“AC” Dispersion Measurement

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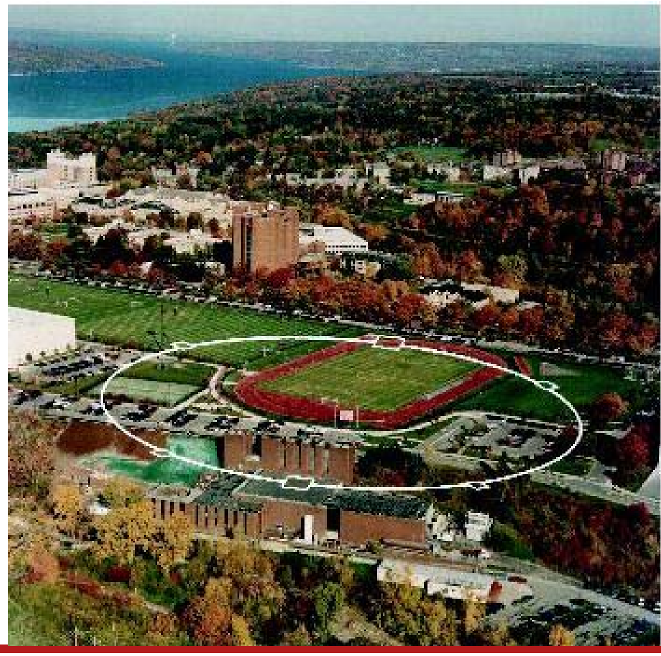


"AC" dispersion measurement

Traditional dispersion measurement
- Measure orbit
- Change ring energy ($\delta E/E = (\delta f_{rf}/f_{rf})/\alpha_p$)
- Measure again. $\eta_x = \Delta x/(\delta E/E)$

"AC" dispersion
- Drive an energy oscillation by modulating the RF phase at the synchrotron tune
- Measure the phase and amplitude of the vertical and horizontal signal at each BPM that is oscillating at the synchrotron tune
- Use the phase and amplitude information to reconstruct vertical and horizontal dispersion

Advantages:
1. Nondestructive
   Use a signal bunch to measure dispersion without disturbing all of the other bunches in the ring
2. Fast - Changing the ring energy via the RF frequency is slow, especially with high Q cavities
3. Better signal to noise (Filter all but signal at synch tune)

"AC" dispersion measurement

"AC" dispersion

Note that dispersion is z-x and z-y coupling

Use transverse coupling formalism to analyze dispersion

Review of transverse coupling formalism and measurement

$T=VUV^{-1}$ (T is 4X4 full turn transport, U is block diagonal)
propagates the phase space vector (x,x′,y,y′)

$$V=\begin{pmatrix}\gamma I & C \\ -C^{+} & \gamma I\end{pmatrix} \qquad \overline{C}=G_b^{-1}CG_a \qquad G=\begin{pmatrix}\sqrt{\beta_a} & 0 \\ -\alpha/\sqrt{\beta_a} & 1/\sqrt{\beta_a}\end{pmatrix}$$

In the absence of coupling, C=0, $V=I_{4X4}$, U=T

AC dispersion measurement

To measure x-y coupling

Drive beam at horizontal [a-mode] (or vertical [b-mode]) frequency.

The beam responds resonantly

Measure x and y amplitude and phase of the a-mode (b-mode) at each BPM

Finite y_{amp} indicates coupling

- The relative phase tells us something about its source

We find that: $\overline{C}_{12} = \frac{y_{amp}}{x_{amp}} \sin(\varphi_y - \varphi_x)$ $\overline{C}_{22} = \frac{y_{amp}}{x_{amp}} \cos(\varphi_y - \varphi_x)$

If we drive the b-mode we can extract $\overline{C}_{11}$

Note: $\overline{C}_{12}$ is insensitive to BPM tilts

If there is no coupling, but a BPM is tilted,

then $y_{amp} \neq 0$, but $\phi_y = \phi_x \rightarrow \overline{C}_{12} = 0$

AC dispersion measurement

To measure x(y) - z coupling

Construct the matrix that propagates x-z motion. Again
$T=VUV^{-1}$ (T is 4X4 full turn transport, U is block diagonal)

But here T propagates the phase space vector (x,x′,l, δ) [or (y,y′,l, δ)]

