Recent development of quasiclassical operator method

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The quasiclassical operator method is described. Application of the method for series of problems are discussed. These are processes in the superposition of plane wave and constant field, radiation in linear colliders, radiation in inhomogeneous field, theory of radiation in oriented crystals, radiation spectra taking into account energy loss.

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Introduction

There are two types of quantum effects at the radiation of high-energy particle in external field. The first one is associated with with the quantization of the motion of the particle in the field. For example, the commutator of velocity components of the relativistic particles in a magnetic field **H** (where energy levels is $\varepsilon = \sqrt{m^2 + 2eH\hbar n} \gg m$) is

$$[v_i, v_k] = \frac{ie\hbar}{\varepsilon^2}\varepsilon_{ikj}H_j, \tag{1}$$

and the uncertainty relation for velocity components reads

$$\Delta v_i \Delta v_k \sim \frac{e\hbar H}{\varepsilon^2} = \frac{H}{H_0\gamma^2} = \frac{\hbar\omega_0}{\varepsilon} \simeq \frac{1}{2n}, \quad H_0 = \frac{m^2}{e\hbar} = \left(\frac{m^2c^3}{\hbar e}\right) = 4.41 \cdot 10^{13}\mathrm{Oe} \tag{2}$$

where we use units where $c = 1$, $\omega_0 = eH/\varepsilon$ is the Larmor frequency, $\gamma = \varepsilon/m$, so that with the energy rise the motion becomes increasingly classical.

The second type of quantum effect is associated with the recoil of the particle when it radiates and is of the order $\hbar\omega/\varepsilon$. Already in the classical limit ($\hbar\omega \ll \varepsilon$) this type is principal since $\omega \sim \omega_0\gamma^3$

The first order matrix element of the photon emission by a charged particle in the external field may be represented in the form

$$U_{fi} = \frac{ie}{2\pi\sqrt{\hbar\omega}} \int dt \int d^3r F^{+}_{fs'}(\mathbf{r}) \exp(i\varepsilon_f t/\hbar)$$
$$(e^* J) \exp[i(\omega t - \mathbf{kr})] \exp(-i\varepsilon_i t/\hbar) F_{is}(\mathbf{r}), \tag{3}$$

where $F_{is}(\mathbf{r})$ is the solution of the wave equation in the given field with the energy ε_i and in the spin state s, e^μ is the photon polarization vector, $k^\mu(\omega, \mathbf{k})$ is the photon 4-momentum, J^μ is the current vector.

For the states with $n \gg 1$ the following approximation may be made

$$\exp(-i\varepsilon_i t/\hbar) F_{is}(\mathbf{r}) = \Psi_s(\mathbf{P}) \exp(-i\mathcal{H}t/\hbar)|i>, \quad P^\mu = i\hbar\partial^\mu - eA^\mu, \tag{4}$$

where $\Psi_s(\mathbf{P})$ is the operator form of the particle wave function in the spin state s in the given field. This form may be obtained from the free wave function via substitution of the variables by the operators: $\mathbf{p} \to \mathbf{P},\ \varepsilon \to \mathcal{H} = \sqrt{\mathbf{P}^2 + m^2}$. In the coordinate representation $|i>$ is the solution of the Klein-Gordon equation in the given field. Substituting Eq.(4) into Eq.(3) and taking into account that the Schrödinger operators, standing between the exponential factors $\exp(\pm i\mathcal{H}t/\hbar)$, convert into the explicitly time-dependent Heisenberg operators of the dynamic variables of the particle in the given field, we obtain the following formula for the matrix element Eq.(3):

$$U_{fi} = \langle f|M|i\rangle\ , \tag{5}$$

where

$$M = \frac{ie}{2\pi\sqrt{\hbar\omega}} \int dt \Psi^+_{s'}(\mathbf{p})\{(e^* J), \exp[i(\omega t - \mathbf{kr}(\mathbf{t}))]\}\Psi_s(\mathbf{p}); \tag{6}$$

here $\mathbf{p}(t), J^\mu(t), \mathbf{r}(t)$ are the Heisenberg operators of the particle momentum, current and coordinates respectively, the brackets {,} denote the symmetrized product of operators. Note, that $\Psi_s(\mathbf{p})\exp(-i\mathcal{H}t/\hbar)\ |i>$ is the operator solution of the wave equation.

We are interested in the transition probability with photon emission summed over final particle state. Using the condition of the completeness of the states $\sum_f |f><f| = 1$ we obtain for

the radiation probability

$$dw_\gamma =< i|M^+M|i > d^3k \tag{7}$$

The next step of calculation is series of transformation of operators ("disentanglement") in Eqs.(7), (6). As a result one obtains

$$M = \frac{e}{2\pi\sqrt{\hbar\omega}}\int_{-\infty}^{\infty} R(t)\exp\left[i\frac{kx(t)}{\varepsilon-\hbar\omega}\right]dt = \frac{e}{2\pi\sqrt{\hbar\omega}}\int_{-\infty}^{\infty} R(t)\exp\left[i\int_0^t \frac{kp(t')}{\varepsilon-\hbar\omega}dt'\right]dt, \tag{8}$$

here $R(t) = R(\mathbf{p}(t))$, $R(\mathbf{p})$ is the matrix element for the free particles depending on the particle spin, $kp = \omega\mathcal{H} - \mathbf{kp}$, $|i >$ is the state vector of the initial particle at the time $t = 0$, and $\mathbf{p}(t)$ is the operator of momentum in the Heisenberg picture.

The very important result of the method is that the recoil at radiation is incorporated into the theory in the *universal* form for any external field. There are two essentially different cases in application of the quasiclassical operator (QO) method. In the first case moving and scattering in a potential can be considered in *classical* terms: phase shifts are large, there is a correspondence between the impact parameter and the momentum transfer. Therefore it is possible to use the version of the QO method where one can substitute classical variables instead of operators in the expression for the process probability (see (BKp67),(BK68), (BKF72), (BLP82)). In the second case, the process of scattering can't be described in classical terms. However, for the process where large angular momentum contributes, one can use the *quasiclassical* approximation for description of scattering including the situation where the phase shifts are small . For example, this formulation of the method must be applied for consideration of radiation from ultrarelativistic particles at scattering on atoms in a media.

For the particle with spin $1/2$ the function $R(t)$ in Eq.(8) written in the two-component form is (below we employ units such that $\hbar = 1$)

$$R(t) = \varphi_{s'}^{+}(A + i\boldsymbol{\sigma}\mathbf{B})\varphi_s, \quad A = \frac{1}{2}\left(1 + \frac{\varepsilon}{\varepsilon'}\right)\mathbf{ev}, \quad \mathbf{B} = \frac{\omega}{2\varepsilon'}\mathbf{e}\times\mathbf{b}, \quad \mathbf{b} = \mathbf{n} - \mathbf{v} + \frac{\mathbf{n}}{\gamma}, \quad (9)$$

where $\mathbf{v} = \mathbf{v}(t)$ is the particle velocity, $\varepsilon' = \varepsilon - \omega, \ \gamma = \varepsilon/m$.

