SUSY Parameter Determination with LHC and ILC Data

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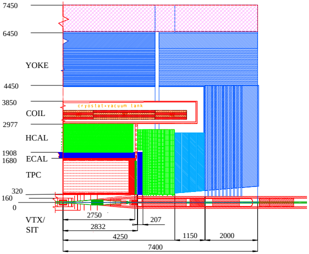
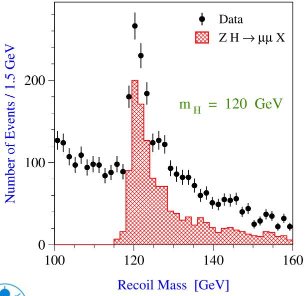
Observables and Parameters





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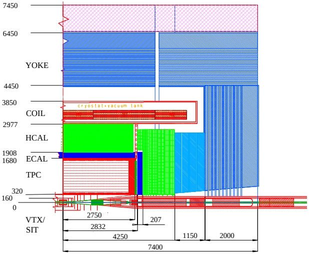
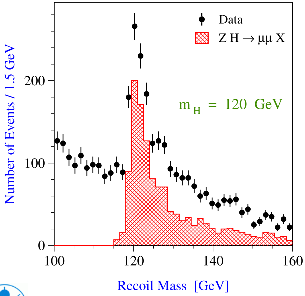
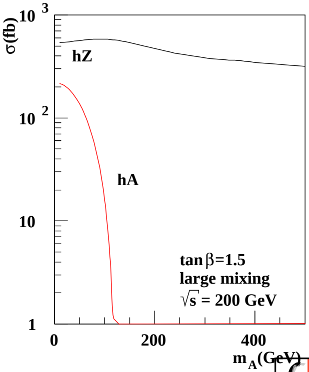
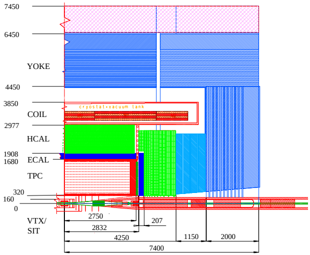


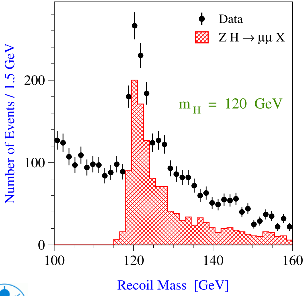


$$
\begin{aligned}
V_{\rm Higgs} \quad = \quad & m^2_{1H}|H_1|^2 + m^2_{2H}|H_2|^2 \\
& - m^2_{12}(\epsilon_{ij}H^i_1 H^j_2 + h.c.) \\
& + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - \\
& |H_2|^2)^2 + \frac{1}{2}g^2|H^*_1 H_2|^2
\end{aligned}
$$



Observables and Parameters

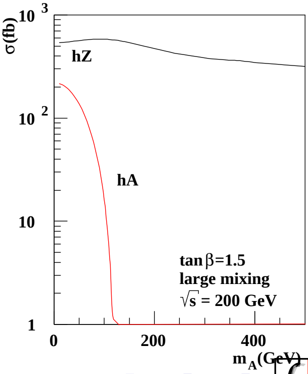




Link:

Parameter Determination

$$
\begin{aligned}
V_{\text{Higgs}} &= m_{1H}^2|H_1|^2 + m_{2H}^2|H_2|^2 \\
&\quad - m_{12}^2(\epsilon_{ij}H_1^i H_2^j + h.c.) \\
&\quad + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - \\
&\quad |H_2|^2)^2 + \frac{1}{2}g^2|H_1^* H_2|^2
\end{aligned}
$$



Aims of Fittino

- Fit the (N)MSSM parameters to the observables from all possible sources: LHC, ILC, Tevatron, B-factories, etc.
- Fit high-scale (mSUGRA, GMSB, AMSB) and low-scale models (MSSM,NMSSM)
- Bottom-up approach
- If the user wishes to test unbiased measurement: no prior knowledge of the parameters at any step
- Provide easy user interface for measurements, parameter definitions and output
- Further Goals:
  - Show that unambiguous parameter determination without human bias is possible
  - Determine precision of parameter measurements
  - Test the necessary experimental and theoretical precision
  - Study comparisons of models: MSSM vs. NMSSM etc.

Observables and other Inputs

Observables:

- Fittino can fit to any combination of the following observables:
    - Masses of SM and MSSM particles
    - Edges in mass spectra
    - Particle widths
    - Branching fractions
    - Cross-sections
    - Any product of cross-sections and/or branching fractions
    - Any ratio of the above
    - LE-Observables: $b \to s\gamma$, $(g-2)_\mu$, relic density, $\Delta m_s/\Delta m_d$, etc.
- Correlations among observables can be specified
- Limits on masses of unobserved particles can be specified

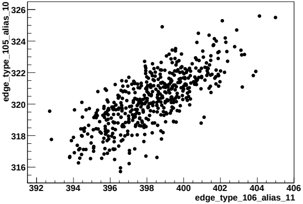
Calculations:

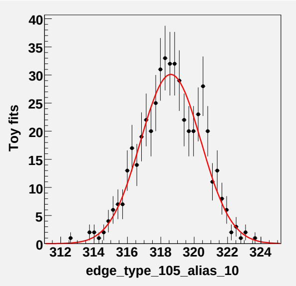
- Use SLHA as an interface to theory codes
- Here: Use SPheno (W. Porod), micrOmegas (Bélanger *et al.*), "mastercode" (Buchmüller *et al.*)

Jump to Adding the ILC

Principles of the MC Toy Method

observables

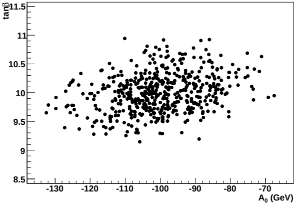


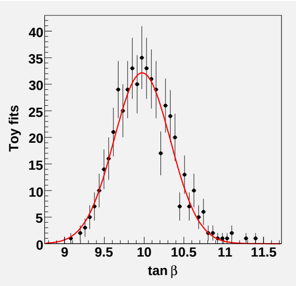


fit

errors given by standard deviation

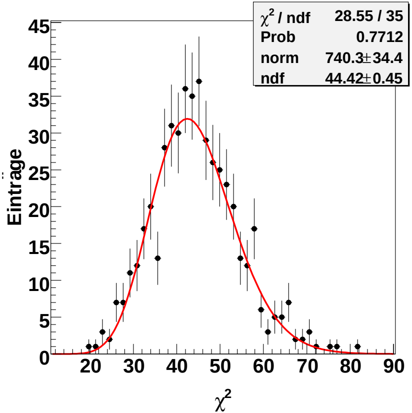
parameters

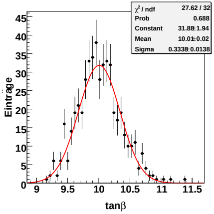


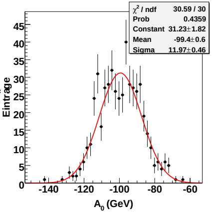


Principles of the MC Toy Method

- "Simple" 4-dim mSUGRA fit to expected LHC data and existing LE-Observables







- Big advantage of Toy fits: χ^2 distribution gives objective criterion for success of the uncertainty analysis
- Here expect $\bar{\chi}^2 = \mathrm{ndf} = 44$, fine within 1σ

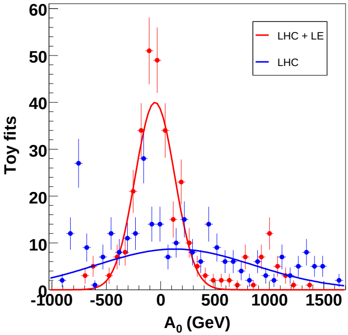
Observables

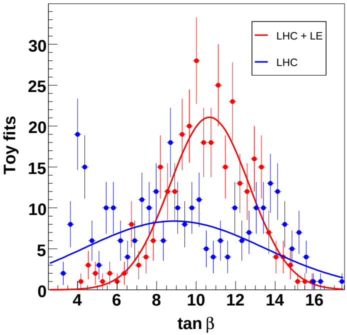
- Realistic estimates (CSC Notes, CMS TDR, own studies) for measurements of masses and edges in mass spectra
- Realistic estimates of measurements of branching fractions and products of branching fractions
- Low Energy observables $(BR(b \to s\gamma), (g-2)_\mu, WMAP, \ldots)$
- Full treatment of known experimental correlations
- In addition test of the effect of theoretical correlations in the running of the mSUGRA parameters from the high to the low scale