As before we can write:

$$V=\begin{pmatrix}\gamma I & C \\ -C^{+} & \gamma I\end{pmatrix} \qquad \overline{C}=G_b^{-1}CG_a \qquad G=\begin{pmatrix}\sqrt{\beta_a} & 0 \\ -\alpha/\sqrt{\beta_a} & 1/\sqrt{\beta_a}\end{pmatrix}$$

Here, the a-mode corresponds to horizontal[vertical] motion
and the b-mode is synchrotron motion
Now C=0 → no coupling of longitudinal and horizontal motion,
That is, zero dispersion

"AC" dispersion measurement

It turns out that n the limit $Q_s \rightarrow 0$

$$C_{12}(z\text{-}x) = \eta_x, \quad C_{22}(z\text{-}x) = \eta'_x$$

$$C_{12}(z\text{-}y) = \eta_y, \quad C_{22}(z\text{-}y) = \eta'_y$$

Drive beam at synchrotron tune (z-mode)
Measure x, y (and z?) amplitude and phase at each BPM

Then as with transverse coupling

$$\overline{C}_{12} = \frac{x_{amp}}{z_{amp}} \sin(\varphi_x - \varphi_z)$$

Is related to the horizontal dispersion according to

$$\overline{C} = G_b^{-1} C G_a \qquad \sim \eta/(\beta_a \beta_b)^{1/2}$$

$$\overline{C}_{12} = \frac{y_{amp}}{z_{amp}} \sin(\varphi_y - \varphi_z)$$

to the vertical dispersion (η_y)

Dispersion

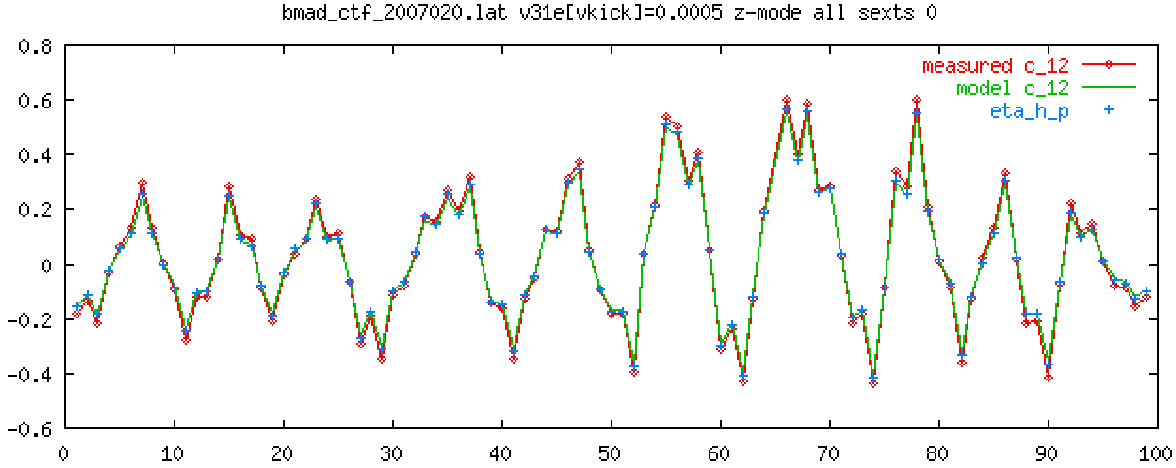
- Simulation of "AC" dispersion measurement

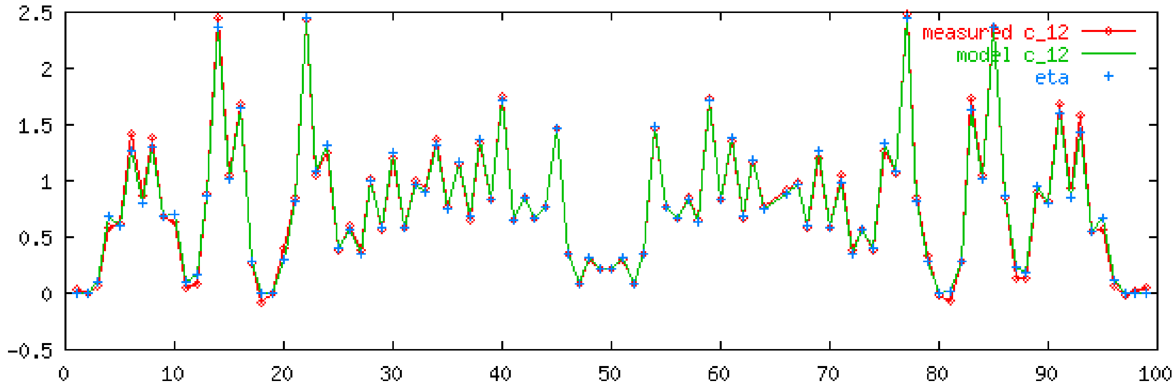
Introduce a vertical kick into CesrTA optics to generate vertical dispersion

