A Model-Independent Approach to Two-Higgs Doublet Model (2HDM) Physics

Howard E. Haber
VLCW06
20 July 2006

This talk is based on work that appears in:

1. S. Davidson and H.E. Haber, "Basis-independent methods for the two-Higgs-doublet model," *Phys. Rev.* **D72**, 035004 (2005). [See also G. Branco, L. Lavoura and J.P. Silva, *CP Violation* (Oxford University Press, Oxford, UK, 1999), chapters 22 and 23.]

2. H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model. II: The significance of $\tan\beta$," arXiv:hep-ph/0602242, *Phys. Rev.* **D74** (2006), in press.

3. J.F. Gunion, H.E. Haber and J. Kalinowski, in preparation.

Outline

- Motivation
  - The MSSM and the two-Higgs-doublet Model
- The general Two-Higgs-Doublet Model (2HDM)
  - Basis-independent techniques
  - The Higgs basis and the Higgs-mass basis
  - Basis-independent (invariant) form of the Higgs couplings
- The Higgs-fermion interaction
  - The significance of $\tan\beta$?
  - Many $\tan\beta$-like parameters?
- The decoupling limit

Motivation

The Higgs sector of the minimal supersymmetric extension of the Standard Model (MSSM) is a constrained 2HDM. However, at one-loop all possible 2HDM interactions allowed by gauge invariance are generated (due to SUSY-breaking interactions).

Thus, the Higgs sector of the MSSM is in reality the most general 2HDM model (albeit with certain relations among the Higgs sector parameters determined by the fundamental parameters of the broken supersymmetric model).

The general 2HDM consists of two identical (hypercharge-one) scalar doublets Φ_1 and Φ_2. One can always redefine the basis, so the parameter $\tan\beta \equiv v_2/v_1$ is not meaningful!

To determine the physical quantities, one must develop basis-independent techniques.

The General Two-Higgs-Doublet Model

Consider the 2HDM potential in a *generic* basis:

$$\begin{aligned}\mathcal{V} = &\, m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 \\ &+ \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ &+ \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}\end{aligned}$$

A basis change consists of a U(2) transformation $\Phi_a \longrightarrow U_{a\bar{b}} \Phi_b$ (and $\Phi_{\bar{a}}^\dagger = \Phi_{\bar{b}}^\dagger U_{b\bar{a}}^\dagger$). Rewrite $\mathcal{V}$ in a U(2)-covariant notation:

$$\mathcal{V} = Y_{a\bar{b}} \Phi_{\bar{a}}^\dagger \Phi_b + \tfrac{1}{2} Z_{a\bar{b}c\bar{d}} (\Phi_{\bar{a}}^\dagger \Phi_b)(\Phi_{\bar{c}}^\dagger \Phi_d)$$

where $Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}}$ and hermiticity implies $Y_{a\bar{b}} = (Y_{b\bar{a}})^*$ and $Z_{a\bar{b}c\bar{d}} = (Z_{b\bar{a}d\bar{c}})^*$. The barred indices help keep track of which indices transform with U and which transform with $U^\dagger$. For example, $Y_{a\bar{b}} \longrightarrow U_{a\bar{c}} Y_{c\bar{d}} U_{d\bar{b}}^\dagger$ and $Z_{a\bar{b}c\bar{d}} \longrightarrow U_{a\bar{e}} U_{f\bar{b}}^\dagger U_{c\bar{g}} U_{h\bar{d}}^\dagger Z_{e\bar{f}g\bar{h}}$.

The most general U(1)$_{\rm EM}$-conserving vacuum expectation value (vev) is:

$$\langle\Phi_a\rangle = \frac{v}{\sqrt{2}}\left(\begin{array}{c} 0 \\ \widehat{v}_a \end{array}\right), \qquad \text{with} \qquad \widehat{v}_a \equiv e^{i\eta}\left(\begin{array}{c} c_\beta \\ s_\beta\, e^{i\xi} \end{array}\right),$$

where $v \equiv 2m_W/g = 246$ GeV. The overall phase η is arbitrary (and can be removed with a U(1)$_{\rm Y}$ hypercharge transformation). If we define the hermitian matrix $V_{a\bar{b}} \equiv \hat{v}_a \hat{v}^*_{\bar{b}}$, then the scalar potential minimum condition is given by the invariant condition:

$${\rm Tr}\,(VY) + \tfrac{1}{2}v^2 Z_{a\bar{b}c\bar{d}} V_{b\bar{a}} V_{d\bar{c}} = 0\,.$$

The orthonormal eigenvectors of $V_{a\bar{b}}$ are $\hat{v}_b$ and $\widehat{w}_b \equiv \widehat{v}^*_{\bar{c}}\,\epsilon_{cb}$ (with $\epsilon_{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = 0$). Note that $\hat{v}^*_{\bar{b}}\hat{w}_b = 0$. Under a U(2) transformation, $\hat{v}_a \longrightarrow U_{a\bar{b}}\hat{v}_b$, but:

$$\widehat{w}_a \longrightarrow ({\rm det}\ U)^{-1}\, U_{a\bar{b}}\,\widehat{w}_b\,,$$

where ${\rm det}\ U \equiv e^{i\chi}$ is a pure phase. That is, $\widehat{w}_a$ is a pseudo-vector with respect to U(2). One can use $\widehat{w}_a$ to construct a proper second-rank tensor: $W_{a\bar{b}} \equiv \hat{w}_a\hat{w}^*_{\bar{b}} \equiv \delta_{a\bar{b}} - V_{a\bar{b}}$.

Remark: U(2) $\cong$ SU(2)$\times$U(1)$_{\rm Y}/\mathbb{Z}_2$. The parameters m^2_{11}, m^2_{22}, m^2_{12}, and $\lambda_1, \ldots, \lambda_7$ are invariant under U(1)$_{\rm Y}$ transformations, but are modified by a "flavor"-SU(2) transformation; whereas $\hat{v}$ transforms under the full U(2) group.

