
The lightest neutralino in the Minimal Non-minimal Supersymmetric Standard Model



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based on

SH, D.J. Miller, G. Moortgat-Pick, R. Nevzorov, M. Trusov,
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Outline

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 - Supersymmetry, MSSM, NMSSM
 - Minimal Non-minimal Supersymmetric Standard Model (MNSSM)
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- Approximate solutions for LSP mass
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Supersymmetry (SUSY)

- Symmetry fermions \leftrightarrow bosons
- “Standard” particles get superpartners with spin $\pm\frac{1}{2}$
- Supersymmetry is broken \Rightarrow soft SUSY parameters
- Motivation: unification of gauge couplings, hierarchy problem

Minimal Supersymmetric Standard Model (MSSM)

- Minimal extension of Standard Model (SM)
- Minimal Higgs sector: 2 doublets
- μ term in superpotential: $W_\mu = \mu \hat{H}_d \hat{H}_u \leftrightarrow \mu$ problem
- Solution: extended Higgs sector

“Simplest” extension of MSSM \rightarrow add **singlet/singlino superfield \hat{S}**
 \rightarrow extended Higgs sector: 2 doublets + 1 singlet
and neutralino sector: 5 neutralinos

Most common model:

Next-to-Minimal Supersymmetric Standard Model (NMSSM)

- Superpotential: $W = \lambda \hat{S} \hat{H}_d \hat{H}_u + \frac{\kappa}{3} \hat{S}^3 + W_{\text{MSSM}}(\mu = 0)$
- Solution of μ problem: $\mu \rightarrow \mu_{\text{eff}} = \lambda \langle S \rangle$
- Larger mass of lightest scalar Higgs possible \Rightarrow less fine-tuning
- $\frac{\kappa}{3} \hat{S}^3$ breaks $U(1)_{\text{PQ}}$, \mathbb{Z}_3 symmetry remains
- Problem: spontaneous breaking of $\mathbb{Z}_3 \leftrightarrow$ domain walls

[Vilenkin, '85; Abel, Sarkar, White, '95]

Minimal Non-minimal Supersymmetric Standard Model (MNSSM)

[Panagiotakopoulos, Tamvakis, '99; Panagiotakopoulos, Pilaftsis, '00]
[Dedes, Hugonie, Moretti, Tamvakis, '00; Menon, Morrissey, Wagner, '04]

- Superpotential: $W = \lambda \hat{S} \hat{H}_d \hat{H}_u + \xi \hat{S} + W_{\text{MSSM}}(\mu = 0)$ ($\xi \lesssim 1 \text{ TeV}$)
- Gravity induced tadpole term $\xi \hat{S}$ breaks $U(1)_{\text{PQ}}$ and \mathbb{Z}_3
→ no domain wall problem
- Solution of μ problem as in NMSSM ($\mu_{\text{eff}} = \lambda \langle S \rangle$)
- Neutralino sector \leftrightarrow NMSSM with $\kappa = 0$
- Chargino sector: like MSSM with $\mu \rightarrow \mu_{\text{eff}}$

MNSSM Neutralino sector

• 5 Parameters (tree-level): $M_1, M_2, \mu_{\text{eff}}, \lambda, \tan \beta$

• (5×5) mass matrix:

(basis $\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}$)

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta & 0 \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta & 0 \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu_{\text{eff}} & -\frac{\lambda v}{\sqrt{2}} s_\beta \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu_{\text{eff}} & 0 & -\frac{\lambda v}{\sqrt{2}} c_\beta \\ 0 & 0 & -\frac{\lambda v}{\sqrt{2}} s_\beta & -\frac{\lambda v}{\sqrt{2}} c_\beta & 0 \end{pmatrix}$$