-Drive synchrotron oscillation by modulating RF at synch tune (In the simulation: apply an energy kick modulated at synch tune) and track for >30k turns

-"Measure" vertical & horizontal amplitudes and phases of synch tune signal at BPMs

- Construct C_{12}





"measured c_{12}" - 30k turn simulation
"model c_{12}" - Model y-z and x-z coupling
"model eta" - Model dispersion

"AC" dispersion measurement

We measure x_{amp} and $\phi_{x,}$ y_{amp} and ϕ_y

But we are unable (so far) to measure z_{amp} and ϕ_z with sufficient resolution

Longitudinal parameters come from the design lattice

(Perhaps with new BPM system we will be able to extract ϕ_z from $2f_s$)

$z_{amp} = \sqrt{(a_z\beta_z)}$, $13\text{m} < \beta_z < 14.6\text{m}$ (CesrTA optics)

$\rightarrow z_{amp}$ is very nearly constant - there is an overall unknown scale (a_z)

$0 < \phi_z < 36°$.

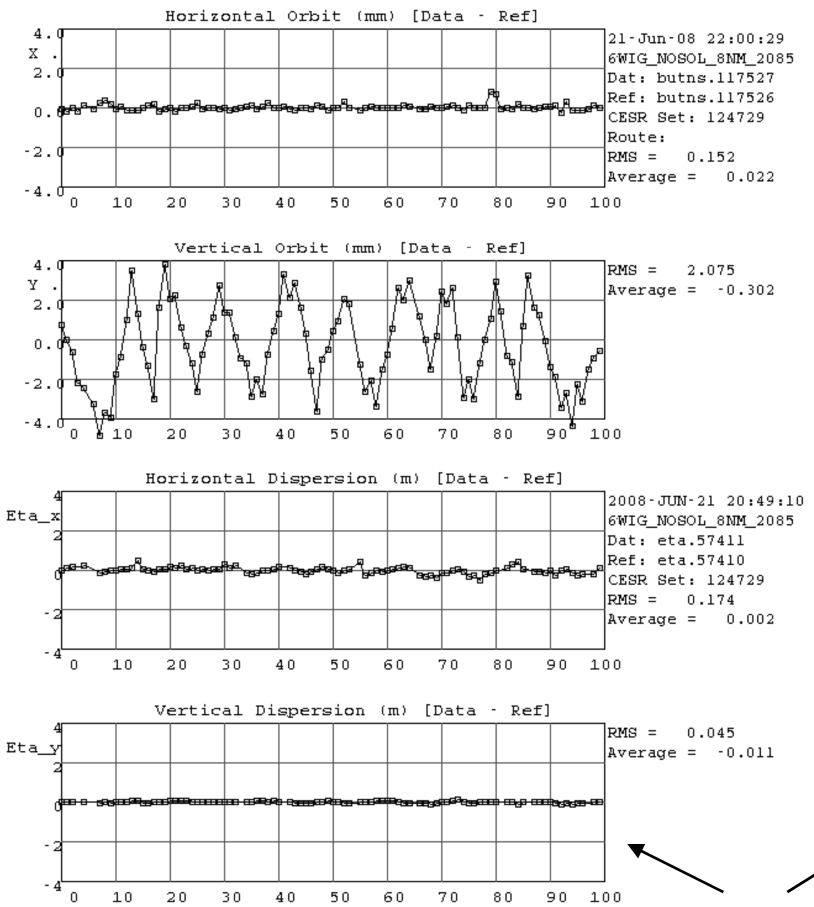
We compute ϕ_z from the design optics at each BPM

- there is an overall unknown phase offset (ϕ_0)