List of observables (excerpt for 1 fb^{-1})

LHC observable (1 fb^{-1})	nominal value (GeV)	uncertainty (GeV) stat.	LES (0.2 %)	JES (5 %)	syst.
m_h	109.1				
m_t	170.9	1.1		1.5	
$m_{\tilde{\chi}_1^\pm}$	179.9				
$m_{\tilde{\ell}_L} - m_{\tilde{\chi}_1^0}$	148.8				6.0
$m_{\tilde{g}} - m_{\tilde{\chi}_1^0}$	510.2				10.0
$m_{\tilde{q}_R} - m_{\tilde{\chi}_1^0}$	533.7	19.6		26.7	10.0
$\langle m_{\tilde{g}} - m_{\tilde{b}_{1,2}} \rangle$	522.6				
$m_{\tilde{g}} - m_{\tilde{b}_1}$	89.0				
$m_{\tilde{g}} - m_{\tilde{b}_2}$	56.7				
$m_{\ell\ell}^{max}$	80.2	1.7	0.16		
$m_{\ell\ell}^{max}$	279.1				
$m_{\tau\tau}^{max}$	83.2	12.6		4.2	8.3
$m_{\ell\ell q}^{max}$	454.3	13.9		11.4	
$m_{\ell q}^{low}$	320.3	7.6		8.0	
$m_{\ell q}^{high}$	398.3	5.2		10.0	
$m_{\ell\ell q}^{thres}$	216.2	26.5		5.4	
$m_{\ell\ell b}^{thres}$	196.4				
m_{tb}^{w}	360.9	43.0		18.0	
$\frac{BR(\tilde{\chi}_2^0 \to \tilde{\ell}\ell) \times BR(\tilde{\ell} \to \tilde{\chi}_1^0 \ell)}{BR(\tilde{\chi}_2^0 \to \tilde{\tau}_1 \tau) \times BR(\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau)}$	0.08	0.009			0.008
$\frac{BR(\tilde{g} \to \tilde{b}_2 b) \times BR(\tilde{b}_2 \to \tilde{\chi}_2^0 b)}{BR(\tilde{g} \to \tilde{b}_1 b) \times BR(\tilde{b}_1 \to \tilde{\chi}_2^0 b)}$	0.16			0.078	

Results of the MC Toy Method

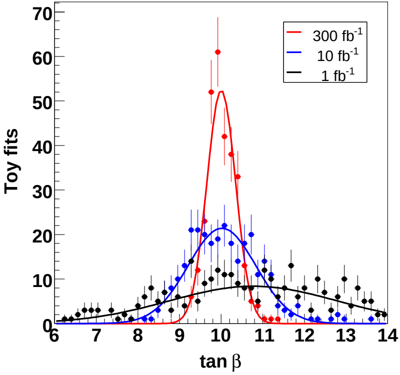


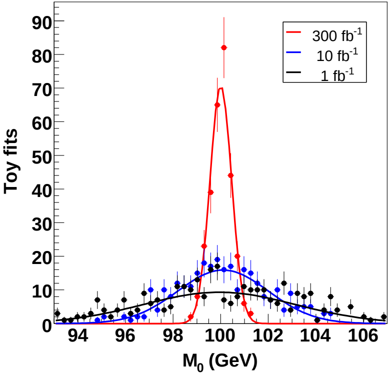


Non-Gaussian behaviour at low luminosities

- at low $\mathcal{L}^{int} = 1\ \text{fb}^{-1}$ deviations from linear approximation
- improvement by constraints from low energy (LE) measurements

Global Fits for Different Luminosities

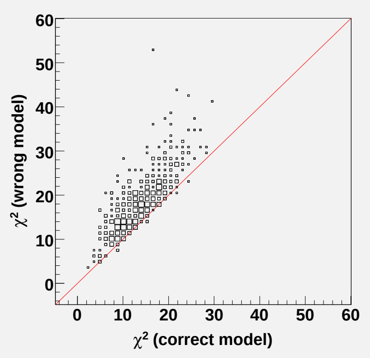


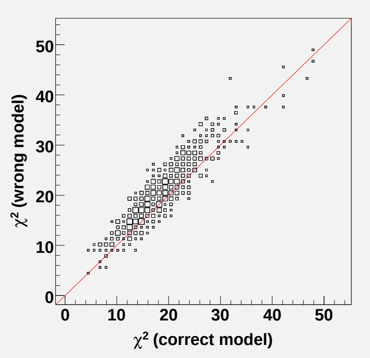


300fb^{-1} : 10.02 ± 0.34 (3.39 %)
10fb^{-1} : 10.01 ± 0.82 (8.19 %)
1fb^{-1} : 10.68 ± 1.98 (18.54 %)

300fb^{-1} : 100.0 ± 0.4 (0.4 %)
10fb^{-1} : 100.3 ± 1.9 (1.9 %)
1fb^{-1} : 99.96 ± 3.15 (3.15 %)

Model Discrimination





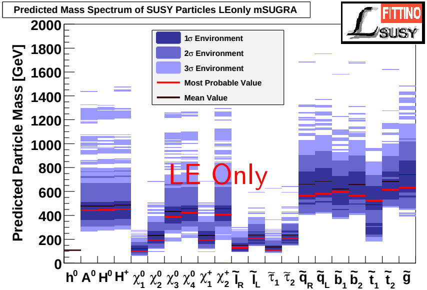
model discrimination (1 fb^{-1})

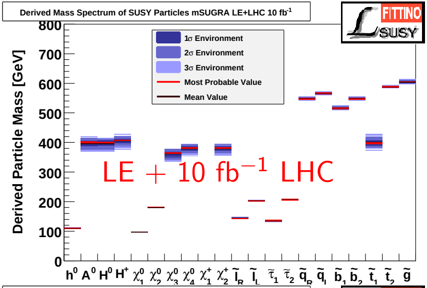
- fit wrong model with signμ = -1
- probability to prefer correct over wrong model: 96 %

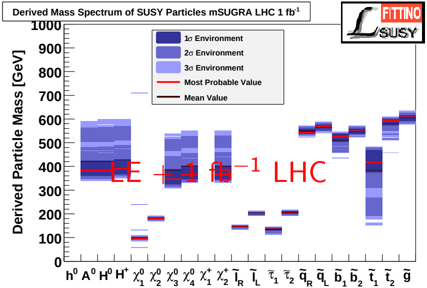
interpretation of mass edge (10 fb^{-1})

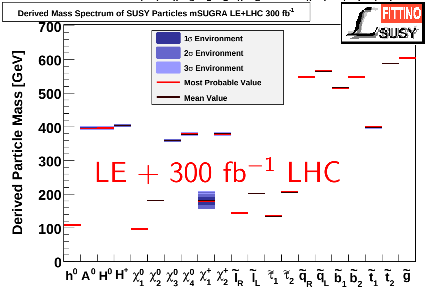
- fit wrongly identified edge in mass spectrum ($m_{\tilde{e}_R} \leftrightarrow m_{\tilde{e}_L}$)
- probability to prefer correct over wrong interpretation: 77 %

Precision of the SUSY Spectrum from LE+LHC

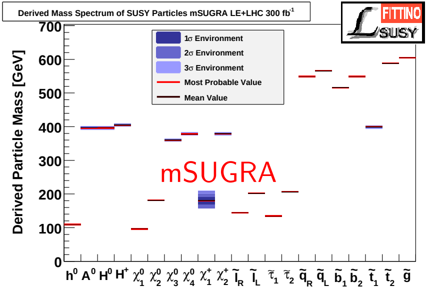


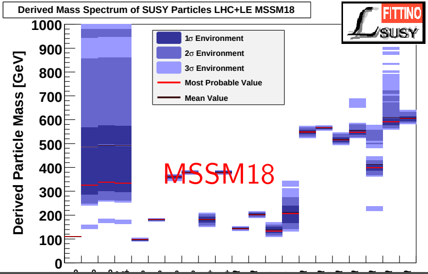






Prediction of Particle Masses: LE+LHC





- The precision of the understanding of the specxtrum looks impressive for LE+LHC with $\mathcal{L}^{int} = 300\ \text{fb}^{-1}$.
- But that heavily depends on the number of degrees of the theory!
- Fitting the corresponding MSSM18 model to the same observables yields much higher uncertainties!
- Imagine what happens for CPV models etc...

