

Constraining SUSY with Electroweak Precision Observables

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Warsaw, 06/2008

based on collaboration with
W. Hollik, A.M. Weber and G. Weiglein

1. Introduction
2. Calculation of the electroweak precision observables
3. Numerical results
4. Conclusions

1. Introduction

Q: How to detect Supersymmetry (or any other BSM)?

A: Two possible ways:

- Search for new SUSY particles

new SUSY particles found
↕
SM ruled out

Problem:

SUSY particles are too heavy for today's colliders, only lower limits of $\mathcal{O}(100 \text{ GeV})$.

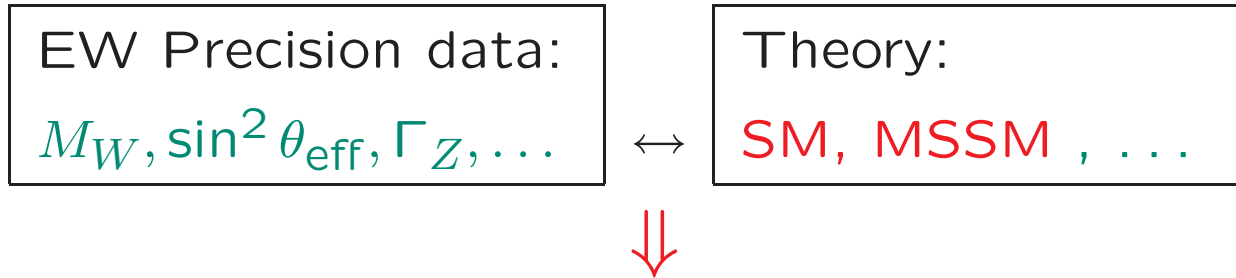
→ waiting for Tevatron (2008/09...?)

→ waiting for LHC (2009/10...?)

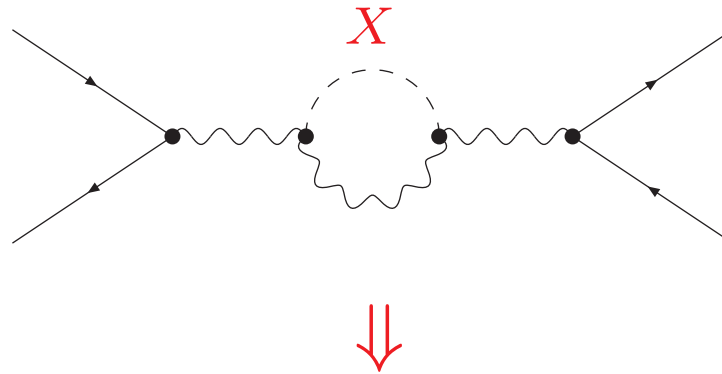
- Search for indirect effects of SUSY
via Precision Observables

Precision Observables (POs):

Comparison of electro-weak precision observables with theory:



Test of theory at quantum level: Sensitivity to loop corrections

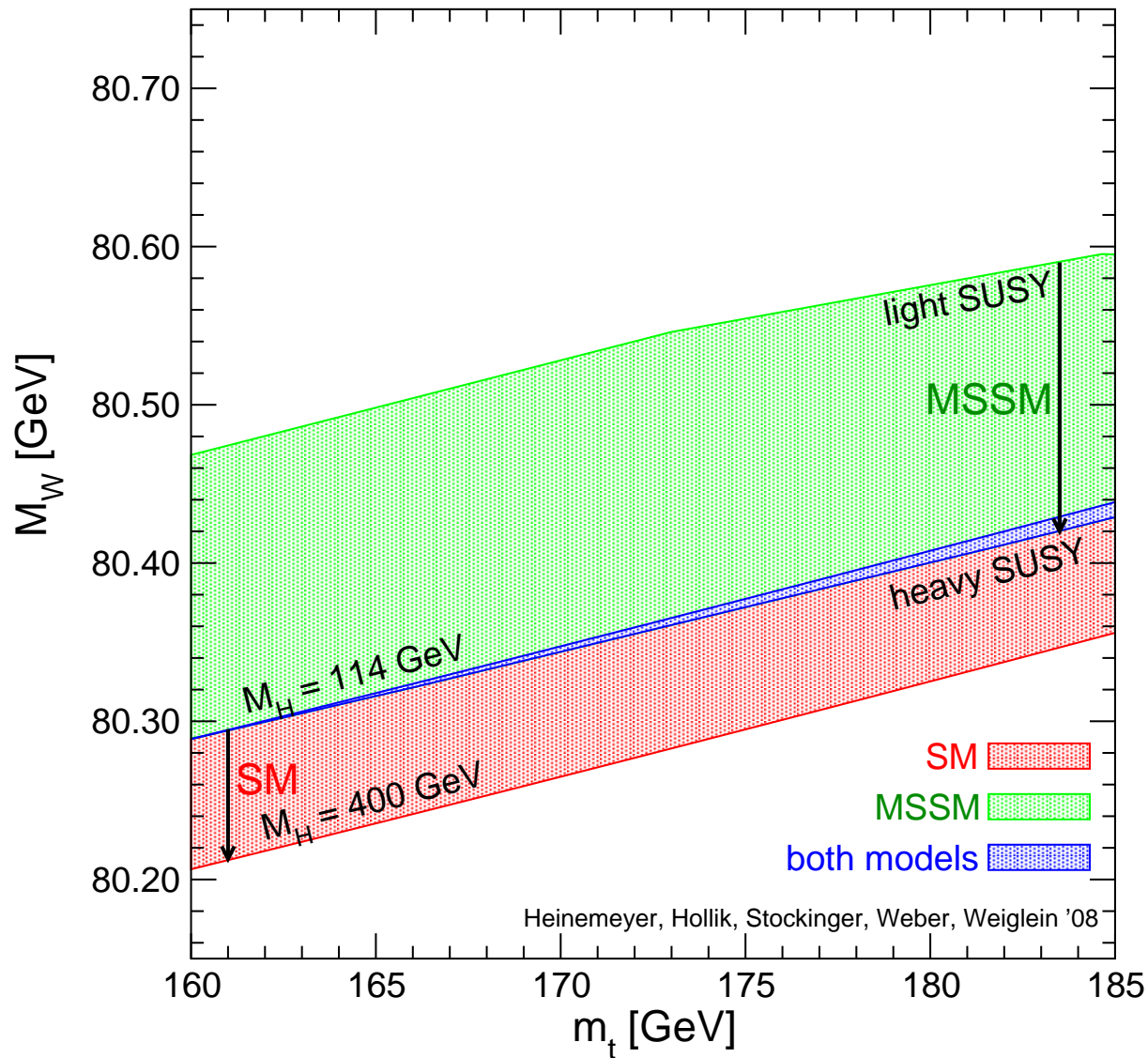


Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

Example: Prediction for M_W in the **SM** and the **MSSM** :

[S.H., W. Hollik, D. Stockinger, A.M. Weber, G. Weiglein '07]



MSSM band:

scan over
SUSY masses

overlap:

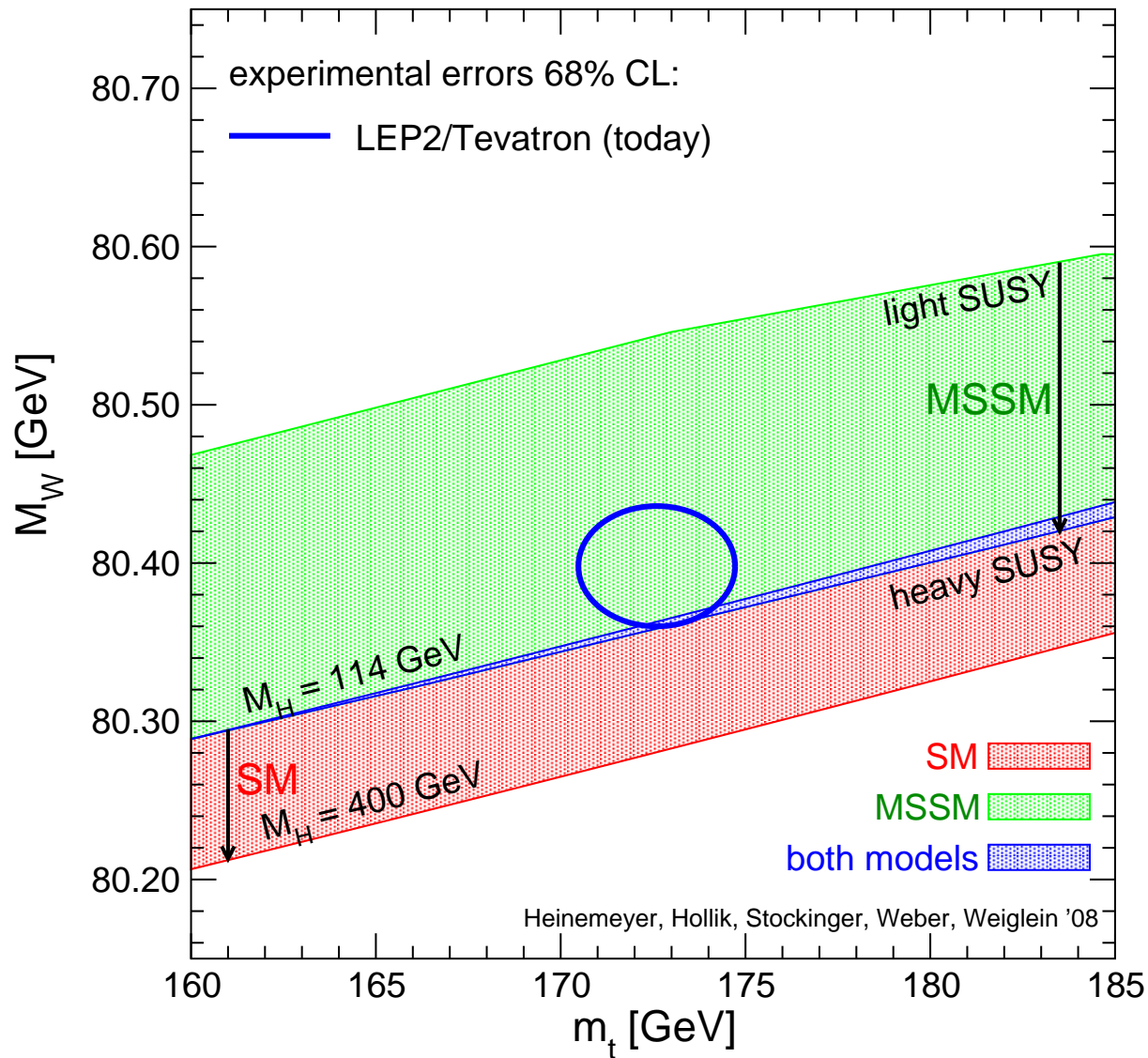
SM is MSSM-like
MSSM is SM-like

SM band:

variation of M_H^{SM}

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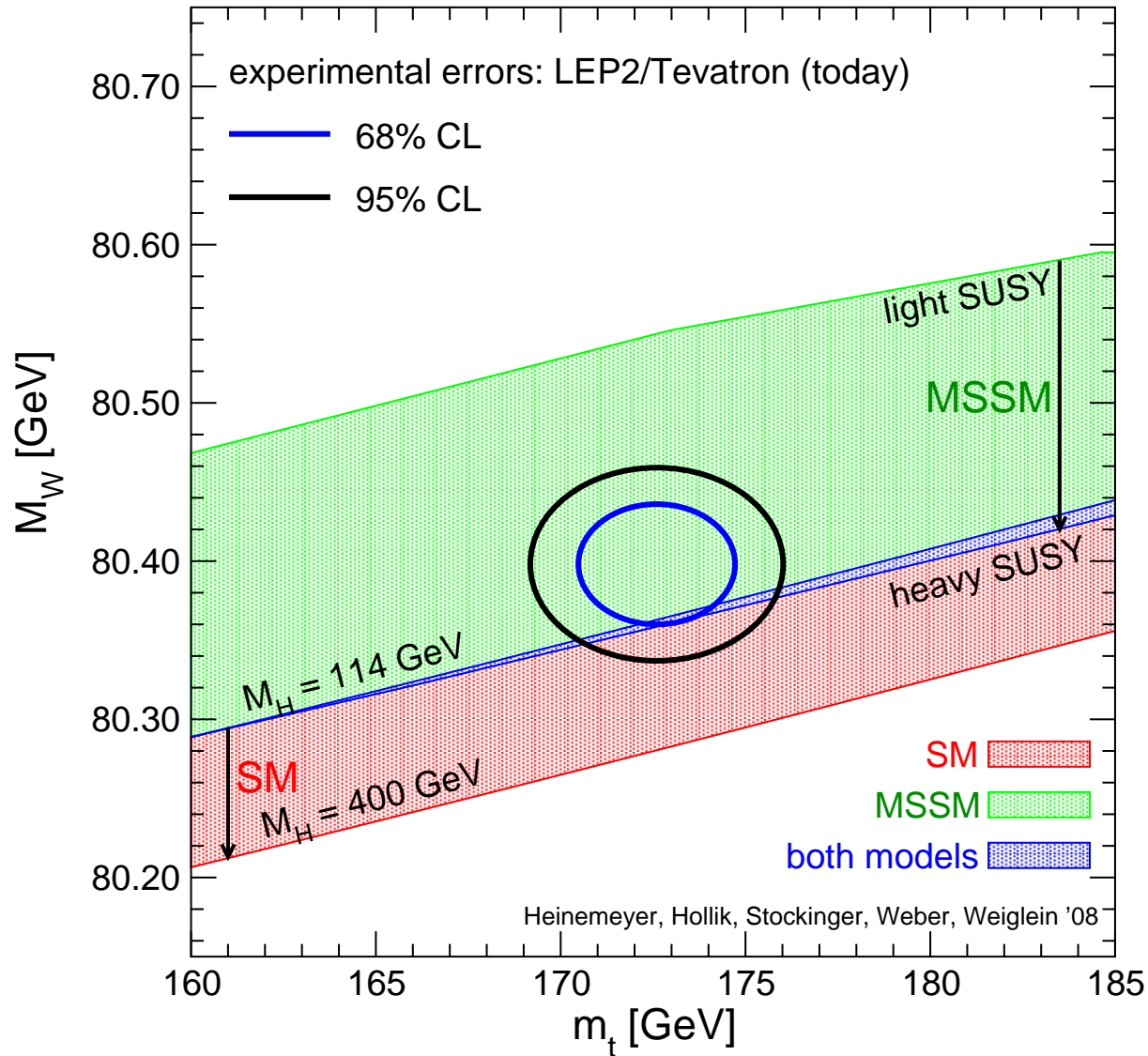
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Example: Prediction for M_W in the **SM** and the **MSSM** :

[S.H., W. Hollik, D. Stockinger, A.M. Weber, G. Weiglein '07]



MSSM band:

scan over
SUSY masses

overlap:

