S, T, U in mHDM

Precision constraints on multi-Higgs-doublet models

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based on work with W. Grimus, L. Lavoura, O.M. Ogreid

arXiv:0711.4011 (JPG); arXiv0802.4353 (NPB)

The famous ρ

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}$$

Tree level relation: $$\rho = 1$$

Relation holds also (at tree level) for more general Higgs sector, as long as only doublets have non-zero vevs

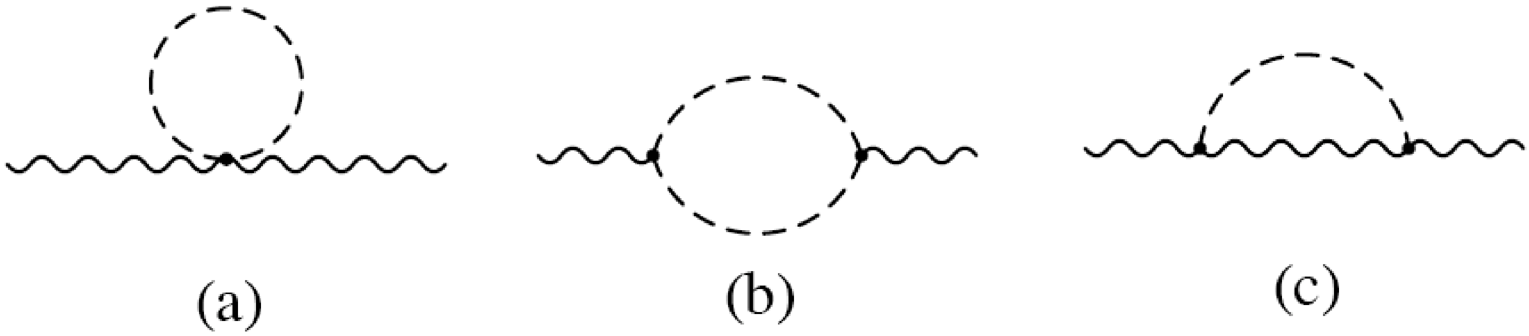
Pioneers on ρ

- D.A. Ross, M.J.G.Veltman, NPB 95 (1975) 135
- M.J.G.Veltman, Acta Phys. Pol. B8 (1977) 475
- M.J.G.Veltman, NPB 123 (1977) 89
- M.B. Einhorn, M.J.G.Veltman, NPB 191 (1981) 146
- S. Bertolini, NPB 272 (1986) 77 2HDM

SM subtraction

$$\rho - 1 = \Delta\rho \simeq \alpha T = \bar{T}$$

Self energies of *W* and *Z* get corrections from scalar sector



Definition of self-energy tensor

$$\Pi^{\mu\nu}_{VV'}(q) = g^{\mu\nu} A_{VV'}\left(q^2\right) + q^\mu q^\nu B_{VV'}\left(q^2\right)$$

Object of interest

$q^2 \approx 0,\ q^2 = m_Z^2$, and $q^2 = m_W^2$

"LEP scales"

Definitions of *S*, *T*, *U*

- M. Peskin & T. Takeuchi, PRL 65 (1990) 964, PRD 46 (1992) 381
- G. Altarelli, R. Barbieri, PL B 253 (1991) 161
- G. Altarelli, R. Barbieri, S. Jadach, NPB 369 (1992) 3
- I. Maksymyk, C.P. Burgess, D. London, PRD 50 (1994) 529
- R. Barbieri, A. Pomarol, R. Rattazzi, A. Strumia, NPB 703 (2004) 127

Recent papers: more observables

Early papers defined *S*, *T*, *U* in terms of
first derivatives of self energy function at the origin

Maksymyk et al define *S*, *T*, *U*, *V*, *W*, *X* in terms of differences
BPRS introduce second derivatives

Maksymyk et al (1994)

$$\frac{\alpha}{4s_W^2c_W^2}S = \frac{A_{ZZ}(m_Z^2) - A_{ZZ}(0)}{m_Z^2} - \left.\frac{\partial A_{\gamma\gamma}(q^2)}{\partial q^2}\right|_{q^2=0} + \frac{c_W^2 - s_W^2}{c_W s_W}\left.\frac{\partial A_{\gamma Z}(q^2)}{\partial q^2}\right|_{q^2=0}$$

$$\alpha T = \frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2},$$

$$\frac{\alpha}{4s_W^2}U = \frac{A_{WW}(m_W^2) - A_{WW}(0)}{m_W^2} - c_W^2\frac{A_{ZZ}(m_Z^2) - A_{ZZ}(0)}{m_Z^2} - s_W^2\left.\frac{\partial A_{\gamma\gamma}(q^2)}{\partial q^2}\right|_{q^2=0} + 2c_W s_W\left.\frac{\partial A_{\gamma Z}(q^2)}{\partial q^2}\right|_{q^2=0},$$