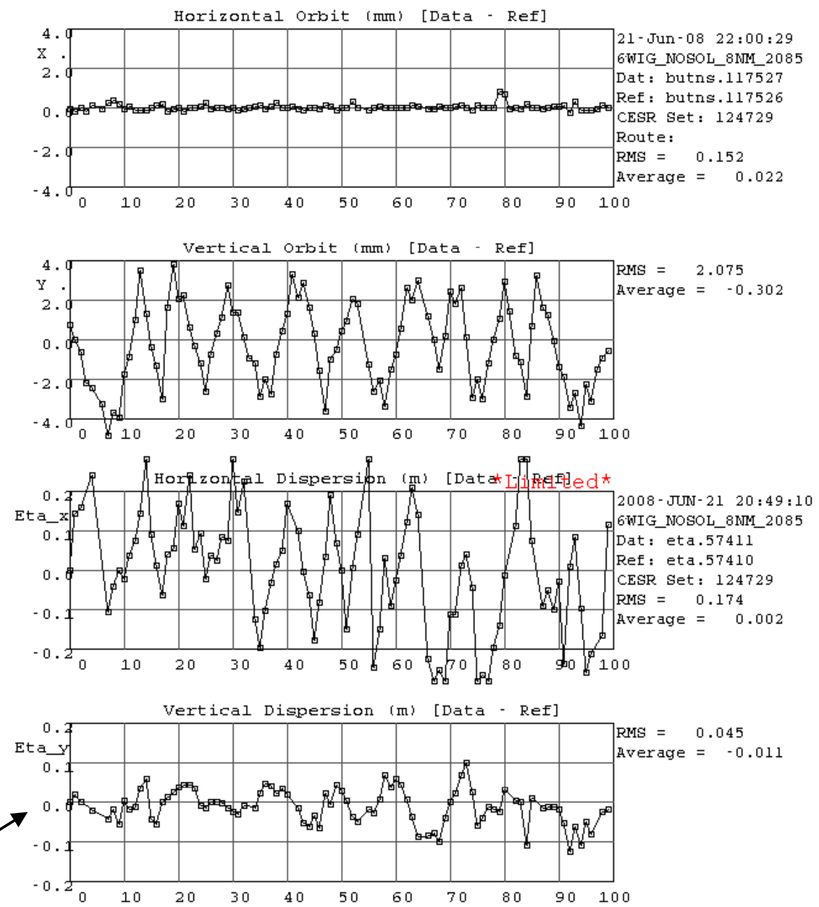
- Determine a_z, ϕ_0 by fitting x - data to model horizontal C_{12}
  - Then use fitted parameters to determine vertical C_{12}

“AC” dispersion measurement

data v31e=200cu
ref: v31e=0



data v31e=200cu
ref: v31e=0

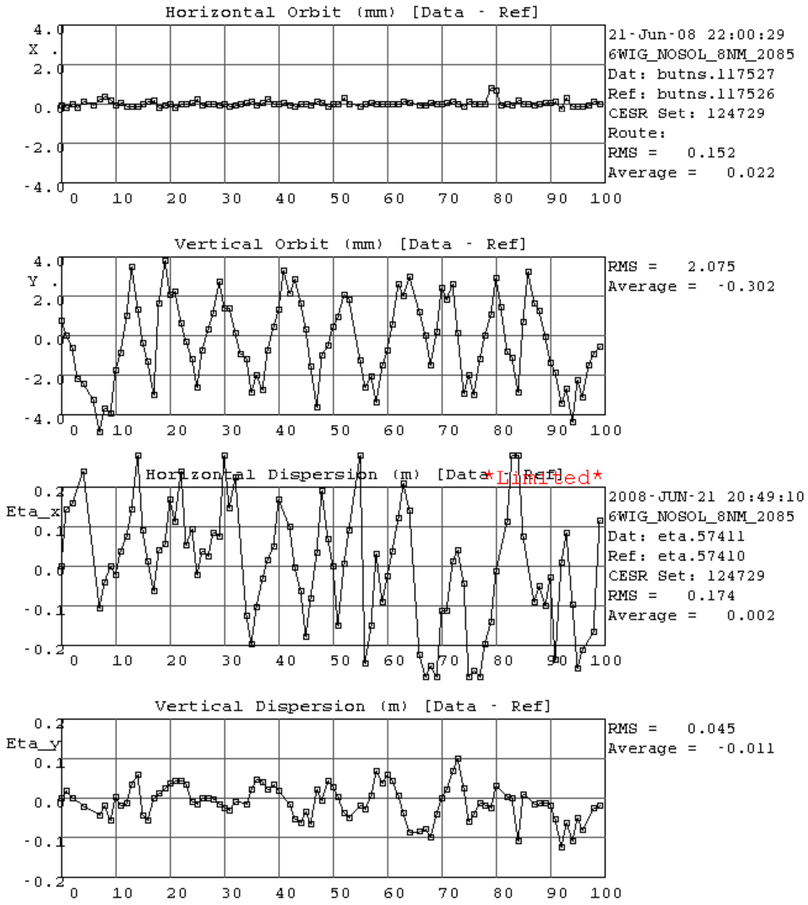


Same data - different scales

Change vertical steering 31e +200cu and measure orbit and “ac”dispersion (data).
Restore vert 31e to zero and remeasure orbit and dispersion (ref)

