Dominant NNLO corrections to W-pair production near threshold

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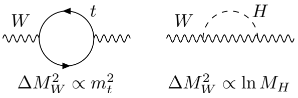
Motivation

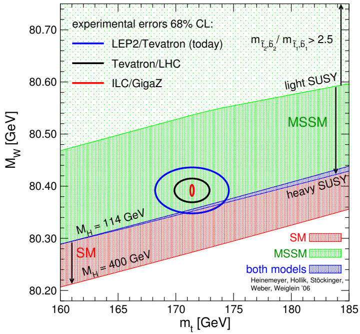
Process of *W*-pair production near threshold has phenomenological relevance for accurate determination of the *W* boson mass at ILC

M_W

- Key observable in the search of virtual-particle effects through EW precision measurements
- Precise determination of M_W and m_t constrains M_H (and New Physics effects)



Threshold scan of *WW* cross section at ILC could reduce the total error on M_W to $\delta M_W \sim 6\text{MeV}$ [Wilson '01] with $\mathcal{L} = 100\text{fb}^{-1}$ (just one year!)



[Heinemeyer et al. '06]

Total theoretical uncertainties must be reduced to 1‰!

Theoretical issues

Instability of W can not be neglected! ($\Gamma_W/M_W \sim 2.5\%$) $\longrightarrow$ Physical process $e^- e^+ \longrightarrow 4f$

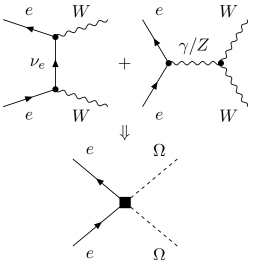
- Systematic inclusion of finite-width effects (may lead to gauge-inv. and unitarity violation)
- Calculation of EW and QCD radiative corrections (difficult for multiparticle final states)

At present two methods give theoretical predictions for four-fermion production near the WW threshold with accuracy better than 1%

1. Complete $O(\alpha)$ $e^- e^+ \longrightarrow 4f$ in Complex Mass Scheme [Denner, Dittmaier, Roth, Wieders '05]
2. Effective Field Theory Approach [Beneke, Chapovsky, Signer, Zanderighi '04]
   - Gauge-invariant expansion around the complex pole (systematization to threshold of the Double Pole Approximation)
   - Computationally simple + final analytic expressions
   - Valid only in the threshold region ($\sqrt{s} \sim 155 - 170$ GeV)

Effective Field Theory Approach

The process is characterized by two well-separated scales: $\Lambda^2 \equiv M_W^2 \gg M_W\Gamma_W \equiv \lambda^2$
$\rightarrow$ Effective Field Theory (EFT) techniques are used to integrate out the large scale M_W^2



Similar techniques used in $t\bar{t}$ production

[Hoang et al. '04, Hoang et al. '07]

- Effective Lagrangian describing long-distance degrees of freedom
  $(k^2 - m_p^2 \lesssim M_W\Gamma_W)$
  - resonant non-relativistic Ws $(k^2 - M_W^2 \sim M_W\Gamma_W)$
  - Coulomb $(k^2 \sim M_W\Gamma_W)$ and soft $(k^2 \sim \Gamma_W^2)$ photons
  - high-energetic external fermions $(k^2 = 0)$
- Matching coefficients determined by short-distance physics
  $(k^2 - m_p^2 \sim M_W^2)$
  - non-resonant Ws $(k^2 - M_W^2 \sim M_W^2)$
  - light degrees of freedom with large virtualities $(k^2 \sim M_W^2)$
  - heavy degrees of freedom (Z boson, Higgs, top quark)

Reorganized expansion of the matrix element in the couplings α_{ew}, α_s and $\frac{\Gamma_W}{M_W}$, $v^2 \equiv \frac{\sqrt{s}-2M_W}{M_W}$

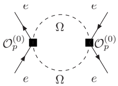
$$\alpha_s^2 \sim \alpha_{ew} \sim \frac{\Gamma_W}{M_W} \sim v^2$$

Collectively referred to as δ!

