DRAFT: Alignment model of ILC LET components – for beam dynamics simulations

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Comments:

Revision on October 3 was a result of comments from Armin Reichold and Ryuhei Sugahara. Some of suggestions by Armin could not be adopted in this version, because there were not enough information to do so.

Revision on October 15 was again a result of comments from Armin Reichold and Ryuhei Sugahara.

We still need numbers for almost all parameters.

Revision on Feb. 08 based on discussions made during LET meeting in Dec. 07 at SLAC with inputs from K. Kubo, D. Schulte, Catherine LeCocq. We also include input from Grzegorz Grzelak following the IWAA 2008 conference.

Revision in April 08 added random walk parameters from LiCAS simulations as determined by G.Grzelak in his IWAA08 paper

1. Introduction

This document describes a method for generating the relevant positions and orientations of all elements of the ILC that are important for low emmitance transport. In the real world the 6D co-ordinates are the result of a global fit to all reference markers and component fiducials. This fit is very complex and requires detailed knowledge of the exact measurement process. We need a simplified method to efficiently generate the required co-ordinates. This method must have the same statistical properties as the real fit. The simplified method will be parametric and parameters that lead to successful beam based alignment can be understood as a requirement for the survey accuracy.

2. Offset errors

Relevant components:

- All magnets, except weak dipole correctors
- Accelerating cavities
- BPMs

“ y ” below represents either horizontal or vertical position offsets from the design line and θ refers to angular offsets.

A. General alignment scheme

Primary reference point will be marked every about L_{pr} (2.5 km, which corresponds to distance between shafts.). The errors of the point in all co-ordinates are assumed to follow a Gaussian distribution. .

Depending on the survey technique to be used, normal reference points will be located every L_r meters (5m ≤ L_r ≤ 50m). The error of the normal reference point can be modeled by a random walk from one of the primary reference points to the next. (see below for detail).

Most of machine components are aligned on girders or cryomodules before being set into the beam line. In the following we refer to cryomodules and girders collectively as supports. The error of the components with respect to their supports is assumed to be Gaussian..

Supports and independent components are aligned with respect to reference points near them. The error of this "stake-out" alignment is assumed to be Gaussian.

Parameters to be defined (horizontal and vertical)

- RMS errors of primary reference points ($y_{P,j}$)
- Parameters of random walk (a_y , t_y , a_θ , t_θ , $\Delta\theta_{systematic}$ and $\Delta y_{systematic}$.)
- RMS errors of supports and independent components w.r.t. reference points (??)
- RMS errors of components w.r.t. supports (??, there may be different errors for different components).
- Initial RMS offset errors of BPM w.r.t. attached magnets. (??, Before beam based measurement, e.g. quad-shunting, which will reduce the error.)

B. Survey network

This section applies to the tunnels for Return line of RTML, the bunch compressor (BC) and the main linac (ML) and describes the use of primary and normal reference points.

Primary reference point will be marked every L_{pr} in the tunnel along a beam line.

Offset error of the j -th primary reference point is:

$$y_{P,j} = G(a_{pr}, t_{pr}) \qquad (1\text{-}1)$$

(Tentative parameter may be a_{pr} = 2-4 mm horizontal and 10mm vertical.)

$G(a,t)$ is a Gaussian random with sigma a and truncated at ta .

We refer to the area between the j -th and j+1-th primary reference points as the j-th region. In the *j*-th region about L_{pr} / L_r normal reference points are located in the survey network. Initially the deviation of the normal reference points from the design line will be a random walk (random offset plus random angle change) with step length l_{step} starting from the j -th primary reference point. Let $y_{0,j,n}$ denote the offset of the normal reference point at the n -th step in the j -th region. The index 0 refers to the position not having been corrected with information from the primary reference points. Let $\theta_{j,n}$ be the angel of the n -th step in the j -th region. The effect of one step on the normal reference points can be expressed as:

$$\begin{aligned}
\theta_{j,n+1} &= \theta_{j,n} + G(a_\theta, t_\theta) + \Delta\theta_{systematic} \\
y_{0,j,n+1} &= y_{0,j,n} + G(a_y, t_y) + l_{step}\theta_{j,n+1} + \Delta y_{systematic} \qquad (0 \le n \le N-1) \qquad (1\text{-}2) \\
y_{0,j,0} &= y_{P,j}
\end{aligned}$$

where a_y , t_y , a_θ and t_θ are statistical error parameters for the random walk and $\Delta\theta_{systematic}$ and $\Delta y_{systematic}$ represent the systematic angle and offset errors, see [1][2][3].

We also need to compute the errors on the co-ordinate offsets from the random walk model. These errors describe how well we know the offsets calculated above. The statistical and systematic errors are separately described as functions of the step number *n* by the following formulae:

$$\sigma_{y,n,stat.} = \sqrt{l_{step}^2 a_\theta^2 \frac{n(n+1)(2n+1)}{6} + a_y^2 \frac{n(n+1)}{2}}$$
$$\sigma_{z,n,stat.} = \sqrt{a_z^2 \frac{n(n+1)}{2}} \qquad (1\text{-}3)$$
$$\sigma_{y,n,syst.} = l_{step} \Delta\theta_{systematic} n \frac{(n+1)}{2} + n\Delta y_{systematic}$$
$$\sigma_{z,n,syst.} = n\Delta z_{systematic}$$

Systematic and statistical errors should be added in quadrature. In practice the systematic errors are going to dominate the error at large distances as they scale quadratically with distance whereas the statistical errors only scale with power 2/3.

In [2] and [3] tentative parameters for the random walk of the LiCAS survey system have been found and are given below. It should be noted that other methods are likely to produce very different parameters.

- Statistical errors
  - $a_{vertical} < 5$ µm; $a_{\theta\text{-}vertical} < 55.4$ nrad
  - $a_{horizontal} < 5$ µm; $a_{\theta\text{-}horizontal} < 25.8$ nrad
  - Note that the transverse displacement errors are compatible with zero withing their errors and negligible compared to the
- Systematic errors
  - $\Delta\theta_{vertical}$ = 58 - 260 nrad $\Delta\theta_{horizontal}$ = 25 – 115 nrad
  - $\Delta y_{vertical}$ = 1.2 - 5.3 µm, $\Delta y_{horizontal}$ = 2.7 – 12.1 µm
- $l_{step} = 25$ m.

When the survey network reaches the next primary reference point, the network has to be corrected to meet the primary reference point. This correction can be done in various ways. A correction proportionally to the distance from the start point leads to kinks at the primary reference points and has been shown to be unsatisfactory for beam based alignment (reference from Kyoshi).

We propose here to use a correction of the random walk co-ordinates proportional to the squared distance from the primary reference marker We also suggest to not force the normal reference co-ordinates to exactly meet the primary reference points but to weight the two sets of co-ordinates with the inverse squared errors.

In the formula below we take N to be the number of steps in the j-th region. Step number $n=0$ corresponds to the j-th primary reference point and $n=N$ corresponds the $j+1$-th primary reference point. It is natural to make L_r / l_{step} integer.

$$y_{j,n} = y_{0,j,n} + \left(y_{0,j,N} - \frac{y_{P,j+1} \big/ \sigma_{y-primary}^2 + y_{0,j,N} \big/ \sigma_{y-0,j,n}^2}{\sigma_{y-primary}^2 + \sigma_{y-0,j,n}^2} \right) \left(s_j(0,n) / s_j(0,N) \right)^2 \quad (1\text{-}4)$$

Where:

- $0 \le n \le N$
- $s_j(m,n)$ is distance between n-th step and m-th step,

or, assuming constant l_{step},

$s_j(m,n) = (n-m) l_{step}$. (1-5)

Figure 1 shows how the correction of the random walk trajectory works. The trajectories across the different sections are continuous both in offset and in angle.