The Higgs basis

Define new Higgs doublet fields:

$$H_1 = (H_1^+\,,\, H_1^0) \equiv \widehat{v}_{\bar{a}}^* \Phi_a\,, \qquad\qquad H_2 = (H_2^+\,,\, H_2^0) \equiv \widehat{w}_{\bar{a}}^* \Phi_a\,.$$

Equivalently, $\Phi_a = H_1 \hat{v}_a + H_2 \hat{w}_a$. Since $\widehat{v}_{\bar{a}}^* \widehat{v}_a = 1$ and $\widehat{v}_{\bar{a}}^* \widehat{w}_a = 0$, it follows that

$$\langle H_1^0 \rangle = \frac{v}{\sqrt{2}}\,, \qquad\qquad \langle H_2^0 \rangle = 0\,.$$

The field H_1 defined above is *invariant*. However, under a U(2) transformation,

$$H_2 \longrightarrow (\det\, U) H_2\,.$$

For example, under the U(2) transformation $U = \mathrm{diag}\,(1\,,\, e^{i\chi})$, one can transform among different Higgs bases that are related by a rephasing of the field H_2. Quantities that are invariant under SU(2) but not under U(2) will henceforth be called *pseudo-invariants*.

If we rewrite the Higgs potential $\mathcal{V}$ in the Higgs basis, we find:

$$\mathcal{V} = Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2$$
$$+ \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1)$$
$$+ \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \left[Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \right] H_1^\dagger H_2 + \text{h.c.} \right\} ,$$

where

$$Y_1 \equiv \mathrm{Tr}\,(YV)\,, \qquad Y_2 \equiv \mathrm{Tr}\,(YW)\,,$$
$$Z_1 \equiv Z_{a\bar{b}c\bar{d}}\, V_{b\bar{a}} V_{d\bar{c}}\,, \qquad Z_2 \equiv Z_{a\bar{b}c\bar{d}}\, W_{b\bar{a}} W_{d\bar{c}}\,,$$
$$Z_3 \equiv Z_{a\bar{b}c\bar{d}}\, V_{b\bar{a}} W_{d\bar{c}}\,, \qquad Z_4 \equiv Z_{a\bar{b}c\bar{d}}\, V_{b\bar{c}} W_{d\bar{a}}$$

are invariant quantities, whereas the following (potentially complex) pseudo-invariants

$$Y_3 \equiv Y_{a\bar{b}}\, \widehat{v}^*_{\bar{a}}\, \widehat{w}_b\,, \qquad Z_5 \equiv Z_{a\bar{b}c\bar{d}}\, \widehat{v}^*_{\bar{a}}\, \widehat{w}_b\, \widehat{v}^*_{\bar{c}}\, \widehat{w}_d\,,$$
$$Z_6 \equiv Z_{a\bar{b}c\bar{d}}\, \widehat{v}^*_{\bar{a}}\, \widehat{v}_b\, \widehat{v}^*_{\bar{c}}\, \widehat{w}_d\,, \qquad Z_7 \equiv Z_{a\bar{b}c\bar{d}}\, \widehat{v}^*_{\bar{a}}\, \widehat{w}_b\, \widehat{w}^*_{\bar{c}}\, \widehat{w}_d\,.$$

transform as $[Y_3, Z_6, Z_7] \to (\det\, U)^{-1} [Y_3, Z_6, Z_7]$ and $Z_5 \to (\det\, U)^{-2} Z_5$.

The invariants and pseudo-invariants in the generic basis are given by:

$$Y_1 = m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - \mathrm{Re}(m_{12}^2 e^{i\xi}) s_{2\beta}\,,$$

$$Y_2 = m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + \mathrm{Re}(m_{12}^2 e^{i\xi}) s_{2\beta}\,,$$

$$Y_3\, e^{i\xi} = \tfrac{1}{2}(m_{22}^2 - m_{11}^2) s_{2\beta} - \mathrm{Re}(m_{12}^2 e^{i\xi}) c_{2\beta} - i\,\mathrm{Im}(m_{12}^2 e^{i\xi})\,,$$

$$Z_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\left[c_\beta^2 \mathrm{Re}(\lambda_6 e^{i\xi}) + s_\beta^2 \mathrm{Re}(\lambda_7 e^{i\xi})\right]\,,$$

$$Z_2 = \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 - 2 s_{2\beta}\left[s_\beta^2 \mathrm{Re}(\lambda_6 e^{i\xi}) + c_\beta^2 \mathrm{Re}(\lambda_7 e^{i\xi})\right]\,,$$

$$Z_3 = \tfrac{1}{4} s_{2\beta}^2\,[\lambda_1 + \lambda_2 - 2\lambda_{345}] + \lambda_3 - s_{2\beta} c_{2\beta} \mathrm{Re}[(\lambda_6 - \lambda_7) e^{i\xi}]\,,$$

$$Z_4 = \tfrac{1}{4} s_{2\beta}^2\,[\lambda_1 + \lambda_2 - 2\lambda_{345}] + \lambda_4 - s_{2\beta} c_{2\beta} \mathrm{Re}[(\lambda_6 - \lambda_7) e^{i\xi}]\,,$$

$$\begin{aligned} Z_5\, e^{2i\xi} &= \tfrac{1}{4} s_{2\beta}^2\,[\lambda_1 + \lambda_2 - 2\lambda_{345}] + \mathrm{Re}(\lambda_5 e^{2i\xi}) + i c_{2\beta} \mathrm{Im}(\lambda_5 e^{2i\xi})\,, \\ &\quad - s_{2\beta} c_{2\beta} \mathrm{Re}[(\lambda_6 - \lambda_7) e^{i\xi}] - i s_{2\beta} \mathrm{Im}[(\lambda_6 - \lambda_7) e^{i\xi})]\,, \end{aligned}$$