Adding the ILC

- We've seen that LHC promises good precision for High Scale SUSY Models with a small number of parameters.
- But who believes that mSUGREA is true? We would like to be able to study the general form of SUSY well to understand EWSB and SUSY breaking in an model independent way.
- Therefore, add ILC.
- In the following, I will show an example using a generic set of possible ILC measurements, mostly based on studies of the TESLA TDR era and collected in hep-ph/0410364
- I would like to propose to collect all the information from our studies, add maybe some conservative estimate for studies we know we can do but didn't do, and then redo that fit.
- We can use all observables listed on Jump to Observables and other Inputs

ILC Measurements (excerpt)

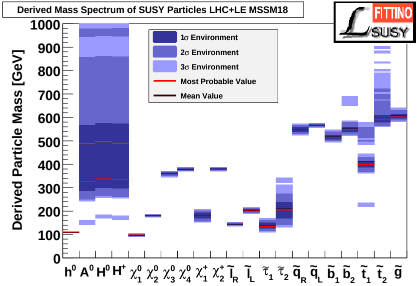
Observable	Value	Stat. Unc.	Syst Unc.
massh0	109.62 GeV	0.05 GeV	0.5 GeV
massA0	397.248 GeV	1.3 GeV	0.6 GeV
massH0	397.618 GeV	1.3 GeV	0.6 GeV
massHplus	405.749 GeV	1.1 GeV	0.6 GeV
massStop1	399.296 GeV	2.0 GeV	5.4 GeV
massSelectronL	202.544 GeV	0.2 GeV	0.4 GeV
massSelectronR	144.08 GeV	0.05 GeV	1.2 GeV
massSnueL	186.286 GeV	0.7 GeV	
massSmuL	202.564 GeV	0.5 GeV	0.4 GeV
massSmuR	144.051 GeV	0.2 GeV	1.2 GeV
massStau1	134.137 GeV	0.3 GeV	0.5 GeV
massStau2	206.768 GeV	1.1 GeV	0.5 GeV
massNeutralino1	97.191 GeV	0.05 GeV	0.4 GeV
massNeutralino3	-360.741 GeV	4.0 GeV	1.1 GeV
massNeutralino4	378.383 GeV	2.3 GeV	1.1 GeV
massChargino1	180.333 GeV	0.55 GeV	1.0 GeV
massChargino2	379.435 GeV	3.0 GeV	3.4 GeV

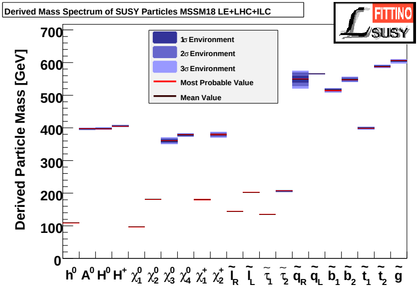
LE+LHC+ILC MSSM18 Fit Results

Now fit the MSSM18 model to LE+LHC+ILC data

Parameter	Nominal value	Fit		LHC Uncertainty	LHC+ILC Uncertainty
MSelectronL	194.31	194.315	±	5.4	0.096
MSelectronR	135.76	135.758	±	3.2	0.117
MStauL	193.52	193.463	±	34.7	0.48
MStauR	133.43	133.448	±	203.5	0.50
MSupL	527.57	527.612	±	3.5	0.88
MSupR	509.14	509.285	±	10.3	10.3
MSbottomR	504.01	504.224	±	20.9	3.7
MStopL	481.69	481.569	±	20.9	2.7
MStopR	409.12	409.237	±	131.7	2.2
TanBeta	10	10.0148	±	2.9	0.39
Mu	355.05	355.015	±	4.4	1.14
Xtau	-3799.88	-3795.12	±	2568.1	61.8
Xtop	-526.62	-526.801	±	192.5	6.98
Xbottom	-4314.33	-4252.11	±	3937.7	783.3
M1	103.15	103.154	±	2.23	0.058
M2	192.95	192.951	±	2.32	0.13
M3	568.87	568.664	±	10.2	2.21
massA0	359.63	360.069	±	529.8	2.5

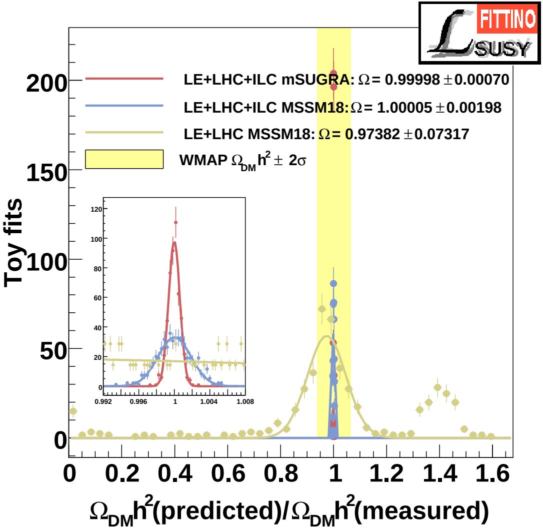
Comparison of the Collider Measurements of Masses





- Tremendous improvement of the Higgs sector
- Strong improvement of the slepton and gaugino sector
- Some improvement in the squark sector
- No bias any more
- Gaussian distributions

Comparison of the Collider Predictions for $\Omega_{DM}h^2$



- Tremendous improvement in the precision of the prediction of quantities like $\Omega_{DM}h^2$

Proposal

- If you agree, let's collect the results of our analyses and repeat that fit.
- I would need all results by February 27th in order to deliver a result until March 15th.

Summary and Outlook

- Fittino provides a flexible and powerful framework for Model Discrimination and Parameter Determination studies
- Model Discrimination is possible (non-trivial, still a lot to come about that)
- Parameter determination is possible for a variety of models (non-trivial!)
- Exploration of the SUSY parameter space with combination of real LE measurements and prospect of LHC results
- Explorations of the general MSSM parameter space with several simplifications with ILC, LHC and LE observables
- Very exciting times are ahead of us if there is SUSY as we know it best: LHC will really find it!
- Still to come:
- In cooperation with phenomenologists: Need to find out more about theoretical uncertainties of the predictions and their correlations!
- More complex model discrimination (NMSSM vs. MSSM, etc)

More Information

- Fittino:
  http://www-flc.desy.de/fittino/
  hep-ph/0412012 Comp. Phys. Comm. 174, Issue 1, (2006), 47-70
  hep-ph/0511006 Eur.Phys.J.C46:533-544,2006
- SPA:
  http://spa.desy.de/spa/
- SPheno:
  http://www-theorie.physik.unizh.ch/~porod/SPheno.html
- More Results from Fittino:
  DESY-THESIS-2004-040
  Another Paper mainly on LE+LHC soon to be published

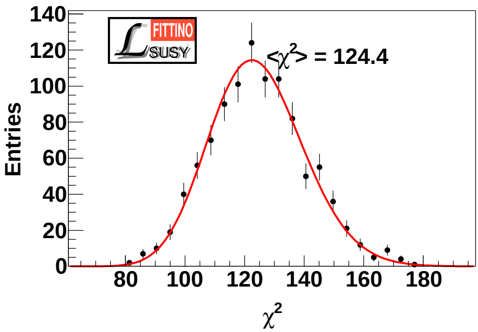
Backup Slides

SPS1a' MSSM Scenario Fit with ILC and LHC Observables

- Observables:
  - SM observables $m_Z, m_W, G_F, m_t, \ldots$
  - Higgs sector masses from 500 GeV and 1 TeV LC
  - All accessible sparticle and gaugino masses from LHC and LC with realistic uncertainties from hep-ph/0410364
  - LC cross sections at 400,500,1000 GeV, polarisation LR, RL, LL and RR
  - h and largest $\tilde{\mathrm{t}}_1$ BR's
- Assumptions for this test:
  - Unification in the first two generations
- Two fits:
  - Theory uncertainty only on m_{h}
  - Theory uncertainty on all masses (hep-ph/0511344) and $2\times$ larger σ uncertainties