Macroscopic case

Substituting Eq.(8) into Eq.(7) we get $(t = (t_1 + t_2)/2,\ \tau = t_2 - t_1)$

$$dW \equiv \frac{dw_\gamma}{dt} = \frac{\alpha}{(2\pi)^2}\frac{d^3k}{\omega}\int R^*\left(t+\frac{\tau}{2}\right)R\left(t-\frac{\tau}{2}\right)$$
$$\times \exp\left\{-i\frac{\varepsilon}{\varepsilon'}\left[kx\left(t+\frac{\tau}{2}\right) - kx\left(t-\frac{\tau}{2}\right)\right]\right\}d\tau. \qquad (10)$$

In a magnetic field, as in the classical theory, the radiation takes place from a small part of the trajectory on which the particle turns on an angle $\sim 1/\gamma$. This means that one can expand the integrand in Eq.(10) on τ powers. One obtains e.g. for the spectral distribution of the radiation intensity

$$\frac{dI}{du} = \frac{\alpha m^2}{\pi\sqrt{3}}\frac{u}{(1+u)^4}\left[(1+(1+u)^2)K_{2/3}(\lambda) - (1+u)\int_\lambda^\infty K_{1/3}(z)dz\right], \qquad (11)$$

where $K_\nu(z)$ is the MacDonald function,

$$u = \frac{\omega}{\varepsilon'}, \quad \lambda = \frac{2u}{3\chi}, \quad \chi = \frac{H}{H_0}\frac{\varepsilon}{m}, \tag{12}$$

here χ is the fundamental parameter. In the limit $\chi \ll 1$ one has classical theory.

Microscopic case

We consider the applicability of the QO method to the problem of radiation from ultrarelativistic particles at potential scattering. In this process the emitted photon and the final electron are moving at a small angle to the initial electron momentum and the large angular momenta $l \gg 1$ contribute. In this situation the quasiclassical scattering theory is applicable.

At very high energy of particles in a media, we can consider the case of complete screening, so that

$$a_s \ll \frac{1}{q_{min}} = \frac{2\varepsilon(\varepsilon-\omega)}{\omega m^2} \equiv l_f, \tag{13}$$

where a_s is the screening radius ($a_s \simeq 111 Z^{-1/3}\lambda_c, \lambda_c = 1/m$), Z is the charge of a nucleus, q_{min} is the minimal momentum transfer which is longitudinal (with respect to the momentum of the initial electron $\mathbf{p}$), l_f is the radiation formation length for a small angle scattering on an isolated atom. Note that in frame of the QO method the radiation problem is solved for the case of arbitrary screening , see (BKF72). The impact parameters ϱ, contributing into the scattering cross section, are small comparing the formation length ($\varrho \leq a_s \ll l_0$) in a screened Coulomb potential. This means that the scattering of ultrarelativistic particles (the virtual electron is close to the mass shell) takes place independently of radiation process (see, (BK69), (BKF72), (BLP82)). Thus, we can present the cross section of radiation as a product of the probability of photon emission with the momentum $\mathbf{k}$ at given momentum transfer $\mathbf{q}_\perp$ ($\mathbf{q}_\perp \mathbf{p} = 0$), and the cross section of particle scattering $d\sigma(\mathbf{q}_\perp)$ with the same momentum transfer $\mathbf{q}_\perp$:

$$d\sigma_\gamma = W_\gamma(\mathbf{q}_\perp, \mathbf{k}) d^3k d\sigma(\mathbf{q}_\perp), \tag{14}$$

where the probability of photon emission is given in Eq.(7).

We show below that in frame of the QO method the probability of radiation $W_\gamma(\mathbf{q}_\perp, \mathbf{k})$ is given by the trajectory of a particle in "the form of an angle" in the *momentum space*

$$\mathbf{p}(t) = \vartheta(-t)\mathbf{p} + \vartheta(t)(\mathbf{p} + \mathbf{q}_\perp), \tag{15}$$

while the cross section $d\sigma(\mathbf{q}_\perp)$ should be taken in the *eikonal* form.

If the formation time of radiation is much longer than the characteristic time of the scattering, one can present the dependence of the operator $\mathbf{p}(t)$ on the time in Eq.(8) as

$$\mathbf{p}(t) = \vartheta(-t)\mathbf{p}(-\infty) + \vartheta(t)\mathbf{p}(\infty),$$
$$\mathbf{p}_\perp(-\infty) \simeq \mathbf{p}_\perp + \int_{-\infty}^{z} \boldsymbol{\nabla}_\varrho V(\boldsymbol{\varrho}, z')dz', \quad \mathbf{p}_\perp(\infty) \simeq \mathbf{p}_\perp - \int_{z}^{\infty} \boldsymbol{\nabla}_\varrho V(\boldsymbol{\varrho}, z')dz'. \quad (16)$$

It should be pointed out that in the case when the scattering process is of nonclassical character the operators $\mathbf{p}(-\infty)$ and $\mathbf{p}(\infty)$ are noncommutative among themselves and, generally speaking, one can't neglect their commutator. But these approximate expressions have *the classical* form. Substituting the "trajectory" (16) in Eq.(8) we obtain

$$M = \frac{ie(\varepsilon - \omega)}{2\pi\sqrt{\omega}} \left[\frac{R(\mathbf{p}(\infty))}{kp(\infty)} - \frac{R(\mathbf{p}(-\infty))}{kp(-\infty)}\right]. \quad (17)$$

For the derivation of the differential cross section of the bremsstrahlung in Eqs.(14), (7) it is necessary to insert the projection operator $|f><f|$ between the operators M^+ and M, and to take $|i>$ and $|f>$ states so that the initial state is the eigenvector of the operator $\mathbf{p}(-\infty)$

and the final state is the eigenvector of the operator $\mathbf{p}(\infty)$:

$$\mathbf{p}(-\infty)|i>=\mathbf{p}_i|i>, \quad |i>=\exp\left[-i\int_{-\infty}^{z} V(\boldsymbol{\varrho},z')dz'\right]|\mathbf{p}_i>,$$
$$\mathbf{p}(\infty)|f>=\mathbf{p}_f|f>, \quad |f>=\exp\left[i\int_{z}^{\infty} V(\boldsymbol{\varrho},z')dz'\right]|\mathbf{p}_f> \qquad (18)$$