SM is MSSM-like
MSSM is SM-like

SM band:

variation of M_H^{SM}

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$$\begin{array}{llll} [u, d, c, s, t, b]_{L,R} & [e, \mu, \tau]_{L,R} & [\nu_{e,\mu,\tau}]_L & \text{Spin } \frac{1}{2} \\ [\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R} & [\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R} & [\tilde{\nu}_{e,\mu,\tau}]_L & \text{Spin } 0 \\ g & \underbrace{W^\pm, H^\pm}_{\text{Spin } 1} & \underbrace{\gamma, Z, H_1^0, H_2^0}_{\text{Spin } 0} & \text{Spin } 1 / \text{Spin } 0 \\ \tilde{g} & \tilde{\chi}_{1,2}^\pm & \tilde{\chi}_{1,2,3,4}^0 & \text{Spin } \frac{1}{2} \end{array}$$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

Problem in the MSSM: many complex parameters

\tilde{t}/\tilde{b} sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ($X_t = A_t - \mu^*/\tan\beta$, $X_b = A_b - \mu^*\tan\beta$):

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix}$$

$$\mathcal{M}_{\tilde{b}}^2 = \begin{pmatrix} M_{\tilde{b}_L}^2 + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M_{\tilde{b}_R}^2 + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m_{\tilde{b}_1}^2 & 0 \\ 0 & m_{\tilde{b}_2}^2 \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

soft SUSY-breaking parameters A_t, A_b also appear in ϕ - \tilde{t}/\tilde{b} couplings

$$SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$$

\Rightarrow relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ + \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

physical states: h^0, H^0, A^0, H^\pm

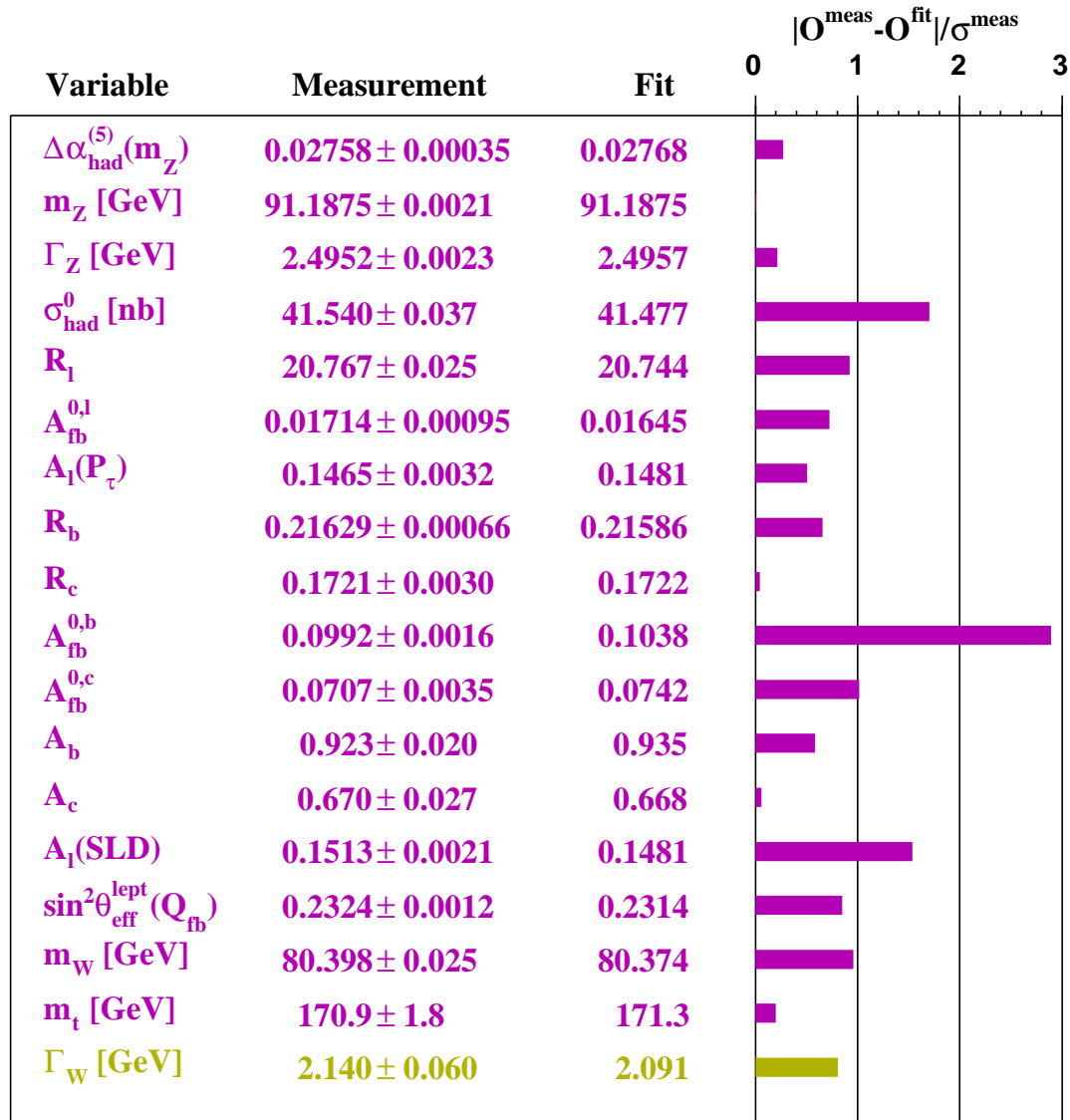
Goldstone bosons: G^0, G^\pm

Input parameters: (to be determined experimentally)

$$\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta)$$

Results for EWPO fit in the SM: [LEPEWWG '08]

Pull distributions:



Probability: 15%

Within the SM: fit for the last unknown parameter: M_H^{SM}

Global fit to all SM data:

[LEPEWWG '08]

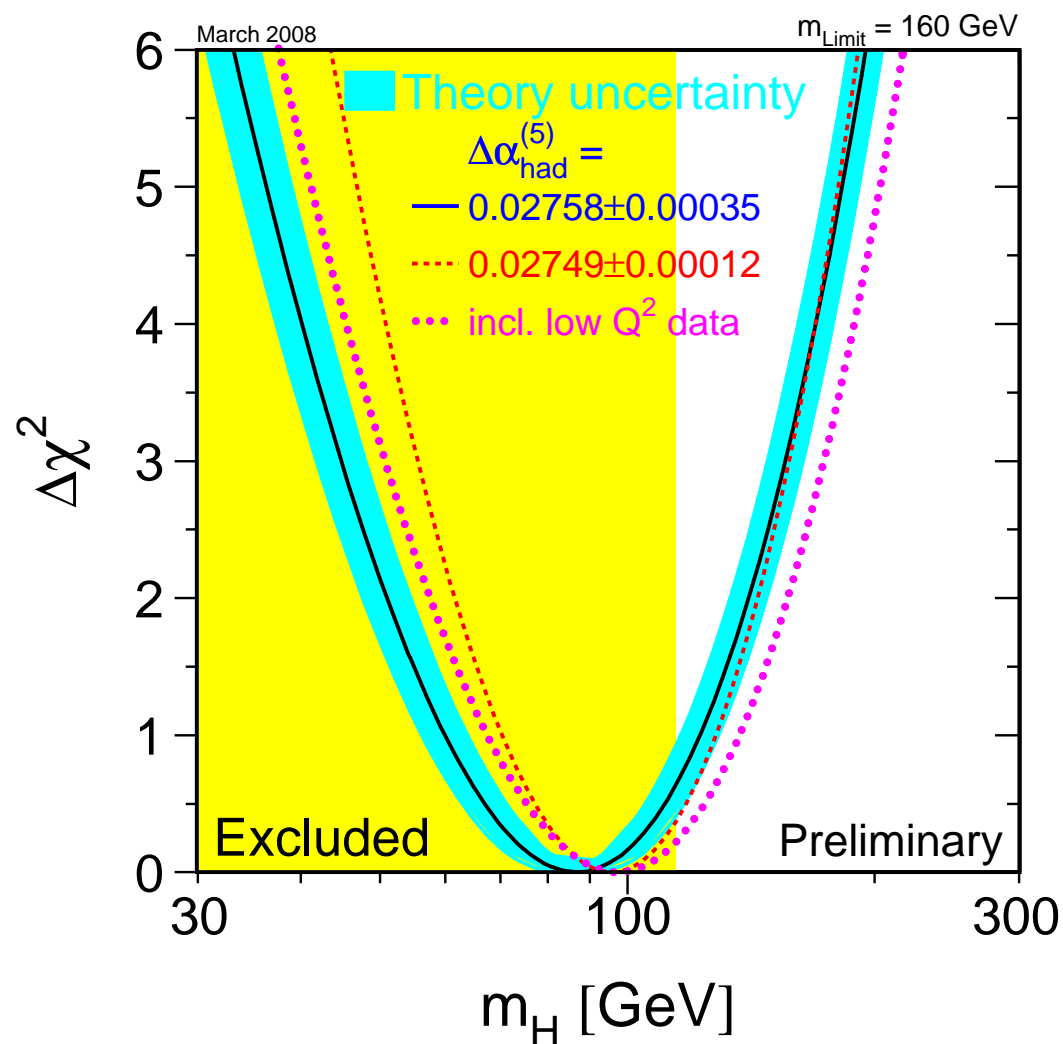
$$\Rightarrow M_H = 87^{+36}_{-27} \text{ GeV}$$

$$M_H < 160 \text{ GeV, 95\% C.L.}$$

Assumption for the fit:

SM incl. Higgs boson

\Rightarrow no confirmation of Higgs mechanism



\Rightarrow Higgs boson seems to be light, $M_H \lesssim 150 \text{ GeV}$

2. Calculation of the electroweak precision observables

$$\mathcal{O} = M_W, \sin^2 \theta_{\text{eff}} (A_{\text{FB}}^{b,c}, A_{\text{LR}}^{e,\mu}), R_l, R_b, \sigma_0^{\text{had}}, \dots$$

Wanted: same precision for MSSM as for the SM

⇒ Combination of SM and MSSM result

→ use **best available SM** result

→ add **all available MSSM** corrections

recalculation of full 1L result, now **incl. full complex phase dependence**

→ subtract double counting

⇒ **decoupling limit ok** ($M_{\text{SUSY}} \rightarrow \infty$)

... but some corrections included only via their **SM part**

⇒ **best solution**

Calculation of M_W , $\sin^2 \theta_{\text{eff}}$, Γ_Z :

1.) Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$

\Updownarrow
loop corrections

2.) Effective mixing angle:

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4 |Q_f|} \left(1 - \text{Re} \frac{g_V^f}{g_A^f} \right)$$

Higher order contributions:

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

3.) Total Z width:

$$\Gamma_Z = \sum_X \Gamma(Z \rightarrow X \bar{X})$$

including higher-order corrections for all the channels

Other observables:

4.) R_b :

$$R_b = \frac{\Gamma_b}{\Gamma_{\text{had}}} = \frac{\Gamma(Z \rightarrow b\bar{b})}{\Gamma(Z \rightarrow \text{hadrons})}$$

5.) R_l :

$$R_l = \frac{\Gamma_{\text{had}}}{\Gamma_l} = \frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow \text{leptons})}$$

6.) hadronic peak cross section σ_{had}^0 :

$$\sigma_{\text{had}}^0 = 12 \pi \frac{\Gamma_e \Gamma_{\text{had}}}{M_Z^2 \Gamma_Z^2}$$

7.) ...

More details about our calculation:

- Complex phases in the squark sector enter only via shift in squark masses (explicit dependence drops out)

$$\begin{aligned}|X_t|^2 &= |A_t|^2 + |\mu \cot \beta|^2 - 2|A_t| \cdot |\mu| \cot \beta \cos(\phi_{A_t} + \phi_\mu) \\ |X_b|^2 &= |A_b|^2 + |\mu \tan \beta|^2 - 2|A_b| \cdot |\mu| \tan \beta \cos(\phi_{A_b} + \phi_\mu)\end{aligned}$$

Only some phase combinations are physical, other phases can be rotated away.

Examples for physical combinations:

$$\begin{aligned}\phi_{A_t} + \phi_\mu \\ \phi_{A_b} + \phi_\mu\end{aligned}$$

- Higgs mass dependence of the two-loop contributions is known to be very strong
⇒ we use *FeynHiggs* (www.feynhiggs.de)
- All one-loop calculations have been performed with *FeynArts* and *FormCalc*
[T. Hahn et al. '00 - '07]

Treatment of the phase dependence beyond one-loop order

Phase dependence at the two-loop level approximated by a simple interpolation based on:

full phase dependence at the one-loop level, $\mathcal{O}^{1L}(\phi)$,

two-loop results for real parameters, $\mathcal{O}^{\text{full}}(0)$, $\mathcal{O}^{\text{full}}(\pi)$

⇒ Two-loop result for complex phase ϕ :

$$\begin{aligned}\mathcal{O}^{\text{full}}(\phi) = \mathcal{O}^{1L}(\phi) &+ \left[\mathcal{O}^{\text{full}}(0) - \mathcal{O}^{1L}(0) \right] \times \frac{1 + \cos \phi}{2} \\ &+ \left[\mathcal{O}^{\text{full}}(\pi) - \mathcal{O}^{1L}(\pi) \right] \times \frac{1 - \cos \phi}{2}\end{aligned}$$

3. Numerical results

Investigation of:

- M_{SUSY} dependence
- m_t dependence
- phase dependence
- special scenarios
- parameter scan
- ...

