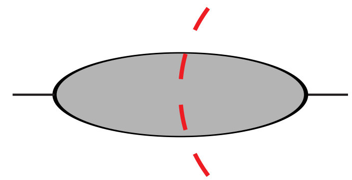
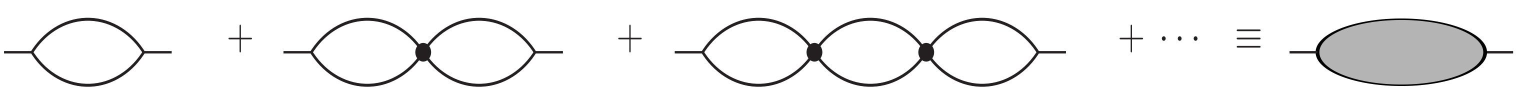


$$\left(-\frac{\nabla^2}{m_t} - \frac{\nabla^4}{4m_t^3} + V(\mathbf{r}) - E\right) G(\mathbf{r},\mathbf{r}',E) = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

where $V(\mathbf{p},\mathbf{p}') = \left[\frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m_t|\mathbf{k}|} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m_t^2\mathbf{k}^2} + \frac{\mathcal{V}_2+2\mathcal{V}_s}{m_t^2}\right], \mathbf{k} = \mathbf{p}-\mathbf{p}'$

LO:



Theory status

QCD effects

$\left(\frac{\alpha_s}{v}\right)^n \sim 1$

$(\alpha_s \ln v)^m \sim 1$

- LL ✓
- NLL ✓
- NNLL $\rightarrow \delta\sigma_{\mathrm{thr}}/\sigma_{\mathrm{thr}} \sim \pm 6\,\%$

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Electroweak effects

$\Gamma_t \sim m_t \alpha \approx E_{\mathrm{kin}} \sim m_t \alpha_s^2$

- LL: $E \rightarrow E + i\Gamma_t$ Fadin, Khoze (JETP Lett. 46, 1987) ✓
- NLL: – NNLO Phase space divergences $\Rightarrow$ NLL RG effects
  – Phase space matching
- NNLL: – real matrixelement corrections
  – imaginary matrixelement corrections $\rightarrow$ interferences
  – NNLL running due to phase space divergences $\rightarrow$ not yet done

Effective theory for unstable quarks

- Replacement rule $E \to E + i\Gamma_t$ at LL
- Complex Wilson coefficients
  $\longrightarrow$ contain interferences at NNLL
  $\longrightarrow$ UV phase space divergences
  $\longrightarrow$ Phase space matching necessary
- Effective Lagrangian non-hermitian
- Cross section from the Optical Theorem using unitarity of the underlying theory

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- Effective Lagrangian non-hermitian
- Cross section from the Optical Theorem using unitarity of the underlying theory

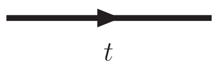
$\Rightarrow$ Contributions of real Wb final states contained in matching conditions of the EFT
$\longrightarrow$ inclusive treatment

$\gg$ Analogy to absorption processes in the Optical Theory

Unstable quarks in vNRQCD

Power counting: $D^0 \sim m_t v^2, \quad \mathbf{p}^2 \sim m_t^2 v^2, \quad \Gamma_t \sim m_t \alpha \sim m_t \alpha_s^2$

Full theory:



$\bar{\psi}(\not{p} - m_t)\psi$

Effective theory:

$$\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^{\dagger}(x)\left[iD^0 - \frac{\mathbf{p}^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3}\right]\psi_{\mathbf{p}}(x)$$

$\sim m_t v^2$

stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

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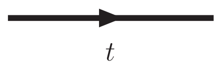

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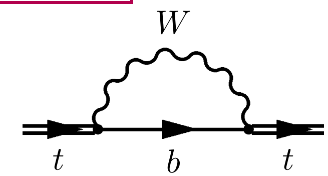

$\operatorname{Im}\Sigma_t = \frac{\Gamma_t}{2}$

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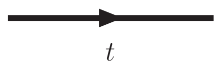

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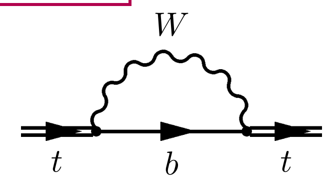
$i\,\Sigma_t =$ 

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$\sim m_t v^2$ $\quad \boxed{v \sim \alpha_s} \quad$ $\sim m_t \alpha_s^2$ $\quad$ $\sim m_t \alpha_s^4$

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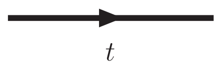

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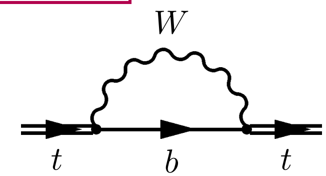

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$\sim m_t v^2$ $\qquad$ $v \sim \alpha_s$ $\qquad$ $\sim m_t \alpha_s^2$ $\qquad$ $\sim m_t \alpha_s^4$

stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

unstable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}}$

NNLL time dilatation correction

Instability beyond LL

Quark bilinear operators:

- Lifetime dilatation at NNLL
- $\mathcal{O}(\alpha_s)$, $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha)$ corrections to Γ_t auf NLL und NNLL

  Jeżabek, Kühn; Blokland, Czarnecki, Ślusarczyk, Tkachov

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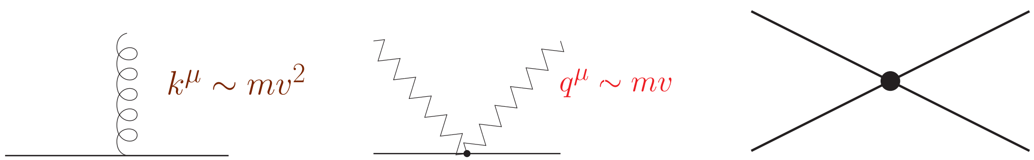
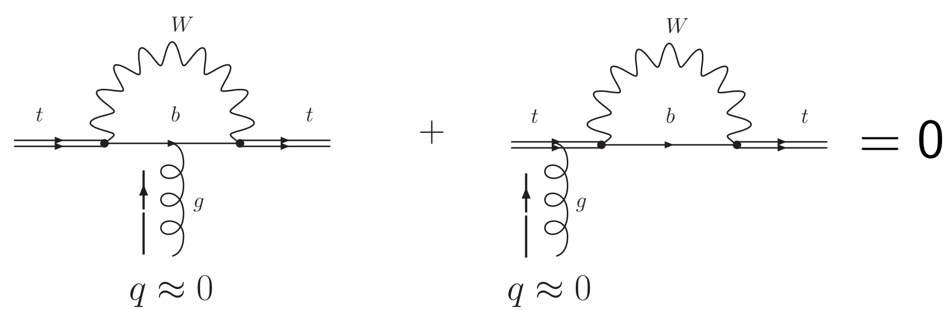
Gluon interactions and potentials:



- Electroweak corrections either beyond NNLL order or contributions to $\sigma_{\rm tot}$ cancel due to gauge invariance

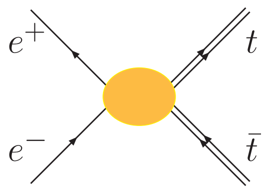
  Melnikov, Yakovlev (Phys. Lett. B324, 1994)
  Fadin, Khoze, Martin, Stirling (1995)



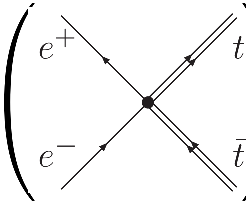
Electroweak matching conditions

Currents:

$$m_t\,\alpha \sim m_t\,\alpha_s^2$$



$$= \left[C_{\rm LL}^{\rm born} + C_{\rm NLL}^{\rm QCD} + C_{\rm NNLL}^{\rm QCD} + C_{\rm NNLL}^{\rm ew} + i\,C_{\rm NNLL}^{\rm int} + \ldots \right] \cdot \Big(\quad \Big)$$



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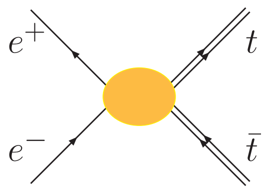

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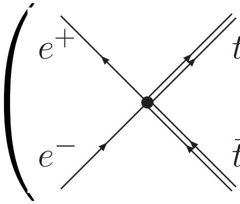

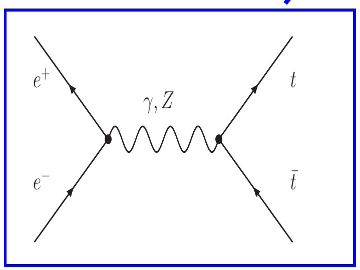

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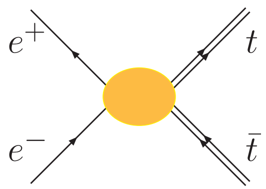

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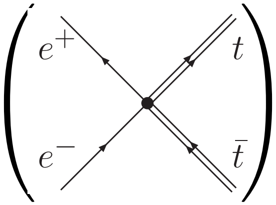
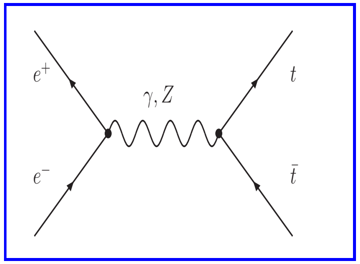
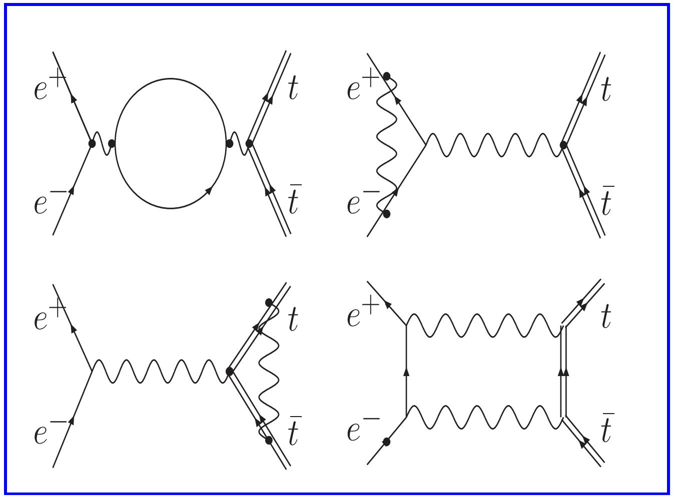






Real parts of electroweak 1-loop diagrams O(α)

(pure QED diagrams not considered)

Grzadkowski, Kühn, Krawczyk, Stuart (Nucl. Phys. B 281,1987)

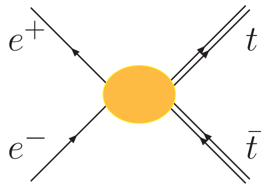
Hoang, CR (Phys. Rev. D 74, 2006)

$\Rightarrow$ Usual hard effects at NNLL

Absorptive matching conditions

Currents:

$m_t\,\alpha \sim m_t\,\alpha_s^2$



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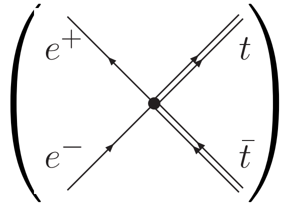
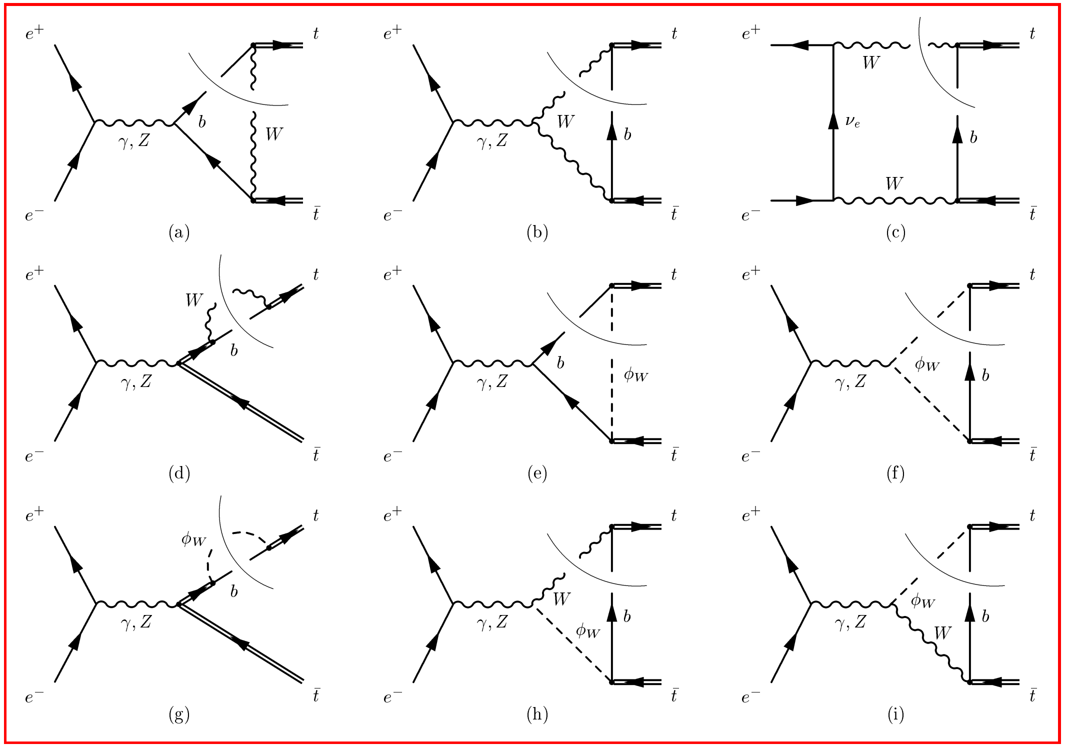