$$\alpha V = \left.\frac{\partial A_{ZZ}(q^2)}{\partial q^2}\right|_{q^2=m_Z^2} - \frac{A_{ZZ}(m_Z^2) - A_{ZZ}(0)}{m_Z^2},$$

$$\alpha W = \left.\frac{\partial A_{WW}(q^2)}{\partial q^2}\right|_{q^2=m_W^2} - \frac{A_{WW}(m_W^2) - A_{WW}(0)}{m_W^2},$$

$$\frac{\alpha}{s_W c_W}X = \left.\frac{\partial A_{\gamma Z}(q^2)}{\partial q^2}\right|_{q^2=0} - \frac{A_{\gamma Z}(m_Z^2)}{m_Z^2}.$$

Main structures:

$$\frac{A_{VV'}\left(q^2\right) - A_{VV'}\left(0\right)}{q^2}$$

$\approx$ first derivative

$$\left.\frac{\partial A_{VV}\left(q^2\right)}{\partial q^2}\right|_{q^2=m_V^2} - \frac{A_{VV}\left(m_V^2\right) - A_{VV}\left(0\right)}{m_V^2}$$

$\approx$ second derivative

Scalar sector: Notation, definitions

Scalars, 3 kinds:

Scalar SU(2) doublets: $\phi_k = \begin{pmatrix} \varphi_k^+ \\ \varphi_k^0 \end{pmatrix} \quad (k = 1, 2, \ldots, n_d)$

Charged SU(2) singlets: $\chi_j^+ \quad (j = 1, 2, \ldots, n_c)$

Neutral SU(2) singlets: $\chi_l^0 \quad (l = 1, 2, \ldots, n_n)$

Neutral fields have vevs:

$$\langle 0 \left| \varphi_k^0 \right| 0 \rangle = \frac{v_k}{\sqrt{2}} \qquad \langle 0 \left| \chi_l^0 \right| 0 \rangle = u_l$$

Scalar sector: Notation, definitions

Expand in terms of mass eigenfields:

Charged fields:

$n = n_d + n_c$

$$\begin{cases} \varphi_k^+ = \sum_{a=1}^{n} \mathcal{U}_{ka} S_a^+ \\ \chi_j^+ = \sum_{a=1}^{n} \mathcal{T}_{ja} S_a^+ \end{cases}$$

$\mathcal{U}:\ n_d \times n$

$\mathcal{T}:\ n_c \times n$

Neutral fields:

$m = 2n_d + n_n$

$$\begin{cases} \varphi_k^{0\prime} = \sum_{b=1}^{m} \mathcal{V}_{kb} S_b^0 \\ \chi_l^{0\prime} = \sum_{b=1}^{m} \mathcal{R}_{lb} S_b^0 \end{cases}$$

$\mathcal{V}:\ n_d \times m$

$\mathcal{R}:\ n_n \times m$

$$\tilde{\mathcal{U}} \equiv \begin{pmatrix} \mathcal{U} \\ \mathcal{T} \end{pmatrix}$$

unitary

$$\tilde{\mathcal{V}} \equiv \begin{pmatrix} \mathrm{Re}\,\mathcal{V} \\ \mathrm{Im}\,\mathcal{V} \\ \mathcal{R} \end{pmatrix}$$

orthogonal

Use unitarity and orthogonality to remove dependence on $\mathcal{T}, \mathcal{R}$

Scalars couple to *W* and *Z* via covariant derivatives

Doublets:

$$D_\mu \phi_k = \begin{pmatrix} \partial_\mu \varphi_k^+ - i\,\frac{g}{\sqrt{2}}\,W_\mu^+ \varphi_k^0 + i\,\frac{g\,(s_W^2 - c_W^2)}{2c_W}\,Z_\mu \varphi_k^+ + ieA_\mu \varphi_k^+ \\ \partial_\mu \varphi_k^0 - i\,\frac{g}{\sqrt{2}}\,W_\mu^- \varphi_k^+ + i\,\frac{g}{2c_W}\,Z_\mu \varphi_k^0 \end{pmatrix}$$