Together with other observables:

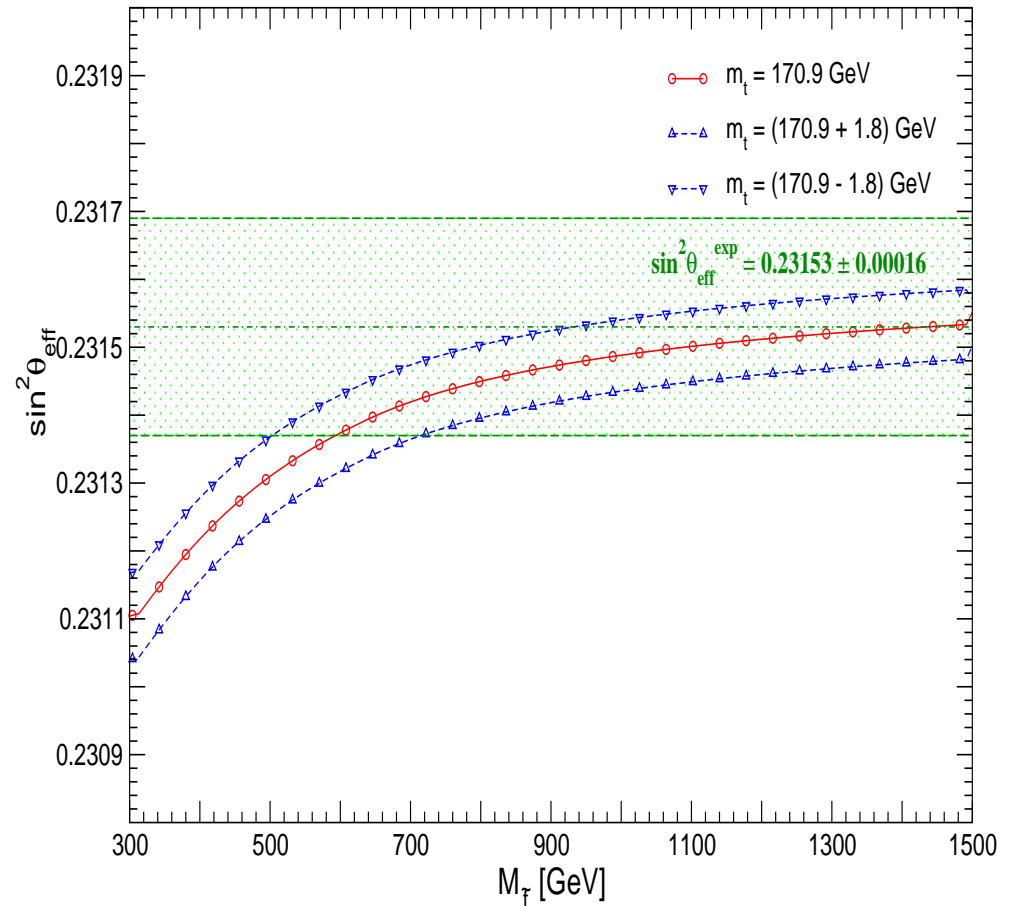
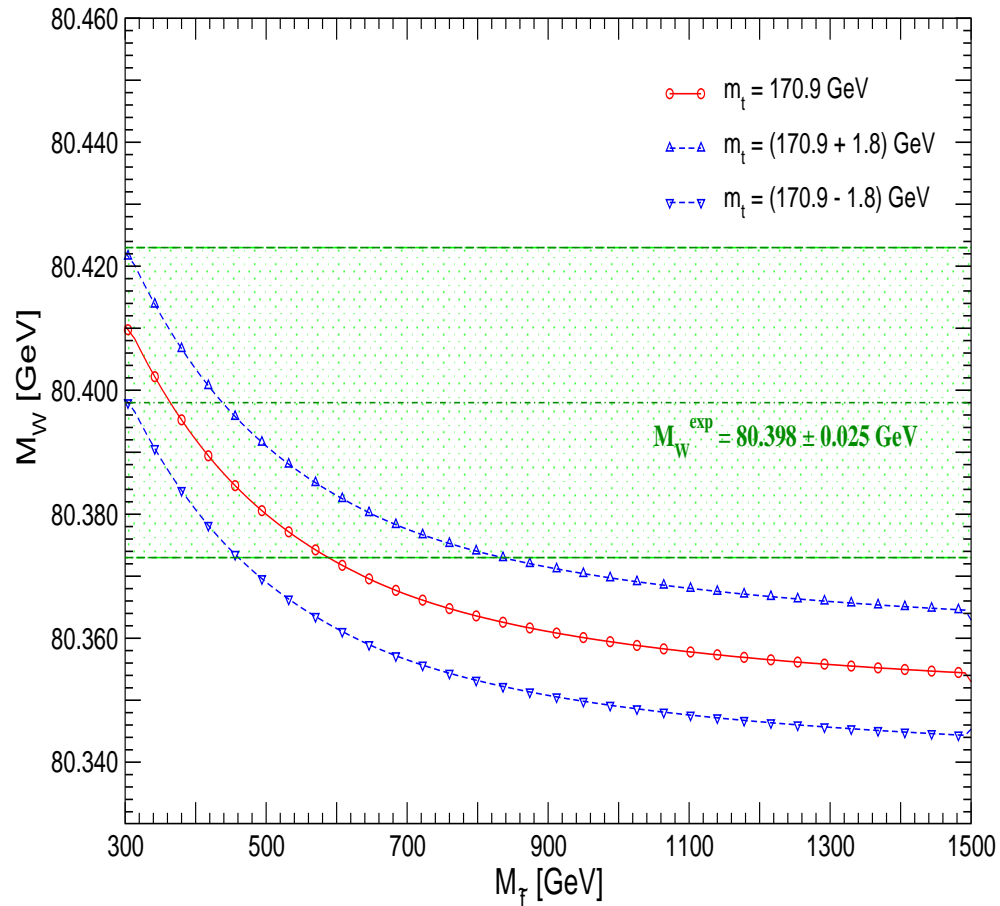
M_h^{SUSY} determination (similar to Blue Band in the SM)

Experimental precision:

observable	central exp. value	$\sigma \equiv \sigma^{\text{today}}$	σ^{LHC}	$\sigma^{\text{ILC/GigaZ}}$
M_W [GeV]	80.398	0.025	0.015	0.007
$\sin^2 \theta_{\text{eff}}$	0.23153	0.00016	0.00020–0.00014	0.000013
Γ_Z [GeV]	2.4952	0.0023	—	0.001
R_l	20.767	0.025	—	0.01
R_b	0.21629	0.00066	—	0.00014
σ_{had}^0	41.540	0.037	—	0.025

⇒ ILC/GigaZ precision yields a very strong improvement

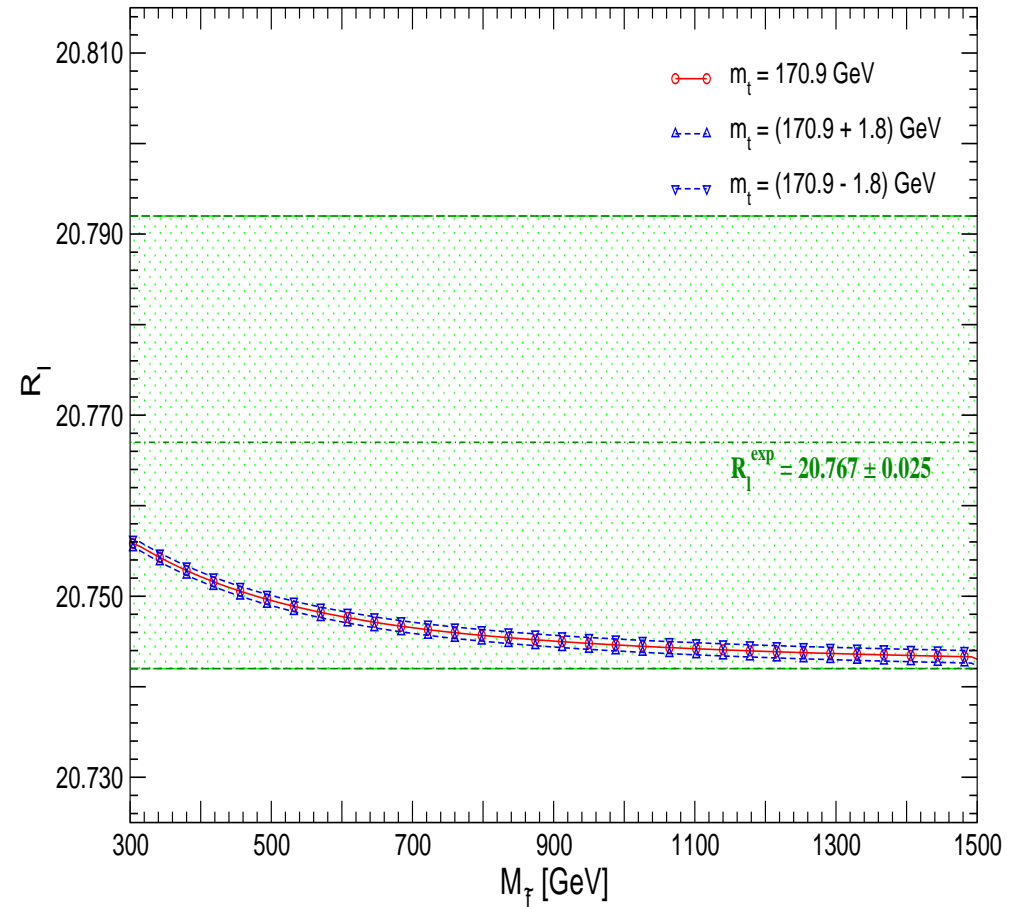
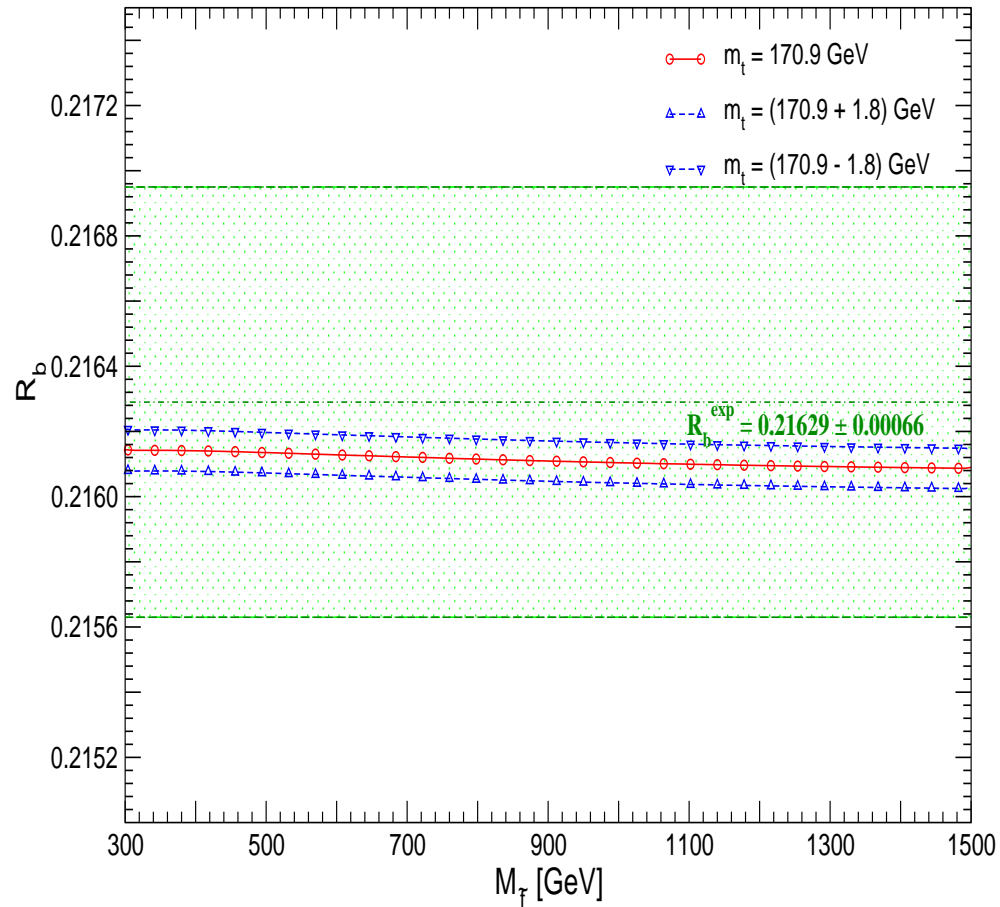
A) M_{SUSY} and m_t dependence (I)



\Rightarrow strong M_{SUSY} dependence

\Rightarrow important m_t dependence

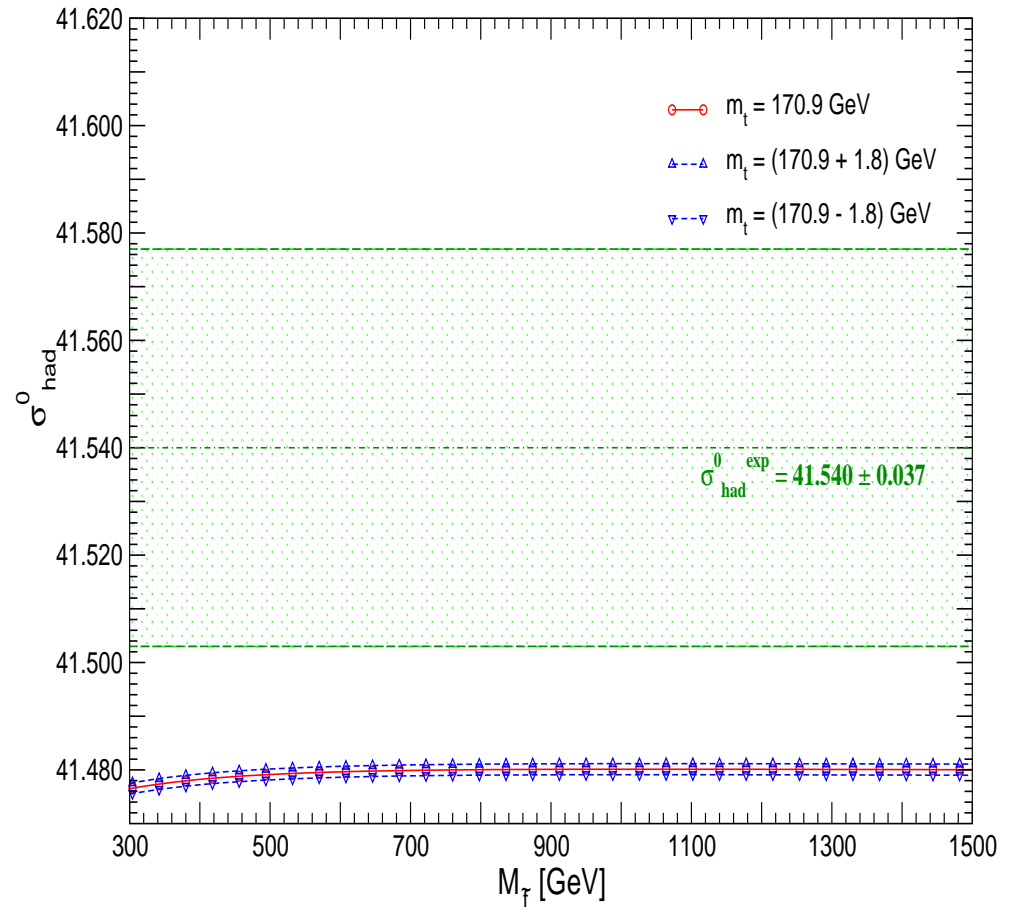
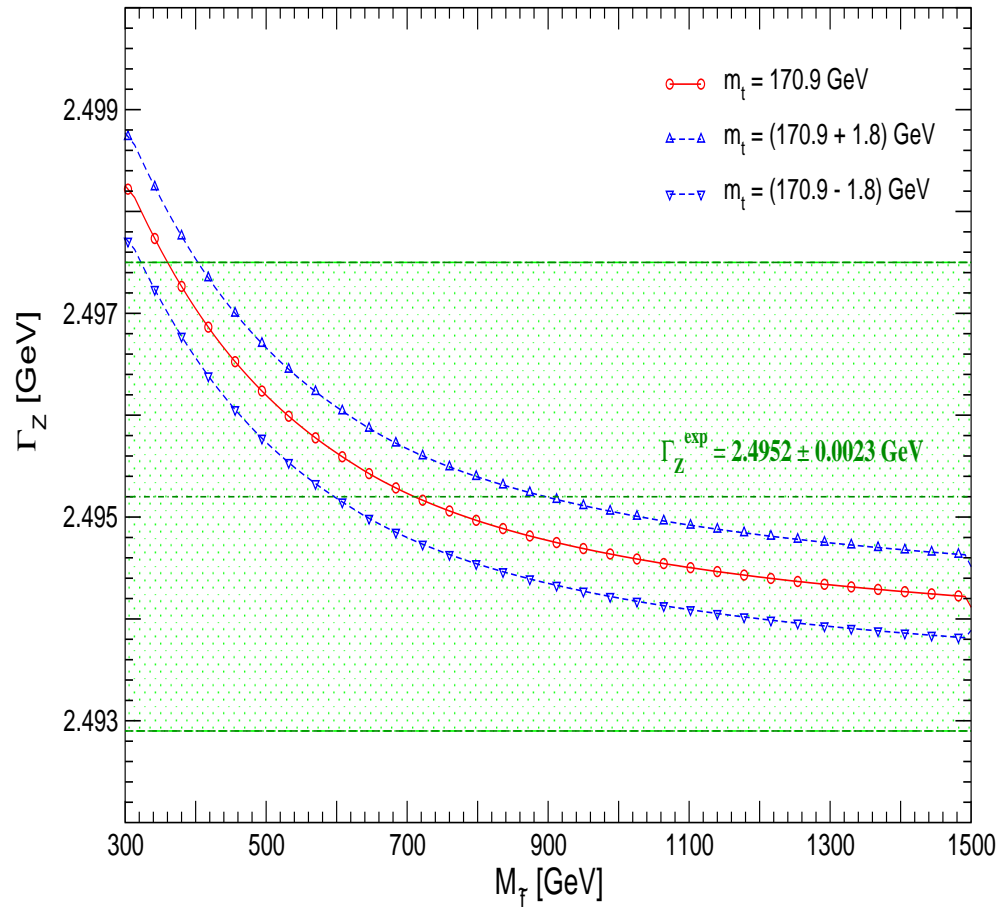
A) M_{SUSY} and m_t dependence (II)



