New Developments in Loop Calculations and Their Implications

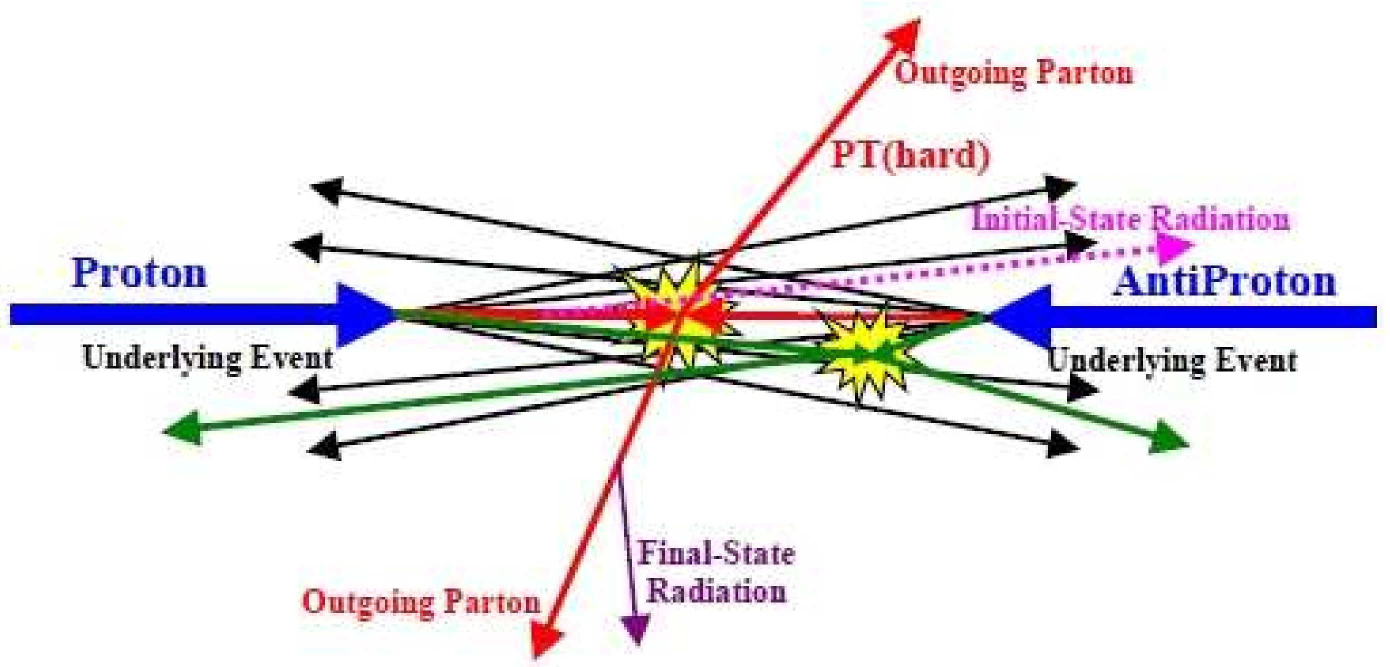
Carola F. Berger

CTP, MIT

ALCPG09, Sept 30th 2009

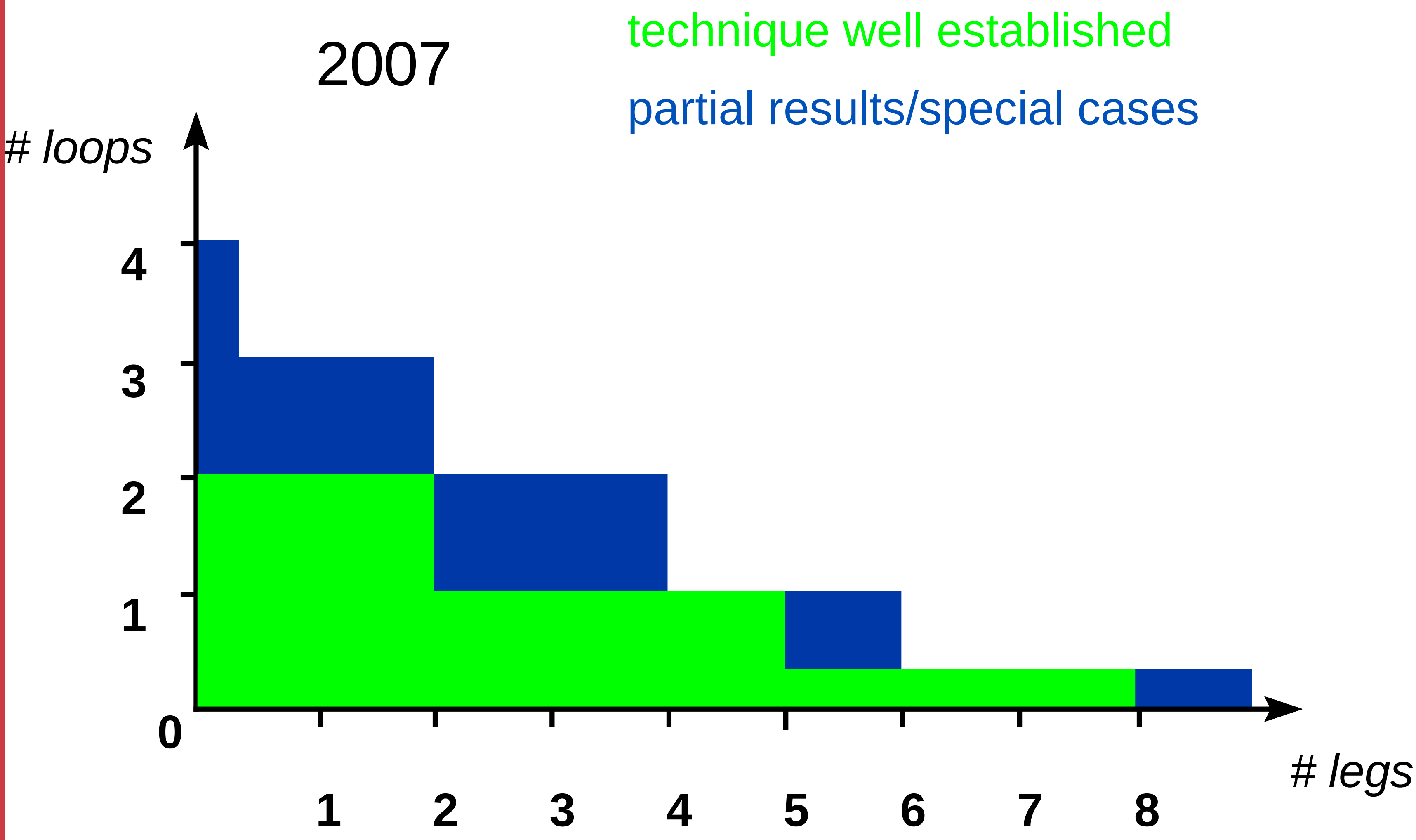
Disclaimer

Perturbative Calculations



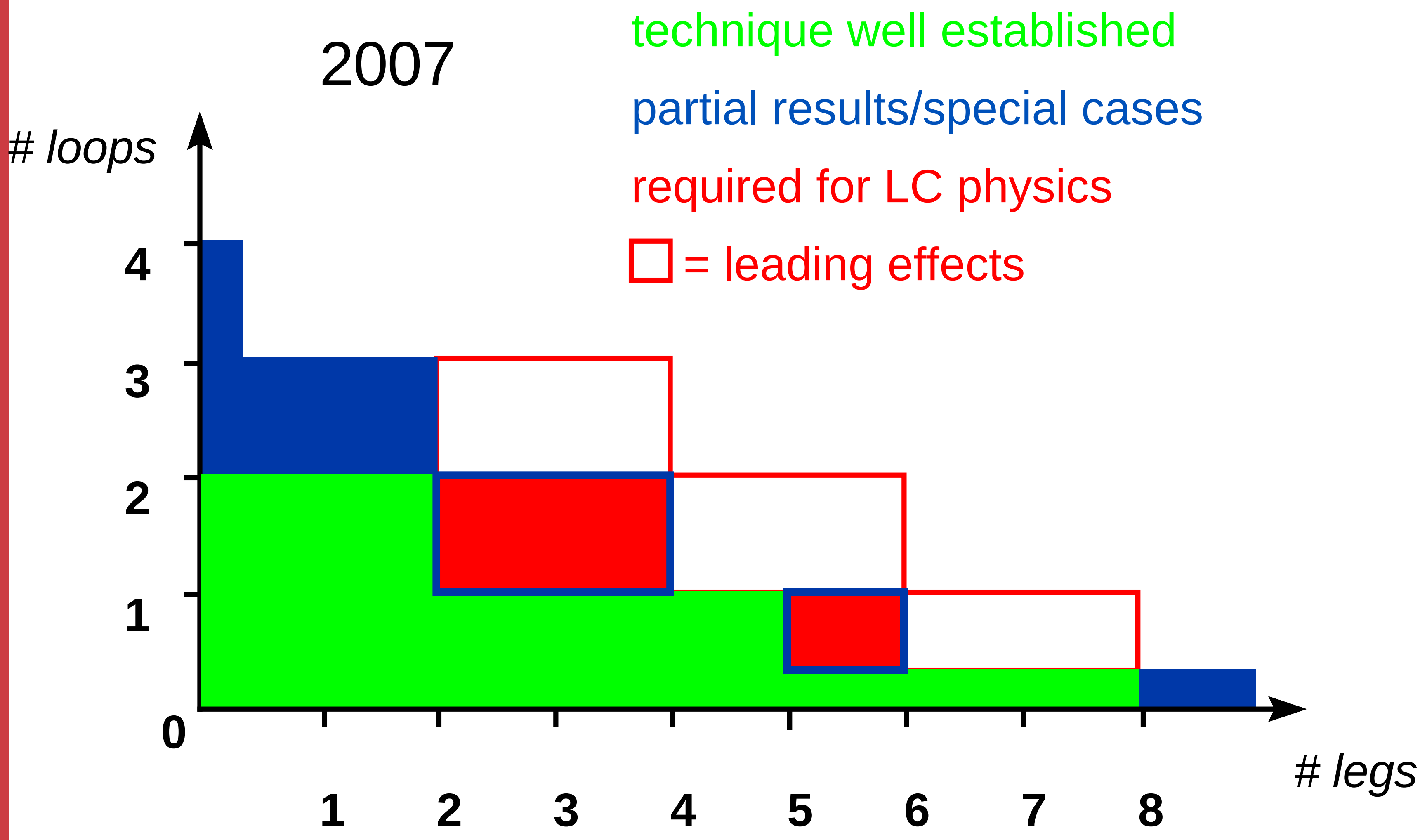
- Parton distribution functions (not for LC)
- Matrix elements $\Longleftarrow$
- Parton showers, resummation
- Monte Carlo models (also for hadronization)

Instead of an Outline



A. Juste at ALCPG07

Instead of an Outline



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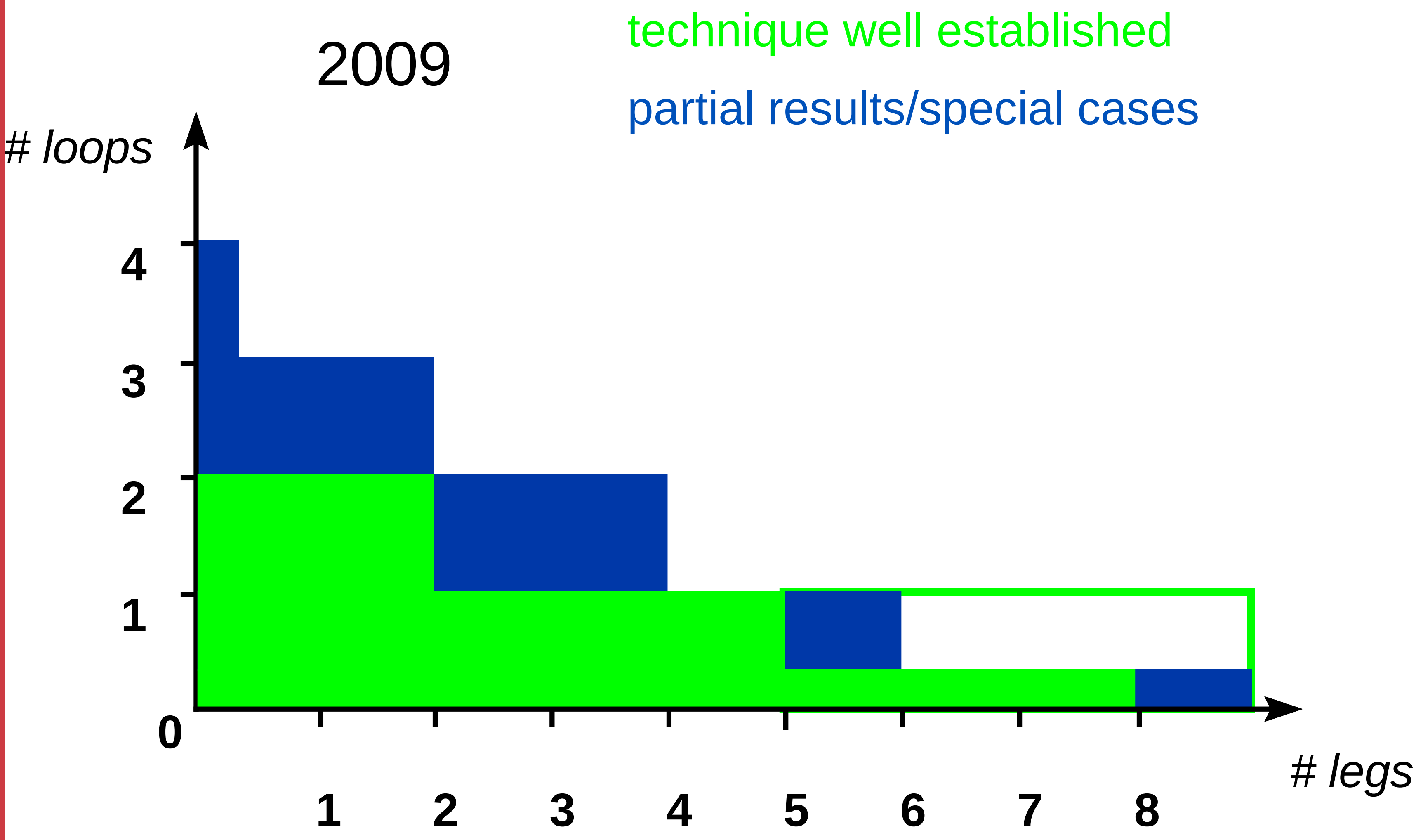

NNLO

NNLO

For certain processes, NNLO is needed

- **when the NLO corrections are large, e.g. Higgs production**
- **for benchmark measurements where experimental errors are small or to facilitate calibration of detectors and determine efficiencies**
- **to minimize PDF and luminosity uncertainties**

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From the updated Les Houches wishlist 2007:

process wanted at/beyond NNLO	
10. $gg \to W^*W^*\mathcal{O}(\alpha^2\alpha_s^3)$	background to Higgs
11. $pp \to t\bar{t}$	benchmark process
12. VBF, $Z/\gamma+$ jet	Higgs couplings, SM benchmark
13. W/Z production at NNLO QCD, NLO EW	SM benchmark

LHC and ILC Processes Known at NNLO

- **(differential) Z, W**

  Anastasiou, Dixon, Melnikov, Petriello; Catani, Cieri, Ferrera, de Florian, Grazzini

- **(differential) Higgs**

  Ravindran, Smith, van Neerven; Kilgore, Harlander; Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello; Anastasiou, Dissertori, Grazzini, Stoeckli, Webber; Catani, Grazzini; Harlander, Ozeren; Pak, Rogal, Steinhauser