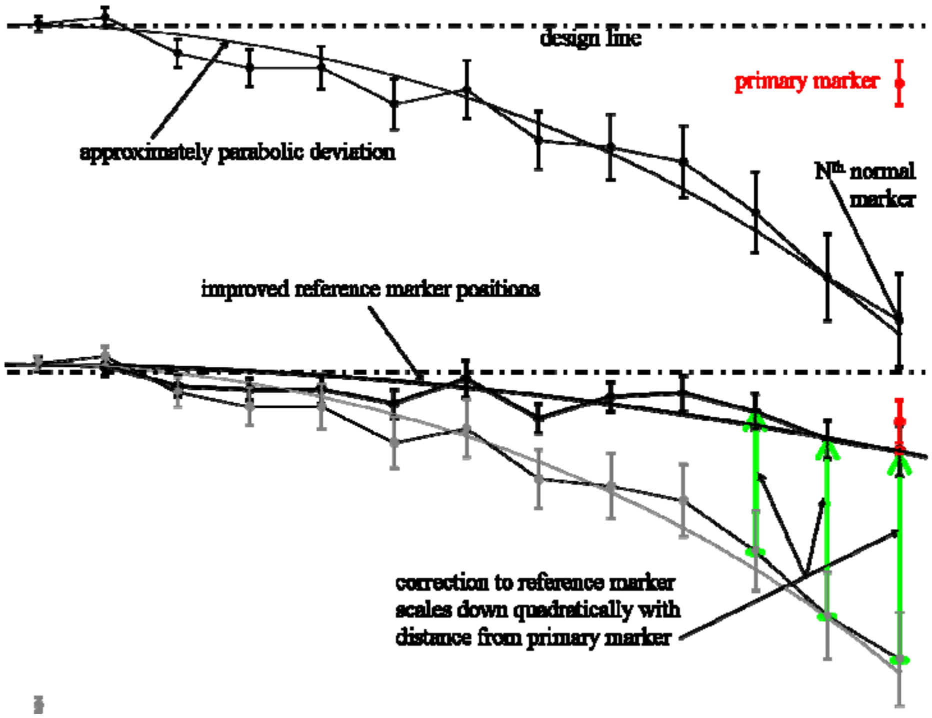

Fig.1, Quadratic correction of random walk trajectory to become compatible with the primary reference markers.

C. Component alignment in cold regions

This section applies to the ML and linac parts of BC.

Cryomodules will be aligned with respect to reference points near them. So, offsets of each cryomodule from the design line are caused by offsets of the reference points plus additional cryomodule alignment errors which are assumed to be Gaussian:

Offset of m-th cryomodule from its design position:

$$y_{cr,m} = y_{R(cr,m)} + G(a_{cr}, t_{cr}) \qquad (1\text{-}5)$$

(Tentative parameter may be taken from [??] a_{cr} = 0.1 mm.)

where $y_{R(cr,m)}$ is derived from misalignment of reference points near the cryomodule.

One possible model for $y_{R(cr,m)}$ will be based on least square fitting, as follows. Assume M reference points are used for the alignment of one module. Assume further that the errors on all these reference points are very similar. Let y_k be the offset of the k-th reference point and s_k the longitudinal position of the k-th reference point ($k = 0,1,,,M-1$) and s_{cr} the average longitudinal position of the fiducial markers on the cryomodule,

$$y_{R(cr,m)} = \bar{y} + (s_{cr} - \bar{s})\frac{\overline{sy} - \bar{s}\,\bar{y}}{\overline{s^2} - \bar{s}^2}, \qquad (1\text{-}6)$$

where bars denote average over used reference points, e.g.,

$$\bar{y} \equiv \frac{1}{M}\sum_{k=0}^{M-1} y_k\,. \qquad (1\text{-}7)$$

Components in each cryomodule have random misalignments with respect to the cryomodule:

Offset of i-th cavity in m-th cryomodule:

$y_{cav,m,i} = y_{cr,m} + (s_{cav,m,i} - s_{cr,m})\theta_{cr,m} + G(a_{cav}, t_{cav})$, (1-8)

(Tentative parameter may be a_{cav} = 0.3 mm.)

where $s_{cav,m,i}$ is the longitudinal position of the (reference of) cavity, $s_{cr,m}$ the longitudinal position of the reference of the cryomodule and $\theta_{cr,m}$ the tilt (yaw or pitch) angle of the cryomodule (see section 3).