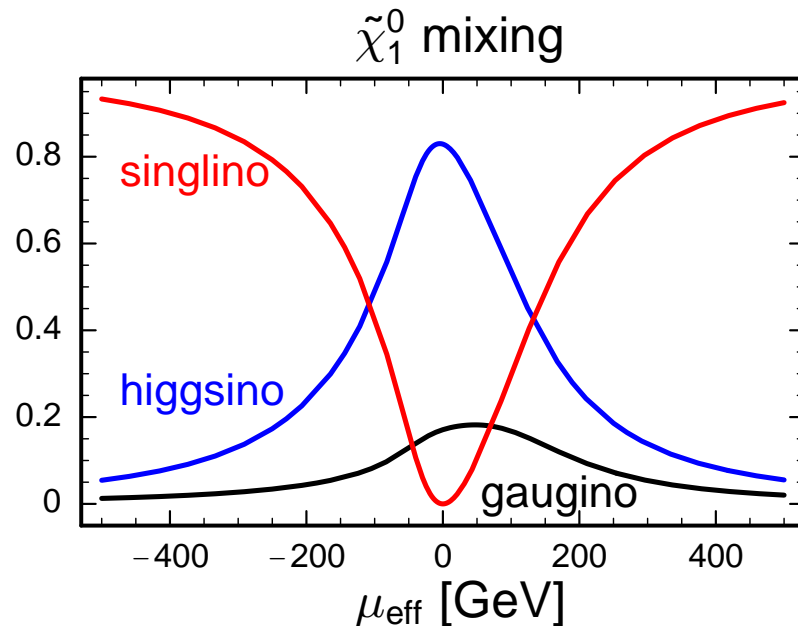
$$s_\beta = \sin \beta, c_\beta = \cos \beta, s_W = \sin \theta_W, c_W = \cos \theta_W$$

$$\mu_{\text{eff}} = \lambda \langle S \rangle, \tan \beta = \frac{v_2}{v_1}, v^2 = v_1^2 + v_2^2, \frac{v_{1,2}}{\sqrt{2}} = \langle H_{d,u} \rangle$$

[NMSSM: $(M_{\tilde{\chi}^0})_{55} = 2\kappa \langle S \rangle$]

MNSSM Neutralino sector

- Perturbativity up to the GUT scale $\Rightarrow \lambda(M_Z) \lesssim 0.7$
- Limits from chargino search $\Rightarrow |M_2|, |\mu_{\text{eff}}| \gtrsim 100 \text{ GeV}$
- GUT relation for gaugino masses $\Rightarrow M_2 = 2M_1$
(assumed in the following)
- $\tilde{\chi}_1^0$ singlino-like, $\tilde{\chi}_{2,3,4,5}^0$ MSSM-like



$$\lambda = 0.7$$

$$\tan \beta = 5$$

$$M_2 = 200 \text{ GeV}$$

$$M_1 = 100 \text{ GeV}$$

Upper bound on $\tilde{\chi}_1^0$ mass

- Consider $M_{\tilde{\chi}^0} M_{\tilde{\chi}^0}^\dagger$ with eigenvalues $|m_{\tilde{\chi}_i^0}|^2$
- In basis $(\tilde{B}, \tilde{W}_3, -\tilde{H}_d^0 s_\beta + \tilde{H}_u^0 c_\beta, \tilde{H}_d^0 c_\beta + \tilde{H}_u^0 s_\beta, \tilde{S})$
 \rightarrow bottom-right 2×2 block of $M_{\tilde{\chi}^0} M_{\tilde{\chi}^0}^\dagger$:

$$\begin{pmatrix} |\mu_{\text{eff}}|^2 + \sigma^2 & \nu^* \mu_{\text{eff}} \\ \nu \mu_{\text{eff}}^* & |\nu|^2 \end{pmatrix}$$

$$\nu = \frac{\lambda v}{\sqrt{2}}, \quad \sigma^2 = M_Z^2 \cos^2 2\beta + |\nu|^2 \sin^2 2\beta$$

- Immediately obvious: $|m_{\tilde{\chi}_1^0}| \leq |\nu| = \frac{\lambda v}{\sqrt{2}}$

(minimal eigenvalue is smaller than smallest diagonal entry)

- $|m_{\tilde{\chi}_1^0}| \rightarrow 0$ for $\lambda \rightarrow 0$

Upper bound on $\tilde{\chi}_1^0$ mass

- Improvement:

$|m_{\tilde{\chi}_1^0}|^2$ also smaller than smallest eigenvalue of sub-matrix:

$$|m_{\tilde{\chi}_1^0}|^2 \leq \frac{1}{2} \left(|\mu_{\text{eff}}|^2 + \sigma^2 + |\nu|^2 - \sqrt{\left(|\mu_{\text{eff}}|^2 + \sigma^2 + |\nu|^2 \right)^2 - 4|\nu|^2\sigma^2} \right)$$

- Upper bound decreasing for increasing $|\mu_{\text{eff}}|$

→ for minimal $|\mu_{\text{eff}}|$ and maximal λ :

$$m_{\tilde{\chi}_1^0} \lesssim 80 \text{ GeV}$$

- For large $|\mu_{\text{eff}}| \gg M_Z$:

$$|m_{\tilde{\chi}_1^0}|^2 \lesssim \frac{|\nu|^2\sigma^2}{\left(|\mu_{\text{eff}}|^2 + \sigma^2 + |\nu|^2 \right)}$$

Approximate solution for $\tilde{\chi}_1^0$ mass

- Characteristic equation (κ : eigenvalues of $M_{\tilde{\chi}^0} \leftrightarrow m_{\tilde{\chi}_i^0}$)

$$\det(M_{\tilde{\chi}^0} - \kappa I) = \left(M_1 M_2 - (M_1 + M_2)\kappa + \kappa^2 \right) \left(\kappa^3 - (\mu_{\text{eff}}^2 + \nu^2)\kappa + \nu^2 \mu_{\text{eff}} \sin 2\beta \right) + M_Z^2 \left(\tilde{M} - \kappa \right) \left(\kappa^2 + \mu_{\text{eff}} \sin 2\beta \kappa - \nu^2 \right) = 0$$

$$\tilde{M} = M_1 c_W^2 + M_2 s_W^2$$

- If $|m_{\tilde{\chi}_1^0}| \ll |m_{\tilde{\chi}_2^0}| \rightarrow$ ignore $\kappa^3, \kappa^4, \kappa^5$ terms

$$\Rightarrow \det(M_{\tilde{\chi}^0} - \kappa I) \approx \kappa^2 - B\kappa + C = 0$$

with

$$B = \frac{M_1 M_2}{M_1 + M_2} + \left(\frac{\nu^2}{\mu_{\text{eff}}^2 + \nu^2} - \frac{M_Z^2}{\mu_{\text{eff}}^2 + \nu^2} \frac{\tilde{M}}{M_1 + M_2} \right) \mu_{\text{eff}} \sin 2\beta - \frac{M_Z^2 \nu^2}{(M_1 + M_2)(\mu_{\text{eff}}^2 + \nu^2)}$$

$$C = \frac{\nu^2}{\mu_{\text{eff}}^2 + \nu^2} \left(\frac{M_1 M_2}{M_1 + M_2} \mu_{\text{eff}} \sin 2\beta - \frac{\tilde{M}}{M_1 + M_2} M_Z^2 \right)$$

Approximate solution for $\tilde{\chi}_1^0$ mass

- Approximate solution

$$|m_{\tilde{\chi}_1^0}| = \text{Min} \left\{ \frac{1}{2} \left| B - \sqrt{B^2 - 4C} \right|, \frac{1}{2} \left| B + \sqrt{B^2 - 4C} \right| \right\}$$

- Decoupling limit

- $\mu_{\text{eff}} \gg M_Z$ or $M_1, M_2 \gg M_Z$, small $\tan \beta \rightarrow B^2 \gg C$

$$\Rightarrow |m_{\tilde{\chi}_1^0}| \simeq \frac{C}{B} \simeq \frac{|\mu_{\text{eff}}| \nu^2 \sin 2\beta}{\mu_{\text{eff}}^2 + \nu^2}$$