Using (18) we deduce for the matrix element of the operator M Eq.(17)

$$M_{fi}=\frac{ie(\varepsilon-\omega)}{2\pi\sqrt{\omega}}\left[\frac{R(\mathbf{p}_f)}{kp_f}-\frac{R(\mathbf{p}_i)}{kp_i}\right]<f|i>,$$
$$<f|i>=<\mathbf{p}_f|\exp\left[-i\int_{-\infty}^{\infty} V(\boldsymbol{\varrho},z)dz\right]|\mathbf{p}_i>$$
$$=\int d^2\varrho\exp\left[i\mathbf{q}_\perp\boldsymbol{\varrho}+i\chi(\boldsymbol{\varrho})\right]2\pi\delta(p_{f\parallel}-p_{i\parallel}), \qquad (19)$$

where

$$\mathbf{q}_\perp = \mathbf{p}_{i\perp} - \mathbf{p}_{f\perp}, \quad \chi(\boldsymbol{\varrho}) = -\int_{-\infty}^{\infty} V(\boldsymbol{\varrho}, z)dz \tag{20}$$

In the centrally symmetric potential we have

$$\chi(\varrho) = -\int_{-\infty}^{\infty} V(\sqrt{\varrho^2 + z^2})dz, \tag{21}$$

We introduce now the notations $\mathbf{p}_i \equiv \mathbf{p},\ \mathbf{p}_f = \mathbf{p}'+\mathbf{k}$ where $\mathbf{p}'$ is the momentum of electron after photon emission. Then neglecting the terms of the order of $q_{\|}$ in the argument of the δ-function in Eq.(19) we have in the region $q_\perp \gg q_{\|}$ which contributes for the potential considered

$$\delta(p_{f\|} - p_{i\|}) \simeq \delta(\varepsilon' + \omega - \varepsilon), \quad \varepsilon' = \sqrt{\mathbf{p}'^2 + m^2}. \tag{22}$$

Using this relation one can express the propagator kp_f through the propagator kp' (up to terms of the order $1/\gamma^2$)

$$kp_f = \omega\sqrt{(\mathbf{p}' + \mathbf{k})^2 + m^2} - \mathbf{k}(\mathbf{p}' + \mathbf{k})$$
$$= \omega\sqrt{\varepsilon^2 - 2kp'} + kp' - \omega\varepsilon \simeq \omega\varepsilon\left(1 - \frac{kp'}{\varepsilon^2}\right) + kp' - \omega\varepsilon = \frac{\varepsilon'}{\varepsilon}kp'. \quad (23)$$

Our final result for the differential probability of radiation in Eq.(14) is therefore

$$W_\gamma(\mathbf{q}_\perp, \mathbf{k}) = \frac{\alpha}{(2\pi)^2}\frac{1}{\omega}\left|\frac{\varepsilon R(\mathbf{p}' + \mathbf{k})}{kp'} - \frac{\varepsilon' R(\mathbf{p})}{kp}\right|^2 \quad (24)$$

Using the Moliere approximation of the atomic potential for the phase $\chi(\varrho)$ in Eq.(21) we have for the spectrum of bremsstrahlung in the case of complete screening

$$\frac{d\sigma}{d\omega} = \frac{4Z^2\alpha^3\varepsilon'}{m^2\varepsilon\omega}\left[\left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon} - \frac{2}{3}\right)\left(\ln(183Z^{-1/3}) - f(Z\alpha)\right) + \frac{1}{9}\right], \quad (25)$$

where

$$f(\xi) = \xi^2 \Sigma_1^\infty \frac{1}{n(n^2+\xi^2)}, . \tag{26}$$

here the function $f(\xi)$ is the Coulomb correction. In the Coulomb field the term $1/9$ in square brackets should be omitted and $\ln(183Z^{-1/3})$ should be substituted by $\ln(1/\delta) - 1/2$, where $\delta = q_{min}/m = \omega m/2\varepsilon\varepsilon'$.

Processes in Plane Wave and Constant Field(BKS91)

Substituting Eqs.(8),(9) into Eq.(7) and performing integration over photon emission angle and summation over the polarization of final particles we obtain for the spectral distribution of radiation the convenient form, in which all cancelations of the leading terms have already carried out:

$$\frac{dw_\gamma}{d\omega} = \frac{i\alpha}{8\pi\gamma^2}\int\int\frac{dt d\tau}{\tau - i0}\left[4 + \beta(\mathbf{\Delta}_1 - \mathbf{\Delta}_2)^2\right]\exp\left\{-\frac{i\tau}{l_\omega}\left(1 + \int_{t_1}^{t_2}\mathbf{\Delta}^2(t')dt'\right)\right\},$$

$$\beta = \left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}\right), \quad \mathbf{\Delta}_1 = \mathbf{\Delta}(t_1), \quad \mathbf{\Delta} = \mathbf{\Delta}(t) = \frac{1}{m}(\mathbf{p}(t) - \boldsymbol{\pi}),$$

$$\boldsymbol{\pi} = \frac{1}{\tau}\int_{t_1}^{t_2}\mathbf{p}(t)dt, \quad l_\omega = \frac{2\varepsilon\varepsilon'}{m^2\omega}, \tag{27}$$

where $\mathbf{p} = \varepsilon\mathbf{v}$ is the momentum of the particle. Obviously, the quantity $\mathbf{\Delta}(t)$ is unaffected by the substitution $\mathbf{p}(t) \to \mathbf{p}(t) + \mathbf{p}_0$, where $\mathbf{p}_0$ is the time-independent momentum. To pursue the analysis we need explicit expressions for the momentum $\mathbf{p}(t)$ and the vector $\mathbf{\Delta}(t)$ in the field under consideration. We will carry out calculations in the frame of reference in which the monochromatic plane wave, with the wave vector $q = q(q_0, \mathbf{q})$, is propagating in the direction $\mathbf{n} = \mathbf{q}/q_0$ opposite to the electron velocity. One can always find a relativistic frame of reference ($\gamma \gg 1$) in which the condition $q_0 \ll \varepsilon$ holds. This is necessary condition if we wish to treat the plane wave as classical. Solving the equation of motion of the particle in the electromagnetic field we get

$$\frac{\mathbf{p}_\perp}{m} = \mathbf{\Omega} t + \boldsymbol{\xi}(t), \quad \boldsymbol{\xi}(t) = \boldsymbol{\xi}_2 \sin(\nu t + \varphi_0) + \boldsymbol{\xi}_1 \cos(\nu t + \varphi_0)$$
$$\mathbf{\Omega} = \frac{e}{m}\mathbf{F}_\perp, \quad \mathbf{F}_\perp = \mathbf{E} - \mathbf{n}(\mathbf{n}\mathbf{E}) + \mathbf{H} \times \mathbf{n}, \quad \nu = 2q_0, \quad \boldsymbol{\xi}\mathbf{n} = 0. \tag{28}$$