\Rightarrow weak M_{SUSY} dependence

\Rightarrow weak m_t dependence

A) M_{SUSY} and m_t dependence (III)



\Rightarrow relevant M_{SUSY} dependence only for Γ_Z

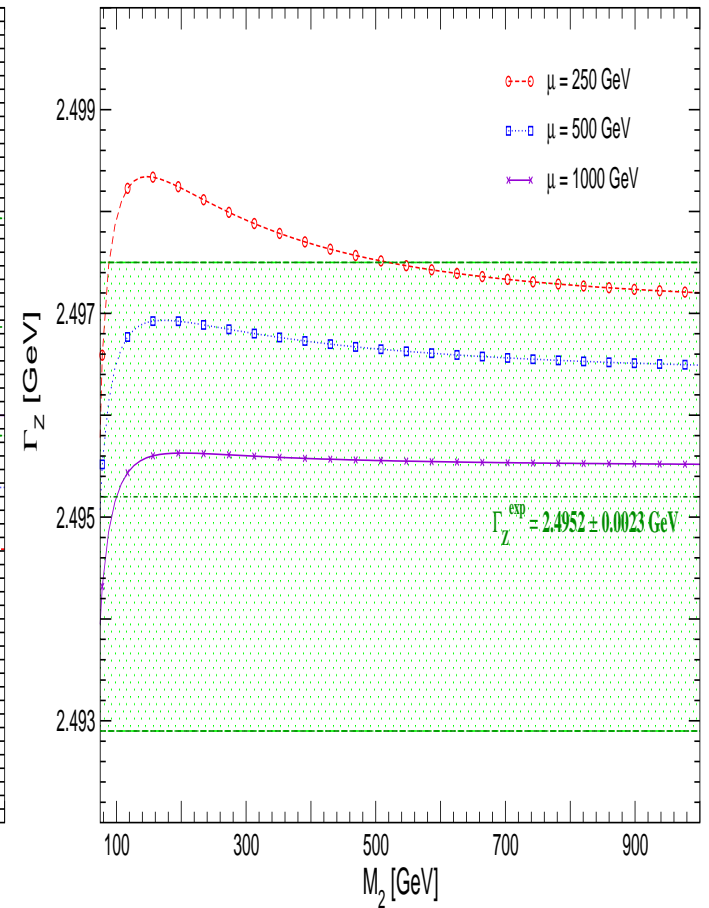
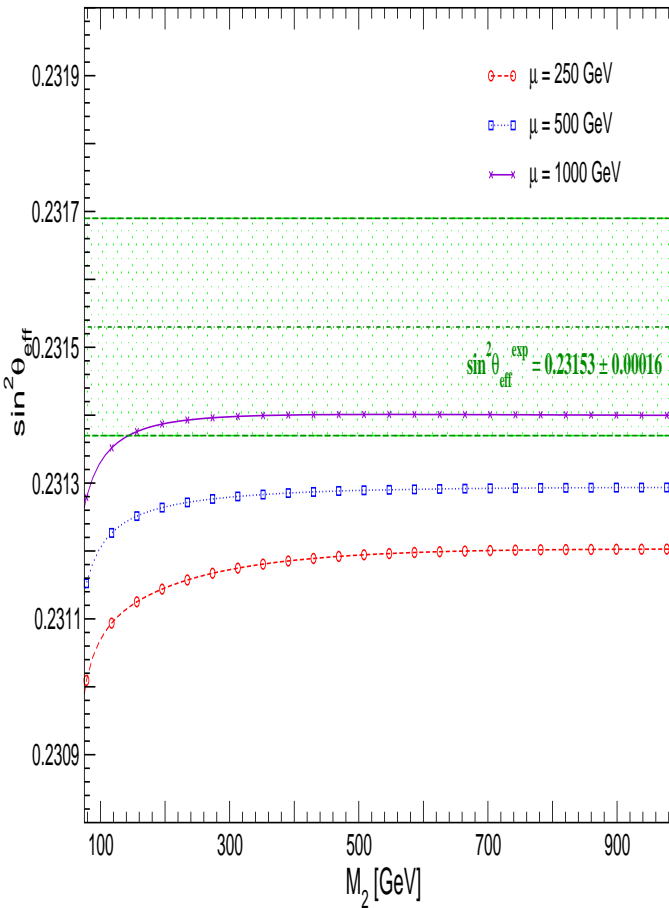
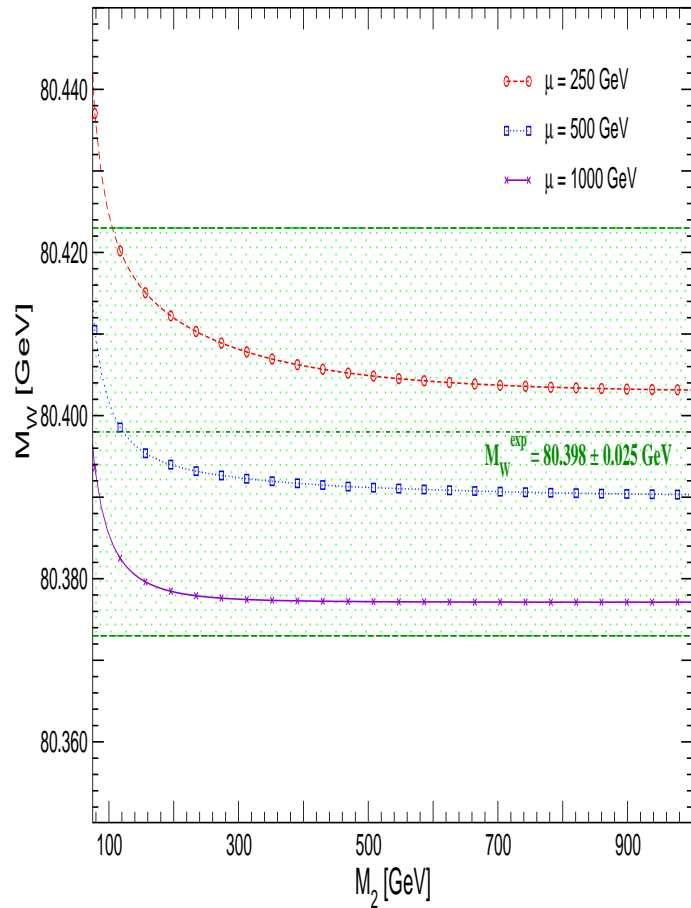
\Rightarrow relevant m_t dependence only for Γ_Z

Parametric uncertainties:

$$\delta m_t^{\text{exp}} = 1.4 \text{ GeV} \Rightarrow \begin{aligned} \delta M_W^{\text{para}, m_t} &= 9 \text{ MeV} \\ \delta \sin^2 \theta_{\text{eff}}^{\text{para}, m_t} &= 4.2 \times 10^{-5} \\ \delta \Gamma_Z^{\text{para}, m_t} &= 0.34 \text{ MeV} \end{aligned}$$

$$\delta(\Delta\alpha_{\text{had}}^{(5)}) = 3.5 \times 10^{-4} \Rightarrow \begin{aligned} \delta M_W^{\text{para}, \Delta\alpha_{\text{had}}^{(5)}} &= 6.3 \text{ MeV} \\ \delta \sin^2 \theta_{\text{eff}}^{\text{para}, \Delta\alpha_{\text{had}}^{(5)}} &= 12 \times 10^{-5} \\ \delta \Gamma_Z^{\text{para}, \Delta\alpha_{\text{had}}^{(5)}} &= 0.32 \text{ MeV} \end{aligned}$$

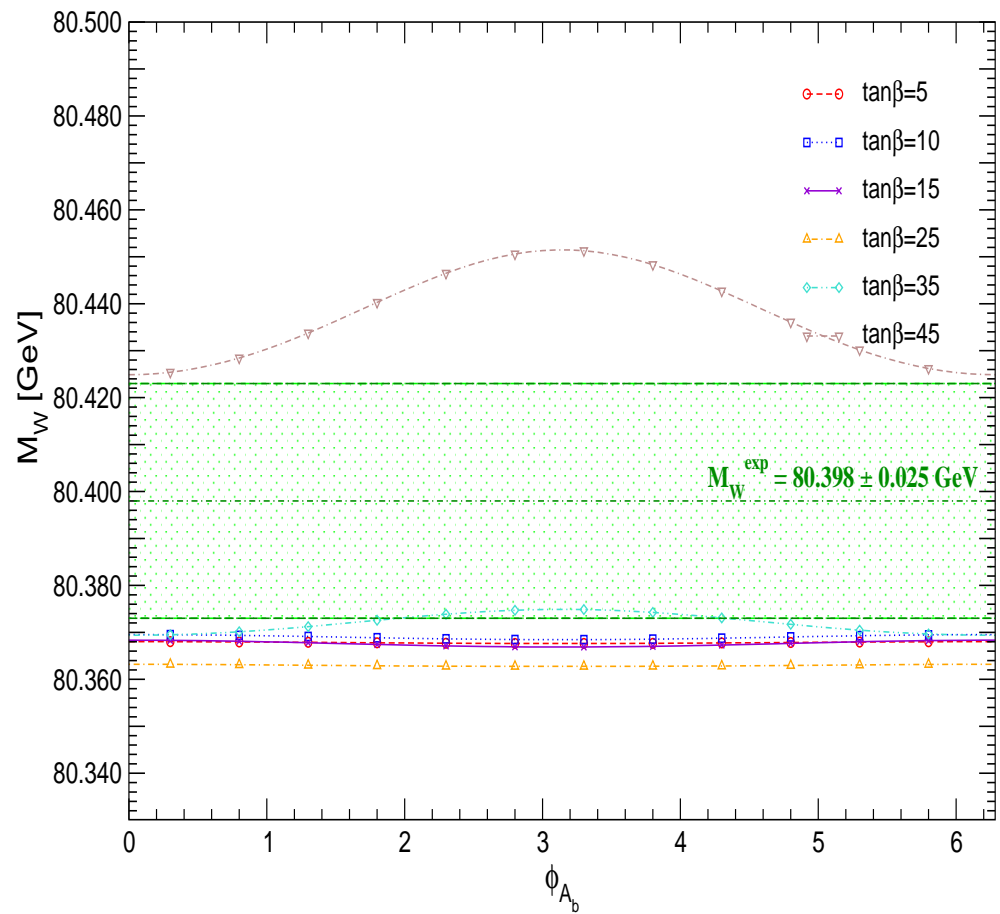
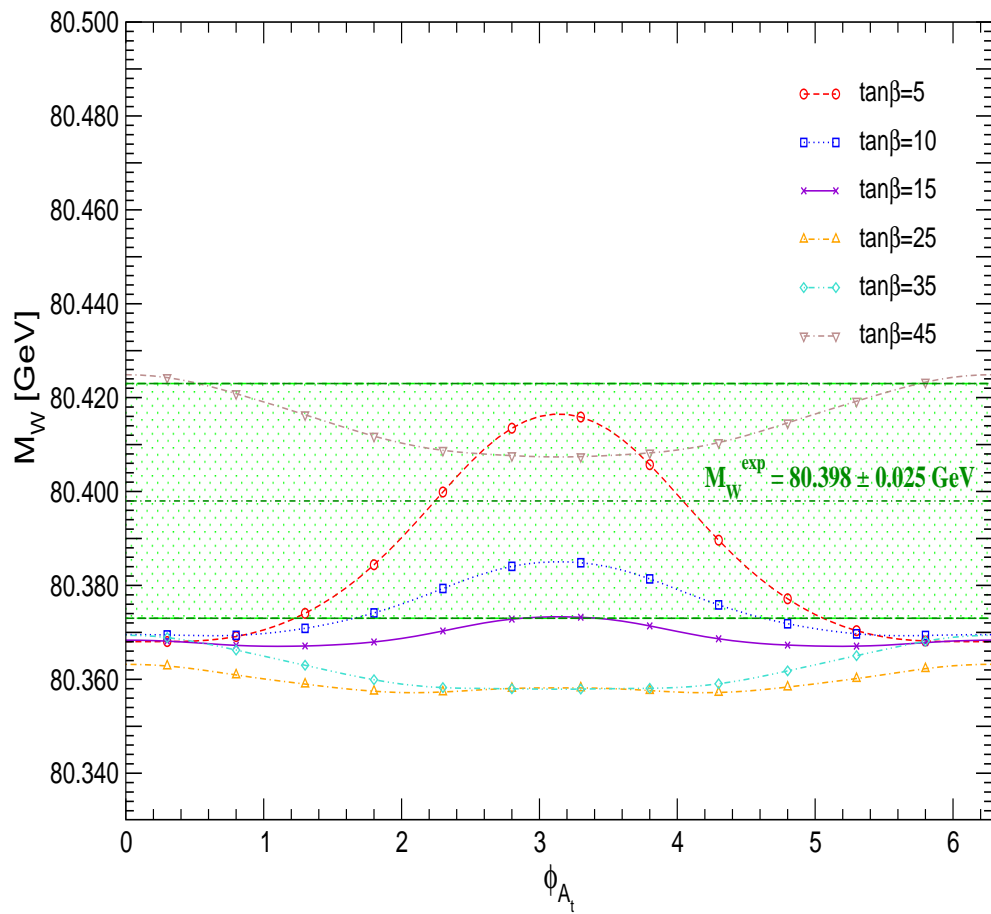
B) Dependence on μ and M_2 :