- **$e^+e^- \to 3$ jets, event shapes**

  Gehrmann-De Ridder, Gehrmann, Glover, Heinrich; Weinzierl

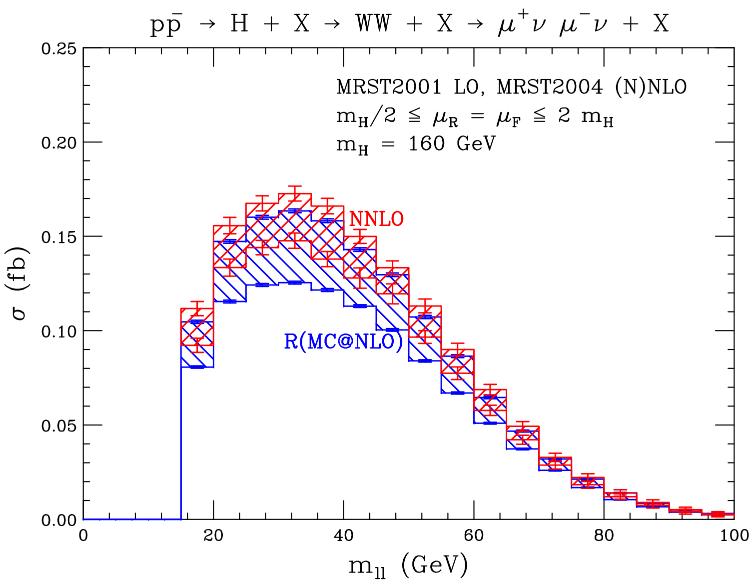
- **DGLAP splitting kernels**

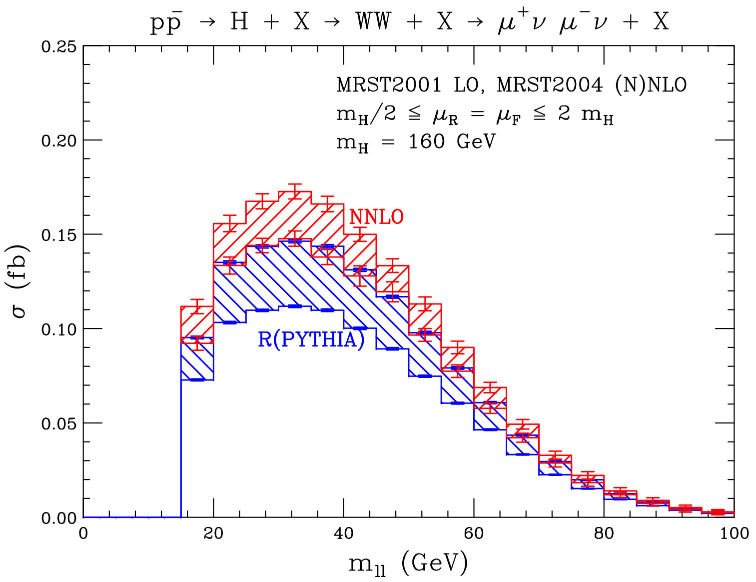
  Moch, Vermaseren, Vogt

- **NNLO parton distributions**

  Martin, Stirling, Thorne, Watt; Alekhin, Blümlein, Klein, Moch; Jimenez-Delgado, Reya

Example of State-of-the-Art NNLO: Higgs





Anastasiou, Dissertori, Grazzini, Stöckli, Webber

Error on α_s from jet observables

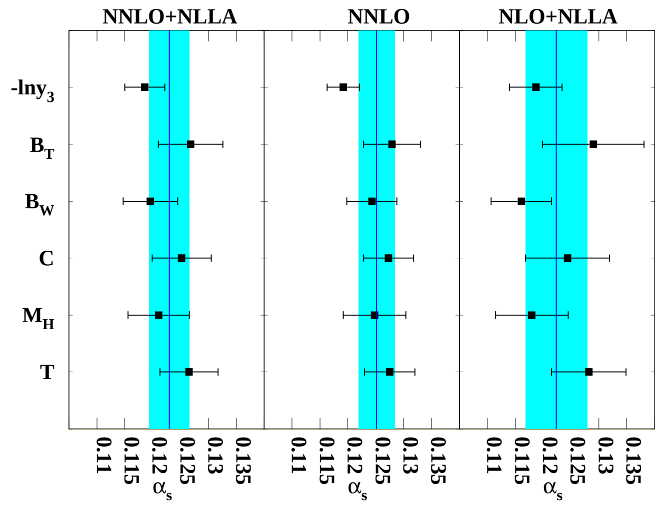
Bethke

$$\alpha_s(M_Z) = 0.121 \pm 0.001(\text{exp}) \pm 0.005(\text{th})$$

Computation of 3-jet event shapes at NNLO

Gehrmann-De Ridder, Gehrmann, Glover, Heinrich; Weinzierl

$\Rightarrow$ extraction of α_s at NNLO+NLLA



$$\alpha_s(M_Z) = 0.1224 \pm 0.0009(\text{stat}) \pm 0.0009(\text{exp}) \pm 0.0012(\text{had}) \pm 0.0035(\text{th})$$

Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Luisoni, Stenzel; Bethke, Kluth, Pahl, Schieck, JADE

NLO

The (In)Famous Wishlist

Les Houches 2005

process wanted at NLO ($V \in \{Z, W, \gamma\}$)	background to
1. $pp \to VV +$ jet	$t\bar{t}H$, new physics
2. $pp \to H + 2$ jets	H production by vector boson fusion (VBF)
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton

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1. $pp \to VV +$ jet	$t\bar{t}H$, new physics
2. $pp \to H + 2$ jets	H production by vector boson fusion (VBF)
	gg: Campbell, Ellis, Zanderighi
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
	VBF: Bozzi, Jäger, Oleari, Zeppenfeld
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton
	ZZZ: Lazopoulos, Melnikov, Petriello
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics

partially completed, via standard methods

The (In)Famous Wishlist

2009

process wanted at NLO	background to
1. $pp \to VV +$ jet	$t\bar{t}H$, new physics Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi
2. $pp \to H + 2$ jets	H in VBF Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$ Bredenstein, Denner Dittmaier, Pozzorini; Bevilaqua, Czakon, Papadopoulos, Pittau, Worek
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$ VBF: Bozzi, Jäger, Oleari, Zeppenfeld
7. $pp \to V + 3$ jets	new physics CFB, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi
8. $pp \to VVV$	SUSY trilepton Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld; Binoth, Ossola, Papadopoulos, Pittau
9. $pp \to b\bar{b}b\bar{b}$	Higgs, new physics

NLO Calculations

Ingredients:

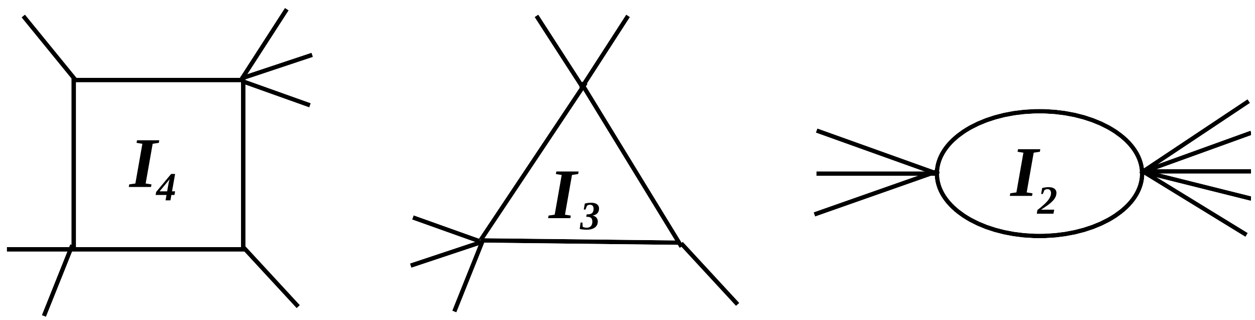
- **One-loop (virtual) matrix elements**
- **Tree-level matrix elements for real emission**
- **Both have IR divergences, which cancel in the full cross section $\Rightarrow$ subtraction terms**
- **Convolution with PDFs (only for hadronic collisions)**
- **Integration over final state phase space (with cuts)**

Bottleneck up until now: 1-loop matrix elements

Any (massless) one-loop integral can be decomposed into

$$\mathcal{M} = \sum_i d_i^D I_{4\,i}^D + \sum_i c_i^D I_{3\,i}^D + \sum_i b_i^D I_{2\,i}^D$$
$$= \sum_i d_i^{D=4} I_{4\,i}^D + \sum_i c_i^{D=4} I_{3\,i}^D + \sum_i b_i^{D=4} I_{2\,i}^D + R$$