Here **E** and **H** are the electric and magnetic fields, both independent on the time, and the orthogonal vectors $\boldsymbol{\xi}_{1,2}$ characterize the intensity $\xi_0^2 = (\xi_1^2 + \xi_2^2)/2$ and the polarization of the wave. The corresponding Stokes parameters are

$$\lambda_3 = \frac{\xi_1^2 - \xi_2^2}{\xi_1^2 + \xi_2^2}, \quad \lambda_2 = \frac{(\boldsymbol{\xi}_1 \times \boldsymbol{\xi}_2)\mathbf{n}}{\xi_0^2}. \tag{29}$$

Performing calculation we obtain the spectral probability of photon emission per unit time

$$\frac{d^2w_\gamma(t)}{dtdy} = \frac{dW_\gamma}{dy} = \frac{i\alpha m^2}{2\pi\varepsilon}\int\limits_{-\infty}^{\infty}\frac{d\tau}{\tau - i0}\left[1 + \left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}\right)m_0\right]\exp(-iu\tau\Phi(\varphi,\tau)). \qquad (30)$$

where

$$y = \frac{\omega}{\varepsilon}, \quad u = \frac{y}{1-y}, \quad m_0 = \chi^2\tau^2 + 2\boldsymbol{\chi}\boldsymbol{\eta}(\varphi)\tau\sin\frac{s\tau}{2} + \xi_0^2(1 - \lambda_3\cos 2\varphi)\sin^2\frac{s\tau}{2},$$

$$\Phi = \frac{1}{3}\chi^2\tau^2 + \frac{8}{s^2\tau}\boldsymbol{\chi}\boldsymbol{\eta}(\varphi)\left(\sin\frac{s\tau}{2} - \frac{s\tau}{2}\cos\frac{s\tau}{2}\right) +$$

$$\xi_0^2\left[1 + \frac{2}{s^2\tau^2}(\cos s\tau - 1) + \frac{\lambda_3}{s\tau}\cos 2\varphi\left(\sin s\tau + \frac{2}{s\tau}(\cos s\tau - 1)\right)\right] + 1$$

$$\boldsymbol{\eta}(\varphi) = \boldsymbol{\xi}_2\cos\varphi - \boldsymbol{\xi}_1\sin\varphi, \quad \varphi = \nu t + \varphi_0, \quad \tau \to 2l_0\tau,\ l_0 = \frac{\varepsilon}{m^2},$$

$$\boldsymbol{\Omega}l_0 = \frac{e}{m}\mathbf{F}_\perp l_0 = \boldsymbol{\chi}, \quad 2\nu l_0 = \frac{4q_0\varepsilon}{m^2} \simeq \frac{2qp}{m^2} = s \qquad (31)$$

If the plane wave is absent ($\boldsymbol{\xi}_1 = \boldsymbol{\xi}_2 = 0$) Eq.(31) turns into the spectral probability in constant field in the quasiclassical approximation. In absence of constant field ($\boldsymbol{\chi} = 0$) Eq.(31) turns into the *exact* spectral probability in the monochromatic plane wave. In the case $\xi_0 \ll 1$ one can expand the exponent in Eq.(31) over $\xi(\eta)$ and retain terms $\propto \xi_0^2$:

$$dW_\gamma = dW_\gamma^F + \xi_0^2 dW_\gamma^\xi, \tag{32}$$

where dW_γ^F is the spectral probability in constant field and dW_γ^ξ is connected with the cross section of the Compton effect

$$d\sigma_c = \frac{8\pi\alpha\varepsilon}{m^2 s} dW_\gamma^\xi. \tag{33}$$

The cross section $d\sigma_c$ describes the photon scattering on electron in the presence of an external field.

Radiation in Linear Colliders(BKS90, BK04)

The particle interaction at beam-beam collision in linear colliders occurs in an electromagnetic field provided by the beams. The magnetic bremsstrahlung mechanism dominates and its characteristics are determined by the value of the quantum parameter $\chi(t)$ dependent on the strength of the incoming beam field at the moment t (the constant field limit) $\chi = \gamma F/H_0$, where $F = |\mathbf{F}|$, $\mathbf{F} = \mathbf{E}_\perp + \mathbf{v} \times \mathbf{H}$, $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields in the laboratory frame, $\mathbf{E}_\perp = \mathbf{E} - \mathbf{v}(\mathbf{vE})$.

The photon radiation length in an external field is

$$l_c(\chi, u) = \lambda_c \frac{H_0}{F} \left(1 + \frac{\chi}{u}\right)^{1/3} = \frac{\lambda_c \gamma}{\chi} \left(1 + \frac{\chi}{u}\right)^{1/3}, \qquad (34)$$

The field of the incoming beam changes very slightly along the formation length l_c, if the condition $l_c \ll \sigma_z$ if satisfied, providing a high accuracy of the magnetic bremsstrahlung approximation.

In the general case, when both polarization of electrons and photons it taken into account, the spectral probability of radiation per unit time has the form

$$\frac{dw_\gamma}{dt} \equiv dW_\gamma(t) = \frac{\alpha}{2\sqrt{3}\pi\gamma^2}\Phi_\gamma^\zeta(1+(\boldsymbol{\lambda}\boldsymbol{\xi}))d\omega; \quad \Phi_\gamma^\zeta = \Phi_\gamma + \frac{\omega}{\varepsilon}(\boldsymbol{\zeta}\mathbf{h})K_{1/3}(z),$$

$$\Phi_\gamma(t) = \beta K_{2/3}(z) - \int_z^\infty K_{1/3}(y)dy, \quad \beta = \frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon} \tag{35}$$

where $z = 2u/3\chi(t)$, $\boldsymbol{\lambda}(\lambda_1, \lambda_2, \lambda_3)$ are the Stokes parameters of emitted photons for the following choice of axes: $\mathbf{e}_1 = (\mathbf{v} \times \mathbf{h})$, $\mathbf{h} = \mathbf{F}^*/F$, $\mathbf{e}_2 = \mathbf{h}$, $\mathbf{F}^* = e/|e|[\mathbf{H}_\perp + (\mathbf{E} \times \mathbf{v})]$, $\boldsymbol{\zeta}$ is the spin vector of the initial electron in its rest frame. The vector $\boldsymbol{\xi}$ determines the mean photon polarization and its components are given by the following expressions:

$$\xi_1 = \frac{\omega(\boldsymbol{\zeta}\mathbf{v}\mathbf{h})}{\varepsilon'\Phi_\gamma^\zeta}K_{1/3}(z), \quad \xi_2 = \frac{(\boldsymbol{\zeta}\mathbf{v})}{\Phi_\gamma^\zeta}\left[\left(\frac{\varepsilon}{\varepsilon'} - \frac{\varepsilon'}{\varepsilon}\right)K_{2/3}(z) - \frac{\omega}{\varepsilon}\int_z^\infty K_{1/3}(y)dy\right],$$

$$\xi_3 = \frac{1}{\Phi_\gamma^\zeta}\left[K_{2/3}(z) + \frac{\omega}{\varepsilon'}(\boldsymbol{\zeta}\mathbf{h})K_{1/3}(z)\right], \tag{36}$$

here $(\boldsymbol{\zeta}\mathbf{v}\mathbf{h}) = \boldsymbol{\zeta}(\mathbf{v}\times\mathbf{h})$.