$$\begin{aligned} Z_6\, e^{i\xi} &= -\tfrac{1}{2} s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta} - i\mathrm{Im}(\lambda_5 e^{2i\xi})\right] + c_\beta c_{3\beta} \mathrm{Re}(\lambda_6 e^{i\xi})\,, \\ &\quad + s_\beta s_{3\beta} \mathrm{Re}(\lambda_7 e^{i\xi}) + i c_\beta^2 \mathrm{Im}(\lambda_6 e^{i\xi}) + i s_\beta^2 \mathrm{Im}(\lambda_7 e^{i\xi})\,, \end{aligned}$$

$$\begin{aligned} Z_7\, e^{i\xi} &= -\tfrac{1}{2} s_{2\beta}\left[\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345} c_{2\beta} + i\mathrm{Im}(\lambda_5 e^{2i\xi})\right] + s_\beta s_{3\beta} \mathrm{Re}(\lambda_6 e^{i\xi}) \\ &\quad + c_\beta c_{3\beta} \mathrm{Re}(\lambda_7 e^{i\xi}) + i s_\beta^2 \mathrm{Im}(\lambda_6 e^{i\xi}) + i c_\beta^2 \mathrm{Im}(\lambda_7 e^{i\xi})\,. \end{aligned}$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \mathrm{Re}(\lambda_5\, e^{2i\xi})$.

The Higgs mass-eigenstate basis

Starting in the Higgs basis,

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + \varphi_1^0 + iG^0\right) \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(\varphi_2^0 + ia^0\right) \end{pmatrix},$$

where φ_1^0, φ_2^0 and a^0 are neutral scalar fields, and H^+ is the physical charged Higgs boson, with mass $m_{H^\pm}^2 = Y_2 + \frac{1}{2}Z_3 v^2$. If the Higgs sector is CP-violating, then φ_1^0, φ_2^0, and A all mix to produce three physical neutral Higgs states of indefinite CP. After employing the potential minimum conditions: $Y_1 = -\frac{1}{2}Z_1 v^2$ and $Y_3 = -\frac{1}{2}Z_6 v^2$, the resulting neutral Higgs squared-mass matrix is:

$$\mathcal{M} = \begin{pmatrix} Z_1 v^2 & \mathrm{Re}(Z_6) v^2 & -\mathrm{Im}(Z_6) v^2 \\ \mathrm{Re}(Z_6) v^2 & Y_2 + \frac{1}{2}\left[Z_3 + Z_4 + \mathrm{Re}(Z_5)\right] v^2 & -\frac{1}{2}\mathrm{Im}(Z_5) v^2 \\ -\mathrm{Im}(Z_6) v^2 & -\frac{1}{2}\mathrm{Im}(Z_5) v^2 & Y_2 + \frac{1}{2}\left[Z_3 + Z_4 - \mathrm{Re}(Z_5)\right] v^2 \end{pmatrix}.$$

Note that Z_7 does not appear above. The real symmetric matrix $\mathcal{M}$ is diagonalized by an orthogonal transformation. That is, $R\mathcal{M}R^T = \mathcal{M}_D = \mathrm{diag}\ (m_1^2\,,\ m_2^2\,,\ m_3^2)$, where $RR^T = I$.

A convenient form for R is:

$$R = R_{12}R_{13}R_{23} = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13} \\ 0 & 1 & 0 \\ s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & -c_{23}s_{12} - c_{12}s_{13}s_{23} & -c_{12}c_{23}s_{13} + s_{12}s_{23} \\ c_{13}s_{12} & c_{12}c_{23} - s_{12}s_{13}s_{23} & -c_{23}s_{12}s_{13} - c_{12}s_{23} \\ s_{13} & c_{13}s_{23} & c_{13}c_{23} \end{pmatrix},$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The neutral Higgs mass eigenstates are denoted by h_1, h_2 and h_3:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \\ a^0 \end{pmatrix}.$$

Since the mass-eigenstates h_i do not depend on the initial basis choice, they are U(2)-invariant fields. We have seen that Higgs basis parameters are either invariant or pseudo-invariant. In particular, one can show that under a U(2) transformation,

$$\theta_{12}\,,\ \theta_{13} \text{ are invariant, and } \quad e^{i\theta_{23}} \to (\det U)^{-1} e^{i\theta_{23}}\,.$$

We can eliminate the middle man by expressing the mass eigenstate neutral Higgs directly in terms of the original shifted neutral fields, $\overline{\Phi}^0_a \equiv \Phi^0_a - v\widehat{v}_a/\sqrt{2}$:

$$h_k = \frac{1}{\sqrt{2}}\left[\overline{\Phi}^{0\,\dagger}_{\bar{a}}(q_{k1}\widehat{v}_a + q_{k2}\widehat{w}_a e^{-i\theta_{23}}) + (q^*_{k1}\widehat{v}^*_{\bar{a}} + q^*_{k2}\widehat{w}^*_{\bar{a}} e^{i\theta_{23}})\overline{\Phi}^0_a\right] ,$$

for $k = 1, \ldots, 4$, where $h_4 = G^0$ and the *invariant* quantities q_{kj} are given by:

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}
4	i	0

Since $\widehat{w}_a e^{-i\theta_{23}}$ is a *proper* U(2)-vector, we see that the mass-eigenstate fields are indeed U(2)-invariant fields. We can now invert the above result to obtain:

$$\Phi_a = \begin{pmatrix} G^+\widehat{v}_a + H^+\widehat{w}_a \\ \dfrac{v}{\sqrt{2}}\widehat{v}_a + \dfrac{1}{\sqrt{2}}\displaystyle\sum_{k=1}^{4}\left(q_{k1}\widehat{v}_a + q_{k2}e^{-i\theta_{23}}\widehat{w}_a\right) h_k \end{pmatrix} .$$

If $\mathrm{Im}\,(Z_5^* Z_6^2) = 0$, then the neutral scalar squared-mass matrix can be transformed into block diagonal form, which contains the squared-mass of a CP-odd neutral mass-eigenstate Higgs field A and a 2×2 sub-matrix that yields the squared-masses of two CP-even neutral mass-eigenstate Higgs fields h and H.