Integrals are known, task is to determine the coefficients



Integrals tabulated in: Bern, Dixon, Dunbar, Kosower; Ellis, Zanderighi

One-Loop Matrix Elements

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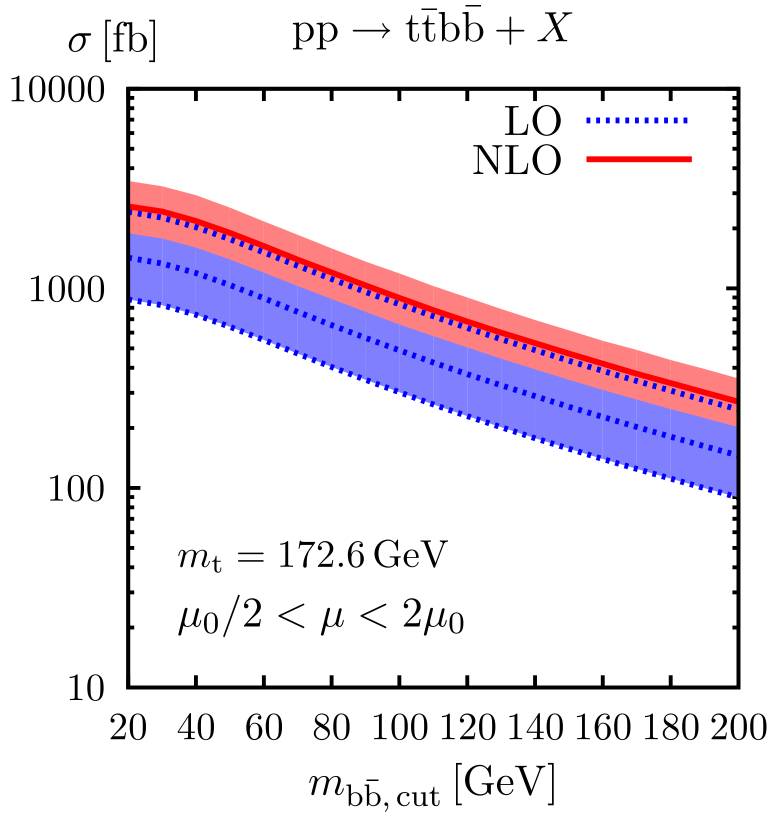
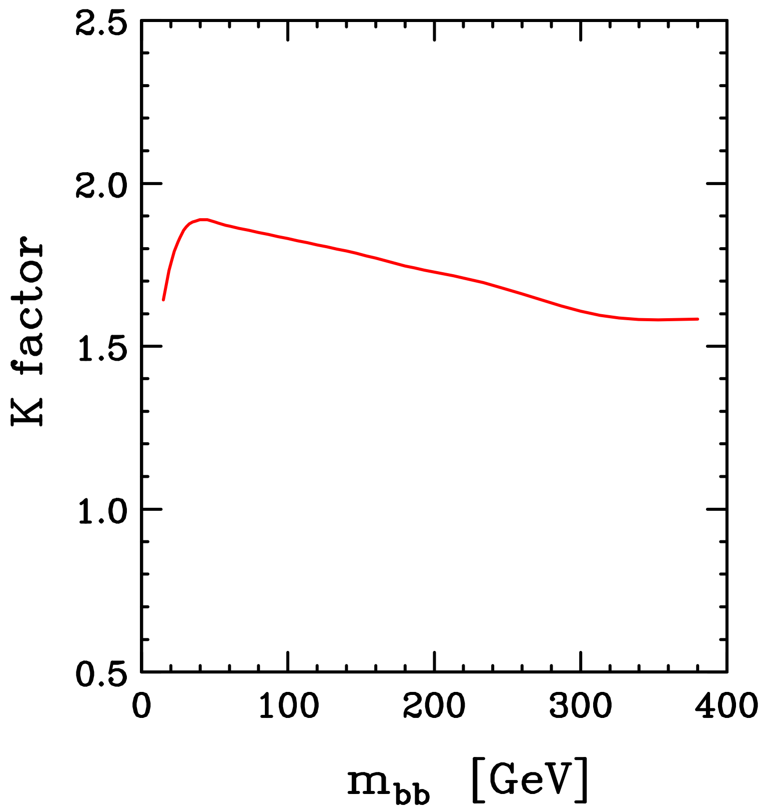
Integrals are known, task is to determine the coefficients

Standard procedure:

- **Generate all Feynman diagrams $\Rightarrow$ many terms**
- **Translate into equations $\Rightarrow$ many more terms**
- **Reduce to known Master integrals $\Rightarrow$ large cancellations between spurious singularities**

Important background to $pp \to t\bar{t}H$, with $H \to b\bar{b}$





left: Bredenstein, Denner, Dittmaier, Pozzorini; right: Bevilaqua, Czakon, Papadopoulos, Pittau, Worek

New Ideas

$$\mathcal{M} = \sum_i d_i^{D=4} I_{4\,i}^D + \sum_i c_i^{D=4} I_{3\,i}^D + \sum_i b_i^{D=4} I_{2\,i}^D + R$$

New Ideas

Introduction

NNLO

NLO

- The LHC Wishlist
- NLO Calculations
- One-Loop Matrix Elements
- $pp \longrightarrow t\bar{t}b\bar{b}$
- New Ideas
- Generalized Unitarity
- From Boxes to Complete Amplitudes

Implications

Conclusions

$$\mathcal{M} = \sum_i d_i^{D=4} I_{4\,i}^D + \sum_i c_i^{D=4} I_{3\,i}^D + \sum_i b_i^{D=4} I_{2\,i}^D + R$$

- **Generalized unitarity**

Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng

$\Rightarrow$ `BlackHat` CFB, Bern, Dixon, Forde, Febres Cordero, Ita, Kosower, Maitre

$\Rightarrow$ `Rocket` Ellis, Giele, Kunszt, Melnikov, Zanderighi

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- **OPP method** Ossola, Papadopoulos, Pittau

  $\Rightarrow$ `CutTools + HELAC`

  Bevilaqua, Czakon, van Hameren, Ossola, Papadopoulos, Pittau, Worek

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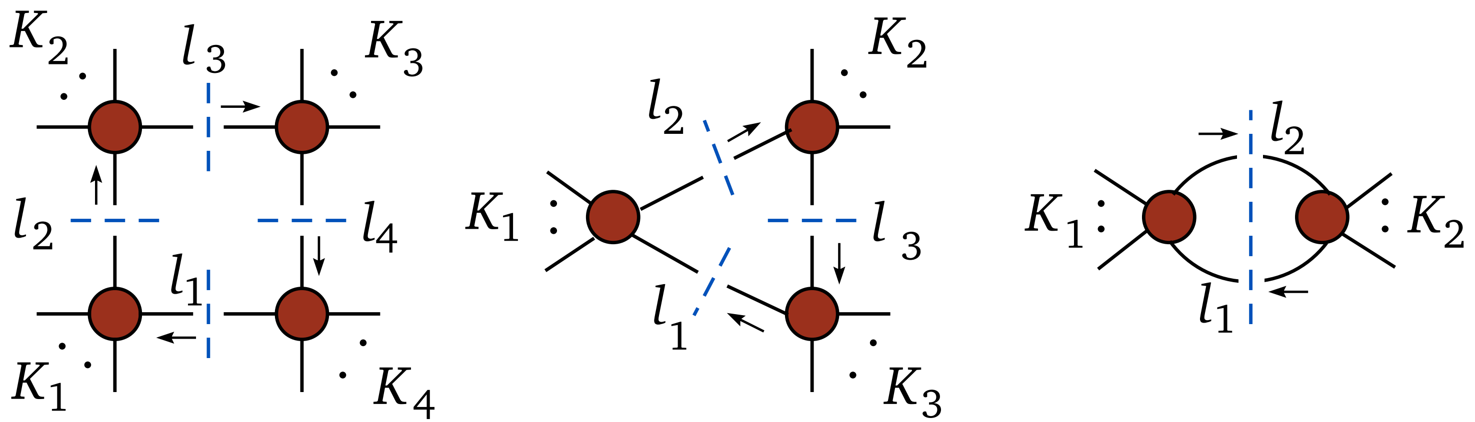
Generalized unitarity and recursion reuse amplitudes, not Feynman diagrams $\Rightarrow$ excellent scaling with number of external legs

Generalized Unitarity

Determine coefficients without doing explicit reduction by generalized unitarity: put internal propagators on-shell

$$\frac{1}{p^2 + i\epsilon} \longrightarrow i\delta^+(p^2)$$

Thus for boxes, the coefficient collapses into a product of 4 tree amplitudes (in $D = 4$) $\left(\int d^4 l \delta^+(l_1^2)\delta^+(l_2^2)\delta^+(l_3^2)\delta^+(l_4^2)\right)$



From Boxes to Complete Amplitudes

Triangle and bubble coefficients are slightly more complicated – left-over integrals (< 4 delta-functions)
$\Rightarrow$ use special parametrization to extract these

- **at integrand level – OPP** — Ossola, Papadopoulos, Pittau
- **or at integral level** — Forde - BlackHat; Rocket