Offset of Quad in m-th cryomodule:

$y_{q,m} = y_{cr,m} + (s_{q,m} - s_{cr,m})\theta_{cr,m} + G(a_q, t_q)$, (1-9)

(Tentative parameter may be taken from [??] as a_q = 0.3 mm.)

where $s_{q,m}$ is the longitudinal position of the magnet.

Assuming quad, correctors and BPM are assembled as one package, BPM is aligned with respect to the attached quad magnet:

Offset of BPM in m-th cryomodule:

$y_{bpm,m} = y_{q,m} + G(a_{bpm}, t_{bpm})$, (1-10)

(Tentative parameter may be a_{bpm} = 0.1 mm.)

Note that this is the initial error, before beam based measurement (e.g. quad-shunting). Beam based measurements will reduce this error. We expect offset error of dipole correctors is not relevant.

D. Component alignment in warm regions.

In warm regions, one magnet or more than one magnet will be on one girder. In analogy to the cryomodules, Girders will be aligned with respect to the nearest reference points. Offsets of each girder arise from the offsets of the reference points plus some additional alignment error:

Misalignment of l-th girder:

$y_{g,l} = y_{R(g,l)} + G(a_g, t_g)$, (1-11)

(Tentative parameter can be taken from [??] a_g = 0.1 mm.)

where $y_{R(g,l)}$ is derived from offsets of the reference points near the girder in the same way as $y_{R(cr,m)}$.Offset of i-th magnet, magnet type-k, on l-th girder:

$y_{k,l,i} = y_{g,l} + (s_{k,l,i} - s_{g,l})\theta_{g,l} + G(a_k, t_k)$, (1-12)

where $s_{k,l,i}$ is the longitudinal position of the (reference of) magnet, $s_{g,l}$ the longitudinal position of the reference of the girder and $\theta_{g,l}$ the tilt (yaw or pitch) angle of the girder (see section 3, where $\theta_{g,l}$ is denoted as $\theta_{k,i}$).

Offset of i-th magnet, magnet type-k, which is not on a girder:

$y_{k,i} = y_{R(k,i)} + G(a_k, t_k)$, (1-13)

(Tentative parameter can be taken from [??] a_k = 0.1 mm.)

where $y_{R(k,i)}$ is given from misalignment of reference points near the magnet as same as $y_{R(cr,m)}$ and $y_{R(g,l)}$.

Offset of BPM which is attached to i-th magnet:

$y_{bpm,l,i} = y_{k,l,i} + G(a_{bpm,k}, t_{bpm,k})$. (1-14)

(Tentative parameter may be $a_{bpm,k}$ = 0.1 mm.)

Note that this is an initial error, before beam based measurement (e.g. quad-shunting). Beam based measurements will reduce this error.

The parameters will depend on type of the magnets. So we write these with index k.

3. *Roll errors (rotation around beam axis)*

We assume that the roll of components (rotation around their beam axis) is measured with respect to gravity using leveling equipment. We assume that changes of the gravitational vertical axis in this direction will be small enough to be neglected. It is of course also possible to relate the roll angle of the components to the reference network which would lead to strong and long distance correlations between the roll errors of different components but this method has to be described and evaluated later.