\Rightarrow strong dependence on μ

\Rightarrow strong dependence on M_2 for small M_2

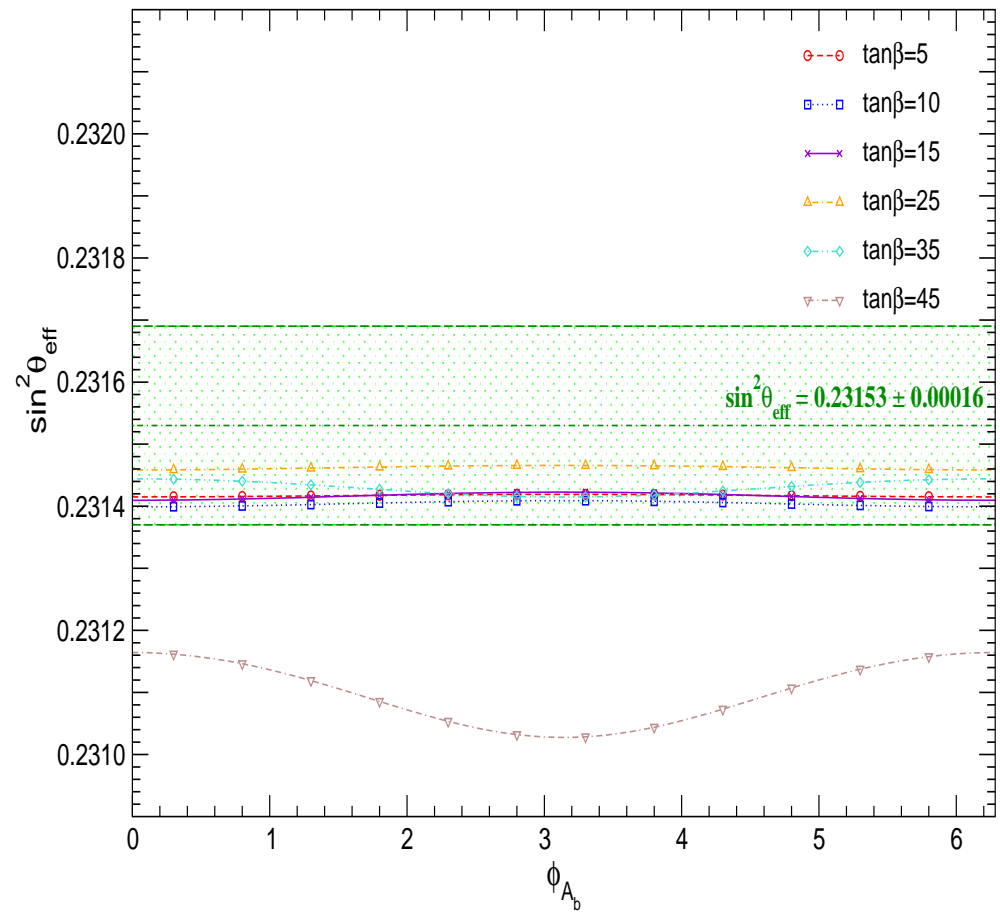
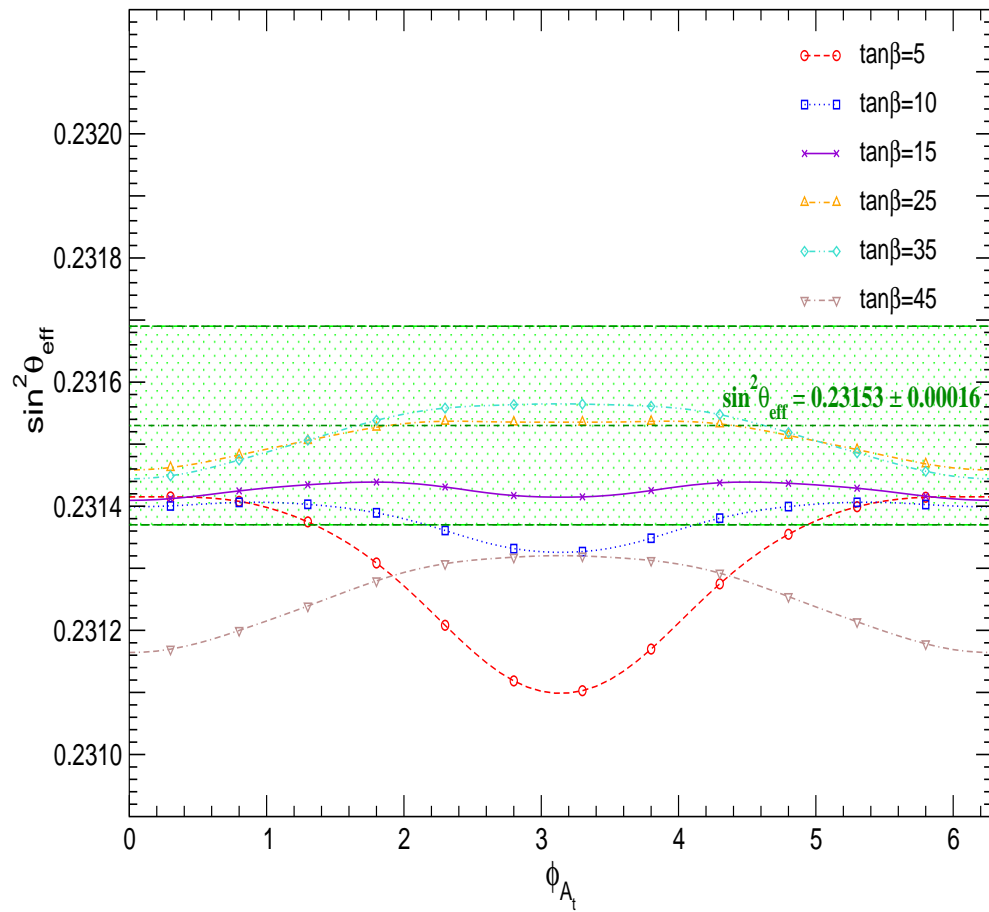
C) Dependence on ϕ_{A_t} and ϕ_{A_b} (for $\phi_\mu = 0$) (I):



\Rightarrow strong dependence on ϕ_{A_t}

\Rightarrow strong dependence on ϕ_{A_b} for large $\tan\beta$ (\rightarrow light sbottom)

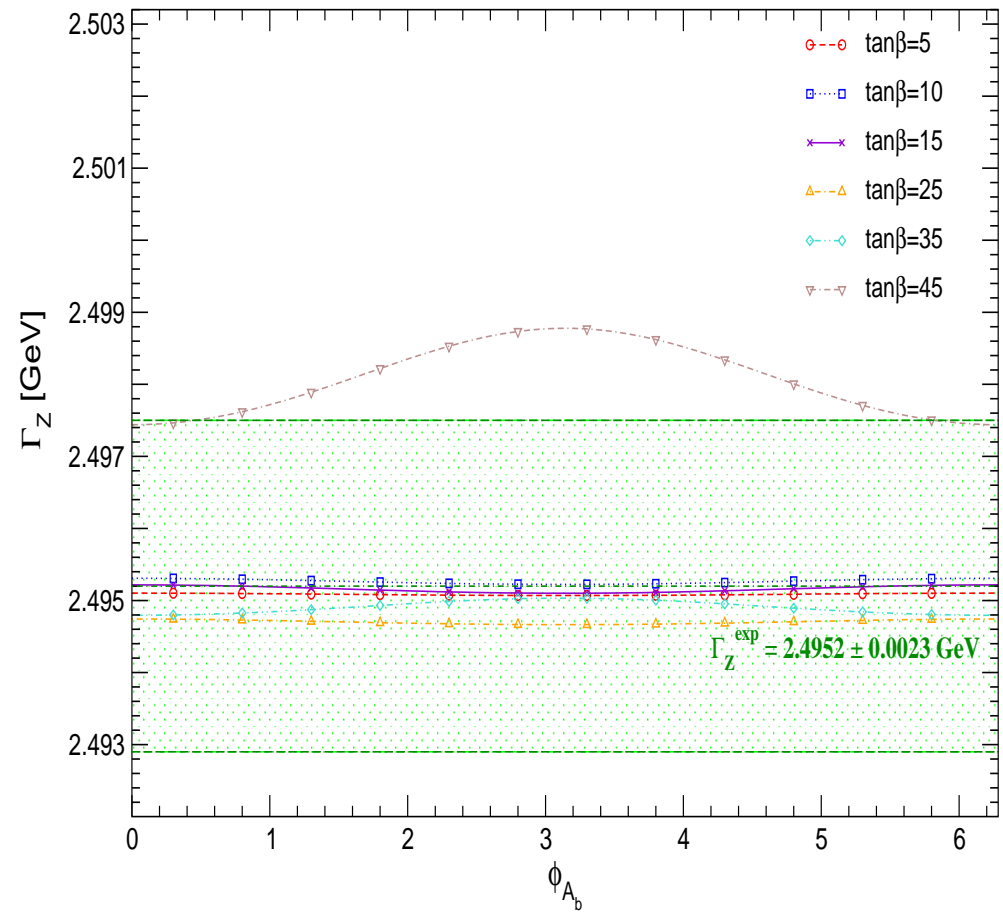
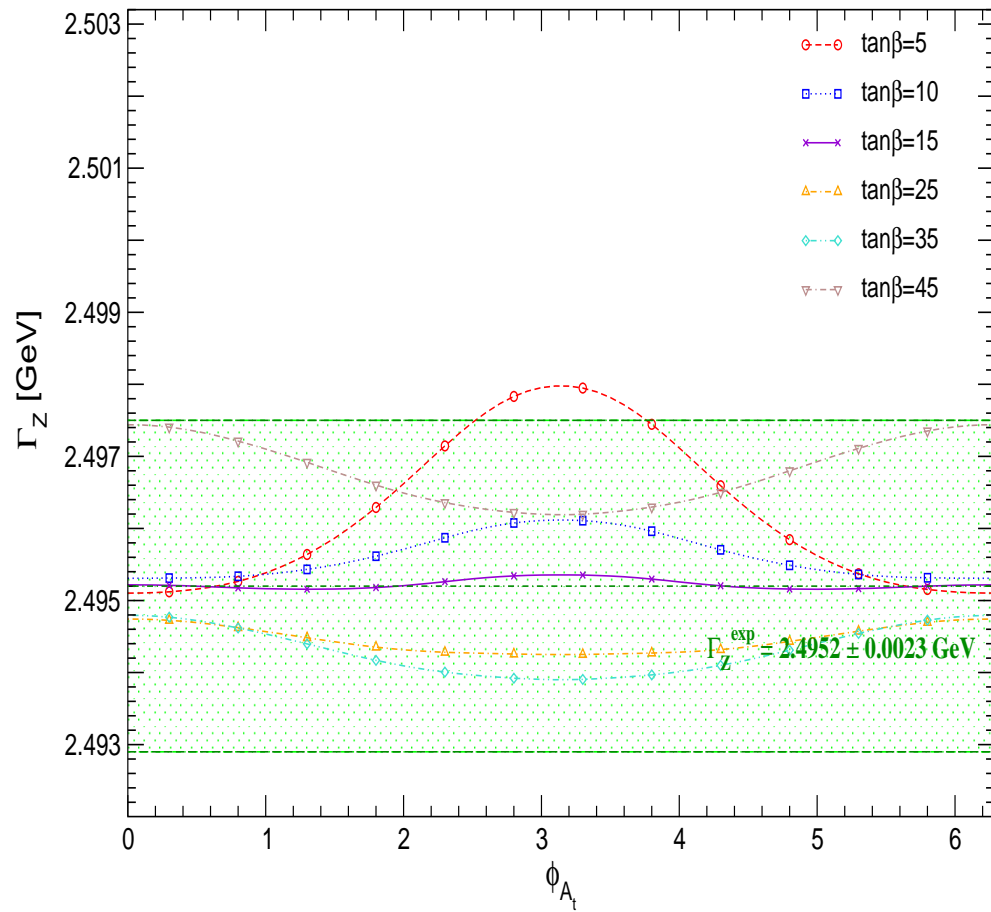
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⇒ strong dependence on ϕ_{A_t}