- ≫ bW cuts of electroweak 1-loop diagrams
- ≫ bW cuts are gauge invariant
- ≫ bW considered as stable particles

⇒ NNLL instability effects



Hoang, CR (Phys. Rev. D 71, 2005)

Interferences

Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\rm tot} = 2\,N_c\ {\rm Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]}$

$$\sigma_{\rm tot} = 2\,N_c\,{\rm Im}\left[C_{\rm LL}^{\rm born}\left(C_{\rm LL}^{\rm born} + 2\,C_{\rm NNLL}^{\rm ew} + 2\,i\,C_{\rm NNLL}^{\rm int}\right)G_{\rm LL} + \ldots\right]$$

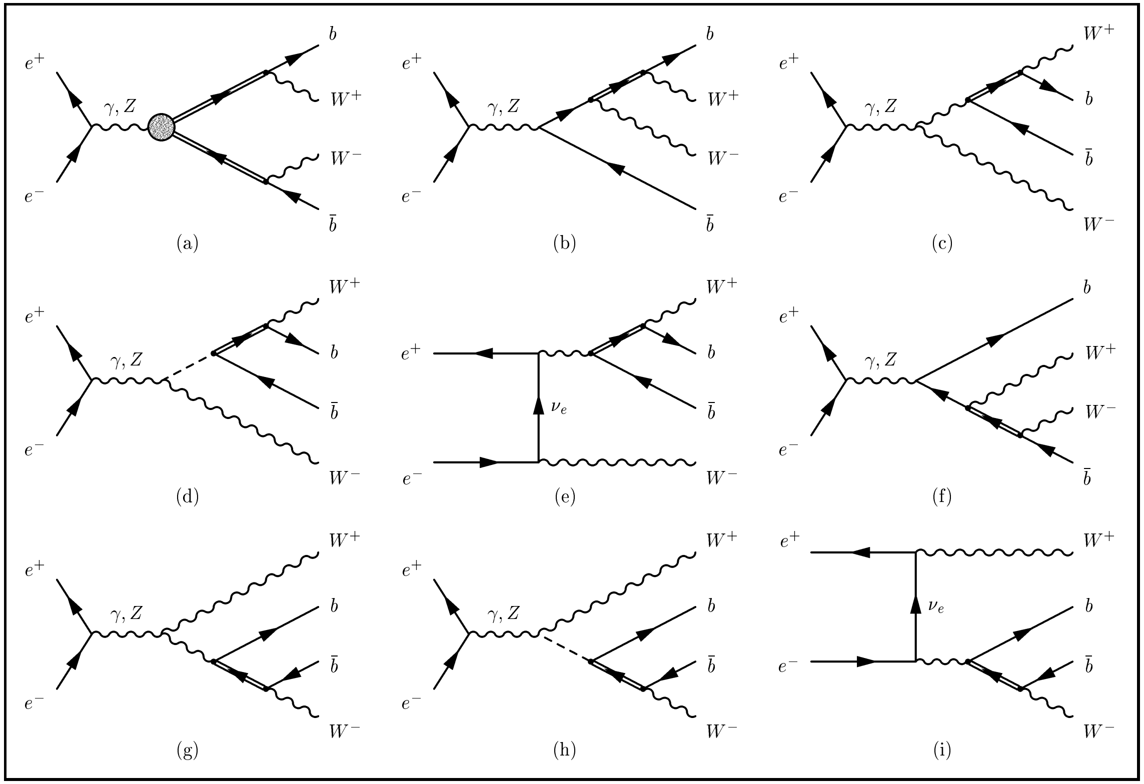
Interferences

Optical Theorem $\Rightarrow$ $\sigma_{\rm tot} = 2\,N_c\ {\rm Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]$

$$\sigma_{\rm tot} = 2\,N_c\,{\rm Im}\left[C_{LL}^{\rm born}\,(C_{LL}^{\rm born} + 2\,C_{NNLL}^{\rm ew} + 2\,i\,C_{NNLL}^{\rm int})\,G_{LL} + \ldots\right]$$

$$= 2\,N_c\left\{\left[(C_{LL}^{\rm born})^2 + 2\,C_{LL}^{\rm born}\,C_{NNLL}^{\rm ew}\right]{\rm Im}[G_{LL}] + \underbrace{2\,C_{LL}^{\rm born}\,C_{NNLL}^{\rm int}\,{\rm Re}[G_{LL}]} + \ldots\right\}$$

- Interference of double- and single-resonant diagrams with $bW^+\bar{b}W^-$ final state



Phase space divergence

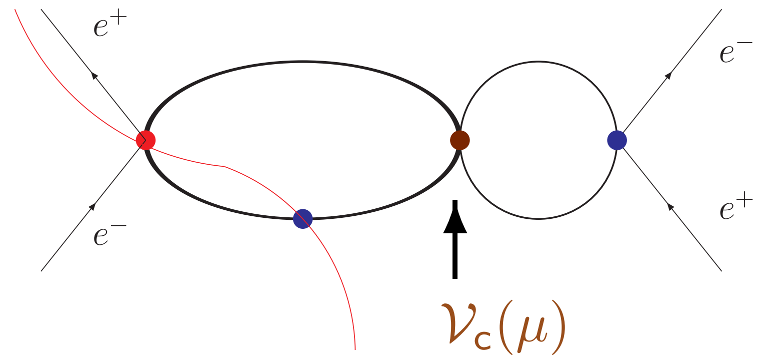
Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\text{tot}} = 2\,N_c\ \text{Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]}$