If $\mathrm{Im}\,(Z_5^* Z_6^2) \neq 0$, we can write $Z_6 \equiv |Z_6| e^{i\theta_6}$. Then the neutral scalar mass-eigenstates do not possess definite CP quantum numbers, and the three invariant mixing angles θ_{12}, θ_{13} and $\phi_6 \equiv \theta_6 - \theta_{23}$ are non-trivial.

The angles θ_{13} and ϕ_6 are determined modulo π from

$$\tan\theta_{13} = \frac{\mathrm{Im}(Z_5\, e^{-2i\theta_{23}})}{2\,\mathrm{Re}(Z_6\, e^{-i\theta_{23}})}\,, \qquad \tan 2\theta_{13} = \frac{2\,\mathrm{Im}(Z_6\, e^{-i\theta_{23}})}{Z_1 - A^2/v^2}\,,$$

where $A^2 \equiv Y_2 + \frac{1}{2}[Z_3 + Z_4 - \mathrm{Re}(Z_5 e^{-2i\theta_{23}})]v^2$. These equations exhibit multiple solutions (modulo π) corresponding to different orderings of the h_k masses. Finally,

$$\tan 2\theta_{12} = \frac{2\cos 2\theta_{13}\,\mathrm{Re}(Z_6\, e^{-i\theta_{23}})}{c_{13}\,[c_{13}^2(A^2/v^2 - Z_1) + \cos 2\theta_{13}\,\mathrm{Re}(Z_5\, e^{-2i\theta_{23}})]}\,.$$

For a given solution of θ_{13} and ϕ_6, the two solutions for θ_{12} (modulo π) correspond to the two possible relative mass orderings of h_1 and h_2.

It is now a simple matter to insert the U(2)-covariant expression for Φ_a in terms of the mass-eigenstate Higgs fields into the Higgs Lagrangian to obtain a U(2)-covariant form for the physical Higgs boson and Goldstone boson interactions. [Note: the Goldstone boson and neutral Higgs fields are invariant fields, whereas $H^\pm \to (\det U)^{\pm 1} H^\pm$.]

The gauge boson–Higgs boson interactions are governed by the following interaction Lagrangians:

$$\begin{aligned}\mathscr{L}_{VVH} &= \left(g m_W W_\mu^+ W^{\mu\,-} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu\right) \mathrm{Re}(q_{k1}) h_k + e m_W A^\mu (W_\mu^+ G^- + W_\mu^- G^+) \\ &\quad - g m_Z s_W^2 Z^\mu (W_\mu^+ G^- + W_\mu^- G^+)\,,\end{aligned}$$

$$\begin{aligned}\mathscr{L}_{VVHH} &= \left[\tfrac{1}{4} g^2 W_\mu^+ W^{\mu\,-} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu\right] \mathrm{Re}(q_{j1}^* q_{k1} + q_{j2}^* q_{k2})\, h_j h_k \\ &\quad + \left[e^2 A_\mu A^\mu + \frac{g^2}{c_W^2}\left(\tfrac{1}{2} - s_W^2\right)^2 Z_\mu Z^\mu + \frac{2ge}{c_W}\left(\tfrac{1}{2} - s_W^2\right) A_\mu Z^\mu\right] (G^+ G^- + H^+ H^-) \\ &\quad + \left\{\left(\tfrac{1}{2} e g A^\mu W_\mu^+ - \frac{g^2 s_W^2}{2c_W} Z^\mu W_\mu^+\right) (q_{k1} G^- + q_{k2}\, e^{-i\theta_{23}} H^-) h_k + \text{h.c.}\right\},\end{aligned}$$

$$\begin{aligned}\mathscr{L}_{VHH} &= \frac{g}{4c_W} \mathrm{Im}(q_{j1} q_{k1}^* + q_{j2} q_{k2}^*) Z^\mu h_j \overleftrightarrow{\partial}_\mu h_k - \tfrac{1}{2} g \left\{ i W_\mu^+ \left[q_{k1} G^- \overleftrightarrow{\partial}^\mu h_k + q_{k2} e^{-i\theta_{23}} H^- \overleftrightarrow{\partial}^\mu h_k\right] + \text{h.c.}\right\} \\ &\quad + \left[ie A^\mu + \frac{ig}{c_W}\left(\tfrac{1}{2} - s_W^2\right) Z^\mu\right] (G^+ \overleftrightarrow{\partial}_\mu G^- + H^+ \overleftrightarrow{\partial}_\mu H^-)\,.\end{aligned}$$

Likewise, the cubic and quartic Higgs couplings are given by (with $h_4 = G^0$):

$$\mathscr{L}_{3h} = -\tfrac{1}{2}v\, h_j h_k h_\ell \Big[q_{j1}q^*_{k1}\mathrm{Re}(q_{\ell 1})Z_1 + q_{j2}q^*_{k2}\,\mathrm{Re}(q_{\ell 1})(Z_3+Z_4) + \mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}Z_5\, e^{-2i\theta_{23}}) + \mathrm{Re}\Big([2q_{j1}+q^*_{j1}]q^*_{k1}q_{\ell 2}Z_6\, e^{-i\theta_{23}}\Big) + \mathrm{Re}(q^*_{j2}q_{k2}q_{\ell 2}Z_7\, e^{-i\theta_{23}})\Big]$$
$$-v\, h_k G^+G^-\Big[\mathrm{Re}(q_{k1})Z_1 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_6)\Big] + v\, h_k H^+H^-\Big[\mathrm{Re}(q_{k1})Z_3 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}}Z_7)\Big]$$
$$-\tfrac{1}{2}v\, h_k \Big\{ G^-H^+\, e^{i\theta_{23}}\Big[q^*_{k2}Z_4 + q_{k2}\, e^{-2i\theta_{23}}Z_5 + 2\mathrm{Re}(q_{k1})Z_6\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\}\,,$$