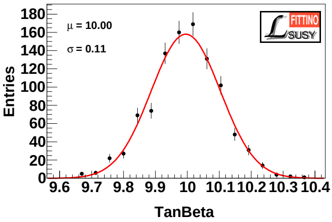
Checking the Results

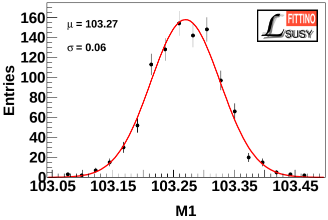
χ^2 distribution:



for n.d.f. $= 126$

Toy Fits:





Theory Input: The SPA Project

- Supersymmetry Parameter Analysis Project
- LHC + ILC: Comprehensive picture of SUSY particles
- Experimental accuracies of the ILC at the per-mille level
- Must be matched by equally precise
  - Theoretical predictions, relevant higher orders + uncertainties
  - Interpretation framework ($\rightarrow$ Fittino)
- SPA provides:
  - Well defined framework for the calculation of
    - masses, mixings
    - couplings
    - branching ratios, production cross-sections
    - widths
  - Framework for parameter extraction and evolution to high scale

The SPA Project

- The SPA convention specifies the framework, on which information is transferred between codes:
  - The masses of the particles are defined as pole masses
  - All parameters are given in $\overline{DR}$ scheme at $\tilde{M} = 1$ TeV
  - ...
- A program base is created, including
  - Scheme translation tools, spectrum calculators, event generators, ...
  - RGE programs
  - Analysis codes, e.g. Fittino, SFitter
  - ...
- A reference point for analyses is chosen based on SPS1a: SPS1a'

The SPA Project

- SUSY parameter measurement
  - Parameter = Observable

    m_{A}

    running or pole-parameter!
  - Parameter measurement on tree-level

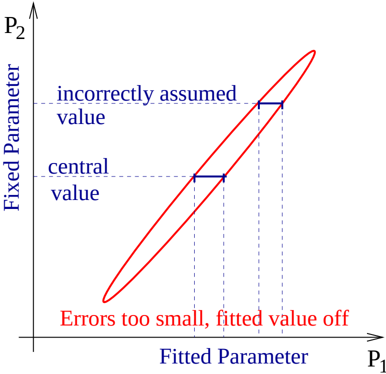
    $M_{\tilde{t}_L} = f(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \cos\theta_t)$

    Caution: loop corrections are not included!
  - Full precision and correlation

    Observable

    $O_i = f(\text{all parameters } P_i)$

- Mutual influence:



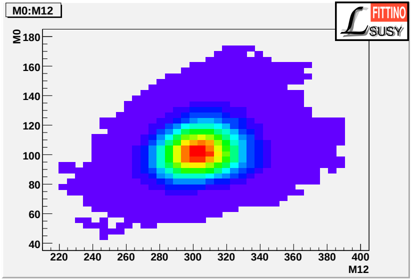
- Loop corrections are much larger than experimental uncertainties:
  maximal $m_{\mathrm{h,tree}} = m_{\mathrm{Z}} \Rightarrow$ maximal $m_{\mathrm{h,loop}} \approx 135$ GeV,
  $\Delta m_{\mathrm{hexp}} = 50\,\mathrm{MeV}$

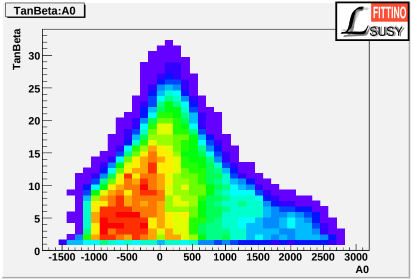
The Parameters of the 'MSSM24'

- General parametrization of minimal SUSY and SUSY breaking: $\mathcal{L}_{\mathrm{soft}}$
- 105 free SUSY parameters
- Assume:
  - No complex phases
  - No mixing between generations (and between q and ℓ)
  - No mixing in first and second generation
- 24 additional parameters are left:
  - Higgs sector: $\tan\beta$, $m_{\mathrm{A\,run}}$
  - Gaugino sector: μ, M_1, M_2, M_3
  - Squark sector: A_q, M_{u_L}, M_{u_R}, M_{d_R}
  - Slepton sector: A_ℓ, M_{ℓ_L}, M_{ℓ_R}
- Understand theory and observables $\Leftrightarrow$ Measure parameters
- Understand SUSY breaking?

Sampling of the Parameter Space: Markov Chains

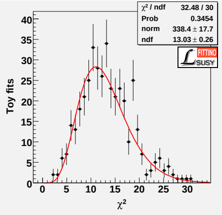
- Only limited information available for first LHC data
- Try to fit High-Scale models such as mSUGRA: 4 parameters, one sign



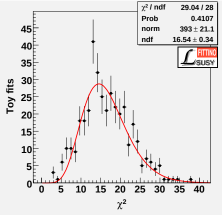


ATLAS Exclusive SUSY measurements $10\,\mathrm{fb}^{-1}$

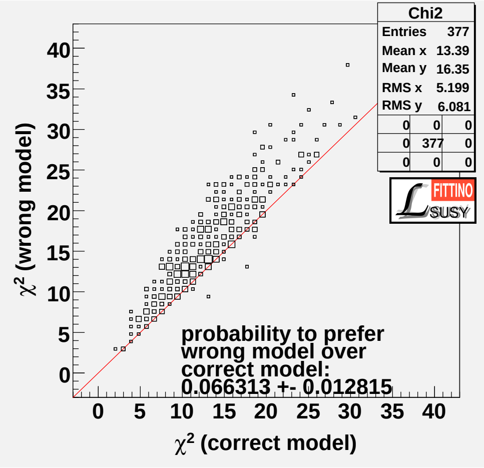
Separation of Different Models



χ^2 for correct model



χ^2 for wrong model

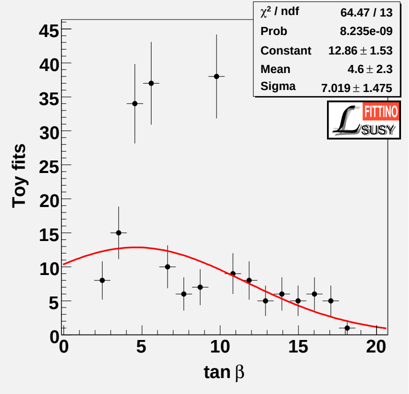


2D distribution of χ^2 values of toy fits with identical smearing for $\mathrm{sign}\mu = \pm 1$

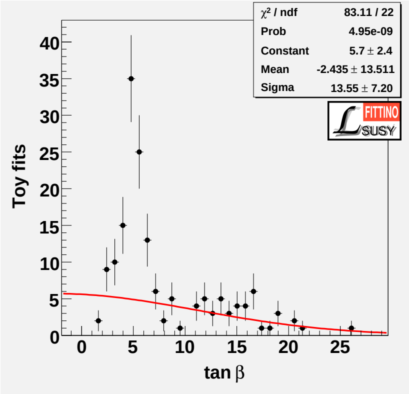
What is the separation power?

Reliable Minimization to Global Minimum

- Use ATLAS CSC Results
- Toy fits to determine uncertainties and correlations of parameters
- Even for a 4-Parameter mSUGRA Minuit is not good enough!



