Rough estimation of power going through HOM coupler of ILC Cavity

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Here we estimate how much power will go through HOM couplers, considering four kinds of fields in a cavity.

1, Accelerating mode
2, Longitudinal higher order mode, single bunch
3, Longitudinal higher order mode, multibunch
4, Dipole (transverse) mode and higher mode

1, Accelerating mode

Leakage power through a HOM coupler depends on external Q-value (Q_{ext}) of the coupler, which can be expressed as:

$$P_{ext} = \frac{V^2}{(R/Q)}\frac{1}{Q_{ext}} \quad , \qquad (1)$$

where V is the voltage (~32 MV), (R/Q) is the R/Q of the mode (~1 kΩ). Then,

$$P_{acc}[W] \sim \frac{1\times 10^{12}}{Q_{ext}} \qquad (2)$$

Specification of Q_{ext} of HOM couplers should determine the limit of this power..

2, Longitudinal higher order mode, single bunch

Energy loss in a cavity due to short range longitudinal wake, for bunch length 0.3 mm, bunch population 2E10, can be calculated from parameters in TESLA-TDR as:

$$\Delta E_{sw} \sim 4.6\times 10^4 \text{ eV/particle} \qquad (3)$$

This includes energy to accelerating mode, which is about 7000 eV/particle. Then, energy loss due to short-range longitudinal higher order mode is

$$\Delta E_{shw} \sim 3.9\times 10^4 \text{ eV/particle}. \qquad (4)$$

Multiplying number of particles per bunch (2E10), number of bunches per pulse (3000) and divided by beam pulselength (0.9 msec) (e = 1.6E-19 C),

$$P_{shw} \sim 400 \text{ W/cavity} \qquad (5)$$

We assume all this power can go out through the HOM couplers, though some part of this power will go out though the input coupler.

3, Longitudinal higher order mode, multibunch

For estimating the power due to this effect, we need to know frequencies, R/Q and Q-values of higher modes. Here, we give formulae to calculate power due to each mode.

Assuming the loaded Q-value is large ($Q_L >> f\,t_b$, f is the frequency of the mode and t_b the bunch spacing), beam-induced voltage of one mode is

$$V_b \approx \frac{(R/Q) Q_L I_b}{\sqrt{1 + 8Q_L^2 (1 - \cos(\omega t_b)) / (\omega t_b)^2}} \quad , \qquad (6)$$

where I_b is the beam current.
Then, power loss due to this mode is

$$P_{lhw} = \frac{V_b^2}{(R/Q) Q_L} \approx \frac{(R/Q) Q_L I_b^2}{1 + 8Q_L^2 (1 - \cos(\omega t_b)) / (\omega t_b)^2} \qquad (7)$$

We should sum up all significant modes and assume all the power will go through HOM couplers

4, Dipole (transverse) mode and higher mode
Power loss due to m-mode scale as x^{4m}, where x is offset of the beam from the cavity center ($m = 1$ for dipole mode).
For realistic beam offset, power loss due to $m \geq 1$ can be neglected.

5. SUMMARY

Field		Result
Accelerating mode	Depend on Q of HOM coupler	Eq. (2)
Short range longitudinal wake	From parameters in TESLA-TDR	~ 400 W/cavity
Long range longitudinal wake	Depend on parameters of modes	Eq. (7)
Dipole and higher mode	Depend on beam - cavity offset	Negligible