- NNLL interference effect

$$\Delta^{\Gamma,1}\sigma_{\text{tot}} = 2\,N_c\left\{2\,C^{\text{born}}_{\text{LL}}\,C^{\text{int}}_{\text{NNLL}}\ \text{Re}[G_{\text{LL}}] + \ldots\right\}$$

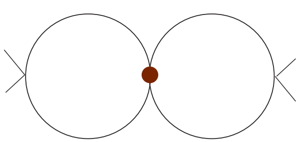
contains logarithmic UV phase space divergence

$$\boxed{C^{\text{int}}_{\text{NNLL}}\,\mathcal{V}_c(\mu)\,\frac{1}{\epsilon}} \quad \sim$$



$$\mathcal{V}_c(\mu) = -4\pi C_F \alpha_s(\mu)$$

from $\mathcal{O}(\alpha_s)$ contribution to Green function



$$= G^{\mathcal{O}(\alpha_s)}_{\text{LL}} = \alpha_s(\mu)\,C_F\,\frac{m_t^2}{4\pi}\left[\frac{1}{4\epsilon} - \ln\left(\frac{-im_t v}{\mu}\right) + \frac{1}{2} - \ln 2\right]$$

Phase space divergence

Optical Theorem $\Rightarrow$ $\sigma_{\mathrm{tot}} = 2\,N_c\,\mathrm{Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]$

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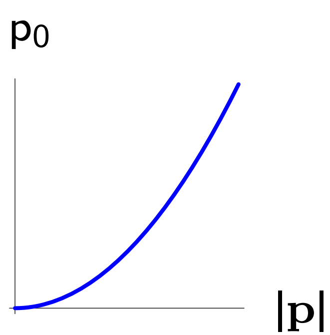
contains logarithmic UV phase space divergence

Phase space:

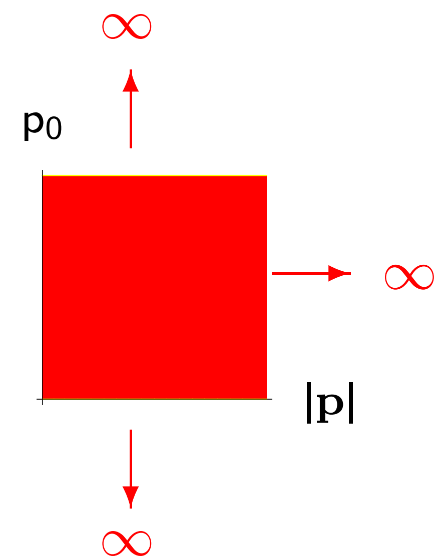
Stable tops

$$\frac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\epsilon} \longrightarrow$$



Unstable tops

$$\frac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}} \longrightarrow$$



Phase space divergence

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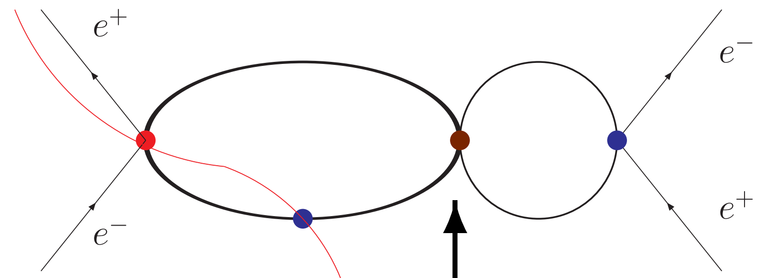
  contains logarithmic UV phase space divergence

$\boxed{C_{\rm NNLL}^{\rm int}\,\mathcal{V}_c(\mu)\,\frac{1}{\epsilon}}$ $\sim$

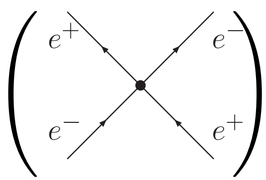


$\mathcal{V}_c(\mu)$

- NLL mixing effect:

  $\Rightarrow$ Anomalous dimension for $(e^+e^-)(e^+e^-)$ operators:

  $i\,C(\mu)\cdot$



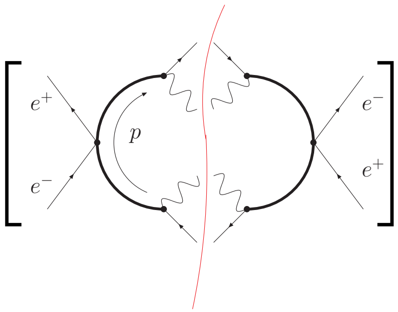
  $\Rightarrow$ RG running leads to correction $\Delta^{\Gamma,2}\sigma_{\rm tot}$

  - $\sqrt{s}$-independent
  - scale-dependent

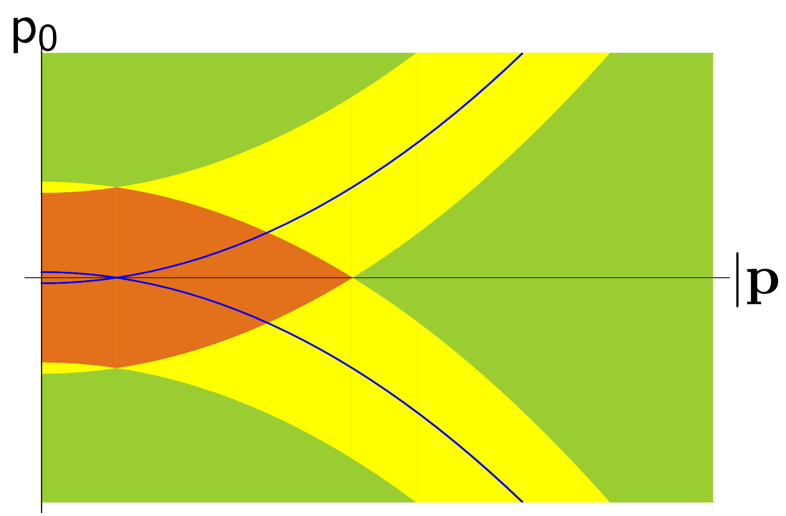
$\gg$ Matching coefficients $C(\mu = m_t, \Lambda)$ from **phase space matching**