Here we consider radiation from unpolarized electrons. The spectral probability of radiation is (35)

$$\frac{dw_\gamma}{d\omega} = \frac{\alpha}{\pi\gamma^2\sqrt{3}}\int_{-\infty}^{\infty}\Phi_\gamma(t)dt. \tag{37}$$

For the Gaussian beams

$$\chi(t) = \chi_0(x,y)\exp(-2t^2/\sigma_z^2), \tag{38}$$

here the function $\chi_0(x,y)$ depends on transverse coordinates.

It turns out that for the Gaussian beams the integration of the spectral probability over time can be carried out in a general form:

$$\frac{dw_\gamma}{du} = \frac{\alpha m\sigma_z}{\pi\gamma\sqrt{6}}\frac{1}{(1+u)^2}\left[\left(1+u+\frac{1}{1+u}\right)\int_1^\infty K_{2/3}(ay)\frac{dy}{y\sqrt{\ln y}}\right.$$
$$\left.-2a\int_1^\infty K_{1/3}(ay)\sqrt{\ln y}\,dy\right], \tag{39}$$

where $a = 2u/3\chi_0$. In the case when $\chi_0 \ll 1$ the main contribution into integral (39) gives the region $y = 1 + \xi, \xi \ll 1$. Taking the integrals over ξ we obtain

$$\frac{dw_\gamma^{CF}}{du} \simeq \frac{\sqrt{3}\alpha m \sigma_z}{4\gamma} \frac{1 + u + u^2}{u(1+u)^3} \chi_0 \exp\left(-\frac{2u}{3\chi_0}\right) \tag{40}$$

For round beams the integration over transverse coordinates is performed with the density

$$n_\perp(\boldsymbol{\varrho}) = \frac{1}{2\pi\sigma_\perp^2} \exp\left(-\frac{\varrho^2}{2\sigma_\perp^2}\right) \tag{41}$$

The parameter $\chi_0(\varrho)$ we present in the form

$$\chi_0(\varrho) = \chi_m \frac{f(x)}{f_0},\ x = \frac{\varrho}{\sigma_\perp}, \quad f(x) = \frac{1}{x}\Big(1 - \exp(-x^2/2)\Big)\,,$$
$$\chi_{rd} = 0.720\alpha N\gamma \frac{\lambda_c^2}{\sigma_z \sigma_\perp}, \quad f'(x_0) = 0,\ f_0 = f(x_0) = 0.451256, \tag{42}$$

where N is the number of electron in the bunch.

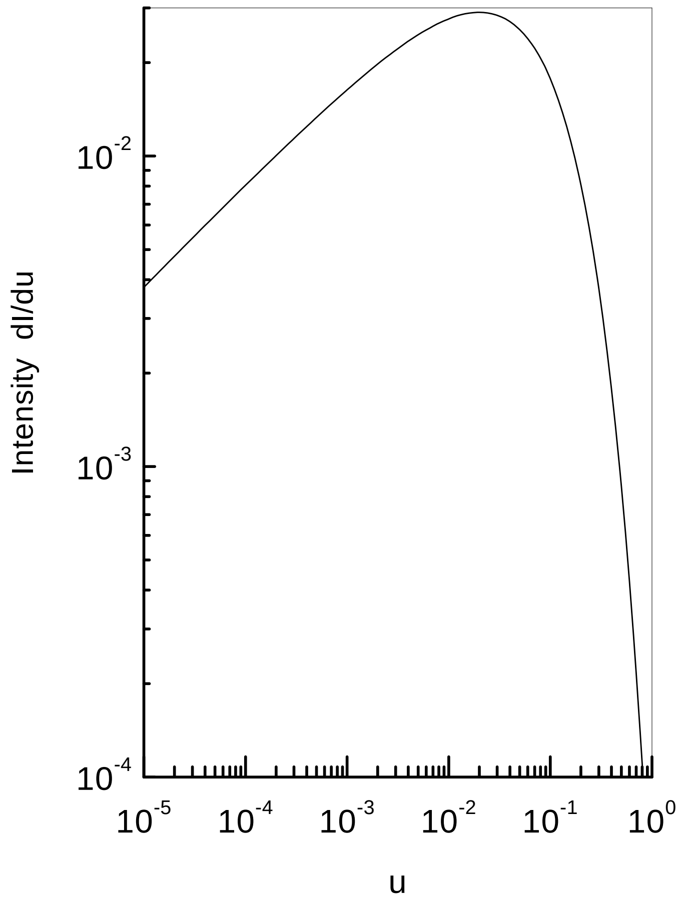

Figure 1: Spectral intensity of radiation of round beams in units $\alpha m^2 \sigma_z$ for χ_{rd}=0.13 calculated according to Eqs.(39),(42)

The Laplace integration of Eq.(40) gives for radiation intensity $dI/du = \varepsilon u/(1+u) dW/du$

$$\frac{dI_{as}}{du} \simeq \alpha m^2 \sigma_z \frac{3}{4}\sqrt{\frac{\pi}{|f_0''|}}\frac{1+u+u^2}{\sqrt{u}(1+u)^4} f_0^{3/2}\chi_{rd}^{3/2} \exp\left(-\frac{2u}{3\chi_m}\right), \tag{43}$$

where $f_0'' = f''(x_0) = -0.271678.$

For the flat beams ($\sigma_x \gg \sigma_y$) the parameter $\boldsymbol{\chi}_0(\boldsymbol{\varrho})$ takes the form

$$\boldsymbol{\chi}_0 = \chi_m \exp\left(-\frac{x^2}{2\sigma_x^2}\right)\left[\mathbf{e}_y \mathrm{erf}\left(\frac{y}{\sqrt{2}\sigma_y}\right) - i\mathbf{e}_x \mathrm{erf}\left(i\frac{x}{\sqrt{2}\sigma_x}\right)\right],\ \chi_m = \frac{2N\alpha\gamma\lambda_c^2}{\sigma_z\sigma_x}, \tag{44}$$

here $\mathrm{erf}(z) = 2/\sqrt{\pi}\int_0^z \exp(-t^2)dt$, $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors along the corresponding axes.