and

$$\mathscr{L}_{4h} = -\tfrac{1}{8}h_j h_k h_l h_m \Big[q_{j1}q_{k1}q^*_{\ell 1}q^*_{m1}Z_1 + q_{j2}q_{k2}q^*_{\ell 2}q^*_{m2}Z_2 + 2q_{j1}q^*_{k1}q_{\ell 2}q^*_{m2}(Z_3+Z_4)$$
$$+2\mathrm{Re}(q^*_{j1}q^*_{k1}q_{\ell 2}q_{m2}Z_5\, e^{-2i\theta_{23}}) + 4\mathrm{Re}(q_{j1}q^*_{k1}q^*_{\ell 1}q_{m2}Z_6\, e^{-i\theta_{23}}) + 4\mathrm{Re}(q^*_{j1}q_{k2}q_{\ell 2}q^*_{m2}Z_7\, e^{-i\theta_{23}})\Big]$$
$$-\tfrac{1}{2}h_j h_k G^+G^-\Big[q_{j1}q^*_{k1}Z_1 + q_{j2}q^*_{k2}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_6\, e^{-i\theta_{23}})\Big]$$
$$-\tfrac{1}{2}h_j h_k H^+H^-\Big[q_{j2}q^*_{k2}Z_2 + q_{j1}q^*_{k1}Z_3 + 2\mathrm{Re}(q_{j1}q_{k2}Z_7\, e^{-i\theta_{23}})\Big]$$
$$-\tfrac{1}{2}h_j h_k\Big\{G^-H^+\, e^{i\theta_{23}}\Big[q_{j1}q^*_{k2}Z_4 + q^*_{j1}q_{k2}Z_5\, e^{-2i\theta_{23}} + q_{j1}q^*_{k1}Z_6\, e^{-i\theta_{23}} + q_{j2}q^*_{k2}Z_7\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\}$$
$$-\tfrac{1}{2}Z_1G^+G^-G^+G^- - \tfrac{1}{2}Z_2H^+H^-H^+H^- - (Z_3+Z_4)G^+G^-H^+H^-$$
$$-\tfrac{1}{2}(Z_5H^+H^+G^-G^- + Z^*_5H^-H^-G^+G^+) - G^+G^-(Z_6H^+G^- + Z^*_6H^-G^+) - H^+H^-(Z_7H^+G^- + Z^*_7H^-G^+)\,.$$

Example: Higgs self-couplings

Lightest neutral Higgs boson cubic self-coupling:

$$g(h_1h_1h_1) = -3v\{Z_1c_{12}^3c_{13}^3 + (Z_3+Z_4)c_{12}c_{13}|s_{123}|^2 + c_{12}c_{13}\,\mathrm{Re}(s_{123}^2 Z_5 e^{2i\theta_{23}})$$
$$-3c_{12}^2c_{13}^2\,\mathrm{Re}(s_{123}Z_6e^{i\theta_{23}}) - |s_{123}|^2\,\mathrm{Re}(s_{123}Z_7e^{i\theta_{23}})\}$$

Lightest neutral Higgs boson quartic self-coupling:

$$g(h_1h_1h_1h_1) = -3\{Z_1c_{12}^4c_{13}^4 + Z_2|s_{123}|^4 + 2(Z_3+Z_4)c_{12}^2c_{13}^2|s_{123}|^2$$
$$+2c_{12}^2c_{13}^2\,\mathrm{Re}(s_{123}^2Z_5e^{2i\theta_{23}}) - 4c_{12}^3c_{13}^3\,\mathrm{Re}(s_{123}Z_6e^{i\theta_{23}})$$
$$-4c_{12}c_{13}|s_{123}|^2\,\mathrm{Re}(s_{123}Z_7e^{i\theta_{23}})\}$$

where $s_{123} \equiv s_{12} + ic_{12}s_{13}$.

Note that these quantities depend on U(2)-invariants. In particular $Z_5e^{-2i\theta_{23}}$, $Z_6e^{-i\theta_{23}}$ and $Z_7e^{-i\theta_{23}}$ are U(2)-invariants!

The Higgs-fermion Yukawa couplings

In the generic basis, the Higgs-fermion Yukawa Lagrangian is:

$$-\mathscr{L}_Y = \overline{Q^0_L}\widetilde{\Phi}_1\eta_1^{U,0}U^0_R + \overline{Q}^0_L\Phi_1(\eta_1^{D,0})^\dagger D^0_R + \overline{Q^0_L}\widetilde{\Phi}_2\eta_2^{U,0}U^0_R + \overline{Q^0_L}\Phi_2(\eta_2^{D,0})^\dagger D^0_R + \text{h.c.}\,,$$

where $\widetilde{\Phi}_i \equiv i\sigma_2\Phi^*_i$, Q^0_L is the weak isospin quark doublet and U^0_R, D^0_R are weak isospin quark singlets in an interaction eigenstate basis, and $\eta_1^{U,0}, \eta_2^{U,0}, \eta_1^{D,0}, \eta_2^{D,0}$ are 3×3 matrices in quark flavor space.