Relevant components:

- All magnets
- BPMs

Parameters to be defined:

- RMS roll offset of cryomodule.
- RMS roll offset of quads w.r.t. cryomodules.
- RMS roll offset of cold BPM and correctors w.r.t. attached quads.
- RMS roll offset of magnets in warm region
- RMS roll offset of warm BPM w.r.t. attached magnets.

A. Cold region

We assume each cryomodule has independent roll offset:

$$\phi_{cr,m} = G(a_{\phi,cr}, t_{\phi,cr}) . \qquad (2\text{-}1)$$

Cold components are aligned w.r.t. the cryomodule before cooldown.

Roll offset of quad magnet in m-th cryomodule:

$$\phi_{q,m} = \phi_{cr,m} + G(a_{\phi,q}, t_{\phi,q}) . \qquad (2\text{-}2)$$

Assuming quadrupole, correctors and BPM are assembled as one package, correctors and BPM are aligned with respect to the attached quadrupole magnet.

Roll of dipole corrector for x and y in m-th cryomodule:

$$\phi_{dx,m} = \phi_{q,m} + G(a_{\phi,dx}, t_{\phi,dx}) , \qquad (2\text{-}3)$$

$$\phi_{dy,m} = \phi_{q,m} + G(a_{\phi,dy}, t_{\phi,dy}) . \qquad (2\text{-}4)$$

Roll of BPM in m-th cryomodule:

$$\phi_{bpm,m} = \phi_{q,m} + G(a_{\phi,bpm}, t_{\phi,bpm}) . \qquad (2\text{-}5)$$

B. Warm region

We assume each warm magnet has an independent roll offset.

Rotation of i-th magnet, magnet type-k :

$$\phi_{k,i} = G(a_{\phi,k}, t_{\phi,k}) . \qquad (2\text{-}6)$$

Rotation of BPM which is attached to i-th magnet:

$$\phi_{bpm,i} = \phi_{k,i} + G(a_{\phi,bpm,k}, t_{\phi,bpm,k}) . \qquad (2\text{-}7)$$

4. Tilt errors (yaw and pitch)

In this section we refer to yaw (rotation around vertical axis) and pitch (rotation around horizontal axis perpendicular to the beam) collectively as tilts and represent these with θ. We assume that tilts are measured with respect to the reference network and not with respect to gravity.

Relevant components:

- Girders (for offset error of components on them)
- Long bending magnets (yaw for vertical bends, pitch for horizontal bends)
- Solenoid magnets
- Accelerating cavities

We consider quadrupoles to have negligible tilt sensitivity.

Parameters to be defined (yaw and pitch) :

- RMS tilt of cryomodule w.r.t. local survey network (reference points near the module).
- RMS tilt of cavities w.r.t. cryomodules.
- RMS tilt of magnets in warm region

A. Cavities

We assume each cryomodule has independent tilt offsets with respect to local survey network:

$$\theta_{cr,m} = \theta_{ref,cr,m} + G(a_{\theta,cr}, t_{\theta,cr}), \qquad (3\text{-}1)$$

where $\theta_{ref,cr,m}$ is the angle of local reference line, which is determined by position errors of reference points near the module. The model for $\theta_{ref,cr,m}$ is similar to that described in formula (1-5) which leads to (3-2) with the usual definitions,

$$\theta_{ref,cr,m} = \frac{\overline{sy} - \overline{s}\,\overline{y}}{\overline{s^2} - \overline{s}^2}, \qquad (3\text{-}2)$$

Cold components are aligned w.r.t. their cryomodule and thus the tilts of i-th cavity in m-th cryomodule are:

$$\theta_{cav,m,i} = \theta_{cr,m} + G(a_{\theta,cav}, t_{\theta,cav}). \qquad (3\text{-}3)$$

B. Girders and Long Magnets

We assume that each girder or long magnet has independent tilt offsets with respect to the local survey line. Tilt offset of i-th girder or magnet of type-k:

$$\theta_{k,i} = \theta_{ref,k,i} + G(a_{\theta,k}, t_{\theta,k}), \qquad (3\text{-}4)$$

$\theta_{ref,k,i}$ is the angle of local reference line, which is determined by offset of reference points near the girder or the magnet, which will be determined in the same way as $\theta_{ref,cr,m}$.

