Dr.-Ing. Mikko Kärkkäinen
Institut für Theorie Elektromagnetischer Felder

TECHNISCHE UNIVERSITÄT DARMSTADT

T E M F

Low-Dispersion Wake Field Calculation Tools

Mikko Kärkkäinen

Technische Universität Darmstadt, Fachbereich Elektrotechnik und Informationstechnik
Schloßgartenstr. 8, 64289 Darmstadt, Germany - URL: www.TEMF.de

New Discrete Scheme

The parameter choice

$$a_r = a_z = \frac{\Delta z^2 + 2\Delta r^2}{2\Delta z^2 + 2\Delta r^2}$$

$$b_r = b_z = \frac{\Delta z^2}{4(\Delta z^2 + \Delta r^2)}$$

satisfies the required conditions if *Δr/ Δz>=1*. The first-order term is discretized according to

$$\frac{\partial \Phi}{\partial r}(i,j) = \frac{1}{2}\frac{\Phi(i,j+1)-\Phi(i,j-1)}{2\Delta r} + \frac{1}{4}\frac{\Phi(i+1,j+1)-\Phi(i+1,j-1)}{2\Delta r} + \frac{1}{4}\frac{\Phi(i-1,j+1)-\Phi(i-1,j-1)}{2\Delta r}$$

and the scheme remains stable with the same time-step.

Dr.-Ing. Mikko Kärkkäinen
Institut für Theorie Elektromagnetischer Felder

Indirect Integration

- The problem
  - How to calculate the wake potential in „infinitely“ long beam pipes ?
- Solution (according to H. Henke)
  - Transformation of the infinite integral into a finite one by using Stokes‘ theorem
  - Requires a solution of a Poisson‘s equation over the arbitrary cross-section of the beam pipe at every time step within the moving window
  - Allows truncating the outgoing beam pipe
- Implementation
  - Direct integration in C-code, post-processing (solutions to Poisson‘s equations) in Matlab

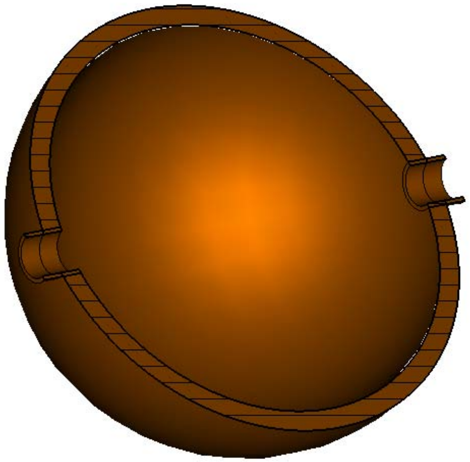
$$W_{\parallel}(0,s) = -\int_{-\infty}^{\infty} E_z\left(z,0,\frac{s+z}{c}\right)dz$$

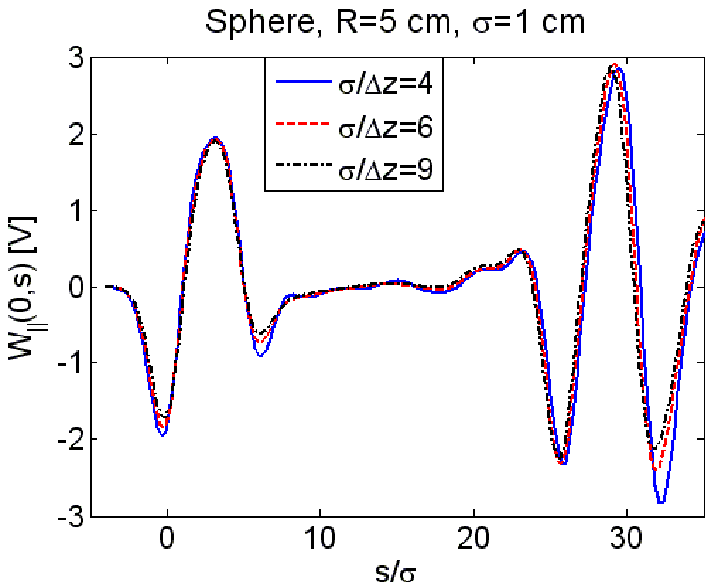
$$\nabla^2\Phi = \frac{\partial E_z}{c\partial t} - \frac{\partial E_z}{\partial z}$$