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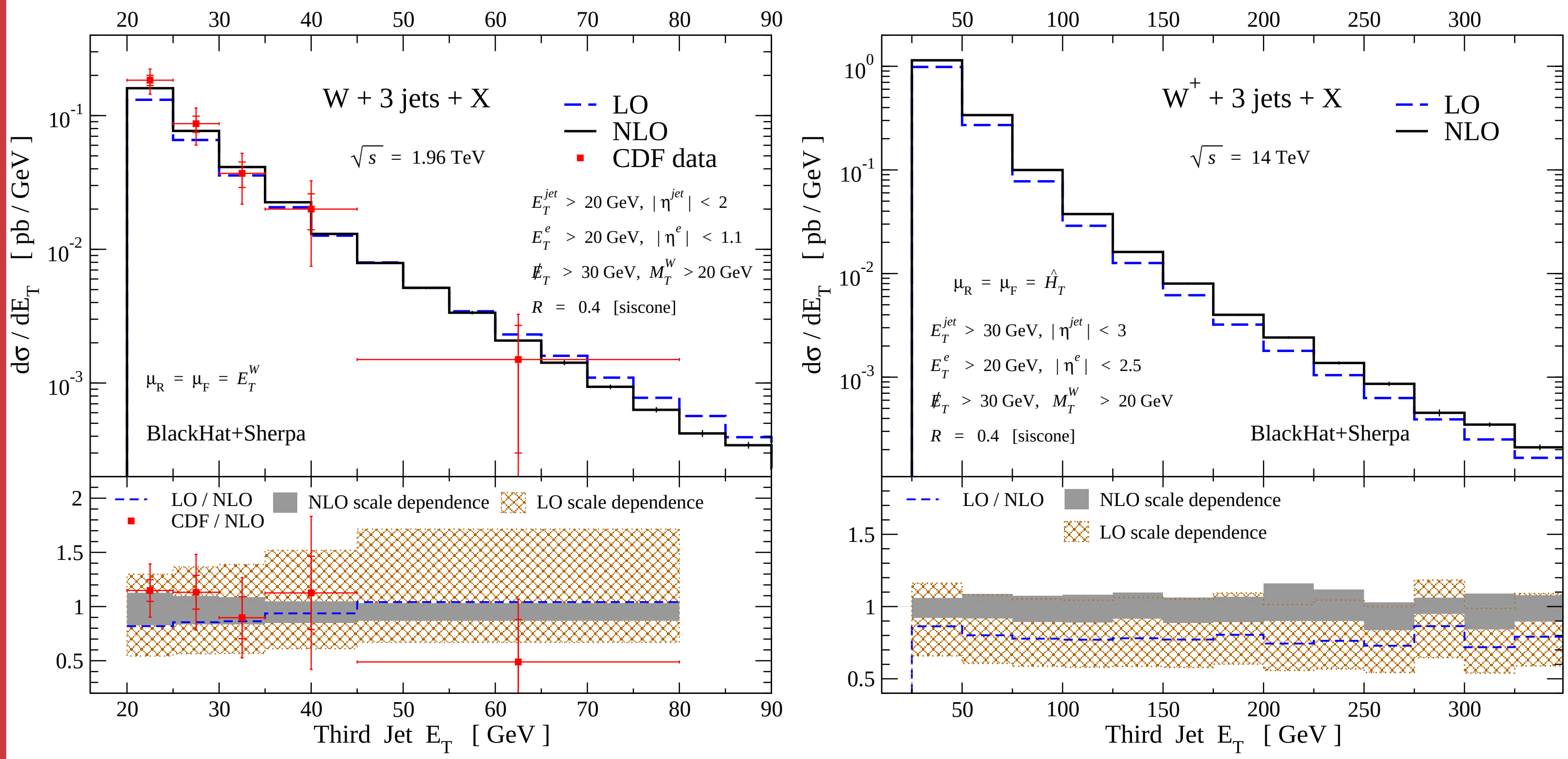
Rational terms:

- **Keep full D-dimensional information in generalized unitarity** — Ellis, Giele, Kunszt, Melnikov, Zanderighi; Badger
- **Rational recursion from lower-point one-loop terms** — CFB, Bern, Dixon, Forde, Kosower
- **Special Feynman rules in OPP approach at integrand level** — van Hameren, Ossola, Papadopoulos, Pittau

Left: W + 3 jets at the Tevatron, comparison to CDF data
Right: W^+ + 3 jets at the LHC (14 TeV)



BlackHat + Sherpa: CFB, Bern, Dixon, Forde, Febres Cordero, Gleisberg, Ita, Kosower, Maitre

Excellent Scaling with External Legs

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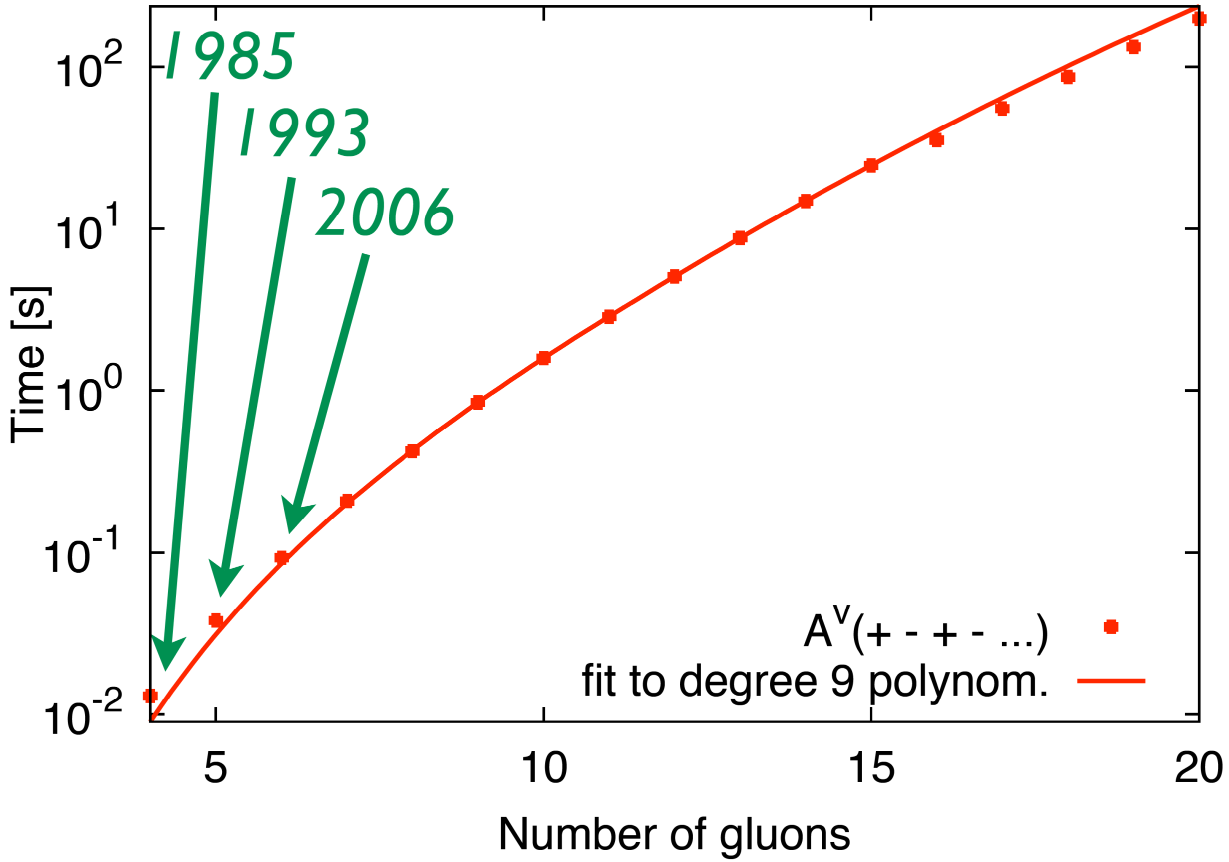