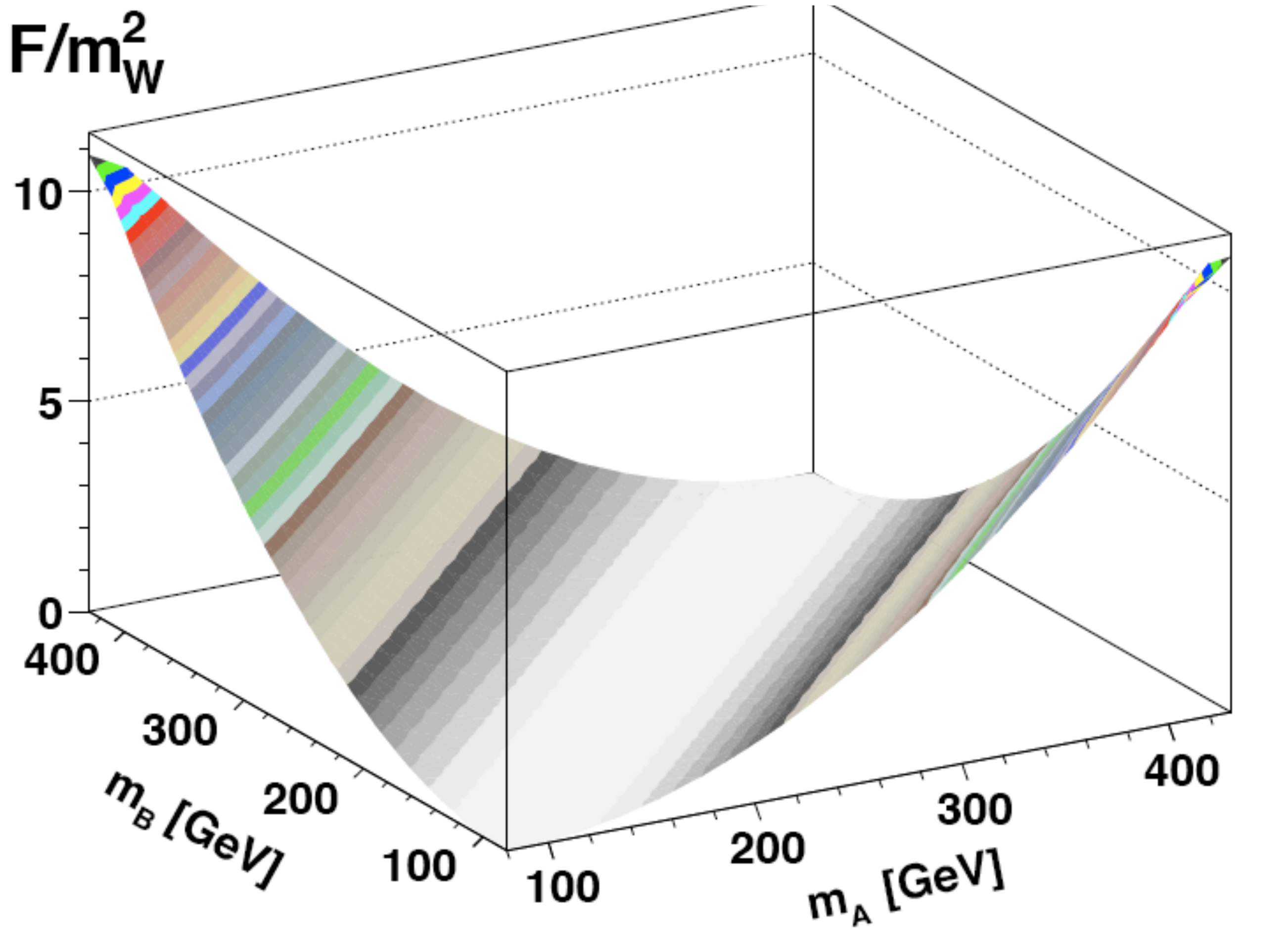
Singlets (charged):

$$D_\mu \chi_j^+ = \partial_\mu \chi_j^+ + i\,\frac{g s_W^2}{c_W}\,Z_\mu \chi_j^+ + ieA_\mu \chi_j^+$$

Singlets (neutral): no coupling

T given by function

$$F\left(m_A^2, m_B^2\right) \equiv \begin{cases} \dfrac{m_A^2 + m_B^2}{2} - \dfrac{m_A^2 m_B^2}{m_A^2 - m_B^2} \ln \dfrac{m_A^2}{m_B^2} & \Leftarrow \; m_A^2 \neq m_B^2 \\ 0 & \Leftarrow \; m_A^2 = m_B^2 \end{cases}$$



General expression for *T*

charged neutral

$$
\begin{aligned}
\bar{T} \; = \; \frac{g^2}{64\pi^2 m_W^2} \Bigg\{ & \sum_{a=2}^{n} \sum_{b=2}^{m} \left| \left(\mathcal{U}^\dagger \mathcal{V} \right)_{ab} \right|^2 F\left(m_a^2, \mu_b^2 \right) \\
& - \sum_{b=2}^{m-1} \sum_{b'=b+1}^{m} \left[\operatorname{Im} \left(\mathcal{V}^\dagger \mathcal{V} \right)_{bb'} \right]^2 F\left(\mu_b^2, \mu_{b'}^2 \right) \\
& -2 \sum_{a=2}^{n-1} \sum_{a'=a+1}^{n} \left| \left(\mathcal{U}^\dagger \mathcal{U} \right)_{aa'} \right|^2 F\left(m_a^2, m_{a'}^2 \right) \\
& +3 \sum_{b=2}^{m} \left[\operatorname{Im} \left(\mathcal{V}^\dagger \mathcal{V} \right)_{1b} \right]^2 \left[F\left(m_Z^2, \mu_b^2 \right) - F\left(m_W^2, \mu_b^2 \right) \right] \\
& -3 \left[F\left(m_Z^2, m_h^2 \right) - F\left(m_W^2, m_h^2 \right) \right] \Bigg\},
\end{aligned}
$$

charged-neutral: positive

neutral-neutral: negative

charged-charged: negative

} vanish in limit $m_W = m_Z$

All information on scalar sector contained in 2 mixing matrices, and masses

(These mixing matrices refer to the SU(2) doublets)