Phase space matching

Hoang, Ruiz-Femenía, CR

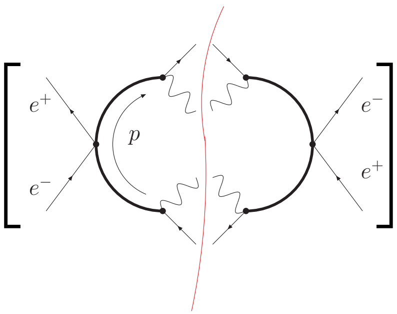
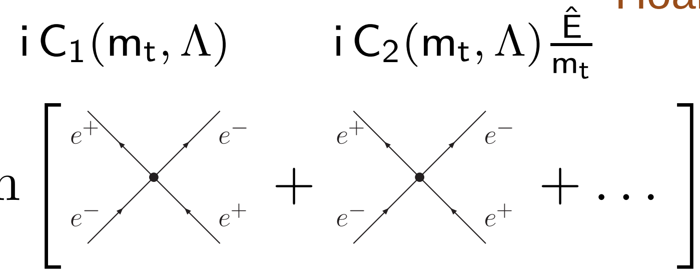
$\sigma_{\text{thr}} \sim$ 

$$\sim \int_{-\infty}^{+\infty} \mathrm{d}p_0 \int_0^{+\infty} \mathrm{d}|\mathbf{p}|\,|\mathbf{p}|^2 \frac{\Gamma_t^2}{\underbrace{|\frac{E}{2} + p_0 - \frac{\mathbf{p}^2}{2m_t}}_{(q^2 - m_t^2)/2m_t} + i\frac{\Gamma_t}{2}|^2 \, |\frac{E}{2} - p_0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}|^2}$$



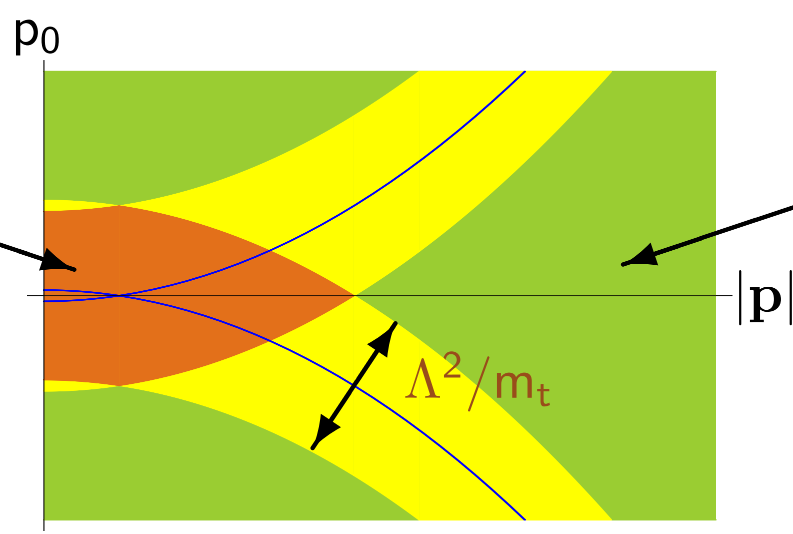
Phase space matching

Hoang, Ruiz-Femenía, CR

$\sigma_{\mathrm{thr},\Lambda} \sim$  $+\ \mathrm{Im}$ 

$$\sim \int dp_0 \int d|\mathbf{p}|\,|\mathbf{p}|^2 \frac{\Gamma_t^2}{\Big|\underbrace{\frac{E}{2}+p_0-\frac{\mathbf{p}^2}{2m_t}}_{(q^2-m_t^2)/2m_t}+i\frac{\Gamma_t}{2}\Big|^2\,\Big|\frac{E}{2}-p_0-\frac{\mathbf{p}^2}{2m_t}+i\frac{\Gamma_t}{2}\Big|^2}$$

$|q^2-m_t^2|<\Lambda^2 \lesssim m_t^2$
double-resonant
non-rel. expansion valid
Λ: cut on inv. mass of top/antitop



unphys. region of the EFT
single-/non-resonant
subtracted by local expansion
Matching conditions:
$C_1(m_t,\Lambda), C_2(m_t,\Lambda), \ldots$

Phase space cut

Experimental cut Λ

- Λ corresponds to the maximal invariant mass of a detected bW pair that is associated with a top quark decay event

$\Rightarrow$ Cross section is differential in exp. parameter Λ:

$$\sigma_{\mathrm{thr}}(\Lambda)$$

Finite imaginary renormalization

Formal treatment: operator product expansion

- Finite imaginary renormalization of the EFT operators
- Phase space effects are contained in Wilson coefficients
- Coefficients are imaginary and proportional to powers of Γ_t or $C_{\rm NNLL}^{\rm int}$
  $\longrightarrow$ top quark instability
- Various contributions characterized by power counting of Λ