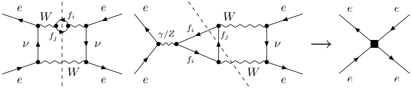
A toy example: the $e^-e^+ \to 4f$ Born cross section

How does the EFT work? → To understand apply it to the calculation of the Born cross section
Extract the total cross section from unitarity cuts of the forward-scattering amplitude

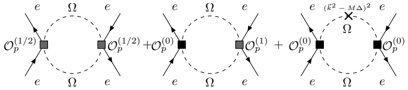
LO ($\sim \alpha_{ew} v$)
From leading-order operators and propagators



$\sqrt{\text{N}}$LO ($\sim \alpha_{ew}^2 \Gamma_W / M_W$)
From non-resonant configurations



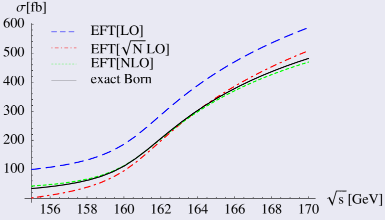
NLO ($\sim \alpha_{ew}^2 v \times (v^2, \Gamma_W/M_W)$)
From higher-dimensional operators



$\sigma_{\text{Born}}(e^-e^+ \to \mu^- \bar{\nu} u \bar{d})$

Comparison with numerical result from Whizard/CompHep

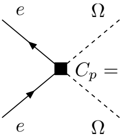
[Kilian, Boos et al. '04, Pukhov et al. '99]



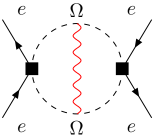
NLO cross section

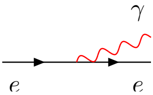
EFT formalism applied to the computation of the total cross section of $e^- e^+ \to \mu^- \bar{\nu} u \bar{d} + X$ up to NLO in δ [Beneke, PF, Schwinn, Signer, Zanderighi '07]

Pragmatic approach: compute the Born cross section with Whizard/CompHep and use the EFT to compute genuine radiative corrections

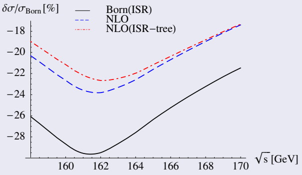


$C_p = C_p^{(0)} + \frac{\alpha}{2\pi} C_p^{(1)} + \ldots$





Relative radiative corrections



Comparison with full $e^- e^+ \to$ 4f calculation

[Denner, Dittmaier, Roth, Wieders '05]

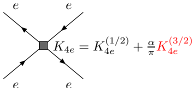
	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d}\, X)$(fb)			
$\sqrt{s}$ [GeV]	Born(ISR)	NLO(EFT)	ee4f	DPA
161	107.06(4)	117.38(4)	118.12(8)	115.48(7)
170	381.0(2)	399.9(2)	401.8(2)	402.1(2)

In the energy range $\sqrt{s} \sim 161 - 170$ difference between the two results $\lesssim 0.6\%$!

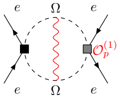
Dominant NNLO corrections

Difference between full $e^-e^+ \to 4f$ calculation and EFT result at NLO due to one-loop corrections $\Delta\sigma_X$ which are subleading in the EFT counting scheme

- $O(\alpha)$ contributions from non-resonant configurations:



- Interference of higher-dimensional EFT operators with Coulomb-photon exchange:



In EFT reorganized expansion these are $\mathrm{N}^{3/2}\mathrm{LO}$ corrections: $\frac{\Delta\sigma_X}{\sigma_{\mathrm{Born}}} \sim \alpha_{ew} v \sim \alpha_{ew}^{3/2} \sim 0.5\%$

Are these all the possible $\mathrm{N}^{3/2}\mathrm{LO}$ corrections? NO!

NNLO diagrams can be enhanced by inverse powers of v: $\frac{\Delta\sigma}{\sigma_{\mathrm{Born}}} \sim \alpha_{ew}^2/v \sim \alpha_{ew}^{3/2} \sim 0.5\%$

(This translates to $\delta M_W^{\mathrm{th}} \sim 5\,\mathrm{MeV}$)

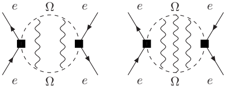
$\Downarrow$

Compute all the $\mathrm{N}^{3/2}\mathrm{LO}$ contributions not already included in the full $e^-e^+ \to 4f$ result!