For *S*, *U*, ..., need functions

$$G(I,J,Q) \equiv -\frac{16}{3} + \frac{5(I+J)}{Q} - \frac{2(I-J)^2}{Q^2} + \frac{3}{Q}\left[\frac{I^2+J^2}{I-J} - \frac{I^2-J^2}{Q} + \frac{(I-J)^3}{3Q^2}\right]\ln\frac{I}{J} + \frac{r}{Q^3}\,f(t,r)$$

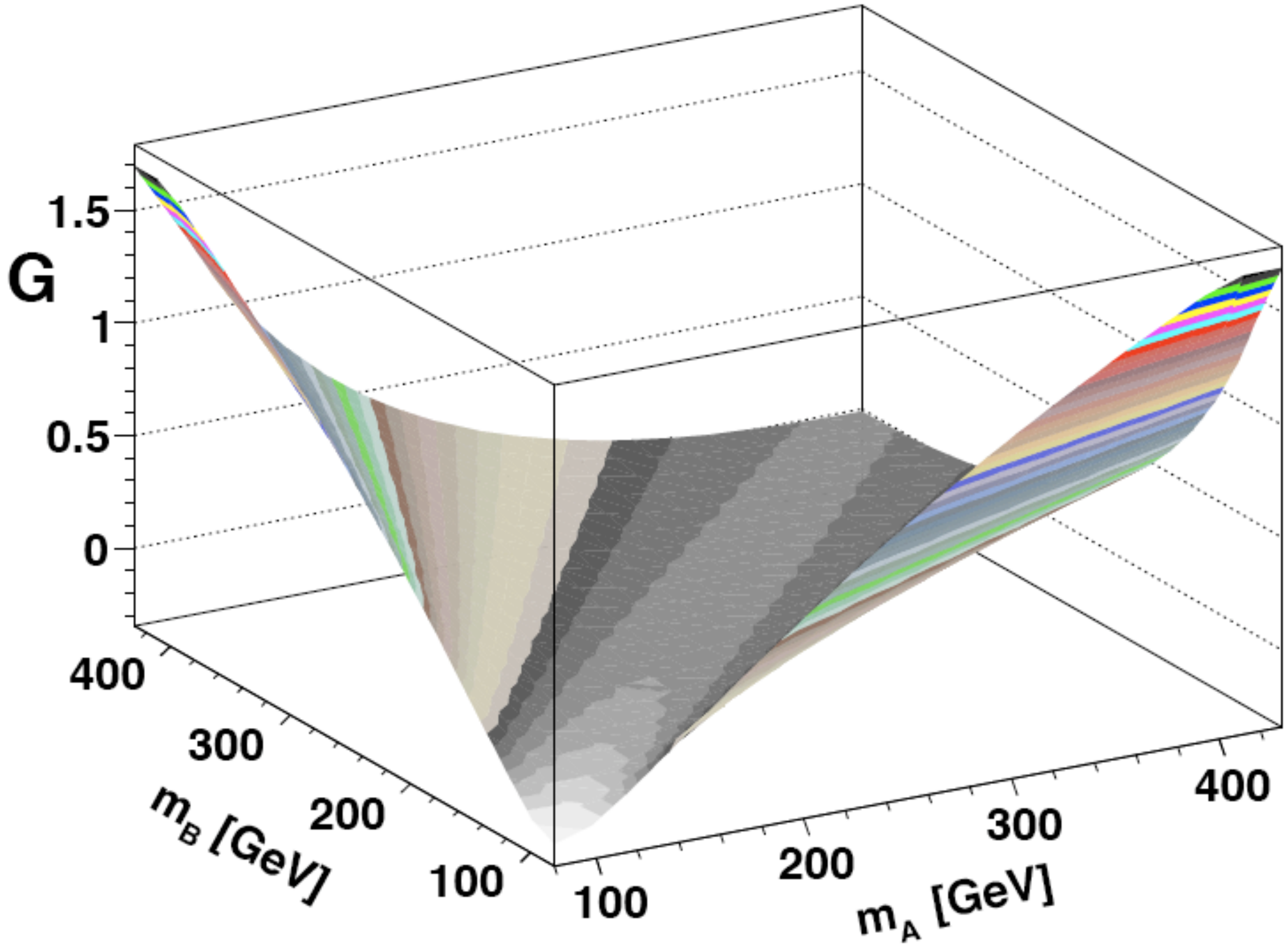
$$H(I,J,Q) \equiv 2 - \frac{9(I+J)}{Q} + \frac{6(I-J)^2}{Q^2} + \frac{3}{Q}\left[-\frac{I^2+J^2}{I-J} + 2\,\frac{I^2-J^2}{Q} - \frac{(I-J)^3}{Q^2}\right]\ln\frac{I}{J} + \left[I+J-\frac{(I-J)^2}{Q}\right]\frac{3\,f(t,r)}{Q^2}.$$