To calculate the asymptotic of radiation intensity for the case $\chi_0 \ll 1$ one has to substitute

$$\chi_0 = |\boldsymbol{\chi}_0| = \chi_m \exp\left(-\frac{x^2}{2\sigma_x^2}\right)\left[\left(\mathrm{erf}\left(\frac{y}{\sqrt{2}\sigma_y}\right)\right)^2 + \left(-i\mathrm{erf}\left(i\frac{x}{\sqrt{2}\sigma_x}\right)\right)^2\right]^{1/2} \tag{45}$$

into Eq.(40) and take integrals over transverse coordinates x, y with the weight

$$n_\perp(x,y) = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right). \tag{46}$$

Integral over x can be taken using the Laplace method, while for integration over y it is convenient to introduce the variable

$$\eta = \frac{2}{\sqrt{\pi}}\int_w^\infty \exp(-t^2)dt, \quad w = \frac{y}{\sqrt{2}\sigma_y}. \tag{47}$$

As a result we obtain for the radiation intensity in the case of flat beams

$$\frac{dI_{fl}}{du} = \frac{9}{8\sqrt{2\left(1-2/\pi\right)}}\alpha m^2 \sigma_z \chi_m^{5/2} \frac{1+u+u^2}{u^{3/2}(1+u)^4} \exp\left(-\frac{2u}{3\chi_m}\right) . \qquad (48)$$

Inhomogeneous Fields

If the field varies slightly on the photon formation length, the vector $\mathbf{\Delta}(t_2)$ in Eq.(27) as well as the exponential factor can be expanded in powers of $t_2 - t_1 = \tau$, with required number of expansion terms.The first terms of the expansion which incorporates the particle acceleration, give the constant field limit, while remaining terms are the correction to this approximation. As the result the spectral intensity of radiation can be presented in the form

$$\frac{dI}{d\omega} = \frac{dI_0}{d\omega} + \frac{dI_c}{d\omega}, \quad \frac{dI_0}{d\omega} = \frac{\alpha m^2\omega}{\sqrt{3}\pi\varepsilon^2}\left[\beta K_{2/3}(z) - \int\limits_z^\infty K_{1/3}(y)dy\right]$$

$$\frac{dI_c}{d\omega} = \frac{\alpha m^2\omega}{\sqrt{3}\pi\varepsilon^2}\Bigg\{-\frac{1}{3\mathbf{b}^4}\Bigg[(\mathbf{b}(\mathbf{V}\boldsymbol{\nabla})^2\mathbf{b})\beta\left[K_{2/3}(z) - \frac{2}{3z}K_{1/3}(z)\right]$$

$$-\frac{1}{10}\left[((\mathbf{V}\boldsymbol{\nabla})\mathbf{b})^2 + 3(\mathbf{b}(\mathbf{V}\boldsymbol{\nabla})^2\mathbf{b})\right]$$

$$\times\left[zK_{1/3}(z) - \frac{4}{3}K_{2/3}(z) + \beta\left(4K_{2/3}(z) - \left(z + \frac{16}{9z}\right)K_{1/3}(z)\right)\right]\Bigg]\Bigg\}, \quad (49)$$

where β is defined in Eq.(35),

$$z = \frac{2m^2\omega}{3\varepsilon\varepsilon'|\mathbf{b}|}, \quad \mathbf{b} = \frac{e\mathbf{F}}{m}, \quad (50)$$

the vector $\mathbf{V}$ is the difference between the velocity of particle and the opposite beam (in the case when the beam shape is not changed during the collision). Here $dI_0/d\omega$ is the intensity spectrum in magnetic bremsstrahlung limit, $dI_c/d\omega$ is the gradient correction.

General Theory of Radiation in Oriented Crystal

Here we consider case $\vartheta_0 \ll V_0/m$. Than the distance of an electron from axis $\boldsymbol{\varrho}$ as well as the transverse field of the axis can be considered as constant over the formation length. For an axial orientation of crystal the ratio of the atom density $n(\varrho)$ in the vicinity of an axis to the mean atom density n_a is

$$\frac{n(x)}{n_a} = \xi(x) = \frac{x_0}{\eta_1} e^{-x/\eta_1}, \quad \varepsilon_0 = \frac{\varepsilon_e}{\xi(0)}, \tag{51}$$

where

$$x_0 = \frac{1}{\pi d n_a a_s^2}, \quad \eta_1 = \frac{2u_1^2}{a_s^2}, \quad x = \frac{\varrho^2}{a_s^2}, \tag{52}$$

Here ϱ is the distance from axis, u_1 is the amplitude of thermal vibration, d is the mean distance between atoms forming the axis, a_s is the effective screening radius of the axis potential

$$U(x) = V_0 \left[\ln\left(1 + \frac{1}{x+\eta}\right) - \ln\left(1 + \frac{1}{x_0+\eta}\right)\right]. \tag{53}$$

The local value of parameters $\chi(x)$, see Eq.(12), which determines the radiation probability in the field Eq.(53) is

$$\chi(x) = -\frac{dU(\varrho)}{d\varrho}\frac{\varepsilon}{m^3} = \chi_s f_a, \quad f_a = \frac{2\sqrt{x}}{(x+\eta)(x+\eta+1)}, \quad \chi_s = \frac{V_0\varepsilon}{m^3 a_s} \equiv \frac{\varepsilon}{\varepsilon_s}. \tag{54}$$

The particular calculation below will be done for tungsten and germanium crystals studied experimentally. The relevant parameters are given in Table 1.

TABLE 1 Parameters of radiation (pair creation) process in the tungsten (the axis $< 111 >$) and germanium (the axis $< 110 >$) crystals for different temperatures T, the energies ε and ω are in GeV

Crystal	T(K)	V_0(eV)	x_0	η_1	η	ε_0	ε_t	$\varepsilon_s(\omega_s)$	$\varepsilon_m(\omega_m)$	h
W	293	413	39.7	0.108	0.115	7.43	0.76	34.8	14.35	0.348
W	100	355	35.7	0.0401	0.0313	3.06	0.35	43.1	8.10	0.612
Ge	100	114.5	19.8	0.064	0.0633	59	0.85	179	51	0.459