(Dominant NNLO corrections)

$N^{3/2}LO$ corrections (I)

1. Double and triple Coulomb-photon exchange



Near threshold exchange of n Coulomb photons ($q_0 \sim \Gamma_W$, $|\vec{q}| \sim \sqrt{M_W \Gamma_W}$) is suppressed by $(\alpha/v)^n \sim \alpha^{n/2}$ compared to the Born cross section

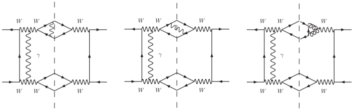
$\Downarrow$

2^{nd} Coulomb correction ($\sim \alpha$) is a NLO effect not included in the full 4f calculation and 3^{rd} Coulomb correction ($\sim \alpha^{3/2}$) is $N^{3/2}LO$!

$$\Delta\sigma = -\frac{\pi\alpha_{ew}^2}{27s}\,\mathrm{Im}\left[-\frac{\alpha^2\pi^2}{12}\sqrt{-\frac{M_W}{\mathcal{E}}} + \frac{\alpha^3\zeta(3)}{4}\frac{M_W}{\mathcal{E}}\right]$$

$$\mathcal{E} = \sqrt{s} - 2M_W + i\Gamma_W$$

2. Interference between first Coulomb and one-loop decay corrections



$$\Delta\sigma = -\left(\frac{\Gamma^{(1,ew)}_{\mu\bar{\nu}}}{\Gamma^{(0)}_{\mu\bar{\nu}}} + \frac{\Gamma^{(1,ew)}_{u\bar{d}}}{\Gamma^{(0)}_{u\bar{d}}}\right)\frac{\pi\alpha_{ew}^2\alpha}{54s}\,\mathrm{Im}\left[\ln\left(-\frac{\mathcal{E}}{M_W}\right)\right]$$

3. Interference between first Coulomb and $O(\alpha)$ self-energy insertion

$$\Pi_T^W(k^2) = M_W^2 \sum_{m,n} \delta^n \Pi^{(m,n)}$$

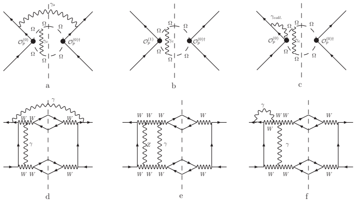
$$m = \text{number loops}, \quad \delta = (k^2 - M_W^2)/M_W^2$$

Leading term $\Pi^{(1,0)}$ resummed in the propagator, subleading terms included perturbatively

$\Rightarrow$ Interference of $\Pi^{(1,1)}$ with first Coulomb contributes at $N^{3/2}LO$

$N^{3/2}LO$ corrections (II)

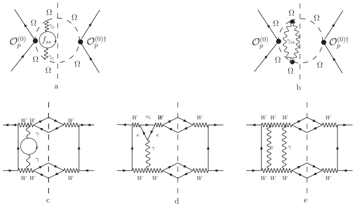
4. Interference of first Coulomb with soft, hard and collinear EW corrections



Large logs of $2M_W/m_e$ subtracted and included in electron structure functions

$$\Delta\sigma = -\frac{\alpha_{ew}^2\alpha^2}{108s}\left\{\left(9+\frac{\pi^2}{2}+2\mathrm{Re}\,c_{p,LR}^{(1),\mathrm{fin}}\right)\times\mathrm{Im}\left[\ln\left(-\frac{\mathcal{E}}{M_W}\right)\right]+2\mathrm{Im}\left[\ln^2\left(-\frac{\mathcal{E}}{M_W}\right)\right]\right\}$$

5. Corrections to the Coulomb potential



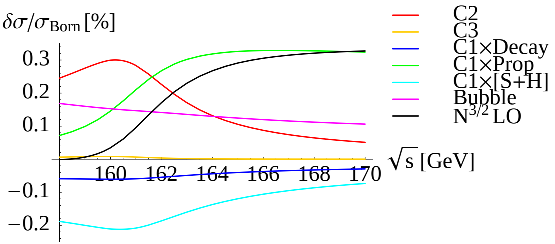
Coulomb potential modified by insertion of semi-soft loops ($q_0 \sim |\vec{q}| \sim \sqrt{M_W\Gamma_W}$)

$$\Delta\sigma = -\frac{\alpha_{ew}^2\alpha^2}{81s}\sum_f C_f Q_f^2\left\{\ln 2c_w\,\mathrm{Im}\left[\ln\left(-\frac{\mathcal{E}}{M_W}\right)\right]+\frac{1}{4}\mathrm{Im}\left[\ln^2\left(-\frac{\mathcal{E}}{M_W}\right)\right]\right\}-\frac{\pi\alpha_{ew}^2\alpha^2}{27s}\Delta_{G_\mu}\,\mathrm{Im}\left[\ln\left(-\frac{\mathcal{E}}{M_W}\right)\right]$$