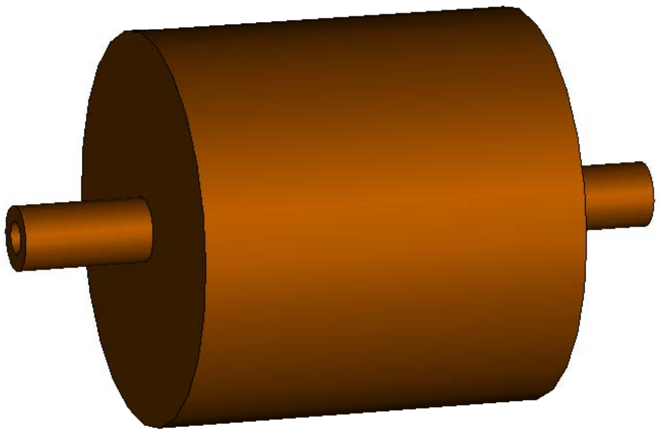
RHS of the Poisson‘s equation is saved within the moving window at the end of the simulation

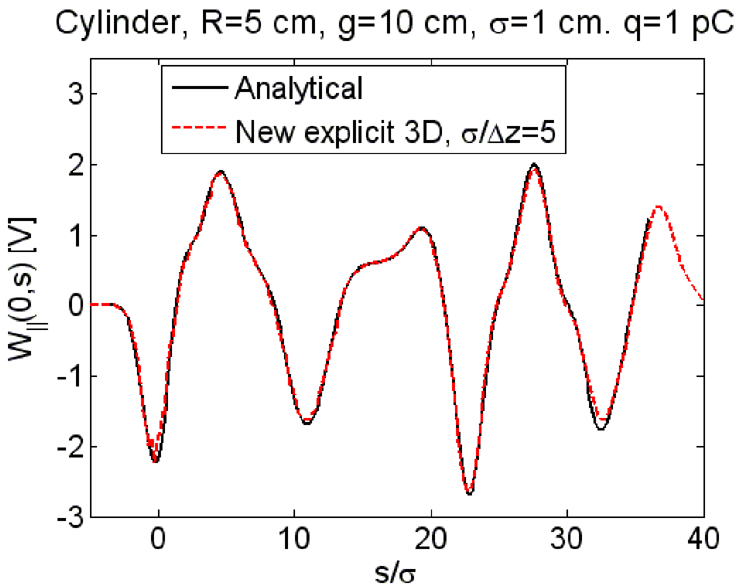
Details of the scheme can be found in EPAC 2006 paper: H. Henke, W. Bruns, „Calculation of wake potentials in general 3D structures“