$$f(t,r) \equiv \begin{cases} \sqrt{r}\,\ln\left|\dfrac{t-\sqrt{r}}{t+\sqrt{r}}\right| & \Leftarrow \quad r>0, \\ 0 & \Leftarrow \quad r=0, \\ 2\sqrt{-r}\,\arctan\dfrac{\sqrt{-r}}{t} & \Leftarrow \quad r<0. \end{cases}$$

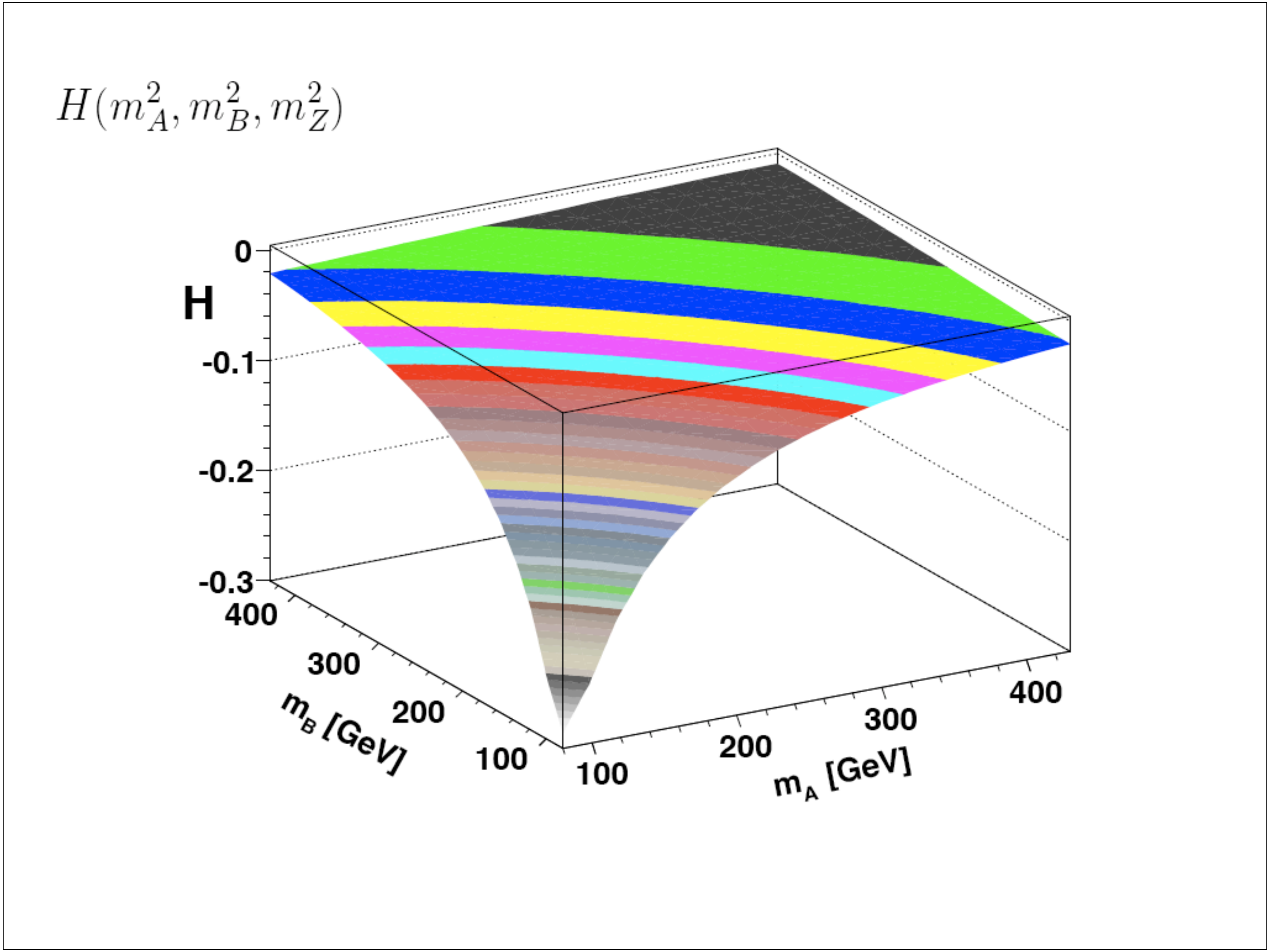
$$t = I + J - Q$$

$$r = Q^2 - 2Q(I+J) + (I-J)^2$$

$G(m_A^2, m_B^2, m_Z^2)$



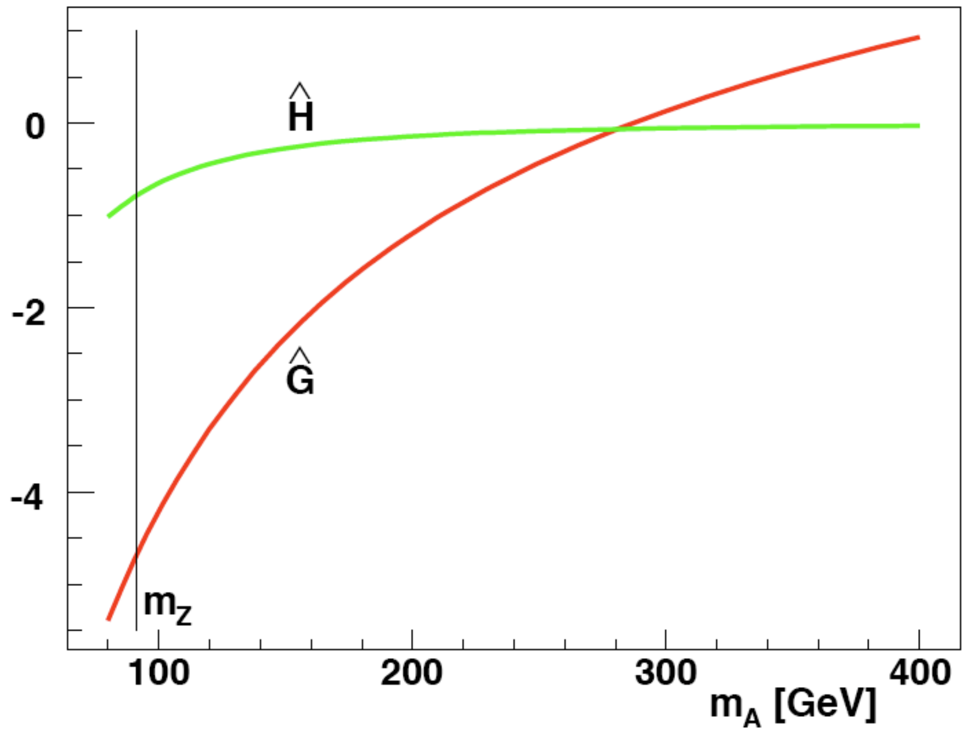
Does not in general vanish when $m_A^2 = m_B^2$



$H(m_A^2, m_B^2, m_Z^2)$
H
0
-0.1
-0.2
-0.3
400
300
200
100
m_B [GeV]
100
200
300
400
m_A [GeV]

$$\hat{G}(I,Q) = -\frac{79}{3} + 9\frac{I}{Q} - 2\frac{I^2}{Q^2} + \left(-10 + 18\frac{I}{Q} - 6\frac{I^2}{Q^2} + \frac{I^3}{Q^3} - 9\frac{I+Q}{I-Q}\right)\ln\frac{I}{Q} + \left(12 - 4\frac{I}{Q} + \frac{I^2}{Q^2}\right)\frac{f(I, I^2 - 4IQ)}{Q}$$

$$\hat{H}(I,Q) = 47 - 21\frac{I}{Q} + 6\frac{I^2}{Q^2} + 3\left(7 - 12\frac{I}{Q} + 5\frac{I^2}{Q^2} - \frac{I^3}{Q^3} + 3\frac{I+Q}{I-Q}\right)\ln\frac{I}{Q} + 3\left(28 - 20\frac{I}{Q} + 7\frac{I^2}{Q^2} - \frac{I^3}{Q^3}\right)\frac{f(I, I^2 - 4IQ)}{I - 4Q}$$