Finite imaginary renormalization

Formal treatment: operator product expansion

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  $\rightarrow$ top quark instability
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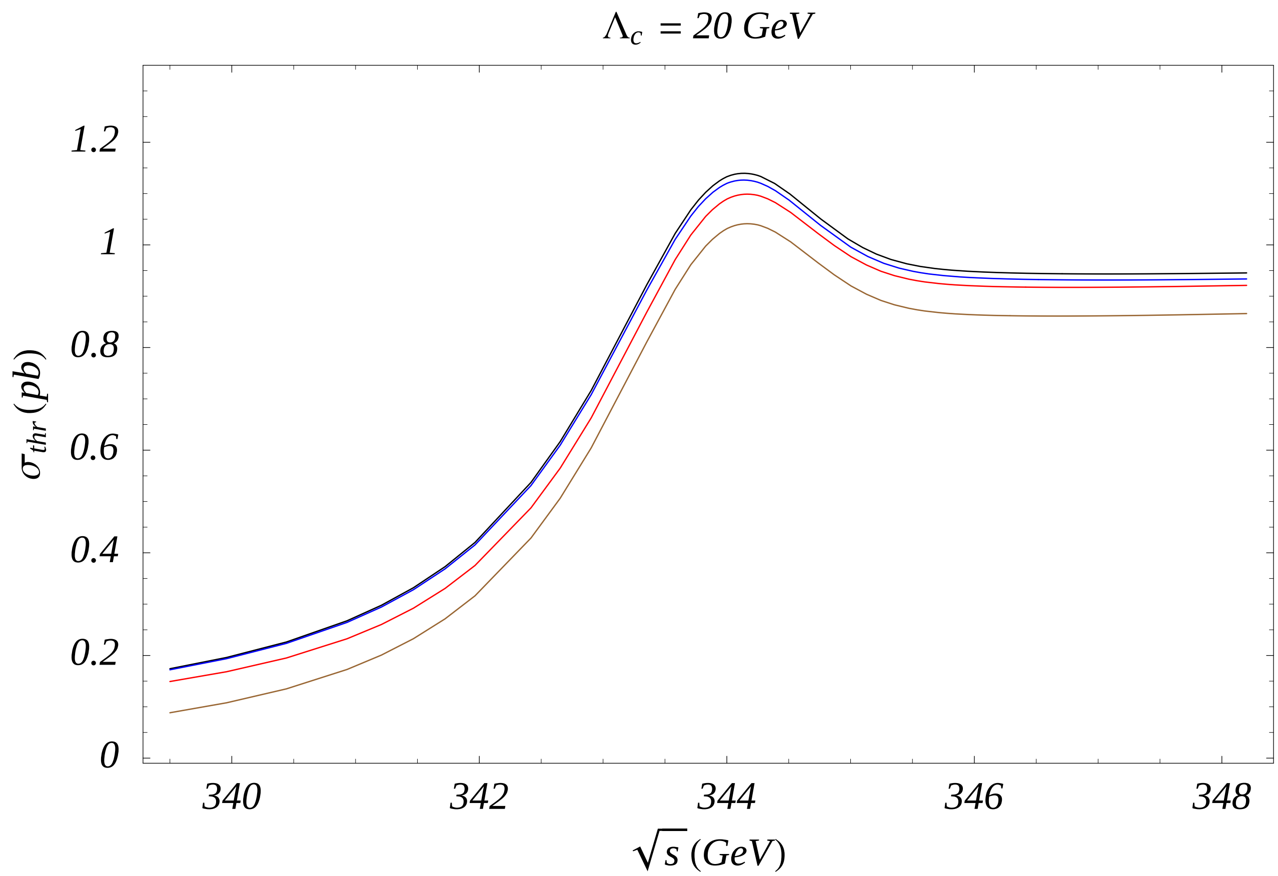
Scaling of the cut: $\boxed{\Lambda^2 \lesssim m_t^2}$

- Power counting breaking: natural scaling $\Lambda^2 \sim 2\, m_t E \sim m_t^2 v^2$
  - – Higher dimension operators not suppressed
  - + However: $\frac{\Lambda}{m_t} < 1$ allows for sufficient numerical suppression
    $\rightarrow$ typical choice $\Lambda \approx 0.6\, m_t$

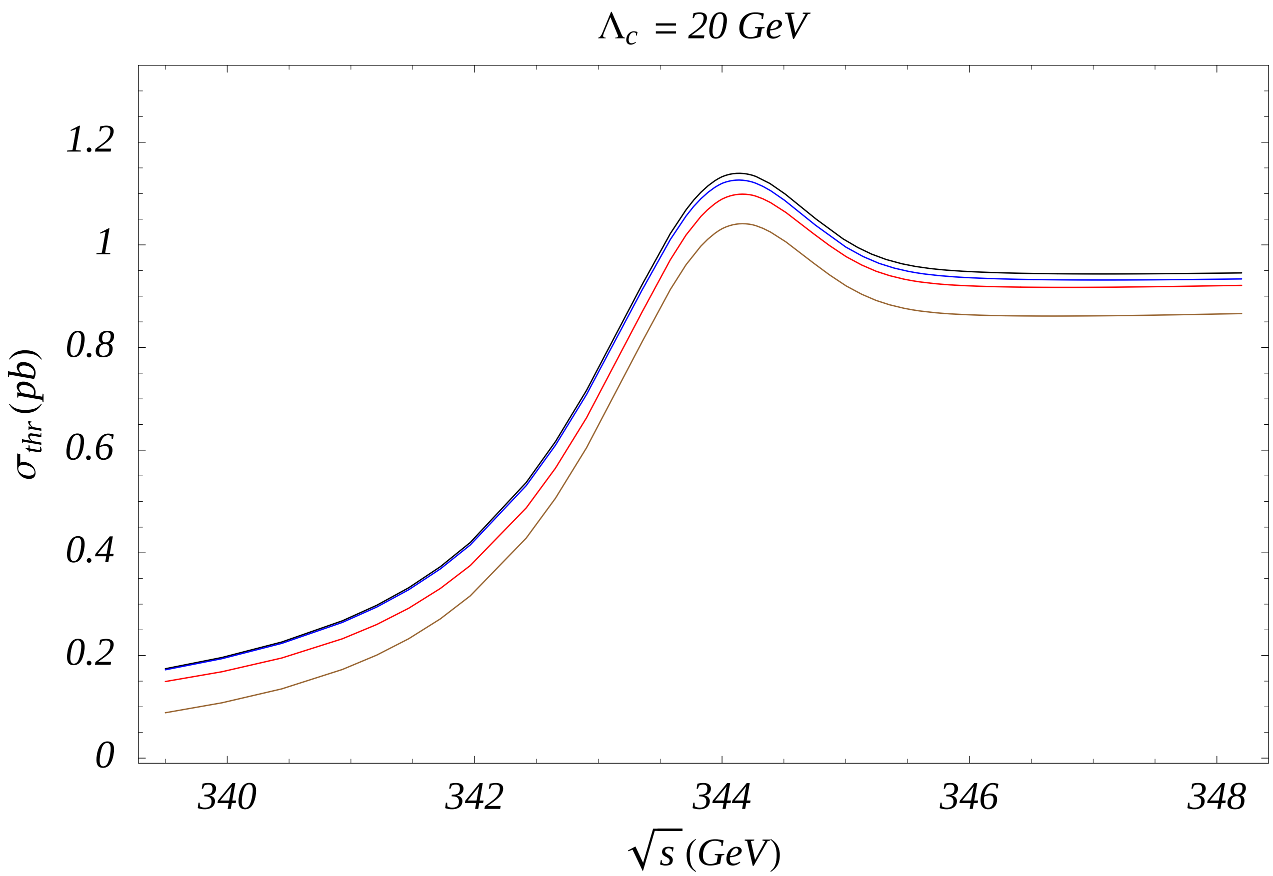
$\Rightarrow$ Mild power counting breaking