Identify the fermion mass eigenstates by employing the appropriate bi-unitary transformation of the quark mass matrices involving unitary matrices V^U_L, V^D_L, V^U_R, V^D_R, where $K \equiv V^U_L V^{D\,\dagger}_L$ is the CKM matrix. Then, define the U(2)-vector $\eta^Q \equiv (\eta^Q_1, \eta^Q_2)$, where

$$\eta^U_a \equiv V^U_L\,\eta^{U,0}_a\,V^{U\,\dagger}_R\,, \qquad\qquad \eta^D_a \equiv V^D_R\,\eta^{D,0}_a\,V^{D\,\dagger}_L\,.$$

In terms of the quark mass-eigenstate fields and the transformed couplings,

$$-\mathscr{L}_{\rm Y} = \overline{Q}_L\widetilde{\Phi}_{\bar a}\eta^U_a U_R + \overline{Q}_L\Phi_a\eta^{D\,\dagger}_{\bar a} D_R + \text{h.c.}$$

We can construct basis-independent couplings by transforming to the Higgs basis.

$$-\mathscr{L}_{\rm Y} = \overline{Q_L}(\widetilde{H}_1\kappa^U + \widetilde{H}_2\rho^U)U_R + \overline{Q}_L(H_1\kappa^{D\,\dagger} + H_2\rho^{D\,\dagger})D_R + {\rm h.c.}\,,$$

where

$$\kappa^Q \equiv \widehat{v}^*_{\bar{a}}\eta^Q_a\,, \qquad\qquad \rho^Q \equiv \widehat{w}^*_{\bar{a}}\eta^Q_a\,.$$

Inverting these equations yields: $\eta^Q_a = \kappa^Q\widehat{v}_a + \rho^Q\widehat{w}_a$. Under a U(2) transformation, κ^Q is invariant, whereas $\rho^Q \to (\det\, U)\rho^Q$.

By construction, κ^U and κ^D are proportional to the (real non-negative) diagonal quark mass matrices M_U and M_D, respectively. In particular,

$$M_U = \frac{v}{\sqrt{2}}\kappa^U = {\rm diag}(m_u\,,\, m_c\,,\, m_t) = V^U_L M^0_U V^{U\,\dagger}_R\,,$$
$$M_D = \frac{v}{\sqrt{2}}\kappa^{D\,\dagger} = {\rm diag}(m_d\,,\, m_s\,,\, m_b) = V^D_L M^0_D V^{D\,\dagger}_R\,,$$

where $M^0_U \equiv (v/\sqrt{2})\widehat{v}^*_{\bar{a}}\,\eta^{U,0}_a$ and $M^0_D \equiv (v/\sqrt{2})\widehat{v}_a\,\eta^{D,0\,\dagger}_{\bar{a}}$. That is, we have chosen the unitary matrices V^U_L, V^U_R, V^D_L and V^D_R such that M_D and M_U are diagonal matrices with real non-negative entries. In contrast, the ρ^Q are independent complex 3×3 matrices.

The final form for the Yukawa couplings of the mass-eigenstate Higgs bosons and the Goldstone bosons to the quarks is:

$$
\begin{aligned}
-\mathscr{L}_Y = & \frac{1}{v}\overline{D}\Big\{ M_D(q_{k1}P_R + q_{k1}^* P_L) + \frac{v}{\sqrt{2}}\left[q_{k2}\,[e^{i\theta_{23}}\rho^D]^\dagger P_R + q_{k2}^*\, e^{i\theta_{23}}\rho^D P_L\right]\Big\} D h_k \\
& + \frac{1}{v}\overline{U}\Big\{ M_U(q_{k1}P_L + q_{k1}^* P_R) + \frac{v}{\sqrt{2}}\left[q_{k2}^*\, e^{i\theta_{23}}\rho^U P_R + q_{k2}\,[e^{i\theta_{23}}\rho^U]^\dagger P_L\right]\Big\} U h_k \\
& + \Big\{ \overline{U}\left[K[\rho^D]^\dagger P_R - [\rho^U]^\dagger K P_L\right] D H^+ + \frac{\sqrt{2}}{v}\overline{U}\,[K M_D P_R - M_U K P_L]\, D G^+ + \text{h.c.}\Big\}\,.
\end{aligned}
$$

By writing $[\rho^Q]^\dagger H^+ = [\rho^Q e^{i\theta_{23}}]^\dagger [e^{i\theta_{23}} H^+]$, we see that the Higgs-fermion Yukawa couplings depend only on invariant quantities: the diagonal quark mass matrices, $\rho^Q e^{i\theta_{23}}$, and the invariant angles θ_{12} and θ_{13}.

The couplings of the neutral Higgs bosons to quark pairs are generically CP-violating as a result of the complexity of the q_{k2} and the fact that the matrices $e^{i\theta_{23}}\rho^Q$ are not generally hermitian or anti-hermitian. $\mathscr{L}_{\rm Y}$ also exhibits Higgs-mediated flavor-changing neutral currents (FCNCs) at tree-level by virtue of the fact that the ρ^Q are not flavor-diagonal. Thus, for a phenomenologically acceptable theory, the off-diagonal elements of ρ^Q must be small.

Conditions for CP-invariance

The general 2HDM is CP-violating. The requirement of a CP-conserving bosonic sector is equivalent to the requirement that the scalar potential is explicitly CP-conserving and that the Higgs vacuum is CP-invariant. The bosonic sector is CP-conserving if and only if:*

$$\mathrm{Im}[Z_6 Z_7^*] = \mathrm{Im}[Z_5^* Z_6^2] = \mathrm{Im}[Z_5^*(Z_6+Z_7)^2] = 0\,.$$

Note that $\mathrm{Im}[Z_5^* Z_6^2]=0$ implies that there is no CP-even/CP-odd scalar mixing in the diagonalization of the neutral scalar squared-mass matrix. Nevertheless, this is not a sufficient condition for CP-conserving Higgs couplings.

Additional constraints arise when the Higgs-fermion couplings are included. If $Z_5[\rho^Q]^2$, $Z_6\rho^Q$ and $Z_7\rho^Q$ $(Q=U,D,E)$ are hermitian matrices, then the couplings of the neutral Higgs bosons to fermion pairs are CP-invariant. Thus, if all the above conditions are satisfied, then the neutral Higgs bosons are eigenstates of CP, and the only possible source of CP-violation in the 2HDM is the unremovable phase in the CKM matrix K that enters via the charged current interactions mediated by either $W^\pm$ or $H^\pm$ exchange.