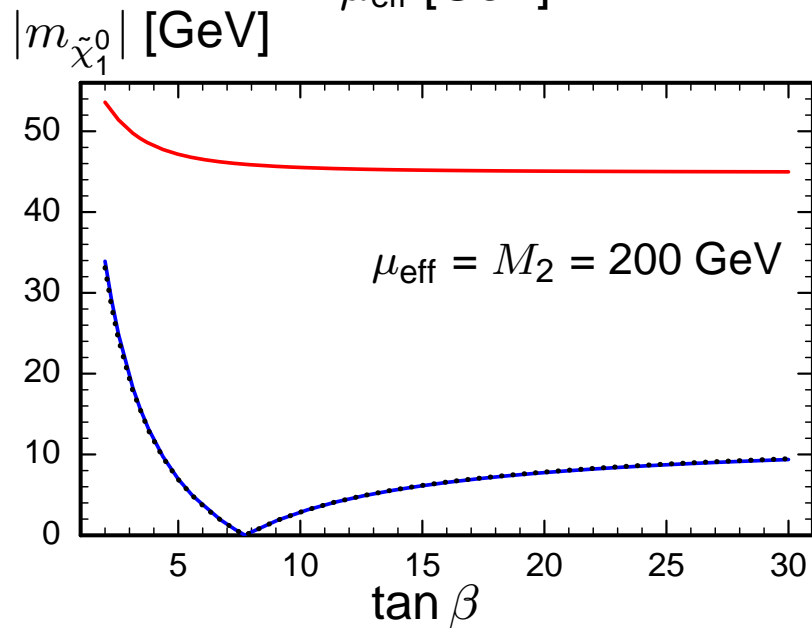
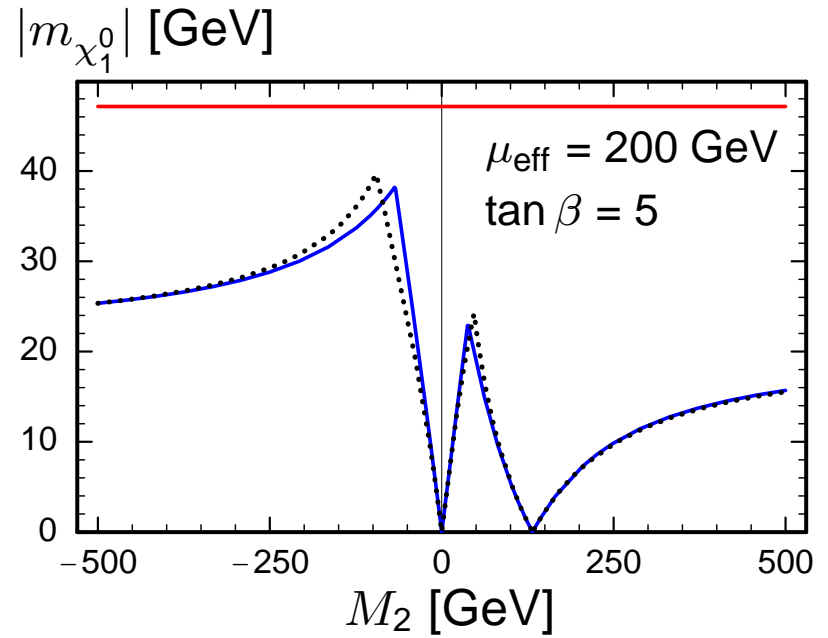
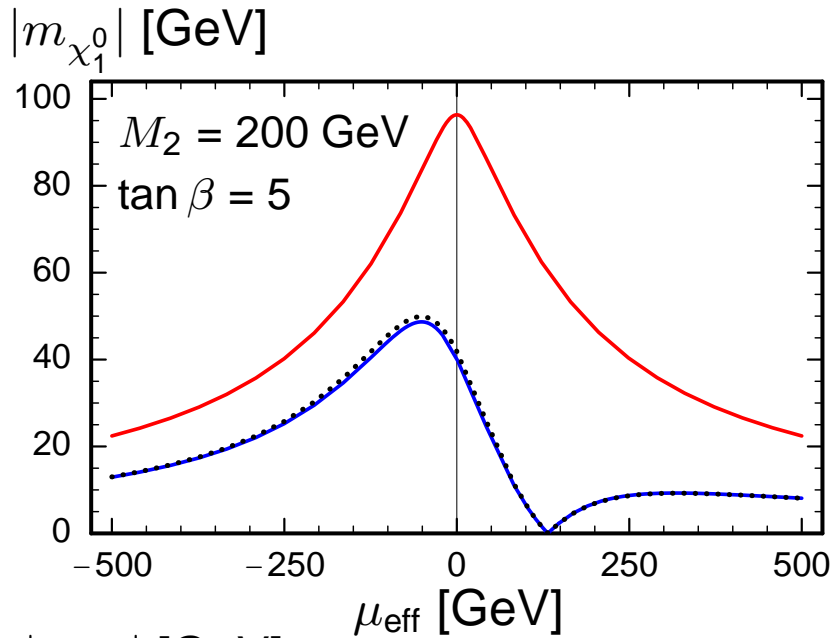
- $\mu_{\text{eff}}, M_1, M_2 \gg M_Z$, large $\tan \beta$