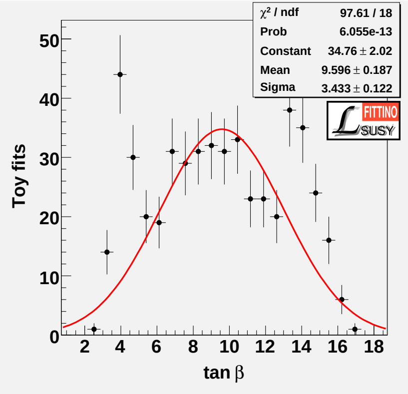
With Minuit only, slight offset in start values (±20 %)



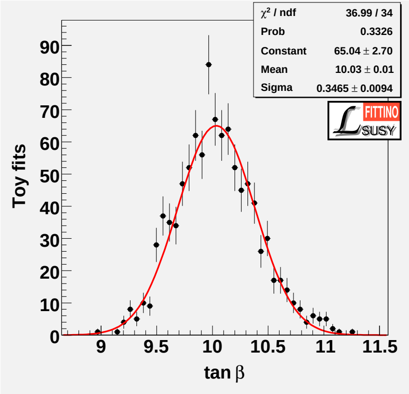
Same start values and toys, with Simulated Annealing

Evolution of LHC and LE Results for mSUGRA

- Use ATLAS CSC, CMS TDR and LE Results



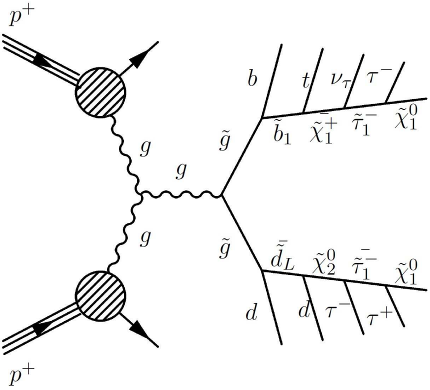
$10\,\mathrm{fb}^{-1}$



$300\,\mathrm{fb}^{-1}$

As in the plots before: Non-Gaussian uncertainties for small number of observables with large uncertainties

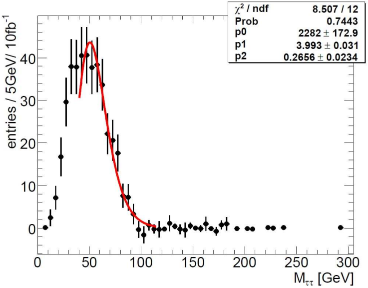
Typical SUSY (SPS1a') Process at LHC

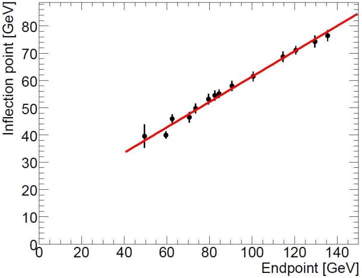


- LSP cannot be detected
- Detect only SM particles:
  - τ leptons (3)
  - jets (at least 4)
- Observable: Inv Mass $m^2_{\tau^+\tau^-}(m^2_{\tilde{\chi}_2^0}, m^2_{\tilde{\tau}_1}, m^2_{\tilde{\chi}_1^0})$

- no SUSY mass reconstructable directly
- only invariant mass reconstruction yield observables $\leftrightarrow$ e.g. one observable: $m^2_{\tau\tau}$ depends on three SUSY masses $(m^2_{\tilde{\chi}_2^0}, m^2_{\tilde{\tau}_1}, m^2_{\tilde{\chi}_1^0})$
- combinatorial background of τs from second decay chain for $m_{\tau\tau}$

Technique to Determine $\tau\tau$ Invariant Mass Edge

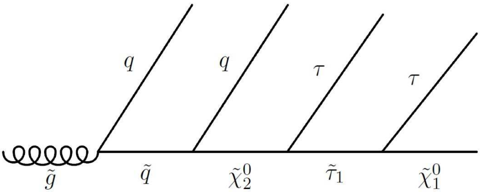




- (left plot): signal plus background distribution after all cuts
- from the parameters of the fitting function, the inflection point is calculated
- with a calibration procedure the calibration curve (right plot) has been calculated (many samples with slightly modified mass sp ectra)
- with the calibration curve the endpoint can be derived from the inflection point

More Mass Edges

- **1** observable $m^2_{\tau\tau}$ depends on **3** SUSY masses $(m^2_{\tilde{\chi}^0_2}, m^2_{\tilde{\tau}_1}, m^2_{\tilde{\chi}^0_1})$



Mass edges:
$m^2_{\tau^+\tau^-}(m^2_{\tilde{\chi}^0_2}, m^2_{\tilde{\tau}_1}, m^2_{\tilde{\chi}^0_1})$
$m^2_{q\tau^+\tau^-}(m^2_{\tilde{q}}, m^2_{\tilde{\chi}^0_2}, m^2_{\tilde{\tau}_1}, m^2_{\tilde{\chi}^0_1})$
$m^2_{q\tau_{near}}(m^2_{\tilde{q}}, m^2_{\tilde{\chi}^0_2}, m^2_{\tilde{\tau}_1})$
$m^2_{q\tau_{far}}(m^2_{\tilde{q}}, m^2_{\tilde{\chi}^0_2}, m^2_{\tilde{\tau}_1}, m^2_{\tilde{\chi}^0_1})$
$m^2_{q\tau_{low}} = min[(m^2_{q\tau_{near}}), (m^2_{q\tau_{far}})]$
$m^2_{q\tau_{high}} = max[(m^2_{q\tau_{near}}), (m^2_{q\tau_{far}})]$

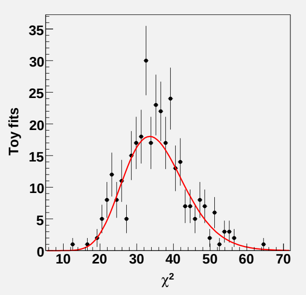
- by adding the squark information
  - **1** additional SUSY mass is needed
  - BUT **3** more observables are gained!!

- τ_{near} and τ_{far} can NOT be distinguished on reco level!!
  - $q\tau_{high}$ and $q\tau_{low}$ edges

- di τ final states $\rightarrow$ **4** possible observables (based on **4** SUSY parameters)

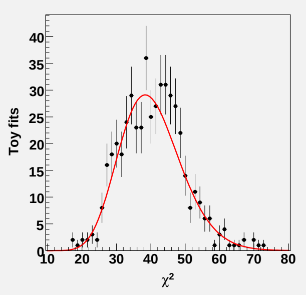
Global Fits for Different Luminosities

$1\ \mathrm{fb}^{-1}$



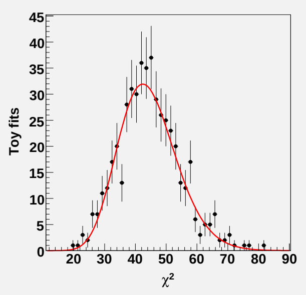
$\mathrm{ndf} = 35.55 \pm 0.52$
(36 expected)

$10\ \mathrm{fb}^{-1}$



$\mathrm{ndf} = 40.38 \pm 0.39$
(41 expected)

$300\ \mathrm{fb}^{-1}$



$\mathrm{ndf} = 44.42 \pm 0.45$
(45 expected)

χ^2 distributions

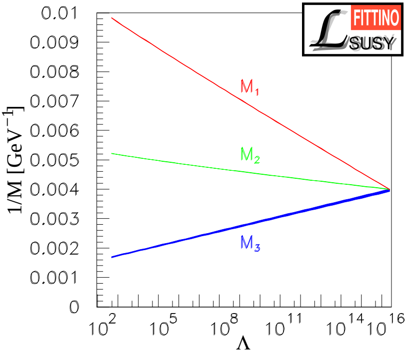
Objective quality criterion: reproduce input number degrees of freedom (ndf)

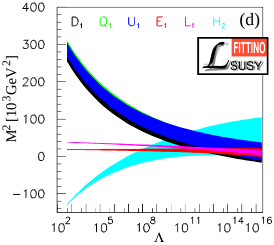
Very Short Example Input File

```
###############################################
###          Fittino input file           ###
###############################################
massh0                   115.237 GeV +- 0.05 GeV +- 0.5 GeV # comment
massTop                  178.0   GeV +- 0.3  GeV
correlationCoefficient   massh0 massTop 0.05                # quatsch
# etc
edge  1 massNeutralino1 massNeutralino2      263.5 GeV +- 1.2 GeV alias 1
sigma ( ee -> Neutralino1 Neutralino2, 1000.,0.8,0.6 ) 7.678 fb +- 2.0 fb
BR ( h0 -> Bottom Bottom~ )   0.8033 +- 0.01 +-(lumiErr) 0.05
BR ( h0 -> Charm Charm~   )   0.05   +- 0.02 +-(lumiErr) 0.01
nofit cos2PhiL                                     0.62865 +- 0.0005
# etc
fitParameter  TanBeta          10.0
fixParameter  Mu               358.6 GeV
universality  MSelectronR MSmuR
# etc
LoopCorrections       on
CalcPullDist          off
```

MSSM24: Evolution to the GUT Scale

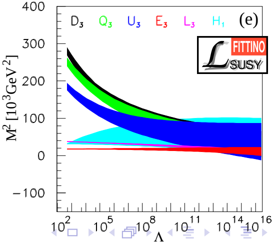
- Fit 20 or 24 parameter MSSM!
- Based on the results of the low-energy parameter fit:





Using the high ILC precision:

Precision test of breaking schemes

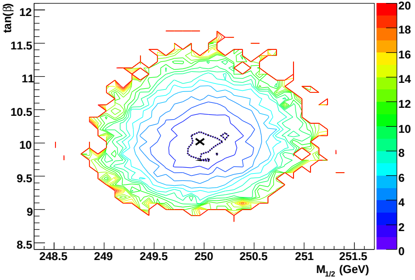


Sampling of the Parameter Space: Markov Chains

- Over the last years, several analyses of the SUSY parameter space with Markov Chains have appeared
- Typically not too much detail about the checks performed on the Markov Chains is given, hence here I'd like to show a few of the pitfalls
- In the first example, we see a 4-dimensional mSUGRA fit and a comparison of the interpretation of the Markov Chains in a Frequentistic or Bayesian way
- In the second example, we see the limits of the sample size or available CPU time for an 18-dimensional MSSM fit

Markov Chains: Bayesian Interpretation

- In order to interprete the Markov Chain, first look for the point with the highest point density $\mathcal{D}_{max} \sim \mathcal{L}$
- Calculate and plot $R = -2\ln\mathcal{D} + 2\ln\mathcal{D}_{max}$
- Then, the line with $\Delta R = 1$ gives the 1dim uncertainty of the parameter
- Results from Bayesian analysis are in the same order as the Pull Fit Results, but differ up to 50 %!

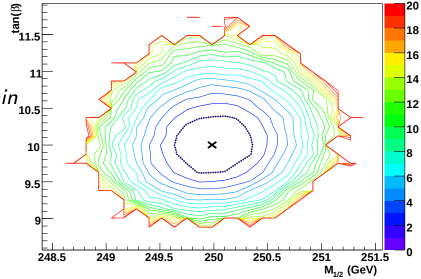


Markov Chains: Frequentist Interpretation

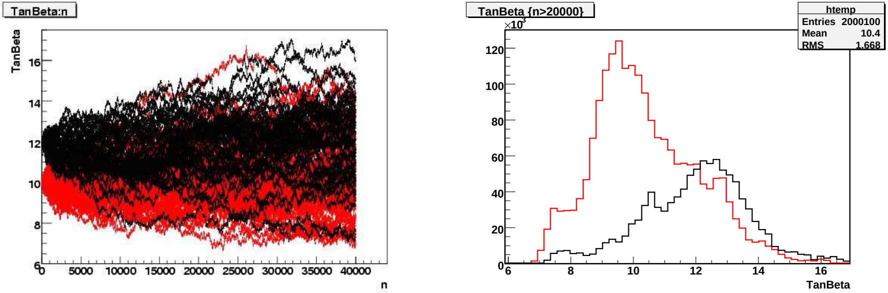
- In order to interprete the Markov Chain, search for the point with the highest $\mathcal{L} = \exp(-\chi^2/2)$ and then look for

$$R = -2\ln\mathcal{L} + 2\ln\mathcal{L}_{max} = \chi^2 - \chi^2_{min}$$

- Instead of plotting the point density, we must scan over all points. Those with $\Delta\chi^2 < 1$ give the 1σ range
- Results from Frequentist analysis corresponds well to Pull Fit Results

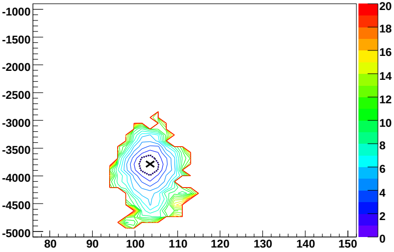


Avoid Distortions from the Starting Point (Problem for Bayes Only)

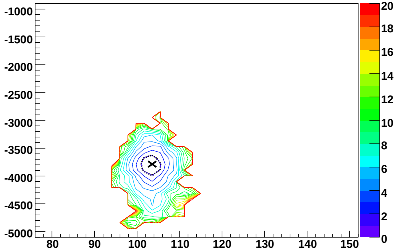


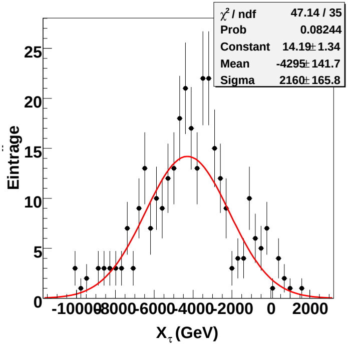
- Before we started that, we were told that waiting for 2000 iterations at the beginning of each markov Chain would be enough to let the Markov Chain settle.
- That's not really the case for a high number of parameters. Even after 20000 iterations (right) there still is a strong difference for different starting values!

Comparison with Toy Fits



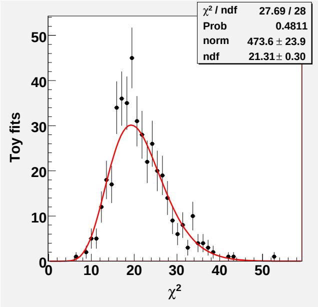
Comparison with Toy Fits

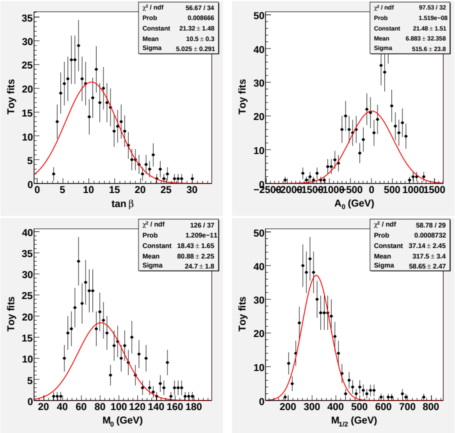




- While the Markov Chains promise to efficiently scan the parameter space, this is only true for sufficiently high statistics!

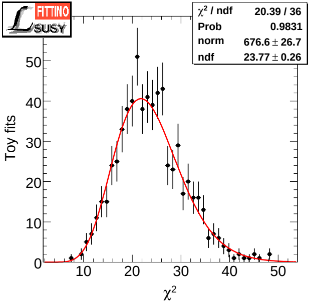
mSUGRA Results

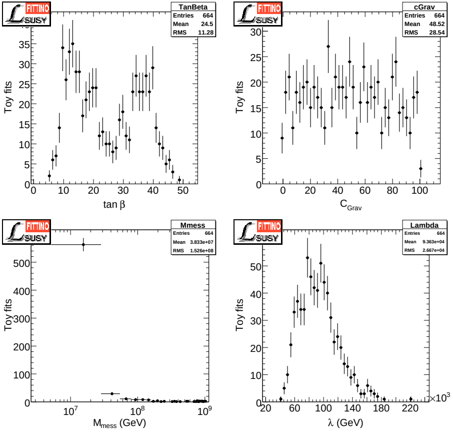




- Best fit for sign$\mu = +1$
- Second best fit for sign$\mu = +1$

GMSB Results





- Best fit for sign$\mu = +1$ and $N_5 = 1$
- Second best fit for sign$\mu = +1$ and $N_5 = 2$

What do we know already?

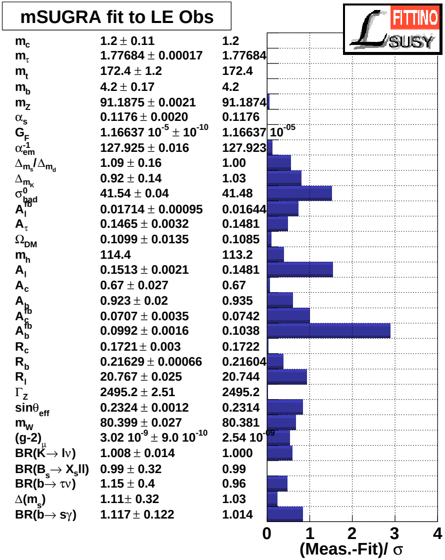
Observable	Experimental Value	Uncertainty stat	syst
α_{em}	127.925 GeV	0.016 GeV	
G_F	1.16637×10^{-5}	0.00001×10^{-5}	
α_s	0.1176 GeV	0.0020 GeV	
m_Z	91.1875 GeV	0.0021 GeV	
m_b	4.20 GeV	0.25 GeV	
m_t	172.4 GeV	1.2 GeV	
m_τ	1.77684 GeV	0.00017 GeV	
m_c	1.27 GeV	0.2 GeV	

That's not everything, luckily . . .