Numerical results



- — Standard vNRQCD cross section (NNLL Coulomb Green function), $\nu = 0.2$
- — plus usual electroweak corrections at NNLL (no QED corr.), $\mathrm{M_H} = 130$ GeV
- — plus absorptive electroweak (instability) effects at NLL and NNLL
- — plus phase space effects at NLO and NNLO (cut: $\Lambda = \sqrt{2\mathrm{m_t} \times 20\,\mathrm{GeV}} \approx 80$ GeV)

Numerical results



— Standard vNRQCD cross section

— plus usual electroweak corrections $\Rightarrow$ change of normalization: $\approx -2\,\%$

— plus absorptive electroweak effects $\Rightarrow$ top mass shift: $\approx$ 50 MeV

— plus phase space effects at NLO and NNLO $\Rightarrow$ change of $\sigma_{\rm thr}$: ≈ -60 fb

Summary

- Threshold scan allows for a precise determination of $\mathrm{m_t, y_t}, \Gamma_\mathrm{t}, \alpha_\mathrm{s}$
- Effective theory important for summation of threshold effects

Top instability leads to

- Complex Wilson coefficients
- UV phase space divergences
- Matching conditions for the $\mathrm{t\bar{t}}$ phase space, dependent on the definition of a “Top pair event”
- Phase space cut leads to mild power counting breaking
- Phase space effects are large: NLO

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Outlook:

- QED contributions: ISR, Coulomb singularities
- NNLL running of $(\mathrm{e^+e^-})(\mathrm{e^+e^-})$ operators
- Investigation of effects due to ultrasoft gluons in phase space matching

Backup slides

vNRQCD Lagrangian

Luke, Manohar, Rothstein (Phys. Rev. D 61, 2000)
Hoang, Stewart (Phys. Rev. D 67, 2003)

$$\mathcal{L} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}$$

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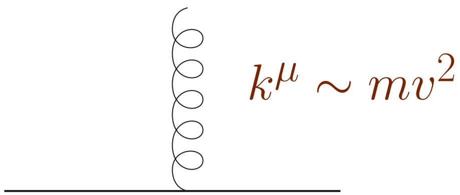
- $\mathcal{L}_{\mathrm{usoft}} = \sum_{\mathbf{p}} \psi_{\mathbf{p}}^{\dagger} \left[\mathrm{i}D^0 - \frac{(\mathbf{p} - \mathrm{i}\mathbf{D})^2}{2\mathrm{m}} + \frac{\mathbf{p}^4}{8\mathrm{m}^3} + \ldots \right] \psi_{\mathbf{p}}$

$$D^{\mu} = \partial^{\mu} + \mathrm{i}g_s A^{\mu}$$



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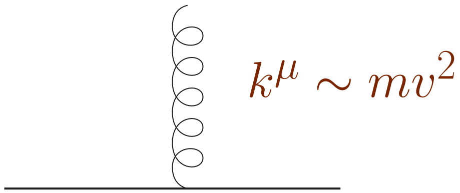
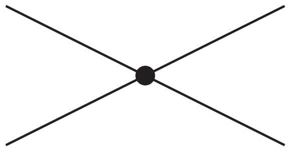
- $$\mathcal{L}_{\rm usoft} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left[\mathrm{i}D^0 - \frac{(\mathbf{p}-\mathrm{i}\mathbf{D})^2}{2\mathrm{m}} + \frac{\mathbf{p}^4}{8\mathrm{m}^3} + \dots \right] \psi_{\mathbf{p}}$$

$$D^\mu = \partial^\mu + \mathrm{i}g_s A^\mu$$



- $$\mathcal{L}_{\rm pot} = -\sum_{\mathbf{p},\mathbf{p}'} \left[\frac{\mathcal{V}_c}{(\mathbf{p}-\mathbf{p}')^2} + \dots \right] \psi^\dagger_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^\dagger_{-\mathbf{p}'} \chi_{-\mathbf{p}}$$



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$$\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm pot} + \mathcal{L}_{\rm soft}$$

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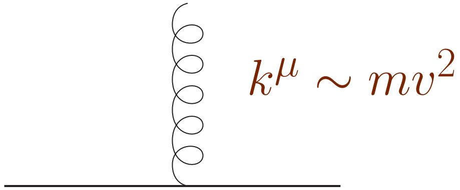
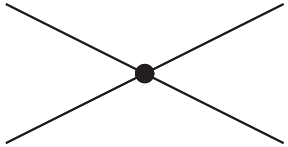
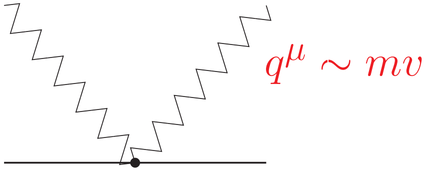
$$\mathrm{D}^\mu = \partial^\mu + \mathrm{i}\mathrm{g}_\mathrm{s}\mathrm{A}^\mu$$



- $$\mathcal{L}_{\rm pot} = -\sum_{\mathbf{p},\mathbf{p}'} \left[\frac{\mathcal{V}_\mathrm{c}}{(\mathbf{p}-\mathbf{p}')^2} + \dots \right] \psi^\dagger_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^\dagger_{-\mathbf{p}'} \chi_{-\mathbf{p}}$$



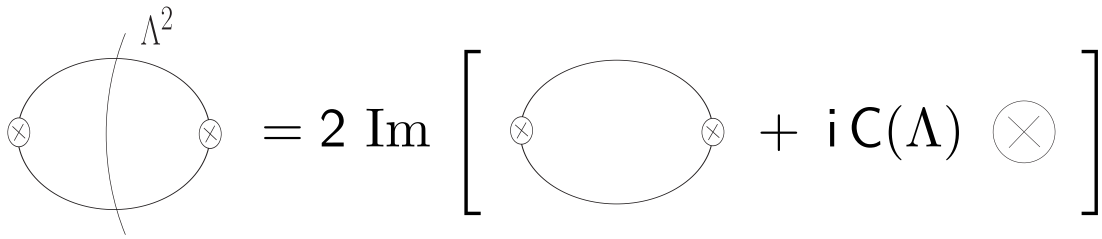
- $$\mathcal{L}_{\rm soft} = -\mathrm{g}_\mathrm{s}^2 \sum_{\mathbf{p},\mathbf{p}',\mathrm{q},\mathrm{q}'} \left[\frac{1}{2} \psi^\dagger_{\mathbf{p}'} [\mathrm{A}^\mu_{\mathrm{q}'}, \mathrm{A}^\nu_{\mathrm{q}}] \mathrm{U}_{\mu\nu} \psi_{\mathbf{p}} + \dots \right]$$