*Since one of the scalar potential minimum conditions yields $Y_3=-\frac{1}{2}Z_6 v^2$, no separate condition involving Y_3 is required.

The significance of $\tan\beta$

So far, $\tan\beta$ has been completely absent from the Higgs couplings. This must be so, since $\tan\beta$ is basis-dependent in a general 2HDM. However, a particular 2HDM may single out a preferred basis, in which case $\tan\beta$ would be promoted to an observable. To simplify the discussion, we focus on a one-generation model, where the Yukawa coupling matrices are simply numbers.

As an example, the MSSM Higgs sector is a type-II 2HDM, *i.e.*, $\eta_1^U = \eta_2^D = 0$. A basis-independent condition for type-II is: $\eta_{\bar{a}}^{D\,*}\eta_a^U = 0$. In the preferred basis, $\hat{v} = (\cos\beta\,,\,\sin\beta\,e^{i\xi})$ and $\hat{w} = (-\sin\beta e^{-i\xi}\,,\,\cos\beta)$. Evaluating $\kappa^Q = \hat{v}^* \cdot \eta^Q$ and $\rho^Q = \hat{w}^* \cdot \eta^Q$ in the preferred basis, it follows that:

$$e^{-i\xi}\tan\beta = -\frac{\rho^{D\,*}}{\kappa^D} = \frac{\kappa^U}{\rho^U}\,,$$

where $\kappa^Q = \sqrt{2}m_Q/v$. These two definitions are consistent if $\kappa^D\kappa^U + \rho^{D\,*}\rho^U = 0$ is satisfied. But this is equivalent to the type-II condition, $\eta_{\bar{a}}^{D\,*}\eta_a^U = 0$.

Since ρ^Q is a pseudo-invariant, we can eliminate ξ by rephasing Φ_2. Hence,

$$\tan\beta = \frac{|\rho^D|}{\kappa^D} = \frac{\kappa^U}{|\rho^U|}\,,$$

with $0 \leq \beta \leq \pi/2$. Indeed, $\tan\beta$ is now a physical parameter, and the $|\rho^Q|$ are no longer independent:

$$|\rho^D| = \frac{\sqrt{2} m_d \tan\beta}{v}\,, \qquad |\rho^U| = \frac{\sqrt{2} m_u \cot\beta}{v}\,.$$

In the more general (type-III) 2HDM, $\tan\beta$ is not a meaningful parameter. Nevertheless, one can introduce three $\tan\beta$-like parameters:†

$$\tan\beta_d \equiv \frac{|\rho^D|}{\kappa^D}\,, \qquad \tan\beta_u \equiv \frac{\kappa^U}{|\rho^U|}\,, \qquad \tan\beta_e \equiv \frac{|\rho^E|}{\kappa^E}\,,$$

the last one corresponding to the Higgs-lepton interaction. In a type-III 2HDM, there is no reason for the three parameters above to coincide.

† Interpretation: In the Higgs basis, up and down-type quarks interact with both Higgs doublets. But, clearly there exists some basis ($i.e.$, a rotation by angle β_u from the Higgs basis) for which only one of the two up-type quark Yukawa couplings is non-vanishing. This defines the physical angle β_u.

The MSSM Higgs sector is a type-III 2HDM

The tree-level Higgs potential of the MSSM satisfies:

$$\lambda_1 = \lambda_2 = -\lambda_{345} = \tfrac{1}{4}(g^2 + g'^2)\,,\, \lambda_4 = -\tfrac{1}{2}g^2\,,\, \lambda_5 = \lambda_6 = \lambda_7 = 0\,.$$

But, one-loop radiative corrections generate corrections to these relations, due to SUSY-breaking. *E.g.*, at one-loop, λ_5, λ_6, $\lambda_7 \neq 0$.

For MSSM Higgs couplings to fermions, Yukawa vertex corrections modify the effective Lagrangian that describes the coupling of the Higgs bosons to the third generation quarks:

$$-\mathcal{L}_{\rm eff} = \epsilon_{ij}\left[(h_b + \delta h_b)\bar{b}_R H_d^i Q_L^j + (h_t + \delta h_t)\bar{t}_R Q_L^i H_u^j\right] + \Delta h_b \bar{b}_R Q_L^k H_u^{k*} + \Delta h_t \bar{t}_R Q_L^k H_d^{k*} + {\rm h.c.}$$

Indeed, this is a general type-III model. For example, in some MSSM parameter regimes (corresponding to large $\tan\beta$ and large supersymmetry-breaking scale compared to v),[‡]

$$\Delta h_b \simeq h_b\left[\frac{2\alpha_s}{3\pi}\mu M_{\tilde{g}}\, I(M^2_{\tilde{b}_1}, M^2_{\tilde{b}_2}, M^2_{\tilde{g}}) + \frac{h_t^2}{16\pi^2}\mu A_t\, I(M^2_{\tilde{t}_1}, M^2_{\tilde{t}_2}, \mu^2)\right]\,.$$

This leads to a modification of the tree-level relation between m_b and h_b. In addition, it leads to a splitting of "effective" $\tan\beta$-like parameters $\tan\beta_b$ and $\tan\beta_t$.

[‡] $I(a,b,c) = [ab\ln(a/b) + bc\ln(b/c) + ca\ln(c/a)]/(a-b)(b-c)(a-c)$.

For illustrative purposes, we neglect CP violation in the following simplified discussion. The tree-level relation between m_b and h_b is modified:

$$m_b = \frac{h_b v}{\sqrt{2}} \cos\beta(1 + \Delta_b) \,,$$

where $\Delta_b \equiv (\Delta h_b/h_b)\tan\beta$. That is, Δ_b is $\tan\beta$-enhanced, and governs the leading one-loop correction to the physical Higgs couplings to third generation quarks. In typical models at large $\tan\beta$, Δ_b can be of order 0.1 or larger and of either sign.