What do we know already? (ctd')

Observable	Experimental Value	Uncertainty stat	syst
$\mathcal{B}(B \to s\gamma)/\mathcal{B}^{SM}(B \to s\gamma)$	1.117	0.076	0.096
$\Delta m_{B_s}/\Delta m_{B_s}^{SM}$	1.11	0.01	0.32
$\mathcal{B}(B_s \to \mu\mu)/SM$	$< 4.7\times10^{-8}$		0.02×10^{-8}
$\mathcal{B}(B \to \tau\nu)/SM$	1.15	0.40	
$\mathcal{B}(B_s \to s\ell\ell)/SM$	0.99	0.32	
$\mathcal{B}(K \to \ell\nu)/SM$	1.008	0.014	
$a_\mu^{exp} - a_\mu^{SM}$	30.2×10^{-10}	8.8×10^{-10}	2.0×10^{-10}
m_W	80.399	0.025	0.010
$\sin^2\theta_{eff}$	0.2324	0.0012	
Γ_Z	2.4952	0.0023	0.001
R_l	20.767	0.025	
R_b	0.21629	0.00066	
R_c	0.1721	0.003	
$A_f(bb)$	0.0992	0.0016	
$A_f(bc)$	0.0707	0.0035	
A_b	0.923	0.020	
A_c	0.670	0.027	
A_l	0.1513	0.0021	
m_h	> 114.4		3.0
Ωh^2	0.1099	0.0062	0.012
A_τ	0.1465	0.0032	
$A_f(bl)$	0.01714	0.00095	
σ_{had}	41.540	0.037	
$\Delta\epsilon_K^{exp}/\Delta\epsilon_K^{SM}$	0.92	0.14	
$\mathcal{B}(K \to \pi\nu\bar{\nu})$	< 4.5	1.0×10^{-10}	
$\mathcal{B}(B_d \to \ell\ell)$	$< 2.3\times10^{-8}$		0.001×10^{-8}
$(\Delta m_{B_s}/\Delta m_{B_s}^{SM})/(\Delta m_{B_d}/\Delta m_{B_d}^{SM})$	1.09	0.01	0.16

mSUGRA Results: Pull

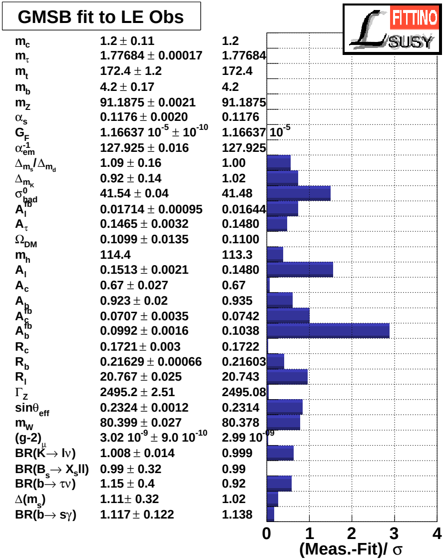


- Show the individual pull of each measured with respect to the best fit result
- Best $\chi^2 = 20.18$ for $ndf = 21$: $\text{Prob}_{\chi^2} = 51\,\%$ $\text{sign}\mu = +1$
- For correct uncertainties: Fit α_{em}, α_s, G_F, m_Z, m_t, m_Z parallel to the SUSY parameters
  makes basically no difference

Parameter	Best Fit		Uncertainty
$\tan\beta$	12.8	±	5.0993
A_0	382.1	±	557.1
M_0	74.0	±	30.8
M_{12}	330.4	±	83.2

remember SPS1a': $M_{1/2} = 250$ GeV, $M_0 = 70$ GeV, $A_0 = -300$ GeV, $\tan\beta = 10$

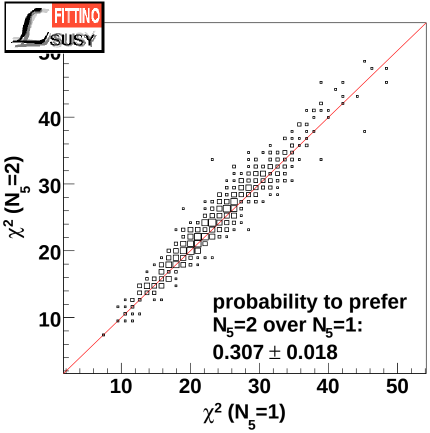
GMSB Results: Pull



- Show the individual pull of each measured with respect to the best fit result
- Best $\chi^2 = 19.36$ for *ndf* $= 21$: $\mathrm{Prob}_{\chi^2} = 56\,\%$ for $N_5 = 1$ and $\mathrm{sign}\mu = +1$
- For correct uncertainties: Fit α_{em}, α_s, G_F, m_Z, m_t, m_Z parallel to the SUSY parameters

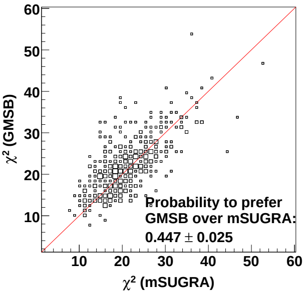
Parameter	Best Fit		Uncertainty
$\tan\beta$	18.4	±	11.3
C_{grav}	88.2	±	28.5
Λ	81428.8	±	26671.0
M_{mess}	95129.4	±	1.52587e+08

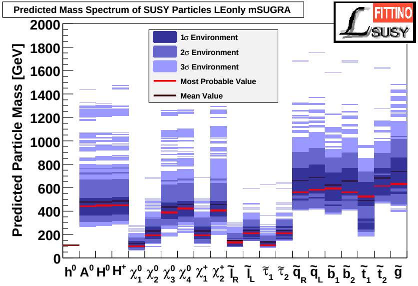
GMSB Results: Digital Parameters

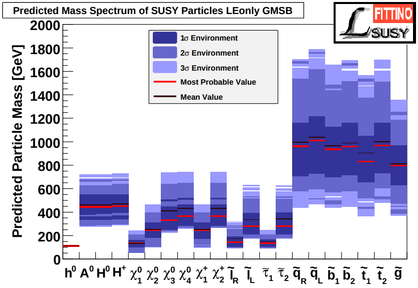


- Perform simultaneous toy fits of different models to the same smeared observables
- With the presently available data: No possibility to distinguish $N_5 = 2$ from $N_5 = 1$

Predictions of Particle Masses: LE only



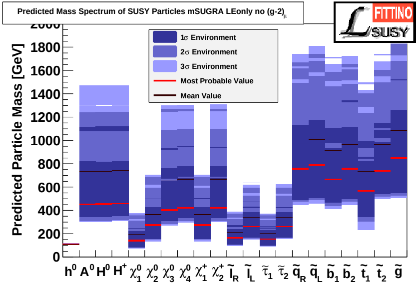


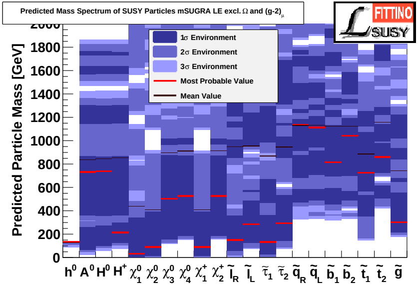


- Well, we can not really distinguish the predictions of GMSB vs. mSUGRA . . .
- But we can predict something!

LE without Omega and $(g-2)_\mu$

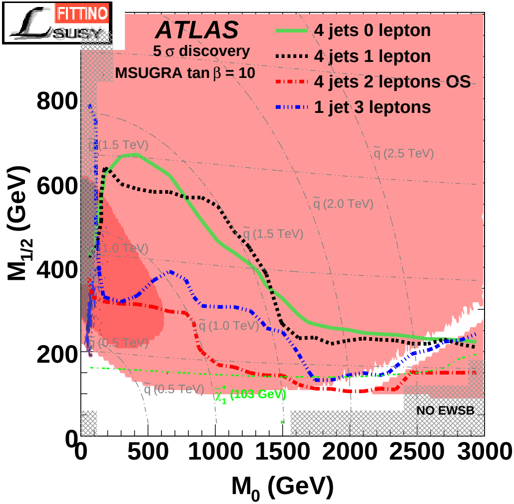
- Some of the observables are critical:
- Ω_{DM} is precisely measured, but who tells us it's due to SUSY?
- The SM predictions for $(g-2)_\mu$ are at least doubtful.