Spherical Cavity & Pillbox



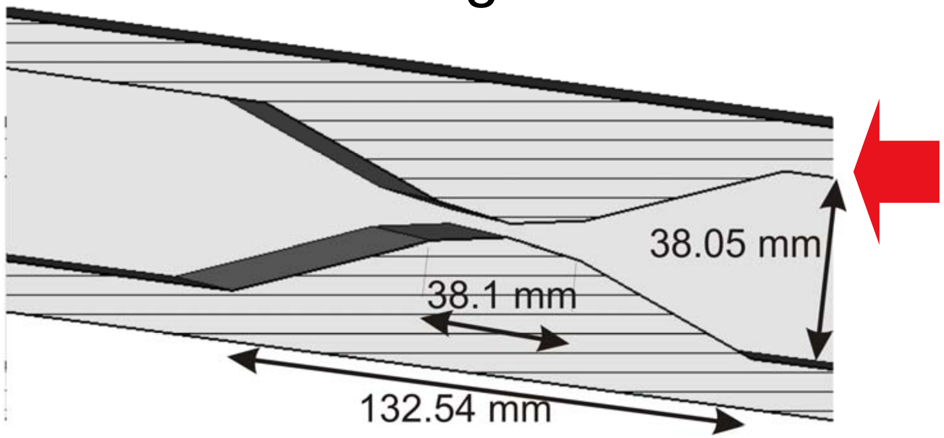




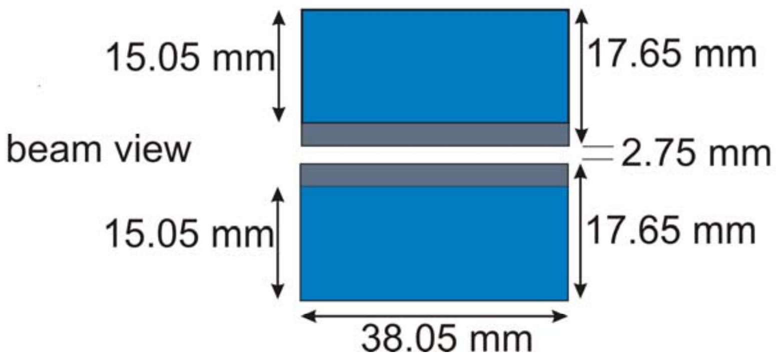


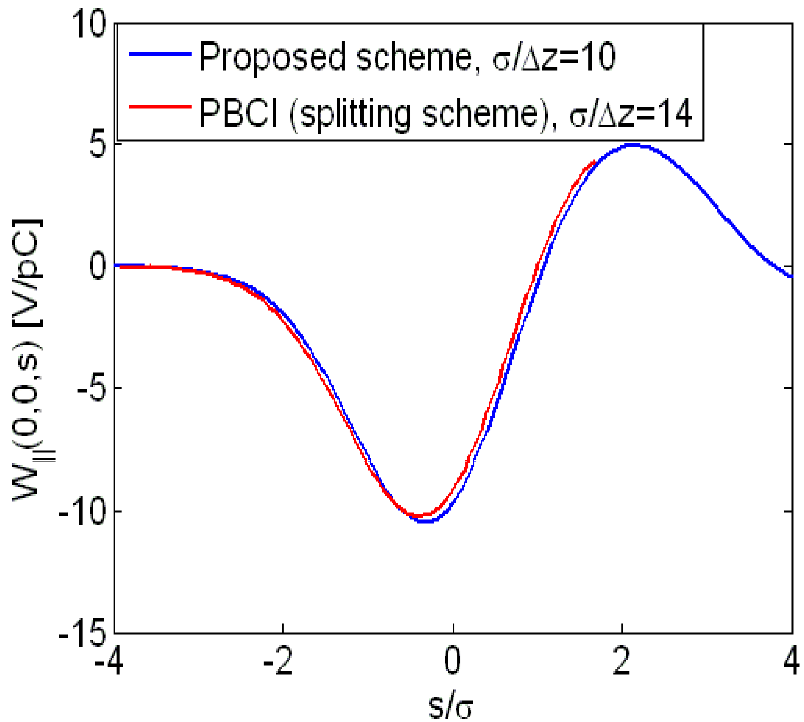
ILC-ESA Collimator Prototype

With a long bunch (1mm) the results are accurate with about 10 cells over the bunch length