"AC" and "DC" measurements

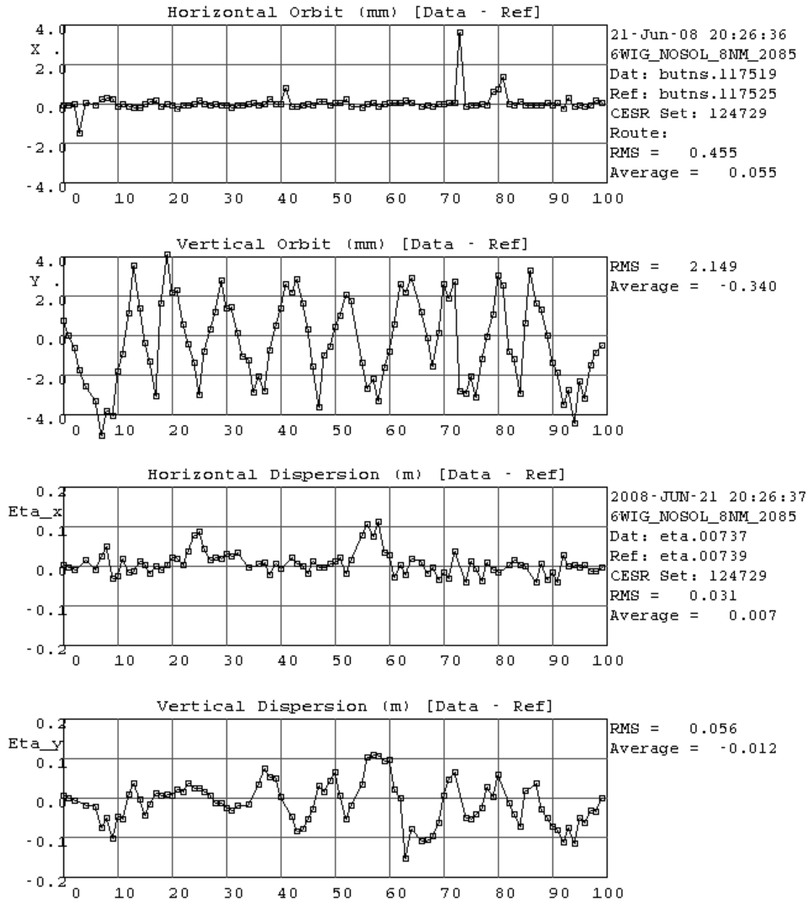
data v31e=200cu
ref: v31e=0



"ac" dispersion

(same as last slide)