Giele, Zanderighi

Implications

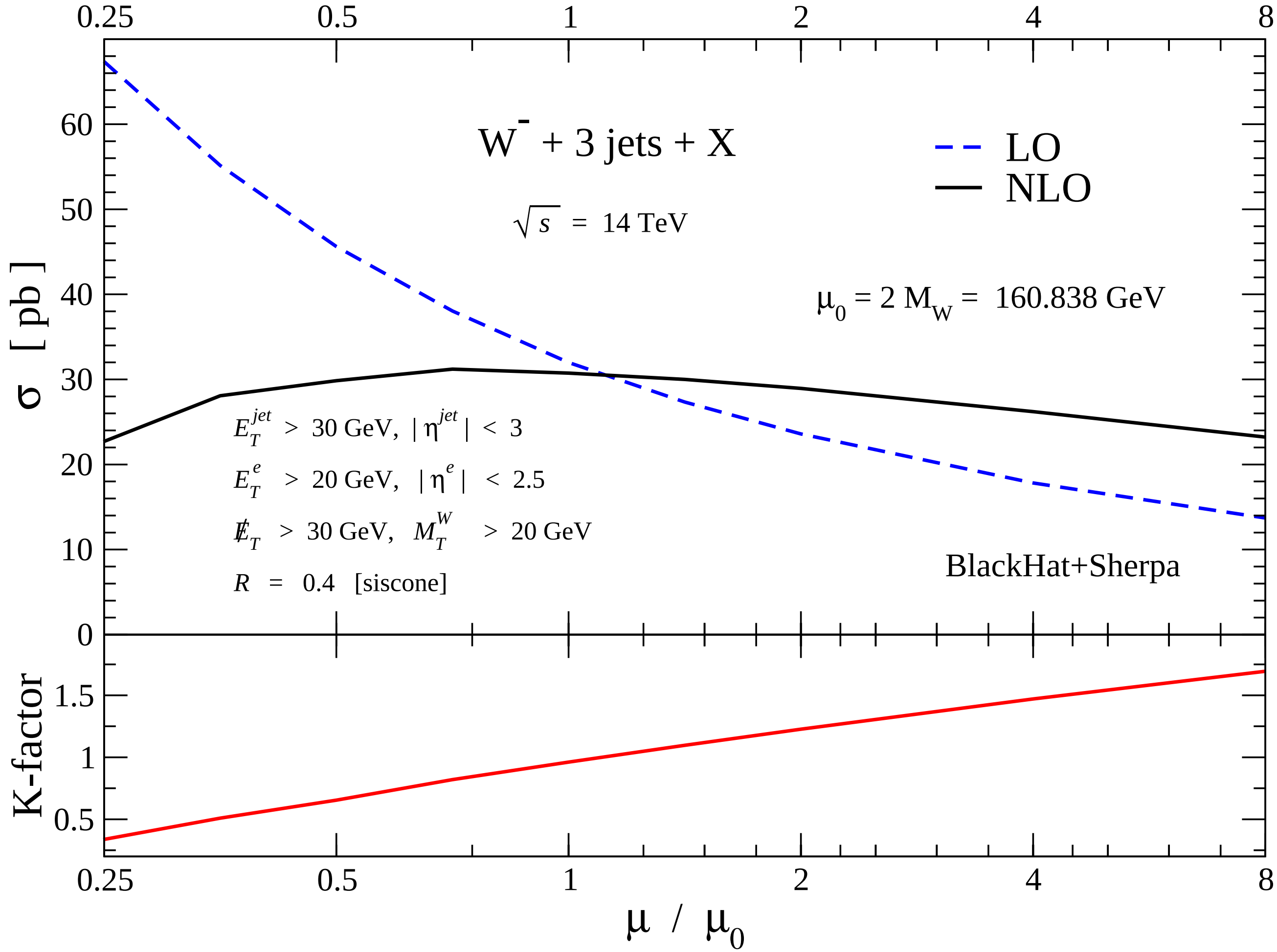
Lessons Learned from NLO: K-Factors

	Typical scales		Tevatron K-factor			LHC K-factor		
Process	μ_0	μ_1	$\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$	$\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$
W	m_W	$2m_W$	1.33	1.31	1.21	1.15	1.05	1.15
W+j	m_W	$p_T^{\rm j}$	1.42	1.20	1.43	1.21	1.32	1.42
W+jj	m_W	$p_T^{\rm j}$	1.16	0.91	1.29	0.89	0.88	1.10
WW+j	m_W	$2m_W$	1.19	1.37	1.26	1.33	1.40	1.42
$t\bar{t}$	m_t	$2m_t$	1.08	1.31	1.24	1.40	1.59	1.48
$t\bar{t}$+j	m_t	$2m_t$	1.13	1.43	1.37	0.97	1.29	1.10
$b\bar{b}$	m_b	$2m_b$	1.20	1.21	2.10	0.98	0.84	2.51
H	m_H	$p_T^{\rm j}$	2.33	–	2.33	1.72	–	2.32
H+j	m_H	$p_T^{\rm j}$	2.02	–	2.13	1.47	–	1.90
H+jj	m_H	$p_T^{\rm j}$	–	–	–	1.15	–	–

- **Large color annihilation** (e. g. $gg \to H$) $\Rightarrow$ **large** K-factor
- **Addition of legs** in final state $\Rightarrow$ **smaller** K-factor

Lessons Learned from NLO: Scales I

Fixed scales are in general not a good idea



BlackHat + Sherpa: CFB, Bern, Dixon, Forde, Febres Cordero, Gleisberg, Ita, Kosower, Maitre

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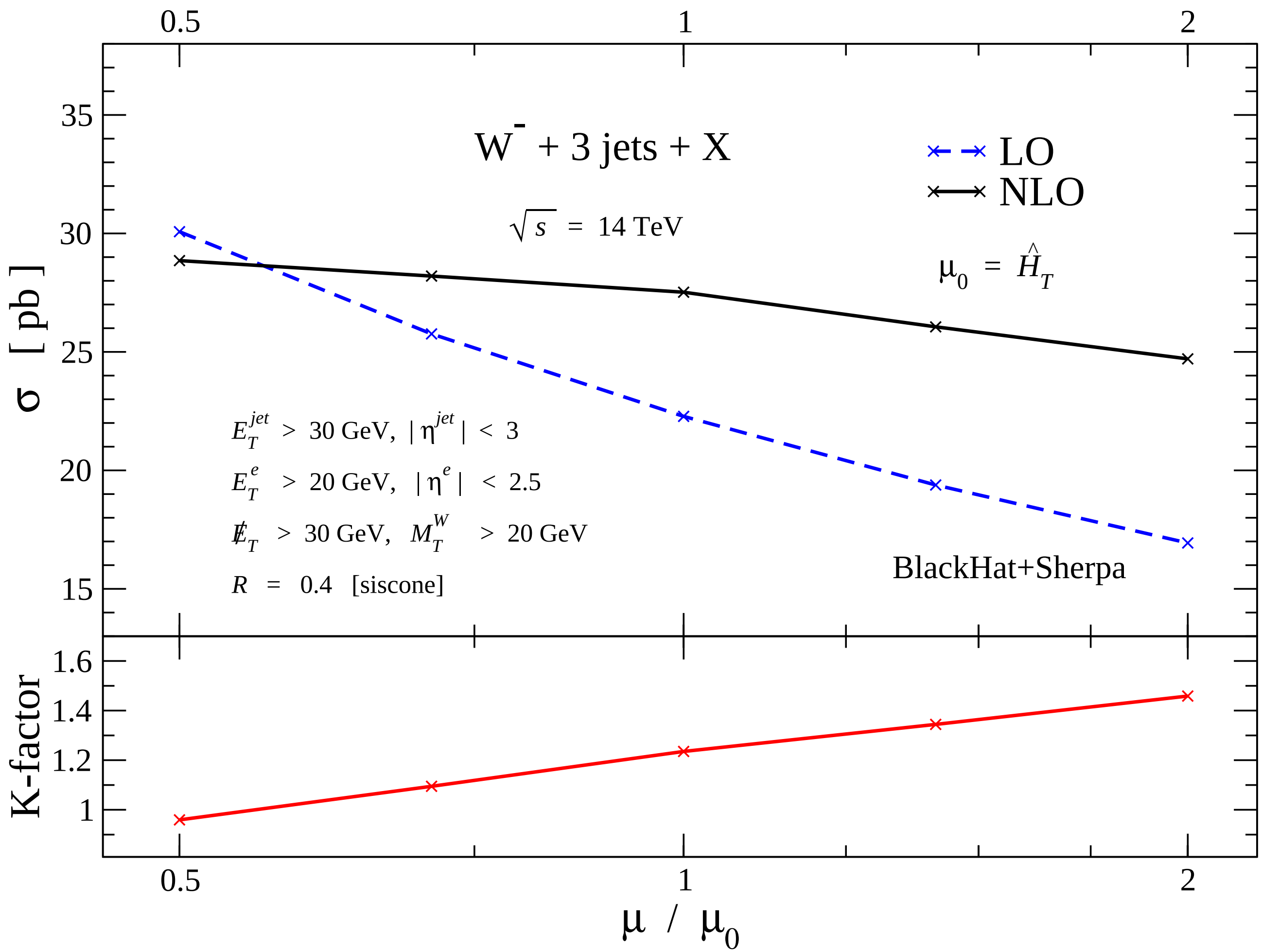