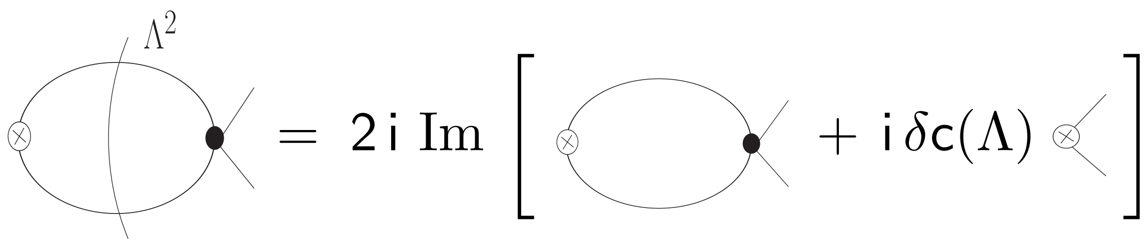
Finite imaginary renormalization

1-loop example:

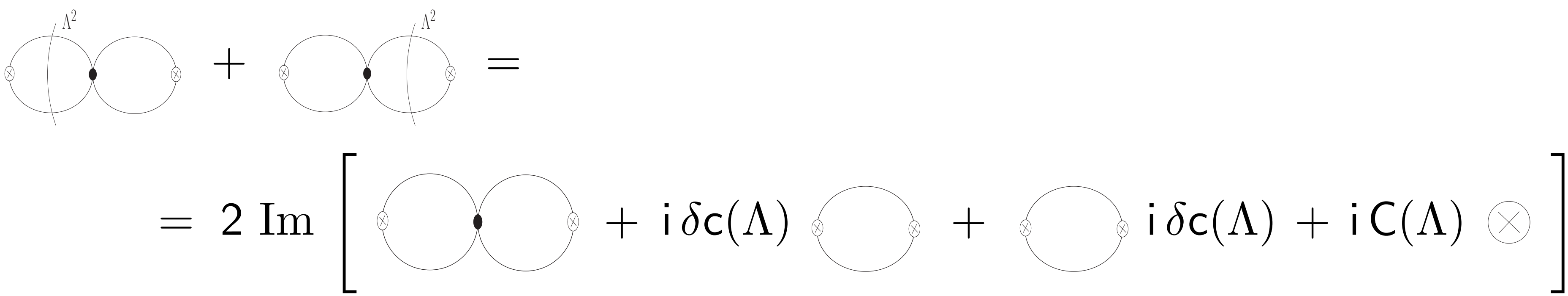
$(e^+e^-)(e^+e^-)$ operator



$(e^+e^-)(t\bar{t})$ production current



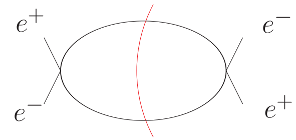
2-loop example: $(e^+e^-)(e^+e^-)$ operator



Calculation of phase space effects

Formal counting $\Lambda^2 \lesssim m_t^2$ numerically $\Lambda = \sqrt{2\, m_t \times 30\,\text{GeV}} \approx 100$ GeV

NLO Leading prod. current



$$\sim i \left[\# \frac{\Gamma_t}{\Lambda} + \frac{\# m_t \hat{E} \Gamma_t + \# m_t \Gamma_t^2}{\Lambda^3} + \ldots \right]$$

$$\sim i\ [-0.014 - 0.00007 + \ldots]$$

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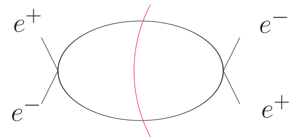

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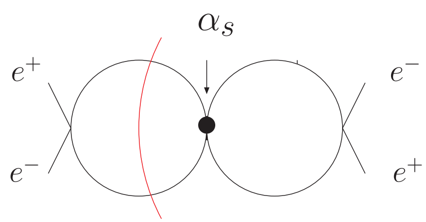
$$\sim i \, [-0.014 - 0.00007 + \ldots]$$

NNLO Coulomb insertion



$$\sim i \left[\# \, \pi \alpha_s \, \frac{m_t \Gamma_t}{\Lambda^2} + \ldots \right]$$

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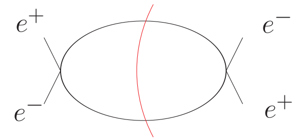

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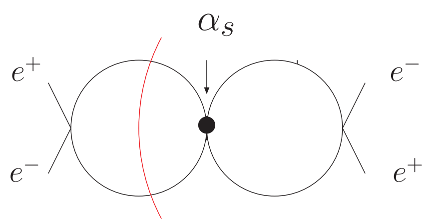

$$\sim \mathrm{i}\left[\#\,\pi\alpha_s\,\frac{m_t\Gamma_t}{\Lambda^2} + \ldots\right]$$

$$\sim \mathrm{i}\,[-0.007 + \ldots]$$

Power counting breaking terms:

Insertions of higher dimension operators, e. g. kinetic energy correction

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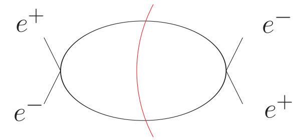

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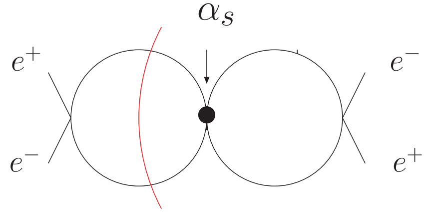

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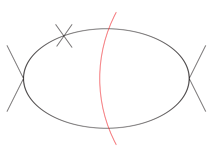
$$\sim i \, [-0.007 + \ldots]$$

Power counting breaking terms:

Insertions of higher dimension operators, e. g. kinetic energy correction

NLO $\frac{\mathbf{p}^4}{8 m_t^3}$



$$\sim i \left[\# \frac{\Gamma_t \Lambda}{m_t^2} + \ldots \right]$$

$$\sim i \, [0.001 + \ldots]$$

$\Rightarrow$ Suppression by *mild* power counting breaking