Beam pipe truncated here, indirect wake calculated by solving Poisson‘s equation

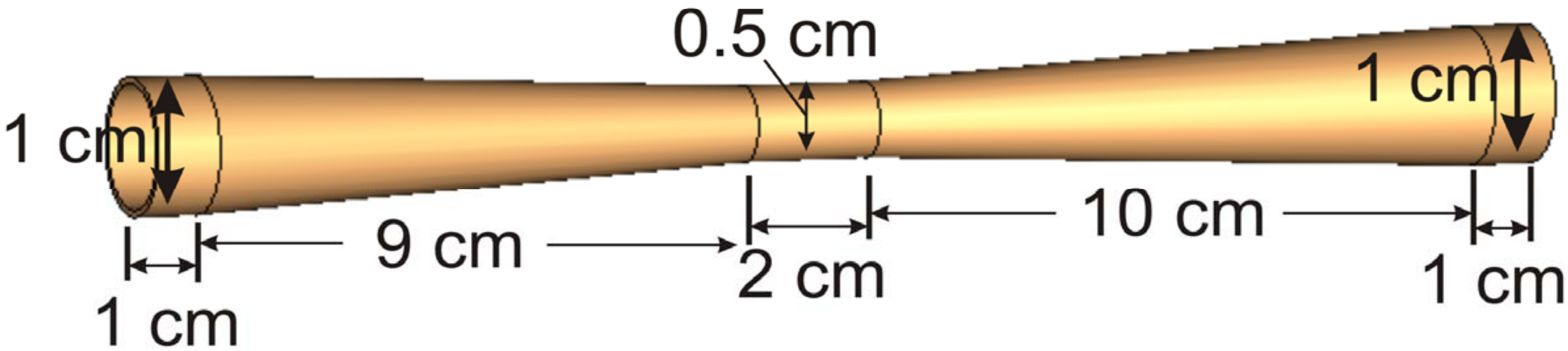


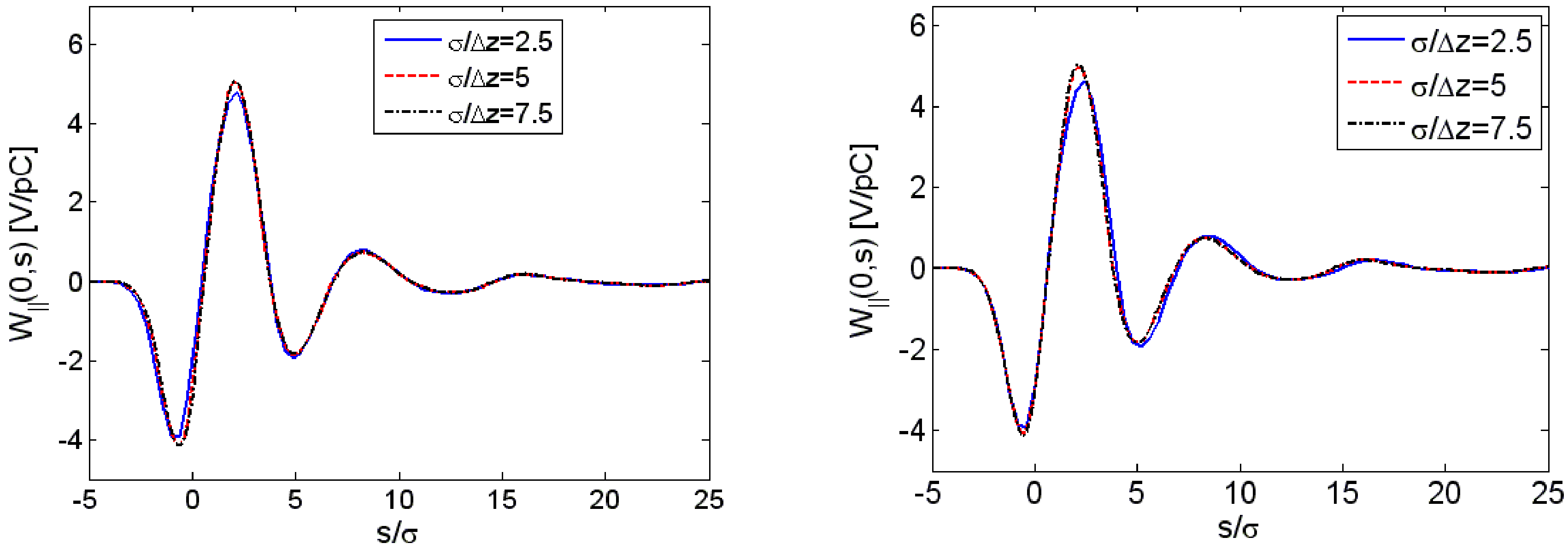


Dr.-Ing. Mikko Kärkkäinen
Institut für Theorie Elektromagnetischer Felder

Conformal Scheme, Collimator

- Faster convergence with conformal schemes.





Proposed explicit 2D scheme

ECHO 2D (implicit scheme)