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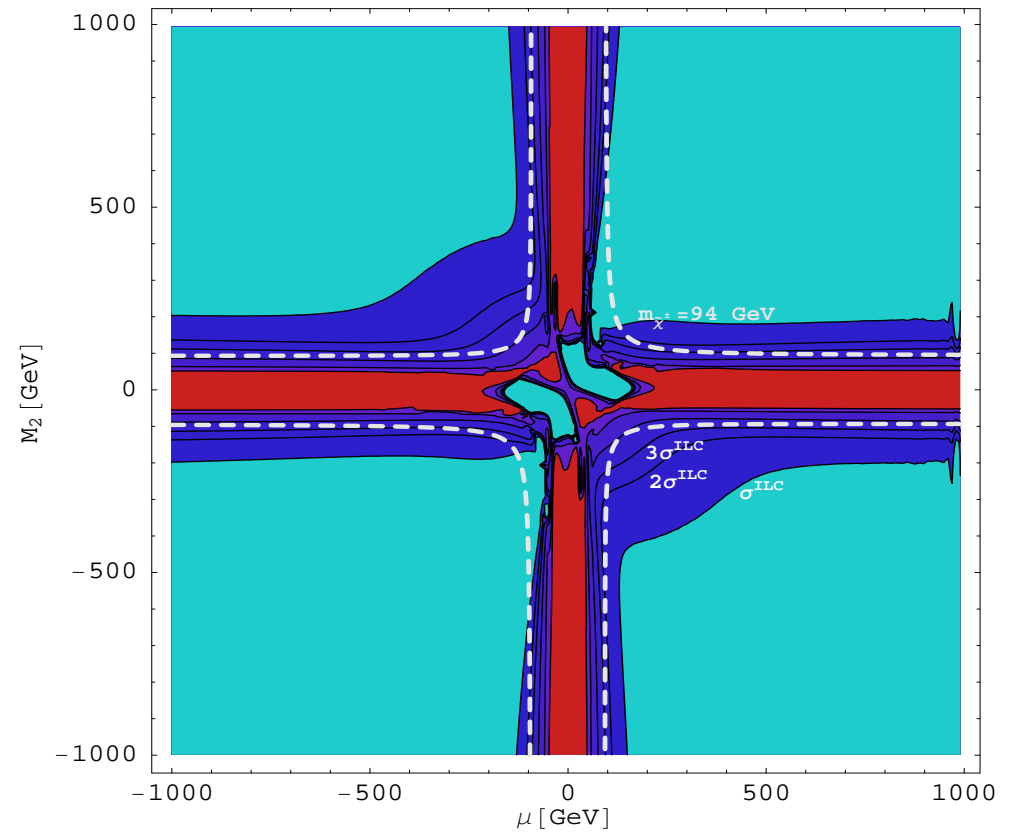
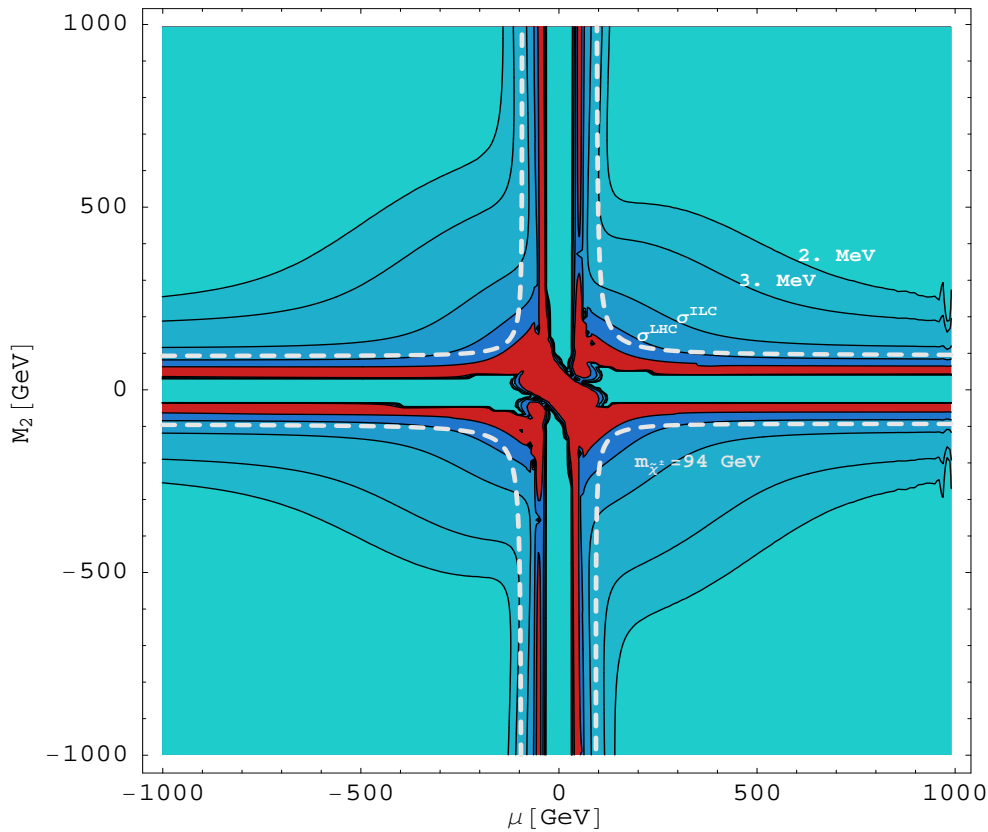
D) M_W and $\sin^2 \theta_{\text{eff}}$ in Split SUSY:

[N. Arkani-Hamed, S. Dimopoulos '04] [G. Giudice, A. Romanino '04]

→ deviations to the SM with $M_H^{\text{SM}} = M_h$:

M_W

$\sin^2 \theta_{\text{eff}}$



⇒ even with ILC precision hardly any effect visible

E) Scenario with no SUSY particles at the LHC:

→ $\sin^2 \theta_{\text{eff}}$ investigation

→ SPS 1a with heavy scalars

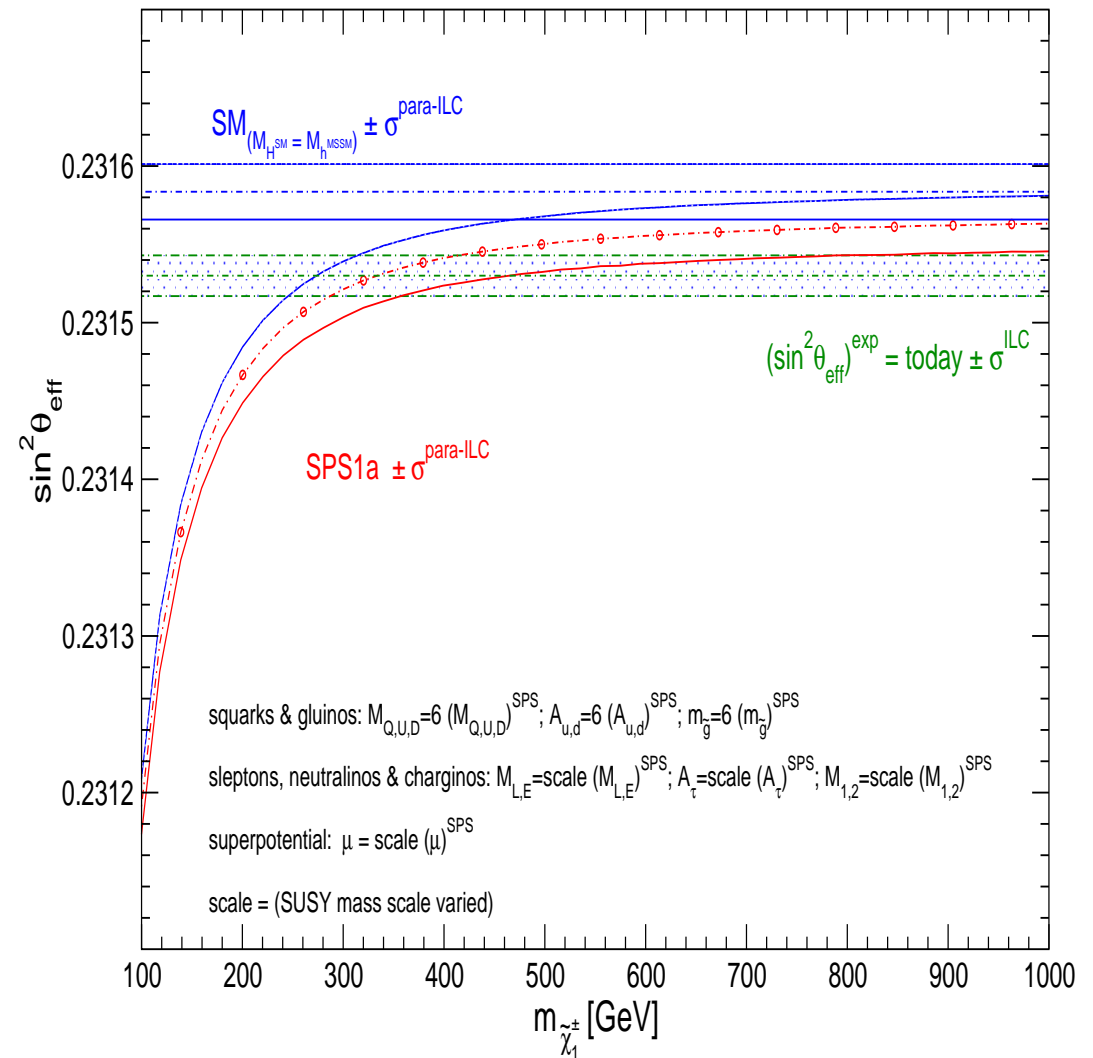
SM prediction

vs.

MSSM (SPS 1a) prediction

vs.

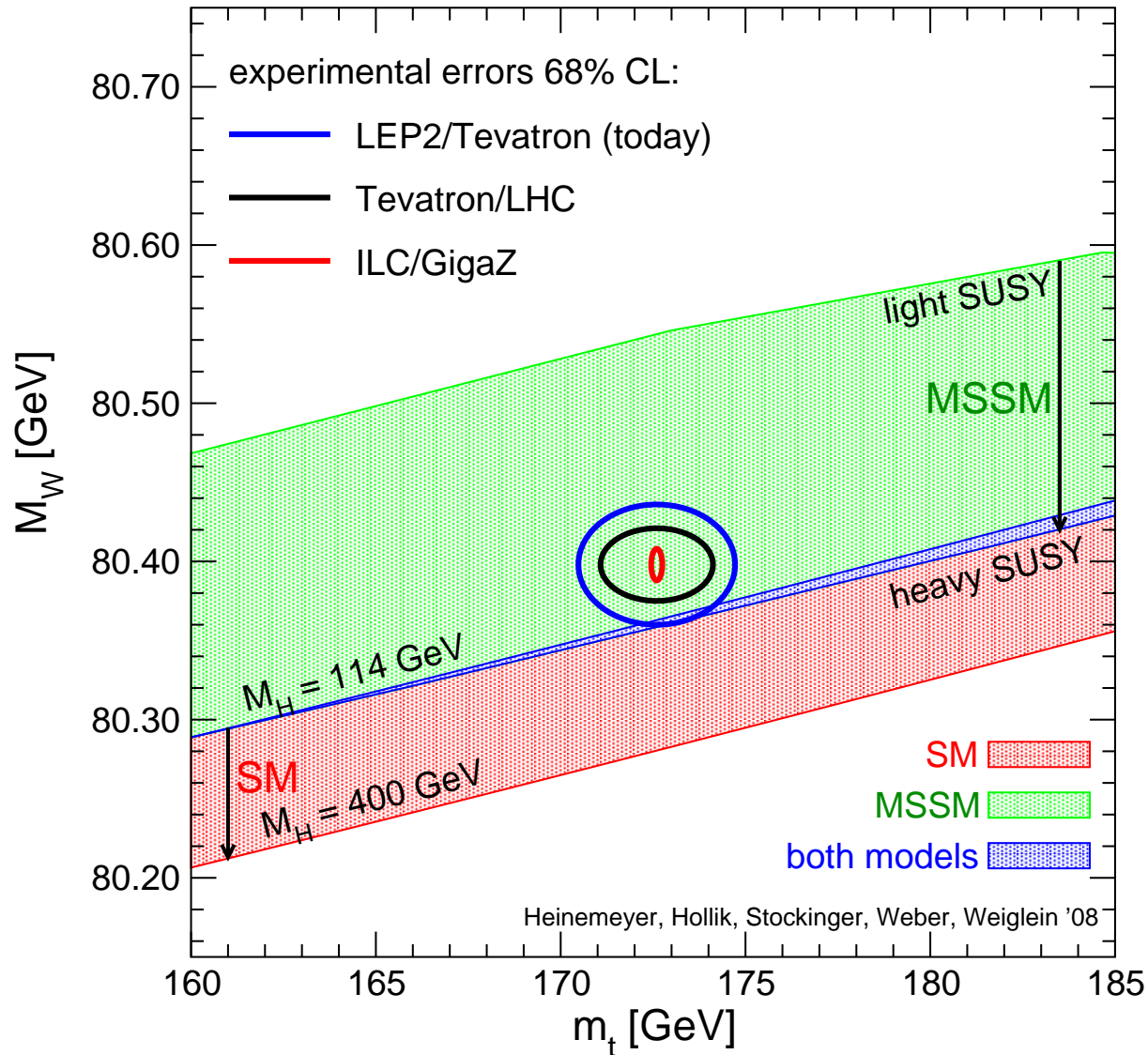
ILC resolution



⇒ the ILC(1000)/GigaZ could detect SUSY directly/indirectly

F) Parameter scans in the (full) MSSM:

Prediction for M_W in the SM and the MSSM :

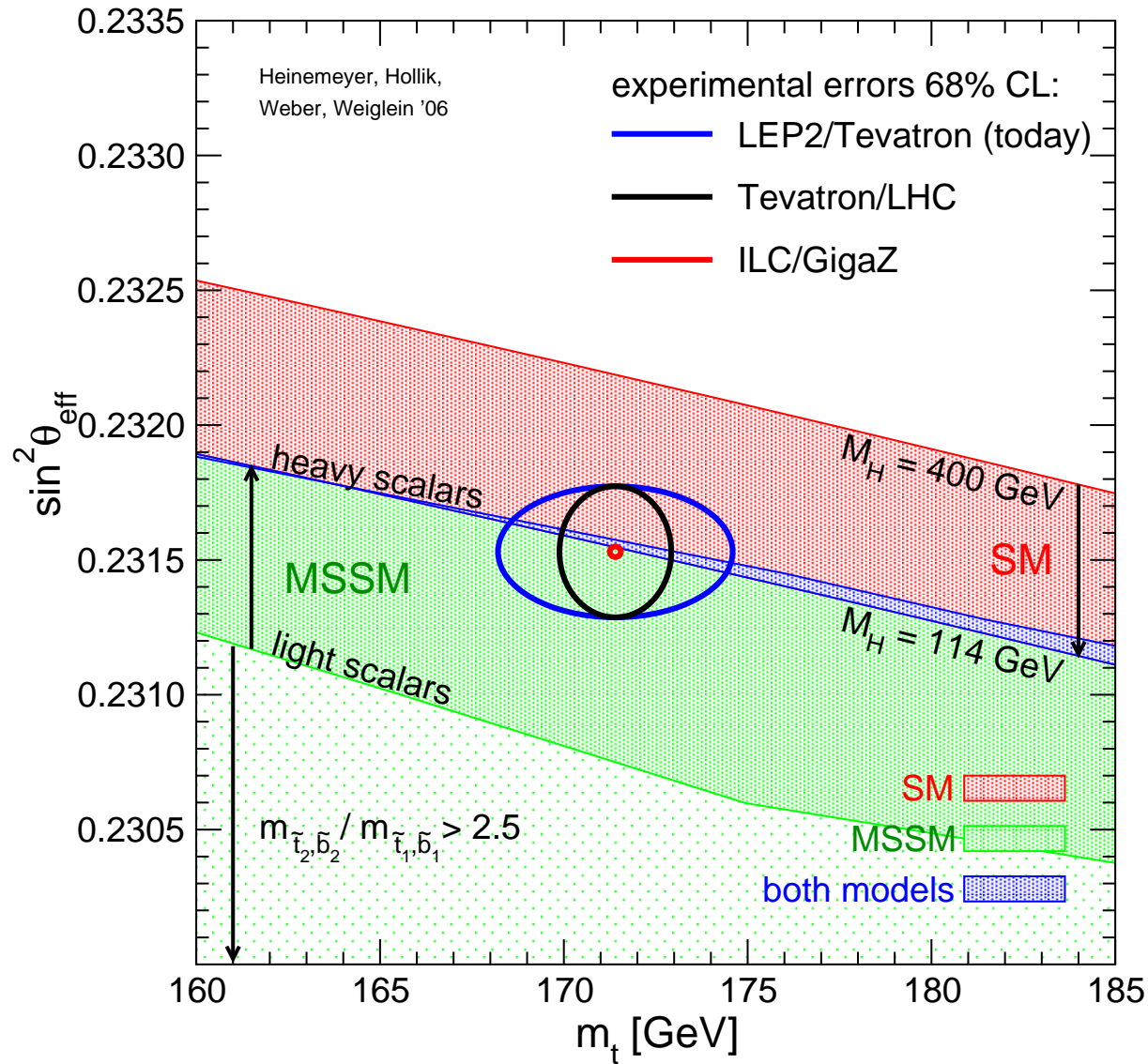


MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of M_H^{SM}

Prediction for $\sin^2 \theta_{\text{eff}}$ in the **SM** and the **MSSM** :