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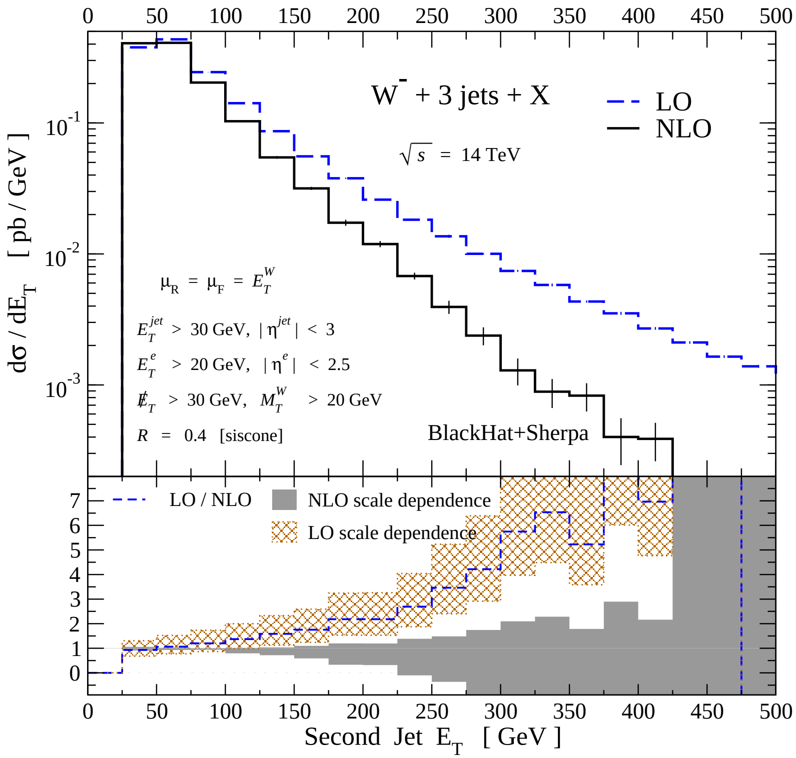
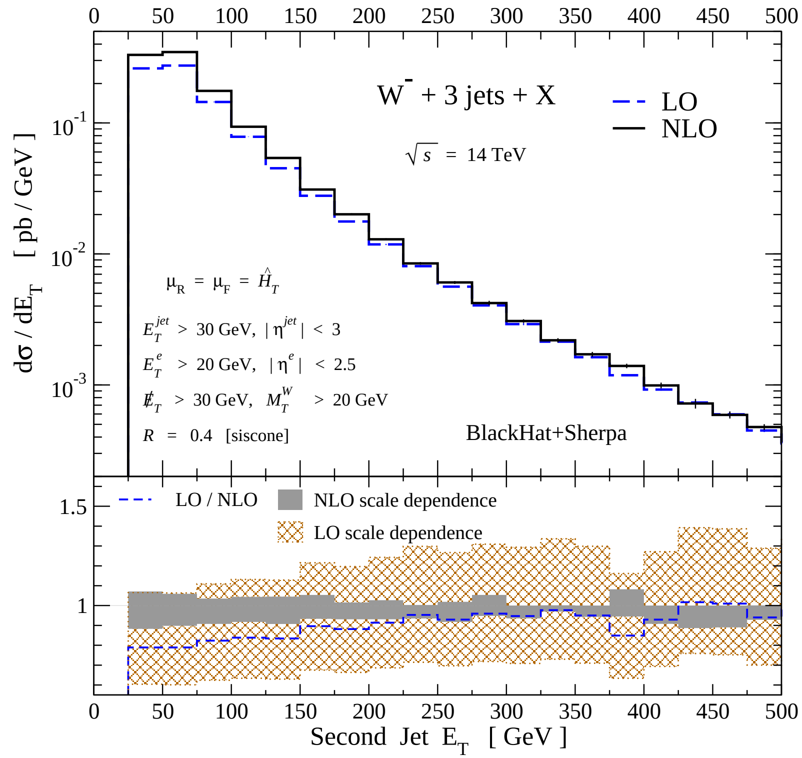
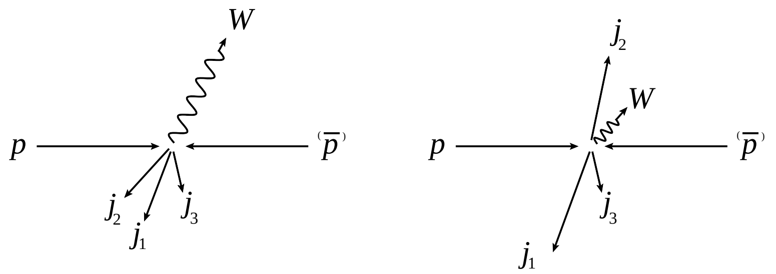
Lessons Learned from NLO: Scales II

Not every dynamical scale is created equal





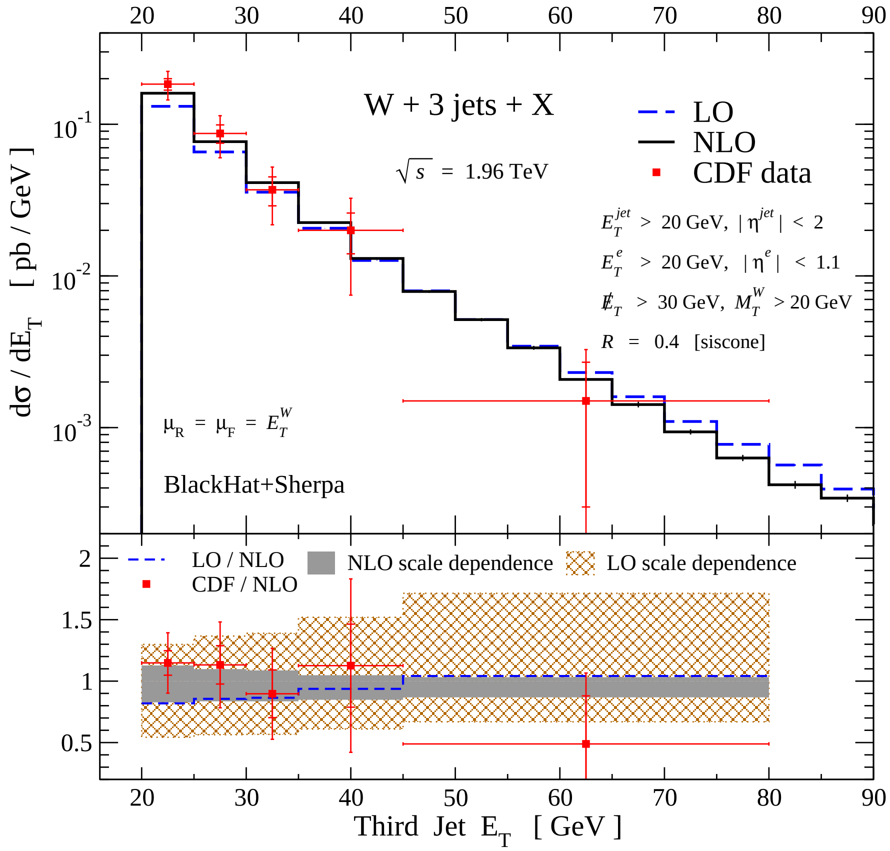


(a) (b)

BlackHat + Sherpa: CFB, Bern, Dixon, Forde, Febres Cordero, Gleisberg, Ita, Kosower, Maitre

Lessons Learned from NLO: IR Safety

This plot actually doesn't make sense:



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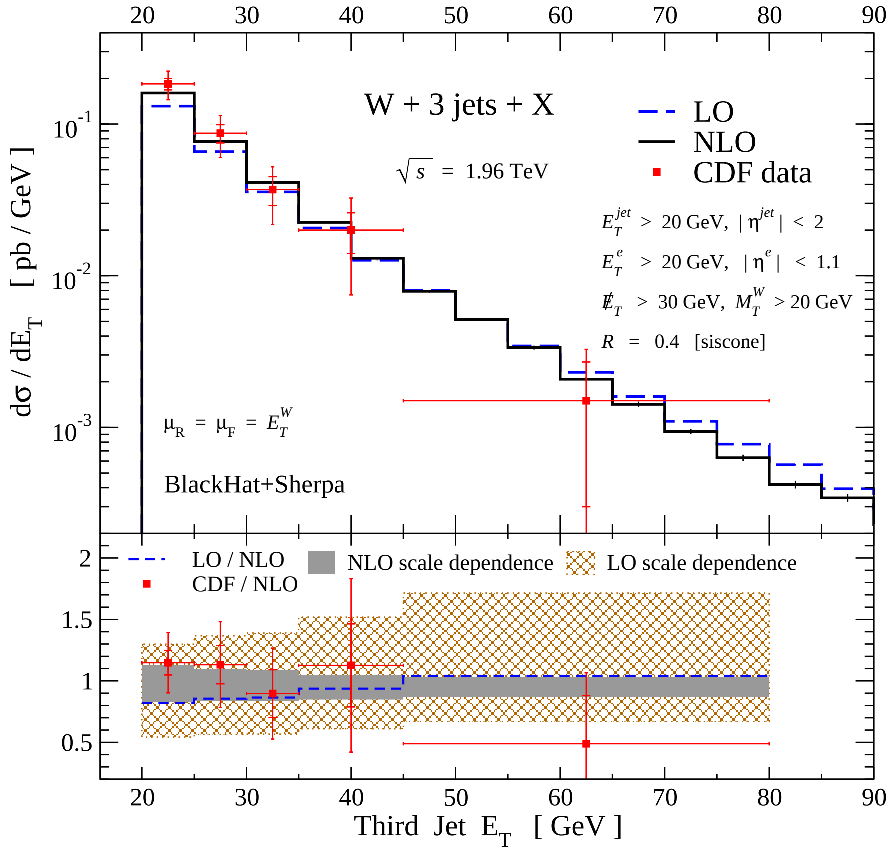
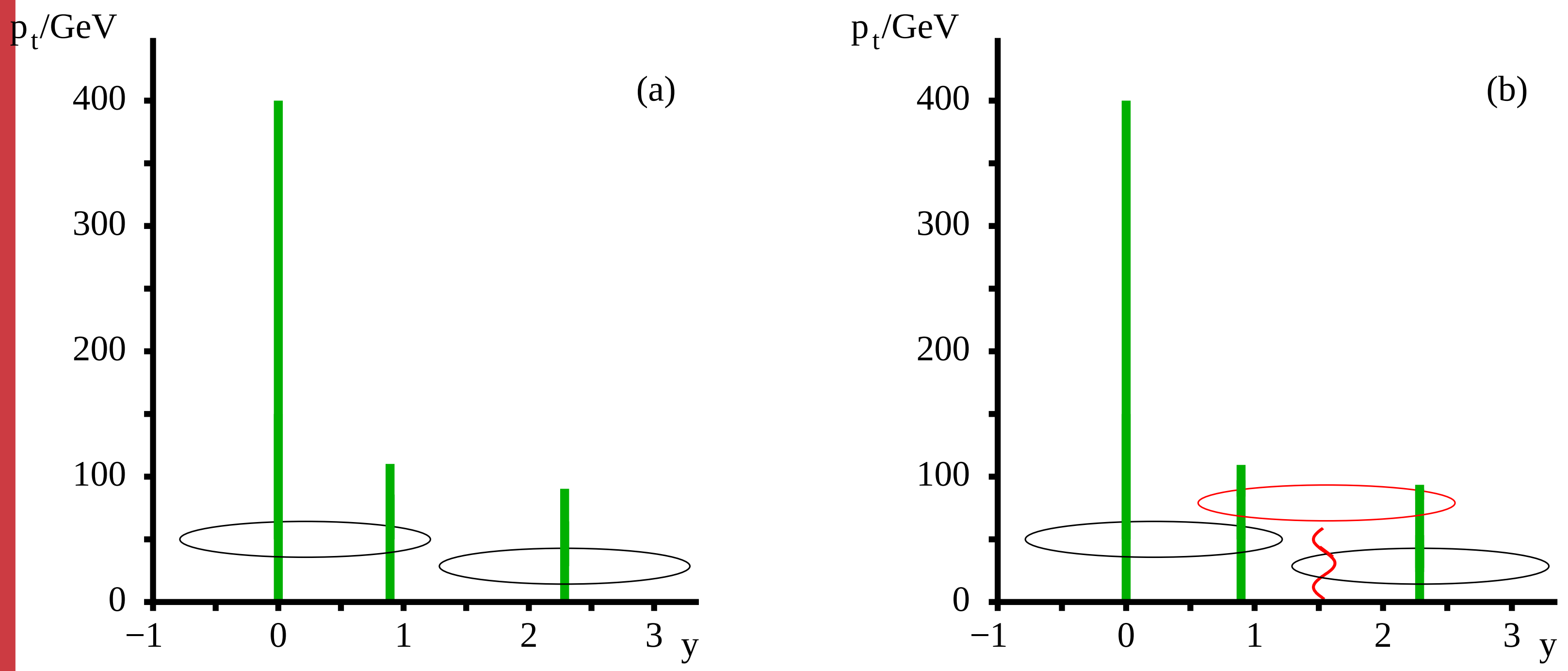


BlackHat + Sherpa: CFB, Bern, Dixon, Forde, Febres Cordero, Gleisberg, Ita, Kosower, Maitre

Comparison of infrared-unsafe JetClu (data) with infrared-safe SISCone (BlackHat+Sherpa)

Lessons Learned from NLO: IR Safety



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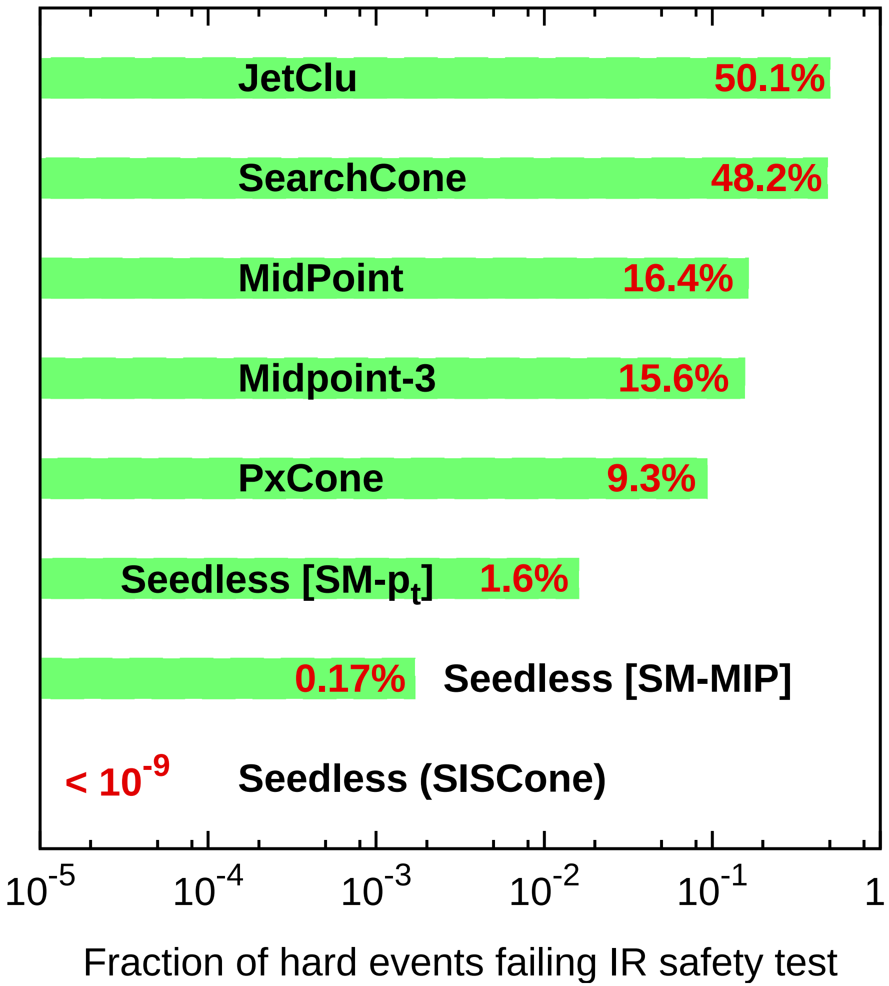

Salam, Soyez

Conclusions and Outlook

- **Progress at NNLO**
  fully differential distributions, several more new calculations soon to be completed
- **Tremendous progress at NLO**
  Feynman diagrams: first $2 \longrightarrow 4$ results
  New methods reuse amplitudes instead of Feynman diagrams via generalized unitarity and recursion, OPP reduction
- **General purpose NLO amplitude codes** being developed, progress toward agreement on common interface at Les Houches 2009
  $\Rightarrow$ event generators incl. parton showers at NLO?
- Lesson learned from NLO calculation for LO simulation: **choose your scale wisely!**
- **New jet algorithms**
  Whichever one you use, please choose an **infrared safe** one!

Omissions

- Parton Distribution Functions
- Shower algorithms, incl. at NLO
- All order conjecture for structure of infrared divergences of amplitudes
- Resummation
- Studies of jet substructure to identify heavy particles
- Omissions from the listed omissions

Outlook

Introduction

NNLO

NLO

Implications

Conclusions
- Conclusions and Outlook
- Omissions
- Outlook