General expression for *S*

charged

$$
\begin{aligned}
\bar{S} \;=\; & \frac{g^2}{384\pi^2 c_W^2} \left\{ \sum_{a=2}^{n} \left[2 s_W^2 - \left(\mathcal{U}^\dagger \mathcal{U} \right)_{aa} \right]^2 G\left(m_a^2, m_a^2, m_Z^2 \right) \right. \\
& + 2 \sum_{a=2}^{n-1} \sum_{a'=a+1}^{n} \left| \left(\mathcal{U}^\dagger \mathcal{U} \right)_{aa'} \right|^2 G\left(m_a^2, m_{a'}^2, m_Z^2 \right) \\
& + \sum_{b=2}^{m-1} \sum_{b'=b+1}^{m} \left[\mathrm{Im} \left(\mathcal{V}^\dagger \mathcal{V} \right)_{bb'} \right]^2 G\left(\mu_b^2, \mu_{b'}^2, m_Z^2 \right) \\
& - 2 \sum_{a=2}^{n} \left(\mathcal{U}^\dagger \mathcal{U} \right)_{aa} \ln m_a^2 + \sum_{b=2}^{m} \left(\mathcal{V}^\dagger \mathcal{V} \right)_{bb} \ln \mu_b^2 - \ln m_h^2 \\
& \left. + \sum_{b=2}^{m} \left[\mathrm{Im} \left(\mathcal{V}^\dagger \mathcal{V} \right)_{1b} \right]^2 \hat{G}\left(\mu_b^2, m_Z^2 \right) - \hat{G}\left(m_h^2, m_Z^2 \right) \right\},
\end{aligned}
$$

neutral

All information on scalar sector contained in 2 mixing matrices, and masses

General expression for *U*

$$
\begin{aligned}
\bar{U} \;=\; & \frac{g^2}{384\pi^2}\left\{\sum_{a=2}^{n}\sum_{b=2}^{m}\left|\left(\mathcal{U}^\dagger\mathcal{V}\right)_{ab}\right|^2 G\left(m_a^2,\mu_b^2,m_W^2\right)\right. \\
& -\sum_{a=2}^{n}\left[2s_W^2-\left(\mathcal{U}^\dagger\mathcal{U}\right)_{aa}\right]^2 G\left(m_a^2,m_a^2,m_Z^2\right) \\
& -2\sum_{a=2}^{n-1}\sum_{a'=a+1}^{n}\left|\left(\mathcal{U}^\dagger\mathcal{U}\right)_{aa'}\right|^2 G\left(m_a^2,m_{a'}^2,m_Z^2\right) \\
& -\sum_{b=2}^{m-1}\sum_{b'=b+1}^{m}\left[\mathrm{Im}\left(\mathcal{V}^\dagger\mathcal{V}\right)_{bb'}\right]^2 G\left(\mu_b^2,\mu_{b'}^2,m_Z^2\right) \\
& +\sum_{b=2}^{m}\left[\mathrm{Im}\left(\mathcal{V}^\dagger\mathcal{V}\right)_{1b}\right]^2\left[\hat{G}\left(\mu_b^2,m_W^2\right)-\hat{G}\left(\mu_b^2,m_Z^2\right)\right] \\
& \left.-\hat{G}\left(m_h^2,m_W^2\right)+\hat{G}\left(m_h^2,m_Z^2\right)\right\}
\end{aligned}
$$

For *V*, *W*, *X*, see arXiv0802.4353 (NPB)

All information on scalar sector contained in 2 mixing matrices, and masses

Application: 2HDM

Higgs basis:

$$\phi_1 = \begin{pmatrix} G^+ \\ (v + H + iG^0)/\sqrt{2} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} S_2^+ \\ (R + iI)/\sqrt{2} \end{pmatrix}$$

Rotate to mass eigenstates:

$$\begin{pmatrix} H \\ R \\ I \end{pmatrix} = O \begin{pmatrix} S_2^0 \\ S_3^0 \\ S_4^0 \end{pmatrix} \qquad \begin{pmatrix} H + iG^0 \\ R + iI \end{pmatrix} = \mathcal{V} \begin{pmatrix} G^0 \\ S_2^0 \\ S_3^0 \\ S_4^0 \end{pmatrix}$$

$$\mathcal{V} = \begin{pmatrix} i & O_{11} & O_{12} & O_{13} \\ 0 & O_{21} + iO_{31} & O_{22} + iO_{32} & O_{23} + iO_{33} \end{pmatrix}$$

$$\begin{aligned} \alpha T = \Delta\rho \;=\; & \frac{g^2}{64\pi^2 m_W^2} \Bigg\{ \sum_{b=2}^{4} \left(1 - O_{1\,b-1}^2\right) F\left(m_2^2, \mu_b^2\right) \\ & - O_{13}^2 F\left(\mu_2^2, \mu_3^2\right) - O_{12}^2 F\left(\mu_2^2, \mu_4^2\right) - O_{11}^2 F\left(\mu_3^2, \mu_4^2\right) \\ & + 3 \sum_{b=2}^{4} O_{1\,b-1}^2 \left[F\left(m_Z^2, \mu_b^2\right) - F\left(m_W^2, \mu_b^2\right) - F\left(m_Z^2, m_h^2\right) + F\left(m_W^2, m_h^2\right) \right] \Bigg\} \end{aligned}$$