5. APPENDIX A

Though the equations (1-2) and (1-4) will be enough for simulations, for further considerations, we can express the offset as:

$$\begin{aligned} y_{j,n} &= \left(1-\frac{n}{N}\right)y_{P,j} + \frac{n}{N}y_{P,j+1} + \left(1-\frac{n}{N}\right)\sum_{i=1}^{n}\Delta_{y,i} - \frac{n}{N}\sum_{i=n+1}^{N}\Delta_{y,i} + l_{step}\frac{n(N-n)}{2}\theta_O \\ &+ l_{step}\left(1-\frac{n}{N}\right)\sum_{i=1}^{n}(1-i)\Delta_{\theta,i} - l_{step}\frac{n}{N}\sum_{i=n+1}^{N}(N+1-i)\Delta_{\theta,i} \qquad (1 \le n \le N), \end{aligned}$$

(A-1)

where $\Delta_{y,i}$ is the random number ($G(a_y, t_y)$) for offset at the i–th step and $\Delta_{\theta,i}$ the random number ($G(a_\theta, t_\theta)$) for angle change at the i–th step.

Assuming the all random numbers are independent and without truncations ($t_{pr}, t_y, t_\theta = \infty$) it can be shown after a little manipulations that the variance of the offset at the n-th offset will be as follows.

$$\begin{aligned} \sigma^2_{y,n} &= \frac{(N-n)^2+n^2}{N^2}a_{pr}^2 + \frac{n(N-n)}{N}a_y^2 + \left(\frac{n(N-n)}{2}\right)^2(l_{step}\theta_O)^2 \\ &+ \left\{\left(1-\frac{n}{N}\right)^2\frac{n(n+1)(2n+1)}{6} + \left(\frac{n}{N}\right)^2\frac{(N-n)(N-n+1)(2N-2n+1)}{6}\right\}(l_{step}a_\theta)^2 \end{aligned} \qquad (A\text{-}2)$$

$$\frac{(N-n)^2+n^2}{N^2}, \quad \frac{n(N-n)}{N^2} \quad \text{and}$$

$$\left\{\left(1-\frac{n}{N}\right)^2 \frac{n(n+1)(2n+1)}{6}+\left(\frac{n}{N}\right)^2 \frac{(N-n)(N-n+1)(2N-2n+1)}{6}\right\}\Big/N^3$$ are shown in Fig. 2 as functions of n/N, assuming $N=500$. Note that their behaviors almost do not depend on N, if N is large. $\sigma_{y,n}$ is shown in Fig. 3 as a function of n/N, assuming $N=500$, $\theta_O=0$ $a_y=0.5\,\mu\text{m}$, $a_\theta=0.1\,\mu\text{rad}$ and $l_{step}=4.5\,\text{m}$. Two lines are from different assumptions; $a_{pr}=1\,\text{mm}$ and $a_{pr}=0$.