data: v31e=+200cu
ref: v31e=0 DC



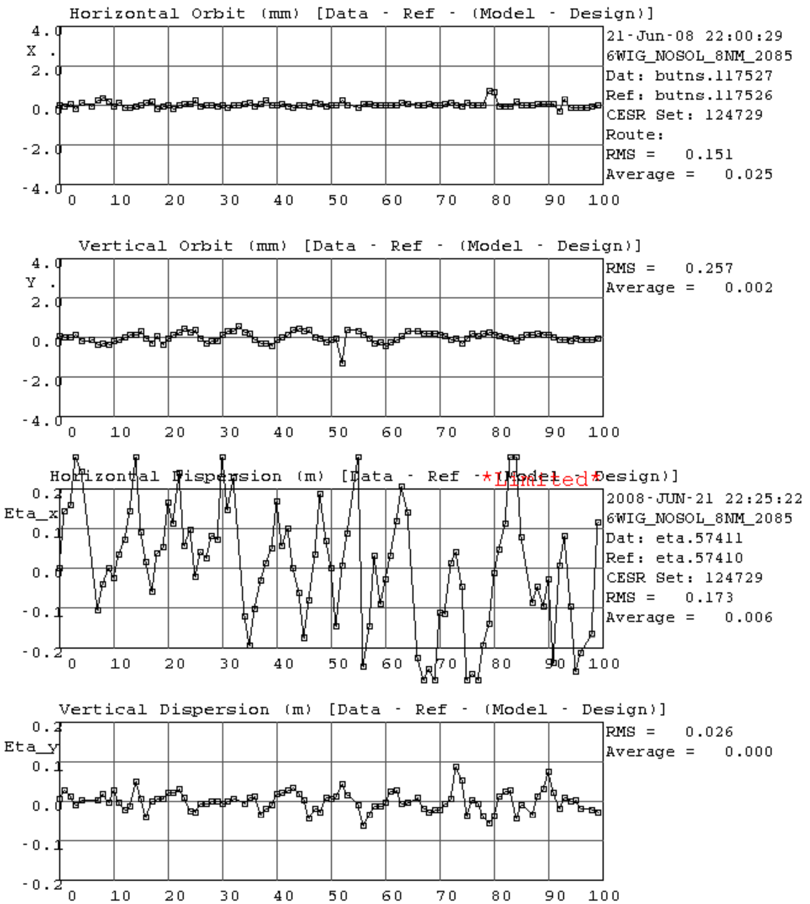
"dc" dispersion

Same steering change with"DC" dispersion measurement

Dispersion modeled and measured

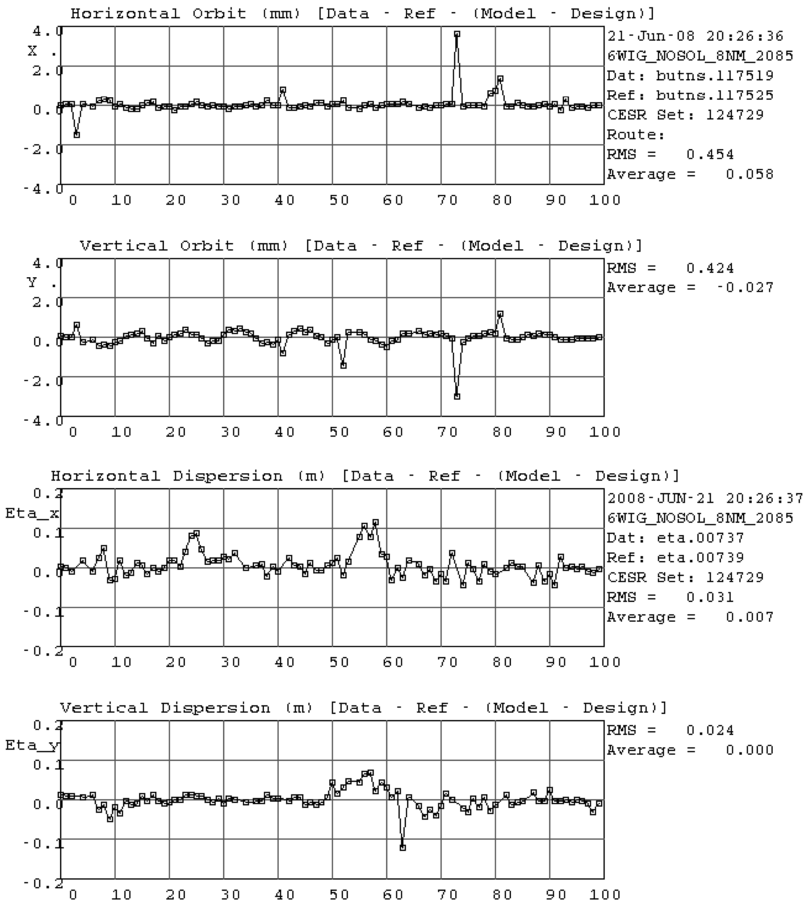
(Data-ref)-model

data: v31e=+200cu
ref v31e=0



"ac" measurement

data: v31e=+200cu
ref: v31e=0 (measurement) – v31e=200cu (model)



"dc" measurement

"Data": measurement with v31e=200cu
"Ref": measurement with v31e=0
"Model": modeled orbit and dispersion with v31e=200cu

AC Dispersion

- ## Conclusions

Dispersion is coupling of longitudinal and transverse motion

Measurement
-Drive synchrotron oscillation by modulating RF at synch tune
-Measure vertical & horizontal amplitudes and phases of signal at synch tune at BPMs

Then
$\{\eta_v/\beta_v\} = (y_{amp}/z_{amp}) \sin(\varphi_y - \varphi_z)$
$\{\eta_h/\beta_h\} = (x_{amp}/z_{amp}) \sin(\varphi_h - \varphi_z)$

Advantages?:

1. Faster (30k turns) (RF frequency does not change)
2. Better signal to noise - (remains to be seen)
   filter all but signal at synch tune
3. Nondestructive (RF frequency does not change)
   Witness bunch?