The spectral intensity of radiation can be presented in the form

$$dI(\varepsilon, y) = \frac{\alpha m^2}{2\pi}\frac{ydy}{1-y}\int\limits_0^{x_0}\frac{dx}{x_0}G(x,y), \quad G(x,y) = \int\limits_0^{\infty}F(x,y,t)dt - r_3\frac{\pi}{4},$$

$$F(x,y,t) = \mathrm{Im}\left\{e^{\varphi_1(t)}\left[r_2\nu_0^2(1+ib)\varphi_2(t) + r_3\varphi_3(t)\right]\right\}, \quad b = \frac{4\chi^2(x)}{u^2\nu_0^2},$$

$$y = \frac{\omega}{\varepsilon}, \quad u = \frac{y}{1-y}, \quad \varphi_1(t) = (i-1)t + b(1+i)(f_2(t) - t),$$

$$\varphi_2(t) = \frac{\sqrt{2}}{\nu_0}\tanh\frac{\nu_0 t}{\sqrt{2}}, \quad \varphi_3(t) = \frac{\sqrt{2}\nu_0}{\sinh(\sqrt{2}\nu_0 t)} \tag{55}$$

where

$$r_2 = 1 + (1-y)^2, \quad r_3 = 2(1-y), \quad \nu_0^2 = \frac{1-y}{y}\frac{\varepsilon}{\varepsilon_c(x)}, \quad \varepsilon_c(x) = \frac{\varepsilon_e(n_a)}{\xi(x)g} = \frac{\varepsilon_0}{g}e^{x/\eta_1},$$

$$\varepsilon_e = \frac{m}{16\pi Z^2\alpha^2\lambda_c^3 n_a L_0}, \quad L_0 = \ln(183Z^{-1/3}) - f(Z\alpha),$$

$$h(z) = -\frac{1}{2}\left[1 + (1+z)e^z\mathrm{Ei}(-z)\right], \quad g = 1 + \frac{1}{L_0}\left[\frac{1}{18} - h\left(\frac{u_1^2}{a^2}\right)\right], \tag{56}$$

The found spectral intensity of radiation contains very reach information. The intensity of coherent radiation $I(\varepsilon) = \int I^{coh}(\varepsilon, y)dy$ is the first term ($\nu_0^2 = 0$) of the decomposition of Eq.(55) over ν_0^2.

$$I^{coh}(\varepsilon) = \int\limits_0^{x_0} I(\chi)\frac{dx}{x_0}. \tag{57}$$

Here $I(\chi)$ is the radiation intensity in constant field (magnetic bremsstrahlung limit). It is convenient to use the following representation for $I(\chi)$

$$I(\chi) = i\frac{\alpha m^2}{2\pi}\int\limits_{\lambda-i\infty}^{\lambda+i\infty}\left(\frac{\chi^2}{3}\right)^s \Gamma\left(1-s\right)\Gamma\left(3s-1\right)\left(2s-1\right)(s^2-s+2)\frac{ds}{\cos\pi s}, \quad \frac{1}{3}<\lambda<1. \tag{58}$$

The second term of the decomposition of Eq.(55)($\propto \nu_0^2$) gives the intensity of incoherent radiation:

$$I^{inc}(\varepsilon) = \frac{\alpha m^2}{60\pi}\frac{\varepsilon}{\varepsilon_0}g\int\limits_0^{x_0} e^{-x/\eta_1}J(\chi)\frac{dx}{x_0}, \tag{59}$$

the new representation of $J(\chi)$ is

$$J(\chi) = \frac{i\pi}{2}\int\limits_{\lambda-i\infty}^{\lambda+i\infty}\frac{\chi^{2s}}{3^s}\frac{\Gamma(1+3s)}{\Gamma(s)}R(s)\frac{ds}{\sin^2\pi s}, \quad -\frac{1}{3}<\lambda<0 \tag{60}$$

where

$$R(s) = 15 + 43s + 31s^2 + 28s^3 + 12s^4. \tag{61}$$

The inverse radiation length in tungsten crystal (axis $< 111 >$) $1/L^{cr}(\varepsilon) = I(\epsilon)/\varepsilon$ Eq.(55), as well as the coherent contribution $1/L^F(\varepsilon) = I^F(\varepsilon)/\varepsilon$ Eq.(57) and the incoherent contribution $1/L^{inc}(\varepsilon) = I^{inc}(\varepsilon)/\varepsilon$ Eq.(59) are shown in Fig.1 for two temperatures T=100 K and T=293 K as a function of incident electron energy ε. One can see that at temperature T=293 K the intensity $I^{coh}(\varepsilon)$ is equal to $I^{inc}(\varepsilon)$ at $\varepsilon \simeq 0.4$ GeV and temperature T=100 K the intensity $I^{coh}(\varepsilon)$ is equal to $I^{inc}(\varepsilon)$ at $\varepsilon \simeq 0.7$ GeV. At higher energies the intensity $I^F(\varepsilon)$ dominates while the intensity $I^{inc}(\varepsilon)$ decreases monotonically.

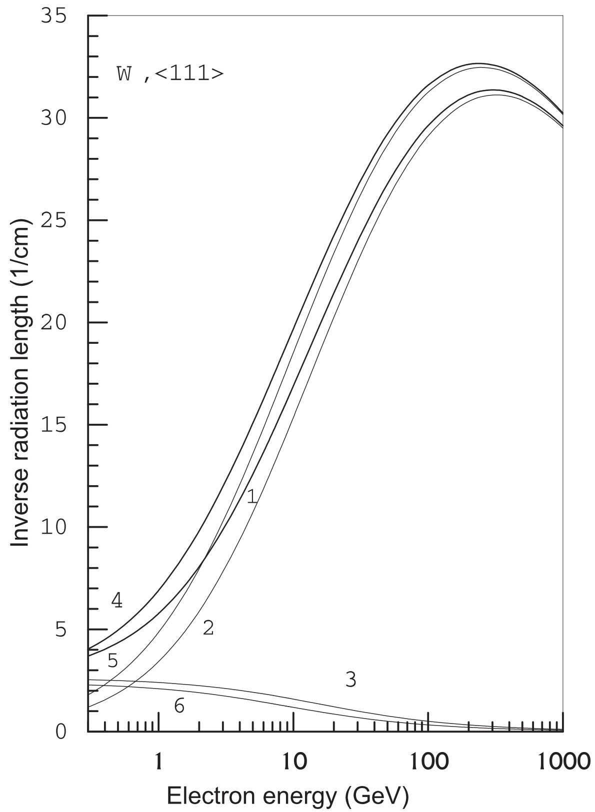

Figure 2: The inverse radiation length in tungsten, axis $< 111 >$ at different temperatures T vs the electron initial energy. Curves 1 and 4 are the total effect: $L^{cr}(\varepsilon)^{-1} = I(\varepsilon)/\varepsilon$ Eq.(55) for T=293 K and T=100 K correspondingly, the curves 2 and 5 give the coherent contribution $I^F(\varepsilon)/\varepsilon$ Eq.(57), the curves 3 and 6 give the incoherent contribution $I^{inc}(\varepsilon)/\varepsilon$ Eq.(59) at corresponding temperatures T.