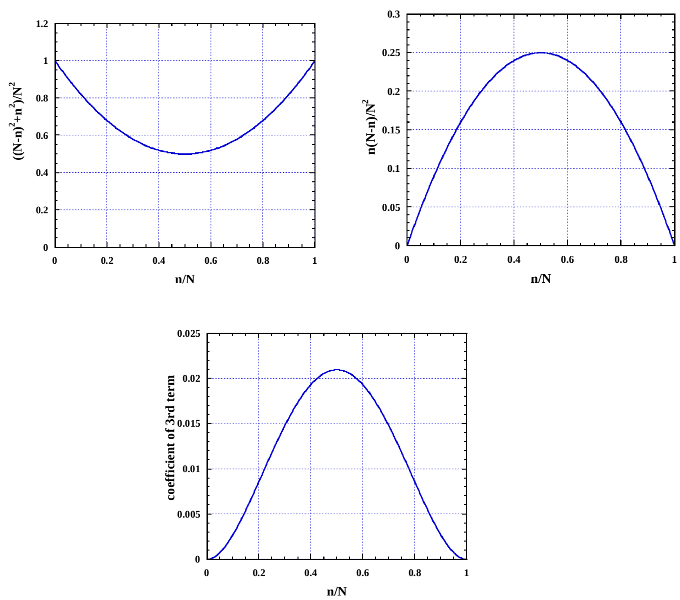

Fig. 2, Coefficients of three terms in Eq. (6) as function of n/N

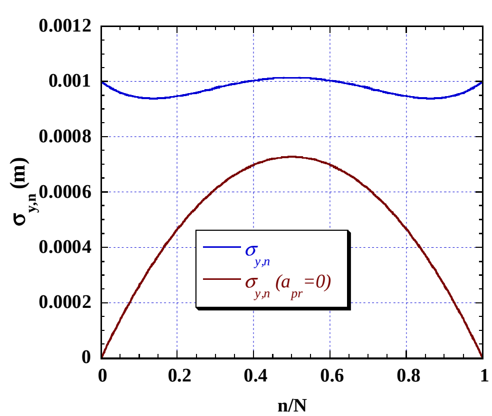

Fig. 3, $\sigma_{y,n}$ ($a_{pr}=1$ mm) and $\sigma_{y,n}$ ($a_{pr}=0$) as function of n/N

6. APPENDIX B

Here, we show the effect of the initial angle of the random walk, $\theta_{j,1}$, disappears after the correction of the accumulated error using a linear correction model. We consider the fact that the quadratic correction model does not have this property to be a problem.

From Eq. (1-2), looking at the term including $\theta_{j,m}$ in $y_{0,j,n}$, ($m=1,\ldots,n$)

$$y_{0,j,n} = l_{step} \sum_{m=1}^{n} (n-m+1)\theta_{j,m} + \text{other terms} \qquad \text{(B-1)}$$

For $n = N$,

$$y_{0,j,N} = l_{step} \sum_{m=1}^{N} (N-m+1)\theta_{j,m} + \text{other terms} \qquad \text{(B-2)}$$

Then, from the Eq. (1-3) and (1-4),

$$\begin{aligned} y_{j,n} &= l_{step} \sum_{m=1}^{n} (n-m+1)\theta_{j,m} - l_{step} \frac{n}{N} \sum_{m=1}^{N} (N-m+1)\theta_{j,m} + \text{other terms} \\ &= -l_{step} \frac{N-n}{N} \sum_{m=1}^{n} (m-1)\theta_{j,m} - l_{step} \frac{n}{N} \sum_{m=n+1}^{N} (N-m+1)\theta_{j,m} + \text{other terms} \end{aligned} \qquad \text{(B-3)}$$

Which shows that the coefficient of $\theta_{j,1}$ vanishes, when looking at he term of $m=1$ in Eq. (B-3).

7. References

[1] Simulation of the performance of the LiCAS train, G. Grzelak et al., Proceedings of the Eighth International Workshop on Accelerator Alignment, CERN, Switzerland/France, October 2004, http://iwaa2004.web.cern.ch/IWAA2004/subsite/PDF/20041007_TS10-3_Grzegorz-Grzelak.pdf

[2] Simulation of the LiCAS Survey System for the ILC, G. Grzelak for the LiCAS collaboration, Proceeding of the 9th International Workshop on Accelerator Alignment, SLAC, USA, September 2006. http://www.slac.stanford.edu/econf/C06092511/papers/TH007.PDF

[3] GLOBAL SIMULATION OF THE LiCAS/RTRS SURVEY SYSTEM FOR THE ILC, Grzegorz Grzelak for the LiCAS collaboration, Proceedings of the 10th International Workshop on Accelerator Alignment, February 11-15, 2008 KEK, Tsukuba, Japan