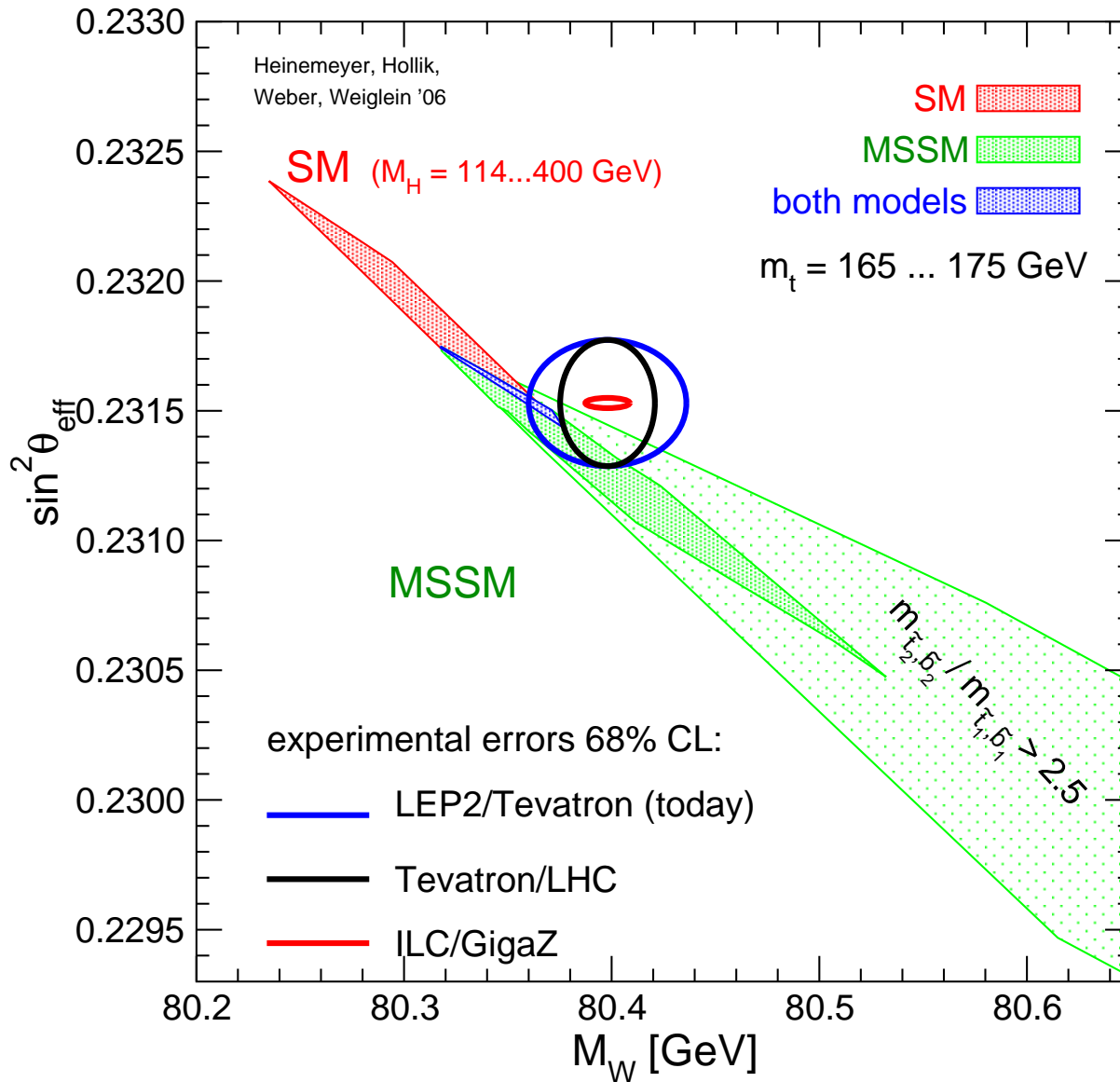
MSSM band:
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SM band:
variation of M_H^{SM}

Prediction for M_W and $\sin^2 \theta_{\text{eff}}$ in the SM and the MSSM :

[S.H., W. Hollik, A.M. Weber, G. Weiglein '07]



MSSM band:
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Prediction of M_h in the CMSSM/mSUGRA

[Buchmüller, Cavanaugh, de Roeck, S.H., Isidori, Paradisi, Ronga, Weber, Weiglein '07]

- combine all electroweak precision data as in the SM
- combine with B physics observables
- combine with CDM and $(g - 2)_\mu$
- include SM parameters with their errors: m_t, \dots
- scan over the full CMSSM parameter space $(m_{1/2}, m_0, A_0, \tan \beta)$

Prediction of M_h in the CMSSM/mSUGRA

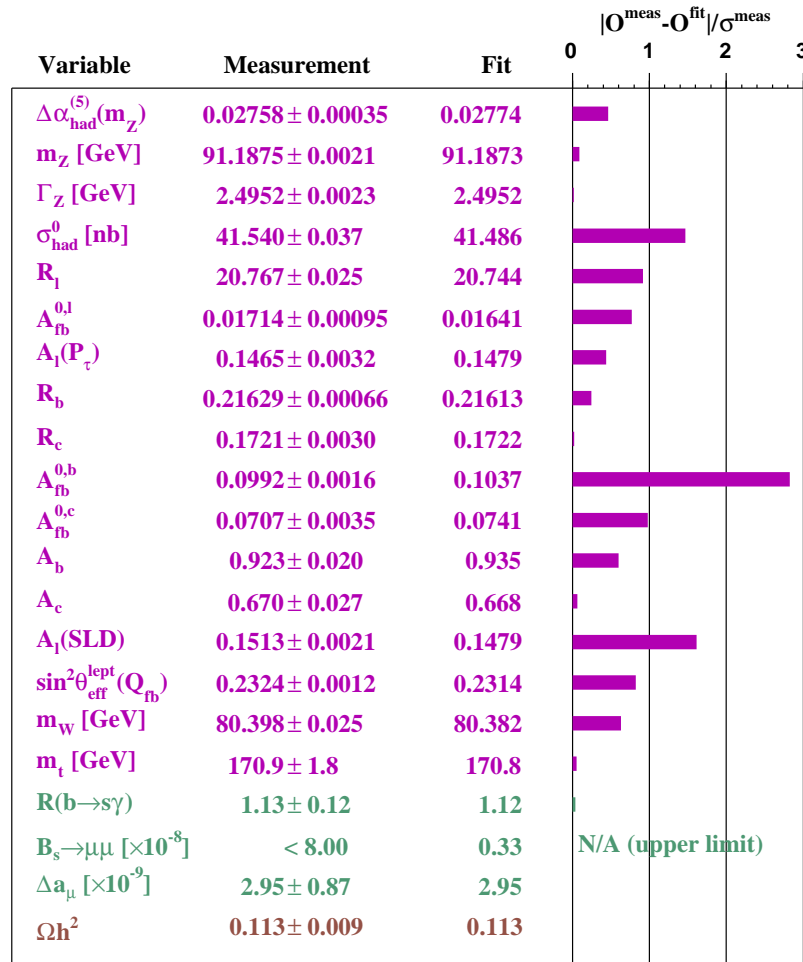
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- combine all electroweak precision data as in the SM
 - combine with B physics observables
 - combine with CDM and $(g - 2)_\mu$
 - include SM parameters with their errors: m_t, \dots
 - scan over the full CMSSM parameter space $(m_{1/2}, m_0, A_0, \tan \beta)$
- ⇒ preferred CMSSM parameters
- ⇒ preferred M_h values
- ⇒ LHC/ILC reach

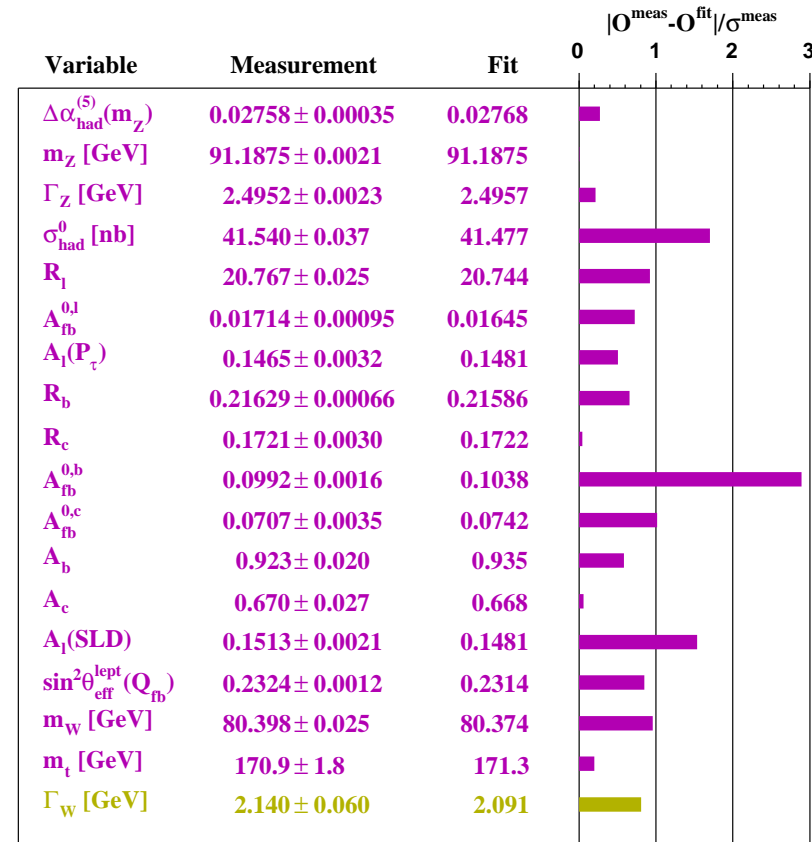
Pull distributions:

[Buchmüller, Cavanaugh, de Roeck, S.H., Isidori, Paradisi, Ronga, Weber, Weiglein '07]

CMSSM



SM

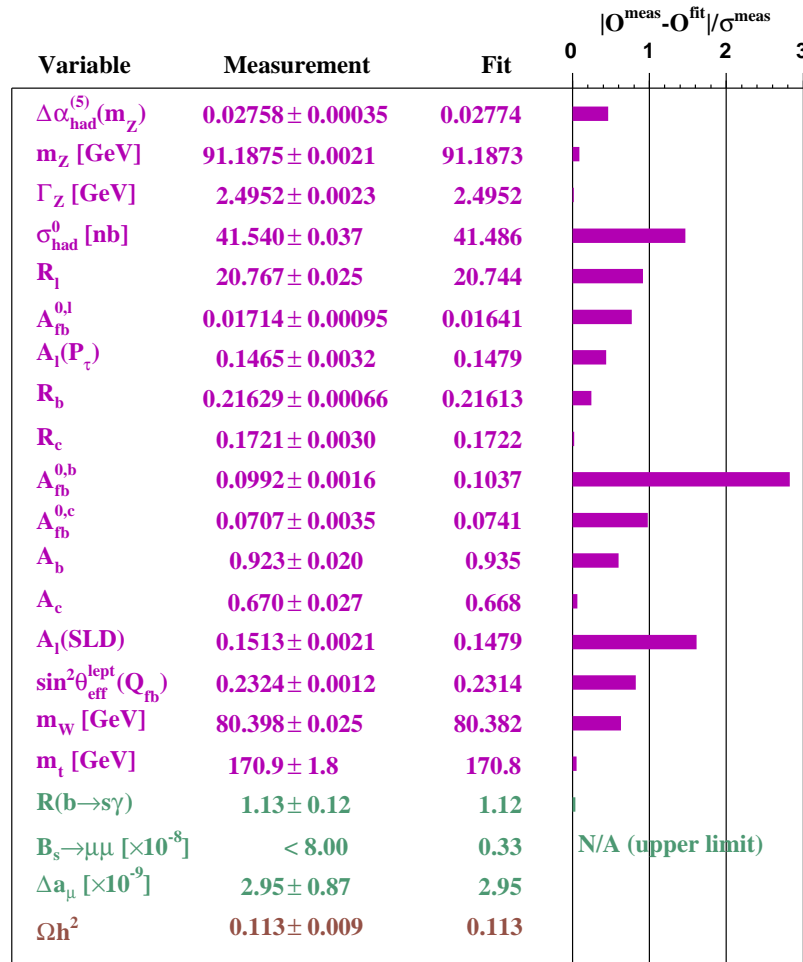


\Rightarrow note the new observables: $BR(b \rightarrow s\gamma)$, $(g - 2)_\mu$, CDM

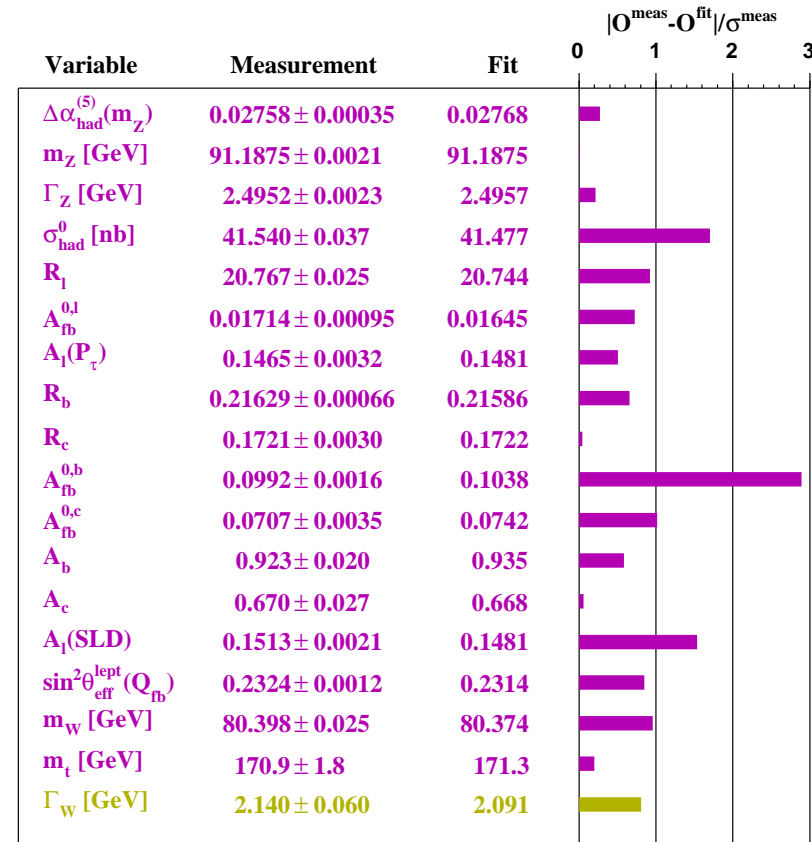
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CMSSM