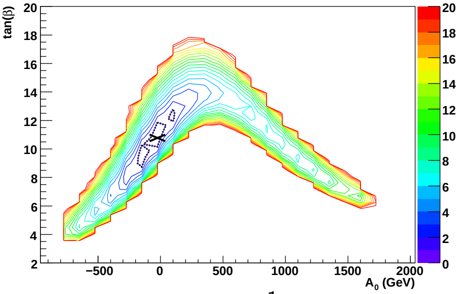


Concrete Prediction for ATLAS

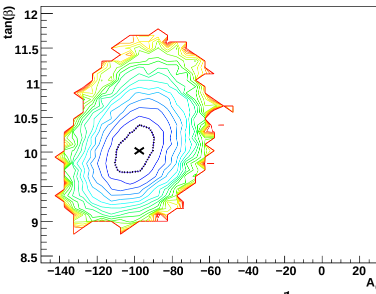
- Overlay the allowed mSUGRA parameter space over the expected sensitivity of ATLAS



Global Fits for Different Luminosities



$1\ \text{fb}^{-1}$



$300\ \text{fb}^{-1}$

Alternative view on parameter space: Likelihood maps

- Markov chain with sampling rate proportional to likelihood L that parameter set is realized in data
- plot $2(\ln(L_0) - \ln(L)) = \chi^2$
- reveals substructure (e.g. second order minima)

LE+LHC MSSM18 Fit Results

Now try to fit the MSSM18 model to LE+LHC data

Parameter	Nominal value	Fit		Uncertainty
MSelectronL	194.31	194.65	±	5.4
MSelectronR	135.76	135.696	±	3.2
MStauL	193.52	192.579	±	34.7
MStauR	133.43	175.633	±	203.5
MSupL	527.57	527.272	±	3.5
MSupR	509.14	508.811	±	10.3
MSbottomR	504.01	510.763	±	20.9
MStopL	481.69	488.458	±	20.9
MStopR	409.12	449.496	±	131.7
TanBeta	10	9.21822	±	2.9
Mu	355.05	353.829	±	4.4
Xtau	-3799.88	-4212.91	±	2568.1
Xtop	-526.62	-535.387	±	192.5
Xbottom	-4314.33	-2197.12	±	3937.7
M1	103.15	103.326	±	2.23
M2	192.95	193.747	±	2.32
M3	568.87	570.274	±	10.2
massA0	359.63	741.11	±	529.8

Overview of some Packages

- Fittino hep-ph/0412012:

  MSSM high and low scale, NMSSM, going to be extended . . .
- SFitter hep-ph/0404282:
  Tilman Plehn, Dirk Zerwas *et al.*
  MSSM high and low scale
- GFitter https://twiki.cern.ch/twiki/bin/view/Gfitter:

  Andreas Höcker *et al.*
  SM, 2HDM, going to be extended . . .

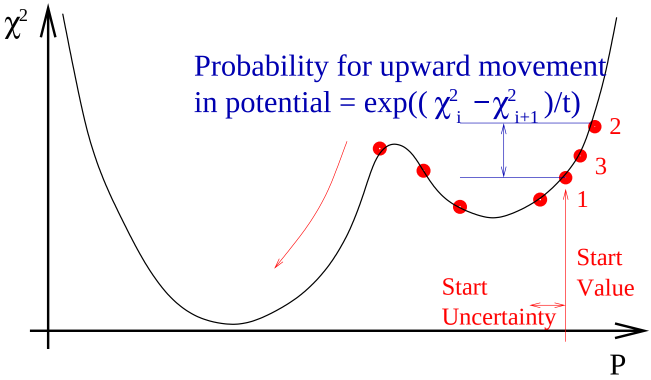
Technologies

- To find the χ^2 minimum
  - Tree-level estimate + MIGRAD in MINUIT:
    fast but unreliable, if not started very close to the true minimum
  - Tree-level estimate + Simulated Annealing:
    not too slow, very reliable detection of the global minimum
- To map the parameter space
  - 1D or 2D scans:
    fast, but no correct treatment of the correlations to fixed parameters
  - Markov Chain:
    n-dim. Probability map of the available parameter space
- To determine the uncertainties
  - MINOS in MINUIT:
    Slow and not very reliable
  - Automatic generation of pull fits:
    Very reliable and (at least on a farm) not slower than MINOS
- To get a feeling for the parameters and observables
  - Visualize the effect of variations of each parameter on observable χ^2's

Markov Chains

- Build a chain of parameter points $\vec{x}_i$ using the Metropolis-Hastings algorithm
- Calculate a likelihood $\mathcal{L}(\vec{x}_i)$ for each point. Here: $\mathcal{L}(\vec{x}_i) = e^{-\chi^2}$
- Randomly pick a new point $\vec{x}_{i+1}$ near $\vec{x}_i$, using a proposal distribution $Q(\vec{x}_{i+1}; \vec{x}_i)$
- Calculate $\rho = \mathcal{L}(\vec{x}_{i+1})/\mathcal{L}(\vec{x}_i)$ for symmetrical $Q(\vec{x}_{i+1}; \vec{x}_i)$
- If $\rho > 1$, accept the new point and add it to the end of the chain
- If $\rho < 1$, accept it with probability ρ, else, add $\vec{x}_i$ again to the chain
- For the efficiency of the algorithm: Optimize $Q(\vec{x}_{i+1}; \vec{x}_i)$, e.g. gaussian distribution around $\vec{x}_i$ with widthes $\vec{\sigma}_i$
- Result: Point density in the chain is proportional to the probability distribution
- Extremely effective sampling: needed number of steps scales with the number of dimensions D instead of n^D

Simulated Annealing



- Works as Markov Chains, but with different potential (Boltzmann) and with reducing the temperature with time
- Aim is not sampling of the parameter space, but reliable convergence towards the global minimum

Models and their Parameters

Standard model (SM)

19 parameters

- 9 fermion masses m_f
- 3 couplings g, $g\prime$, g_S
- Higgs mass m_H, VEV v
- strong CP phase θ_{QCD}
- 3 CKM angles, 1 CKM phase

Effective SM Parameters

6 parameters

- m_t (sometimes also m_b)
- m_Z
- α_s, α_{em}, G_F
- Higgs mass m_H

MSSM-24

24 parameters

- 15 sfermion masses $m_{\tilde{f}}$
- 3 trilinear couplings A_τ, A_b, A_t
- 3 gaugino masses M_1, M_2, M_3
- pseudoscalar Higgs mass m_{A^0}
- Higgsino mass parameter μ
- ratio of 2 Higgs VEVs $\tan\beta = \frac{v_1}{v_2}$

MSSM-18

18 parameters

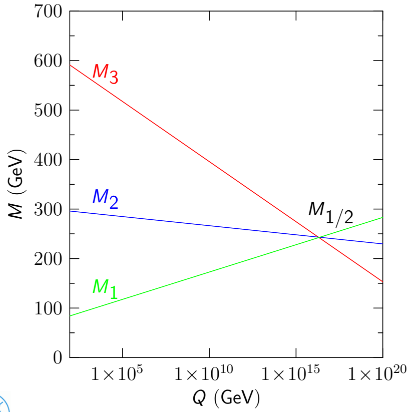
- unify $M_{\tilde{q}_L} = M_{\tilde{u}_L} = \cdots = M_{\tilde{c}_L}$
- unify $M_{\tilde{q}R} = M_{\tilde{u}_R} = \cdots = M_{\tilde{c}_R}$
- unify $M_{\tilde{\ell}_L} = M_{\tilde{e}_L} = M_{\tilde{\mu}_L}$
- unify $M_{\tilde{\ell}_R} = M_{\tilde{e}_R} = M_{\tilde{\mu}_R}$

Unified Models

Unification of parameters at high energies

example

universal gaugino mass $M_{1/2}$



unified model: mSUGRA

4 parameters

- universal gaugino mass $M_{1/2}$
- universal scalar mass M_0
- universal trilinear coupling A_0
- $\tan\beta$

assumed model point: SPS 1a

$M_{1/2} = 250$ GeV, $M_0 = 100$ GeV,
$A_0 = -100$ GeV, $\tan\beta = 10$, $\text{sign}\mu = 1$