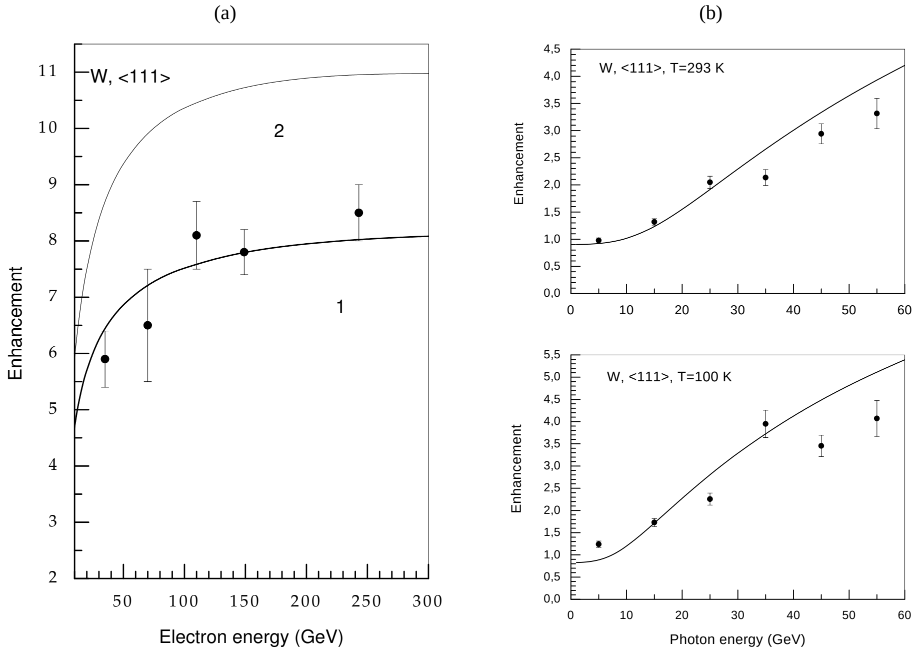

Figure 3: Comparison of theory and experiment.(a) Enhancement of radiation intensity (the ratio L^{BM}/L^{ef}) in tungsten, axis $< 111 >$, T=293 K. The curve 1 is for the target with thickness $l = 200\ \mu m$, where the energy loss was taken into account. The curve 2 is for a considerably more thinner target, where one can neglect the energy loss ($L^{ef} \to L^{cr}$). The data are from $K.Kirshbom,\ et\ al,\ 2001$.

(b) Enhancement of the probability of pair creation in tungsten for different temperatures, axis $< 111 >$. The data are from $K.Kirshbom,\ et\ al,\ 1998$.

Back-reaction: spectra of radiation taking into account energy loss in oriented crystals

The crystal radiation length $L(\varepsilon) = \varepsilon/I(\varepsilon)$, $I(\varepsilon)$ is the intensity of electron radiation, and the pair creation length $L_{pr}(\omega) = 1/W(\omega)$, $W(\omega)$ is the pair creation probability, are the function of energy in oriented crystals.

We consider the case when the target thickness l is of the order $l \sim L(\varepsilon)$ in the intermediate energy region. Here we will neglect the energy dispersion. On this assumption the energy loss equation acquires the form

$$dt = \frac{L(\varepsilon)}{\varepsilon}d\varepsilon, \quad t(\varepsilon, \varepsilon_0) = \int\limits_{\varepsilon}^{\varepsilon_0} \frac{dx}{x}L(x), \quad \varepsilon = \varepsilon(\varepsilon_0, t) \tag{62}$$

Now the photon spectral distribution taking into account the energy loss can be written in the form

$$\omega\frac{dn_\gamma}{d\omega} = \int\limits_0^l \frac{dI(\varepsilon(\varepsilon_0,t),\omega)}{d\omega}\vartheta(\varepsilon(\varepsilon_0,t)-\omega)dt$$

$$= \int\limits_{\varepsilon_l}^{\varepsilon_0} \frac{L(\varepsilon)}{\varepsilon}\frac{dI(\varepsilon,\omega)}{d\omega}\vartheta(\varepsilon-\omega)d\varepsilon, \quad \varepsilon_l = \varepsilon(\varepsilon_0, l), \tag{63}$$

where $dI(\varepsilon,\omega)/d\omega$ is radiation intensity spectral distribution (see Eq.(55)), $\varepsilon_l = \varepsilon(\varepsilon_0, l)$ is the electron energy after traversing the thickness l by the electron with the initial energy ε_0. The result of calculation for tungsten crystal is shown in Fig.4.

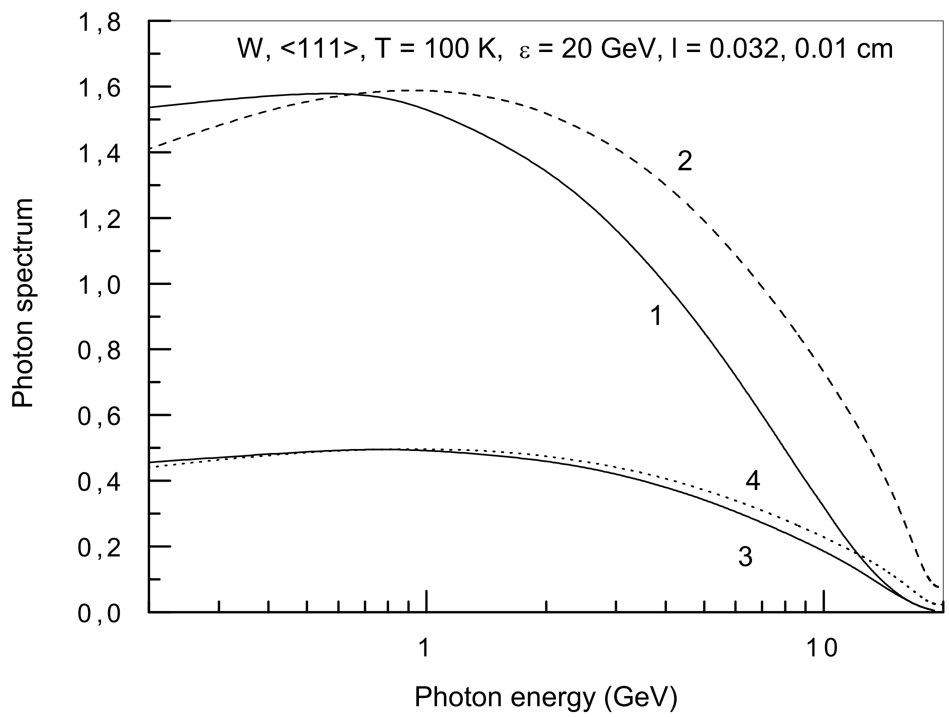

Figure 4: The spectral distribution of radiation at the initial electron energy $\varepsilon_0 = 20$ GeV in the tungsten crystal, axis $< 111 >$, T$=$100 K in two targets with thickness $l = 0.032$ cm$= 0.77\ L = 0.16\ L_{pr}$ (curves 1 and 2) and $l = 0.01$ cm$= 0.24\ L = 0.093\ L_{pr}$ (curves 3 and 4) vs the photon energy ω. The curves 2 and 4 are calculated according to Eq.(55), while the curves 1 and 3 are calculated according to Eq.(63) which takes into account the electron energy loss.

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