SM

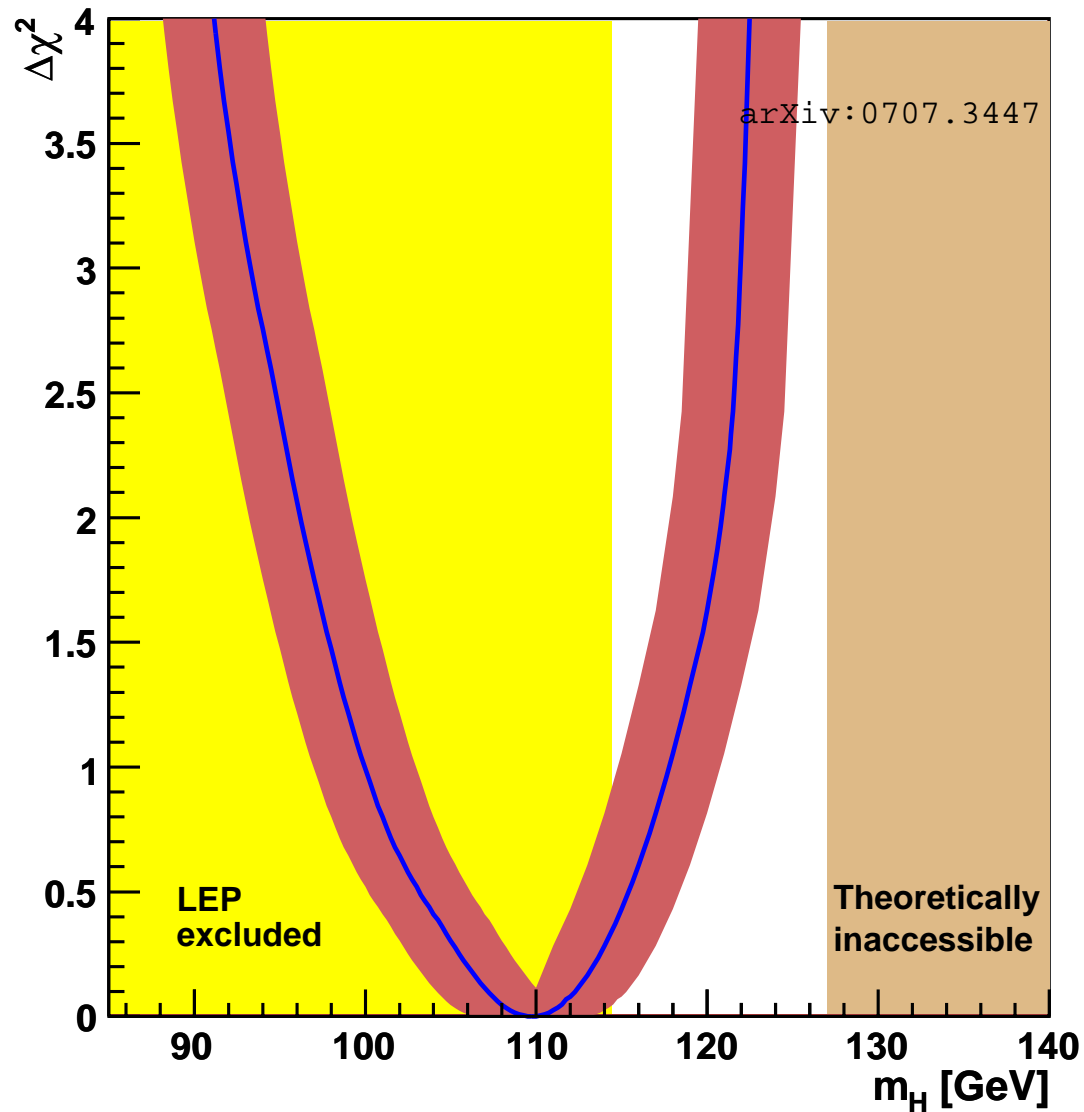


Probabilities: 24% / 20%

15% / 15% (incl. / excl. M_h)

Red band plot:

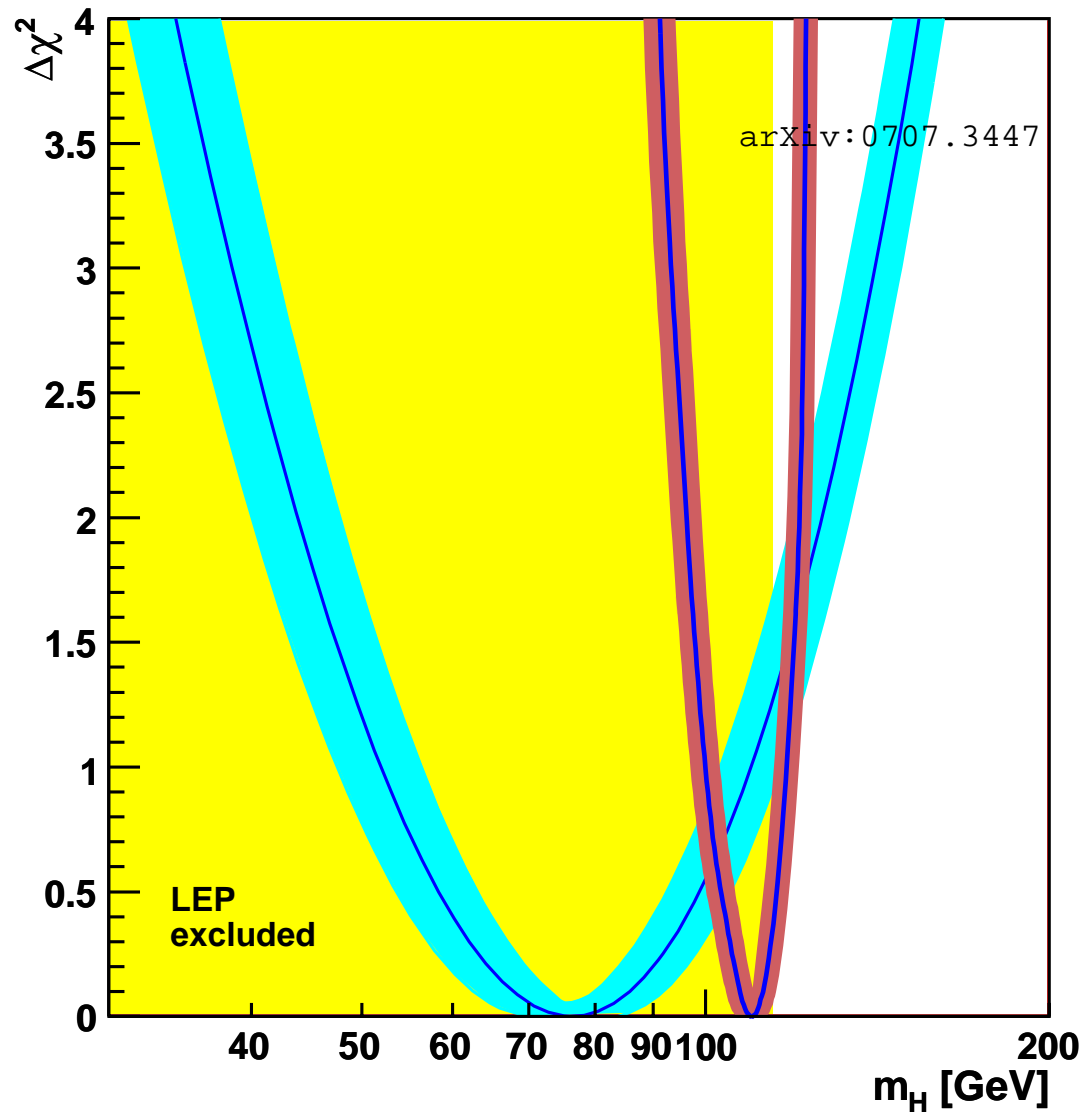
[Buchmüller, Cavanaugh, de Roeck, S.H., Isidori, Paradisi, Ronga, Weber, Weiglein '07]



$$M_h = 110_{-10}^{+8} (\text{exp}) \pm 3(\text{theo}) \text{ GeV}$$

Blue/Red band plot:

[Buchmüller, Cavanaugh, de Rooek, S.H., Isidori, Paradisi, Ronga, Weber, Weiglein '07]



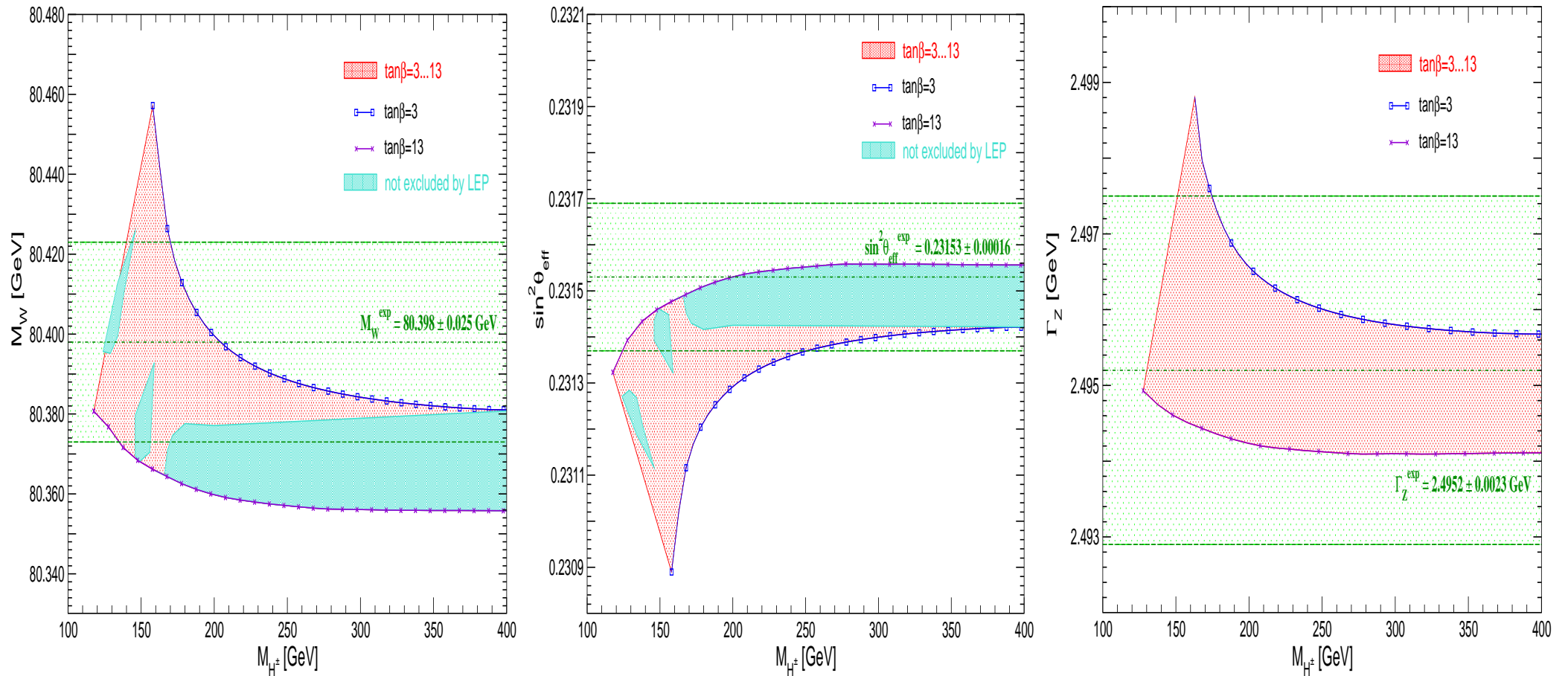
CMSSM (despite its simplicity) is better than the SM

4. Conclusinos

- Precision observables
 - can give valuable information about the “true” Lagrangian
 - can provide bounds on SUSY parameter space
- SM: Blue band plot: $\Rightarrow M_H^{SM} = 87_{-27}^{+36}$ GeV (too light for LEP bounds?)
- electroweak precision observables (EWPO):
 $\mathcal{O} = M_W, \sin^2 \theta_{\text{eff}} (A_{\text{FB}}^{b,c}, A_{\text{LR}}^{e,\mu}), R_l, R_b, \sigma_0^{\text{had}}, \dots$
- best MSSM prediction = full (available) SM result
+ all existing MSSM corrections
e.g. full 1L incl. complex phases
– double counting
- SUSY dependencies:
 - strong dependence only for $M_W, \sin^2 \theta_{\text{eff}}, \Gamma_Z$
 - strong dependence on $M_{\text{SUSY}}, \mu, M_2, m_t, \dots$
 - strong dependence on ϕ_{A_t}
 - strong dependence on ϕ_{A_b} for large $\tan \beta$
- CMSSM/mSUGRA: Red band plot: $\Rightarrow M_h^{\text{CMSSM}} = 110 \pm 8 \pm 3$ GeV

Back-up

Can EWPO close the CPX holes?



⇒ **No!** EWPO do not have a strong impact on CPX holes
 CPX holes: ZZh_1 coupling small ⇒ small change in EWPO

Remaining theoretical (intrinsic) uncertainties of M_W :

[J. Haestier, S.H., D. Stöckinger, G. Weiglein '05]

[S.H., W. Hollik, D. Stöckinger, A.M. Weber, G. Weiglein '06]

Estimate missing SUSY corrections order by order:

- $\mathcal{O}(\alpha_t^2, \alpha_t \alpha_b, \alpha_b^2)$: beyond existing leading contributions
- $\mathcal{O}(\alpha \alpha_s)$: beyond $\Delta\rho$ approx.
- $\mathcal{O}(\alpha \alpha_s^2)$
- $\mathcal{O}(\alpha^2 \alpha_s)$
- $\mathcal{O}(\alpha^3)$
- missing phase dependence at two-loop

⇒ evaluate for $M_{\text{SUSY}} = 300, 500, 1000 \text{ GeV}$

Combine with SM uncertainty: $\delta M_W^{\text{SM, intr.}} = 4 \text{ MeV}$

$$\delta M_W^{\text{SUSY, intr.}} = 5 - 11 \text{ MeV}$$

(depending on M_{SUSY})