Numerical impact of $N^{3/2}LO$ corrections (preliminary)

Dominant NNLO corrections in the G_μ-scheme ($\alpha = \sqrt{2}G_\mu M_W^2 s_w^2/\pi$)

$M_W = 80.377\,\text{GeV}$ $\Gamma_W = 2.04483\,\text{GeV}$ $M_Z = 91.188\,\text{GeV}$
$m_t = 174.2\,\text{GeV}$ $M_H = 115\,\text{GeV}$



	$\sigma(e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}X)$(fb)							
$\sqrt{s}$ [GeV]	Born	$\Delta\sigma^{(3/2)}_{C3}$	$\Delta\sigma^{(3/2)}_{C1\times Dec}$	$\Delta\sigma^{(3/2)}_{C1\times Prop}$	$\Delta\sigma^{(3/2)}_{C1\times[S+H]}$	$\Delta\sigma^{(3/2)}_{Bub}$	$\Delta\sigma^{(3/2)}_{tot}$	$\Delta\sigma^{(1)}_{C2}$
161	154.19(6)	0.010	-0.091	0.324	-0.321	0.226	0.147 [+0.9 ‰]	0.437 [+2.8 ‰]
170	481.7(2)	0.000	-0.142	1.562	-0.354	0.511	1.577 [+3.3 ‰]	0.246 [+0.5 ‰]

Implementation of cuts

Measurement of total cross section not realistic! Invariant-mass cuts and angular cuts are applied experimentally to suppress background and by the geometry of the detector.

LEP II
[L3 Collaboration '97]

Cut	$\sigma_{\rm tree}(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d})$(fb)	$\sigma_{\rm cut}/\sigma_{\rm tot}$
-	154.18 (5)	
$E_\mu > 20$ GeV	153.71 (5)	99.69 (5) %
$M_{\mu\nu} > 55$ GeV, 40 GeV $< M_{jj} <$ 120 GeV	150.60 (5)	97.68 (5) %
$\theta_{\mu j} > 15$	149.35 (5)	96.87 (5) %
$\mid\cos\theta_\nu\mid < 0.95$	148.28 (5)	96.17 (5) %
all	140.03 (5)	90.82 (5) %

How to implement the cuts in the EFT?

Invariant-mass cuts

Include $\theta(\Lambda^2 - |k^2 - Mw^2|)$ in all integrals and expand according to scaling of k^2 ($\Lambda^2 \sim M_W^2$)

- Resonant regions: $k^2 - Mw^2 \sim M_W \Gamma_W \to$ $\theta(\Lambda^2 - |k^2 - Mw^2|) \sim \theta(\Lambda^2) = 1$. No modification!
- Non-resonant regions: $k^2 - Mw^2 \sim M_W^2 \to$ Cuts must be taken into account (modification of the four-electron operators).

	$\sigma_{\rm Born}^{\rm Whizard}(161{\rm GeV})$	$\sigma_{\rm Born}^{\rm EFT}(161{\rm GeV})$
no cuts	154.18(5)	154.02
inv. mass. cuts	150.61(5)	150.71

Angular cuts

Implementation of angular cuts more complicated. Need EFT containing explicitly the final states!.

Observation: Coulomb and soft interactions do not modify the polarizations and angular distributions of the final states significantly ($|\vec{p}|_{\rm soft}/|\vec{p}|_{\rm ext} \sim \Gamma/M$)

$\Downarrow$

$$\Delta\sigma_{\rm cut}^{(3/2)} \sim \kappa \Delta\sigma_{\rm tot}^{(3/2)} \qquad \kappa = \frac{\sigma_{\rm cut}^{\rm Born}}{\sigma_{\rm tot}^{\rm Born}}$$

Error of the approximation well below 1‰

Conclusions

- The EFT approach allows a straightforward identification and evaluation of the dominant NNLO corrections near threshold
- The total $N^{3/2}LO$ correction is $\sim 1‰$ at threshold and grows to $\sim 3‰$ at $\sqrt{s} = 170\,\text{GeV}$
- Combining the full 4f NLO result and the dominant NNLO corrections the remaining theoretical uncertainty is estimated to be $\delta M_W^{\text{th}} \lesssim 1\,\text{MeV}$ (for the partonic cross section!)
- But NLL resummation of the electron structure function must be addressed! (estimated $\delta M_W^{\text{th}} \sim 30\text{MeV}$[Beneke, PF, Schwinn, Signer, Zanderighi '07])