$$\Rightarrow |m_{\tilde{\chi}_1^0}| \rightarrow \frac{\nu^2 M_Z^2}{\mu_{\text{eff}}^2 + \nu^2} \left| \frac{\tilde{M}}{M_1 M_2} \right|$$

$\rightarrow |m_{\tilde{\chi}_1^0}|$ decreases with increasing $\mu_{\text{eff}}, M_1, M_2$

$\rightarrow |m_{\tilde{\chi}_1^0}| \sim \nu^2 \sim \lambda^2$ for $\lambda \rightarrow 0$ and $\mu_{\text{eff}} = \lambda \langle S \rangle = \text{const}$

Numerical results



$\lambda = 0.7, M_1 = 0.5M_2$

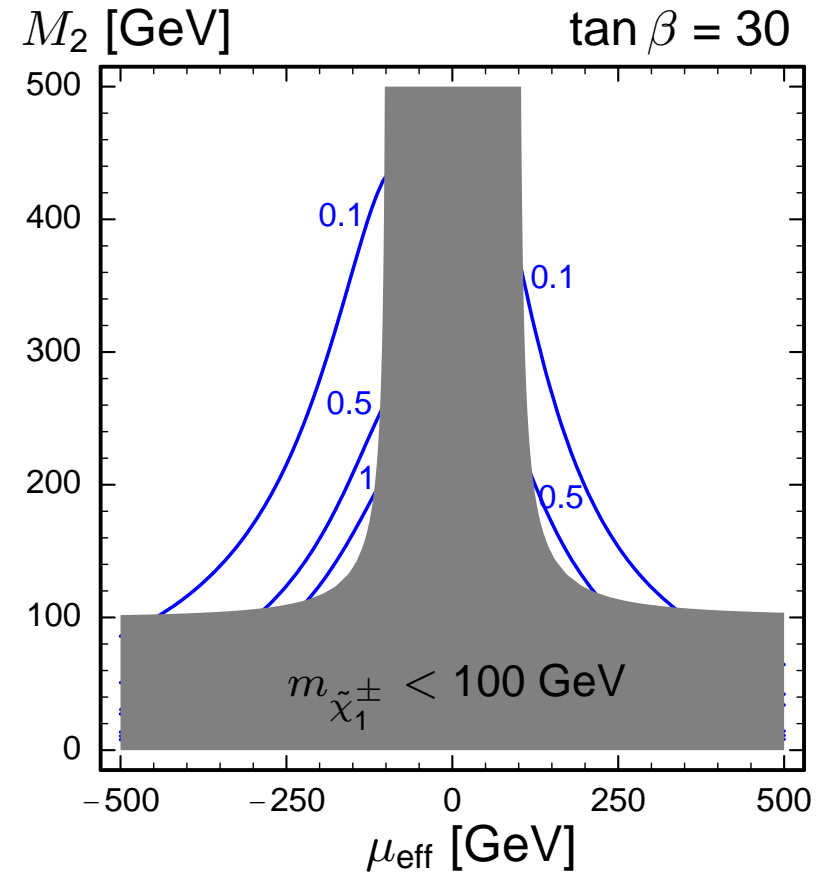
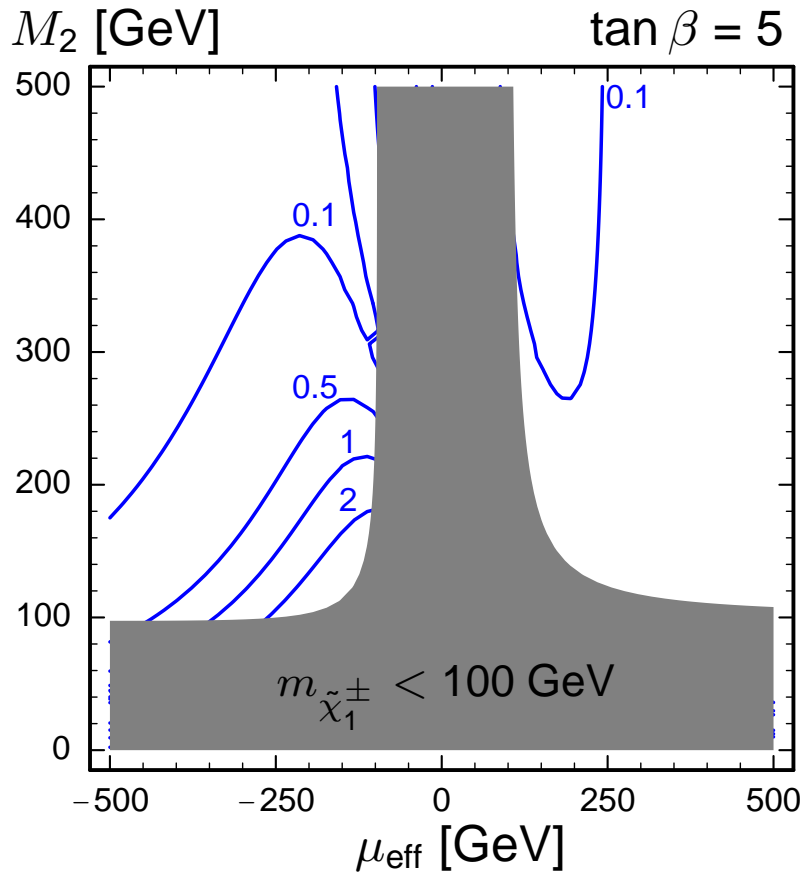
red: upper bound

dotted: approximate solution

blue: exact $|m_{\tilde{\chi}_1^0}|$

Numerical results

Contours of difference between approximate and exact solution [GeV]