In the approximation scheme above (keeping only the $\tan\beta$-enhanced terms),

$$\tan\beta_b \equiv \frac{v\rho^D}{\sqrt{2}\, m_b} \simeq \frac{\tan\beta}{1+\Delta_b} \,, \qquad \tan\beta_t \equiv \frac{\sqrt{2}\, m_t}{v\rho^U} \simeq \frac{\tan\beta}{1-\tan\beta\,(\Delta h_t/h_t)} \,.$$

Thus, supersymmetry-breaking loop-effects can yield observable differences between $\tan\beta$-like parameters that are defined in terms of basis-independent quantities. In particular, the leading one-loop $\tan\beta$-enhanced corrections are automatically incorporated into:

$$g_{Ab\bar{b}} = \frac{m_b}{v}\tan\beta_b \,, \qquad g_{At\bar{t}} = \frac{m_t}{v}\cot\beta_t \,.$$

The decoupling limit

The decoupling limit corresponds to the limiting case in which one of the two Higgs doublets of the 2HDM receives a very large mass and is therefore decoupled from the theory. This can be achieved by assuming that $Y_2 \gg v^2$ and $|Z_i| \lesssim \mathcal{O}(1)$ [for all i]. The effective low energy theory is a one-Higgs-doublet model that corresponds to the Higgs sector of the Standard Model. We shall order the neutral scalar masses according to $m_1 \ll m_{2,3}$ and define the invariant Higgs mixing angles accordingly. Thus, we expect one light CP-even Higgs boson, h_1, with couplings identical (up to small corrections) to those of the Standard Model (SM) Higgs boson. In particular,[§]

$$m_1^2 = Z_1 v^2 + \mathcal{O}\left(\frac{|Z_6|v^2}{m_3^2}\right)\,, \qquad m_3^2 = A^2 + \mathcal{O}\left(\frac{|Z_6|v^2}{m_3^2}\right)\,,$$

$$m_2^2 = A^2 + v^2\,\mathrm{Re}(Z_5 e^{-2i\theta_{23}}) + \mathcal{O}\left(\frac{|Z_6|v^2}{m_3^2}\right)\,, \qquad m_{H^\pm}^2 = Y_2 + \tfrac{1}{2}Z_3 v^2\,.$$

Hence, $m_1 \ll m_2 \simeq m_3 \simeq m_{H^\pm}$.

[§]Recall that: $A^2 \equiv Y_2 + \frac{1}{2}[Z_3 + Z_4 - \mathrm{Re}(Z_5 e^{-2i\theta_{23}})]v^2$.

Finally, the invariant mixing angles are given by:

$$s_{12} = \frac{v^2 \mathrm{Re}(Z_6\, e^{-i\theta_{23}})}{m_2^2 - m_1^2} \left[1 + \mathcal{O}\left(\frac{|Z_6|^2 v^4}{m_3^4}\right)\right] \ll 1\,,$$

$$s_{13} = \frac{-v^2 \mathrm{Im}(Z_6\, e^{-i\theta_{23}})}{m_3^2 - m_1^2} \left[1 + \mathcal{O}\left(\frac{|Z_6|^2 v^4}{m_3^4}\right)\right] \ll 1\,,$$

$$\mathrm{Im}(Z_5\, e^{-2i\theta_{23}}) = \frac{-v^2\, \mathrm{Im}[(Z_6\, e^{-i\theta_{23}})^2]}{m_3^2 - m_1^2} \left[1 + \mathcal{O}\left(\frac{|Z_6|^2 v^4}{m_3^4}\right)\right] \ll 1\,.$$

In the *exact* decoupling limit, these quantities are all zero. However, the identity:¶

$$\mathrm{Im}(Z_5^* Z_6^2) = 2\,\mathrm{Re}(Z_5 e^{-2i\theta_{23}})\,\mathrm{Re}(Z_6\, e^{-i\theta_{23}})\,\mathrm{Im}(Z_6\, e^{-i\theta_{23}}) - \mathrm{Im}(Z_5 e^{-2i\theta_{23}}) \left\{[\mathrm{Re}(Z_6\, e^{-i\theta_{23}})]^2 - [\mathrm{Im}(Z_6\, e^{-i\theta_{23}})]^2\right\}\,.$$

implies that $\mathrm{Im}(Z_5^* Z_6^2)$ need not be particularly small in the decoupling limit. Therefore in the decoupling limit, the properties of h_1 approach those of the neutral CP-even Standard Model Higgs boson. In contrast, h_2 and h_3 are states of indefinite CP (*i.e.*, strongly-mixed linear combinations of H and A).

¶Another identity, $\mathrm{Im}(Z_5^* Z_6^2)\, v^6 = 2 s_{13} c_{13}^2 s_{12} c_{12}\, (m_2^2 - m_1^2)(m_3^2 - m_1^2)(m_3^2 - m_2^2)$, yields the same conclusion.

Lessons and future work

• If phenomena consistent with the 2HDM are found, we will not know a priori the underlying structure that governs the model. In this case, one needs a model-independent analysis of the data that allows for the most general CP-violating Model-III.

• Instead of claiming that you have measured $\tan\beta$ (which can only be done in the context of a simplified version of the model), measure the physical parameters of the model. Examples include the $\tan\beta$-like parameters introduced in the one-generation model. (For three generations, the formalism becomes more complicated. However, one has good reason to assume that the third generation quark–Higgs Yukawa couplings dominate.)

• Which $\tan\beta$-like parameters will be measured in precision Higgs studies at the ILC? How can one best treat the full three-generation model to one-loop order? What simplifications can be exploited in the MSSM?

• Compute the one-loop radiative corrections to various Higgs processes in terms of the physical $\tan\beta$-like parameters in the MSSM.