SM part subtracted out

Limit of $m_W = m_Z$

$$\Delta\rho \simeq \frac{g^2}{64\pi^2 m_W^2}\Big\{(1-O_{11}^2)F(m_2^2,\mu_2^2) + (1-O_{12}^2)F(m_2^2,\mu_3^2) + (1-O_{13}^2)F(m_2^2,\mu_4^2) - O_{13}^2 F\left(\mu_2^2,\mu_3^2\right) - O_{12}^2 F\left(\mu_2^2,\mu_4^2\right) - O_{11}^2 F\left(\mu_3^2,\mu_4^2\right)\Big\}$$

charged-neutral: positive

neutral-neutral: negative

Recall:

m_2	$\mu_2, \quad \mu_3, \quad \mu_4$
charged	neutral

Partial degeneracy: $m_2 = \mu_3 = \mu_4$

$$\Delta\rho \simeq \frac{g^2}{64\pi^2 m_W^2}\Big\{(1-O_{11}^2)F(m_2^2,\mu_2^2) - O_{13}^2 F\left(\mu_2^2,m_2^2\right) - O_{12}^2 F\left(\mu_2^2,m_2^2\right)\Big\}$$

$$= \frac{g^2}{64\pi^2 m_W^2}(1-O_{11}^2-O_{12}^2-O_{13}^2)F(m_2^2,\mu_2^2) = 0$$

Limit of CP conservation

$$\begin{pmatrix} H \\ R \\ I \end{pmatrix} = O \begin{pmatrix} S_2^0 \\ S_3^0 \\ S_4^0 \end{pmatrix} \qquad \begin{pmatrix} H + iG^0 \\ R + iI \end{pmatrix} = \mathcal{V} \begin{pmatrix} G^0 \\ S_2^0 \\ S_3^0 \\ S_4^0 \end{pmatrix}$$

$$O_{13} = 0$$

$$\Delta\rho \simeq \frac{g^2}{64\pi^2 m_W^2} \Big\{ (1 - O_{11}^2) F(m_2^2, \mu_2^2) + (1 - O_{12}^2) F(m_2^2, \mu_3^2) + F(m_2^2, \mu_4^2) - O_{12}^2 F\left(\mu_2^2, \mu_4^2\right) - O_{11}^2 F\left(\mu_3^2, \mu_4^2\right) \Big\}$$

Impose also custodial symmetry: $m_2 = \mu_4$

Use: $O_{11}^2 + O_{12}^2 = 1$

$$\Delta\rho \simeq 0$$

Twisted custodial symmetry:

J.-M. Gerard, M. Herquet, PRL 98 (2007) 251802

$$m_2 = \mu_3$$

Allows for a light pseudoscalar

$$\Delta\rho \simeq \frac{g^2}{64\pi^2 m_W^2} O_{12}^2 \left\{ F(m_2^2, \mu_2^2) + F(m_2^2, \mu_4^2) - F(\mu_2^2, \mu_4^2) \right\}$$

Does not "automatically" vanish

But it can be made small by a suitable choice of masses

Inert or dark scalar:

E. Ma, PRD 73 (2006) 077301

R. Barbieri, L. Hall, V.S. Rychkov, PRD 74 (2006) 015007

SM Higgs: μ_2

$$\Delta\rho = \frac{g^2}{64\pi^2 m_W^2}\left[\sum_{b=3}^{4} F\left(m_2^2, \mu_b^2\right) - F\left(\mu_3^2, \mu_4^2\right)\right]$$

Vanishes in limits $m_2 = \mu_3$ or $m_2 = \mu_4$

Barbieri, Hall, Rychkov take $\mu_2 = 400 - 600 \text{ GeV}$ "naturalness" argument

Makes $\Delta\rho < 0$

Compensate by $m_2 \sim 170 \text{ GeV}$ $\mu_3 < \mu_4 < m_2$

DM

2HDM cont.

$$\bar{S} = \frac{g^2}{384\pi^2 c_W^2}\Big\{(1-2s_W^2)^2\, G(m_2^2, m_2^2, m_Z^2)$$
$$+ O_{13}^2\, G(\mu_2^2, \mu_3^2, m_Z^2) + O_{12}^2\, G(\mu_2^2, \mu_4^2, m_Z^2) + O_{11}^2\, G(\mu_3^2, \mu_4^2, m_Z^2)$$
$$+ \log\frac{\mu_2^2\mu_3^2\mu_4^2}{m_2^4\, m_h^2}$$
$$+ O_{11}^2\, \hat{G}(\mu_2^2, m_Z^2) + O_{12}^2\, \hat{G}(\mu_3^2, m_Z^2) + O_{13}^2\, \hat{G}(\mu_4^2, m_Z^2)$$
$$- \hat{G}(m_h^2, m_Z^2)\Big\}$$

a bit more complicated

Summary

- *S*, *T*, *U*, *V*, *W*, *X* calculated for general scalar sector, consisting of doublets and singlets
- Result expressed in terms of two mixing matrices and simple functions of masses:

  T: one function

  S, *U*: 2 functions
- In general (*S*, *T*, *U*, *V*, *W*, *X*): 5 functions of masses