$$\lambda = 0.7, M_1 = 0.5M_2$$

→ difference < 1 GeV in most of allowed parameter space

Production of $\tilde{\chi}_1^0$ at ILC

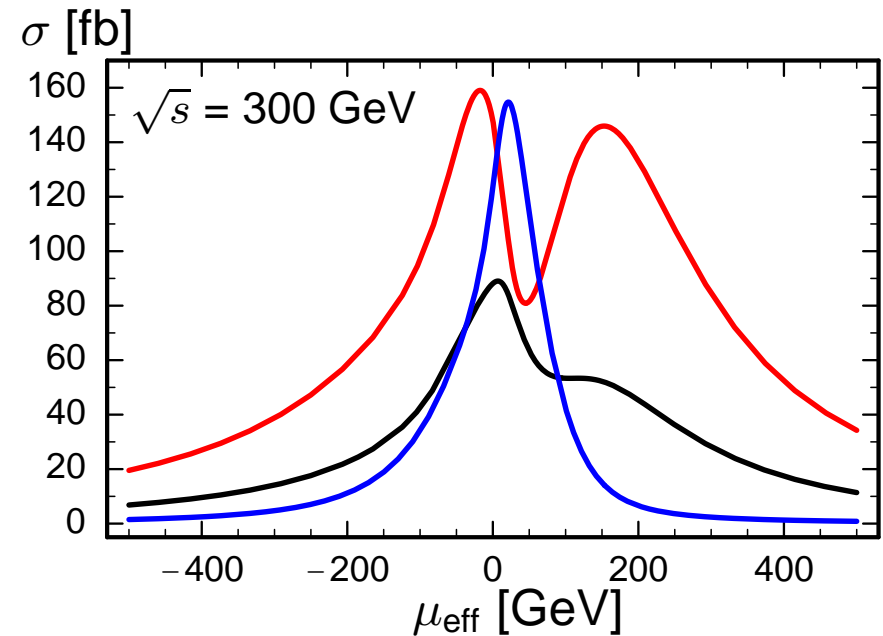
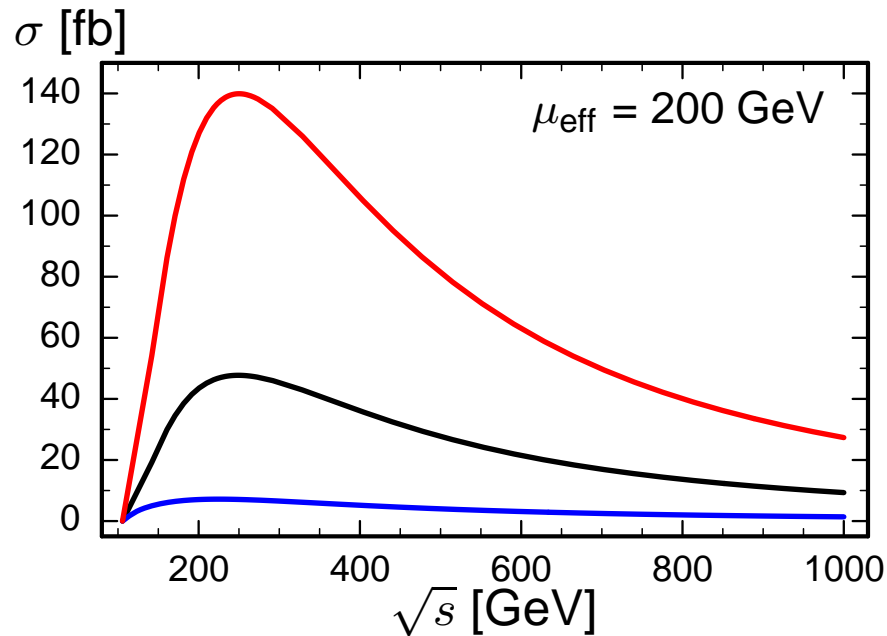
- Singlino-like neutralinos visible at ILC up to high purity $\sim 99\%$

[Franke, SH, '01, '02]

- $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$ in MNSSM

for $\lambda = 0.7$, $\tan\beta = 5$, $M_2 = 200$ GeV, $M_1 = 100$ GeV, $m_{\tilde{e}_L} = 200$ GeV, $m_{\tilde{e}_R} = 150$ GeV

beam polarizations $(P_{e^-}, P_{e^+}) = (0, 0)$, $(90\%, -60\%)$, $(-90\%, 60\%)$



Conclusions and outlook

- Lightest neutralino $\tilde{\chi}_1^0$ in MNSSM
 - predominantly singlino in most of allowed parameter space
- Upper bound and approximate solution for $m_{\tilde{\chi}_1^0}$ in MNSSM
 - $|m_{\tilde{\chi}_1^0}| \lesssim 80 \text{ GeV}$ (for large $\lambda \sim 0.7$ and small $M_1, M_2, \mu_{\text{eff}}, \tan\beta$)
 - $|m_{\tilde{\chi}_1^0}|$ decreasing with increasing $M_1, M_2, \mu_{\text{eff}}$ or decreasing λ
- Visible at ILC in large regions of parameter space
- Cosmological implications
 - Correct relic density through Z and (singlet) Higgs exchange
[Menon, Morrissey, Wagner, '04]
 - Analogue singlino dark matter in NMSSM
[e.g. Bélanger, Boudjema, Hugonie, Pukhov, Semenov, '05]