4 SOLENOID HOMEWORK

1. Consider a 200 MeV/c particle starting on the axis with a transverse momentum of 20 MeV/c in an axial solenoidal field of 3.33 T.
   (a) What is its motion in the lab frame and out to what transverse distance from the axis does it get.
   (b) What is the distance along the axis before it first returns to that axis?
   (c) What is the wavelength λ in the Larmor frame?
   (d) what is the lattice parameter $\beta_\perp$ for that particle

Answers

(a) It's motion is helical with radius in the lab frame from eq.25 of $\rho = \frac{[pc/e]_\perp}{B\ c} = \frac{20\ 10^6}{3\ 10^8\ 3.33} = 2\ 10^{-2}$ m, or 2 cm. The helix pitch angle is 0.1 radians

Its furthest out will be at $2 \times \rho = 4$ cm.

(b) This distance is the helix period in the lab frame from eq.30 $\lambda_{\rm helix} = 2\pi\rho\,\frac{p_z}{p_\perp} = 2\ \pi\ 0.02\ \frac{200}{20} = 1.256$ m

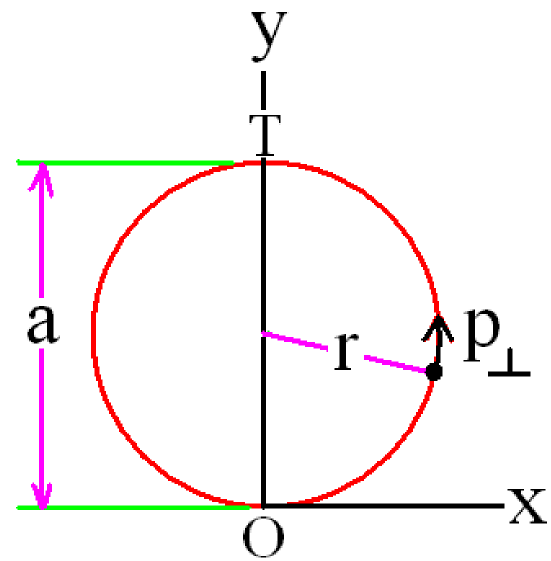
(c) From eq.31 $\lambda_{\rm Larmor} = 2\ \lambda_{\rm helix} = 2.512$ m

(d) From eq.32: $\beta_\perp = \frac{\lambda_{\rm Larmor}}{2\pi} = \frac{2\ [pc/e]_z}{B\ c} = 0.4$ m

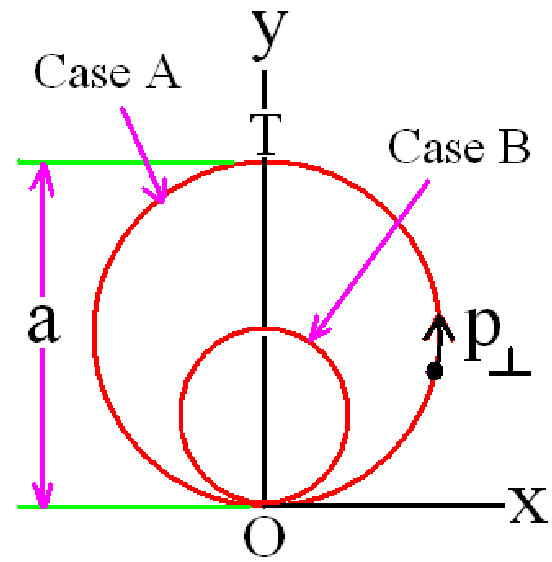
2. Consider again a 200 MeV/c particle starting on the axis with a transverse momentum of 20 MeV/c in an axial solenoidal field of 3.33 T. After a distance
A) corresponding to 1/2 a helix rotation, or
B) corresponding to a full helix rotation determine,
the field abruptly doubles to 6.66 T. In the two cases determine:

(a) The shape of the motion projected onto the x, y plane

(b) The following length in z for one helix rotation (λ_{helix})



Answers



(a) In case A) the field doubles as the muon is at the top (T) of its rotation. At that point the particle receives a kick (eq.19) of

$$\Delta p_{\perp} \;=\; ac\frac{\Delta B_z}{[pc/e]_z} \;=\; 20MeV/c$$

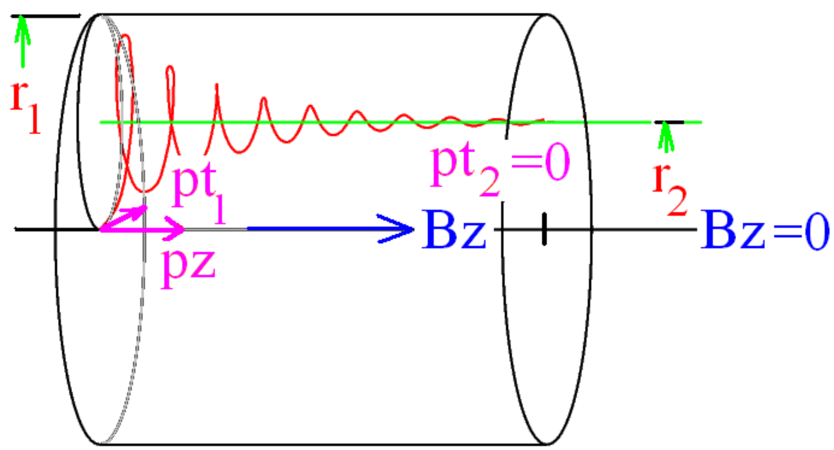
so now $p_{\perp} \;=\; 2\times\; 20 \;=\; 40$ MeV/c
At the same time the axial field doubles, so the transverse bending radius remains the same $=$ 2 cm

In case B The field doubles when the particle is on the magnet axis and there is no transvers momentum kick, and the transverse momentum remains at 20 MeV/c. Now with double the axial field, the transverse radius is halved to 1 cm.

(b) For both cases A and B, the longitudinal momentum is approximately unchanged, and the wavelengths (eq.30) $\propto\; 1/B_z$ the helix wavelength is halved to $1.256/2 \;=\; 0.628$ m.

7 IONIZATION COOLING HOMEWORK

1. Again consider a solenoid with $B_z = 3.33$ T and a muon with starting on axis with $p_t = 20$ MeV/c and $p_z = 200$ MeV/c. Imagine an ideal transverse cooling system with continuous energy loss and re-acceleration so that all transverse momenta are reduced to near zero, then the above particle will settle at half its maximum distance from the axis $r_2 = r_1/2$ and pass straight down the field lines at p_z.



(a) What now is its motion in the Larmor frame?

(b) If now the field B_z suddenly stops, what is the further motion of the muon?

(c) Taking the initial phase space to be $\Phi_1 = pt_1\ r_1$, what is the final phase space if it is defined as $\Phi_2 = pt_3\ r_3$?

(d) This (from Bush's Law) is not good. What could one do to avoid it ?

Answers

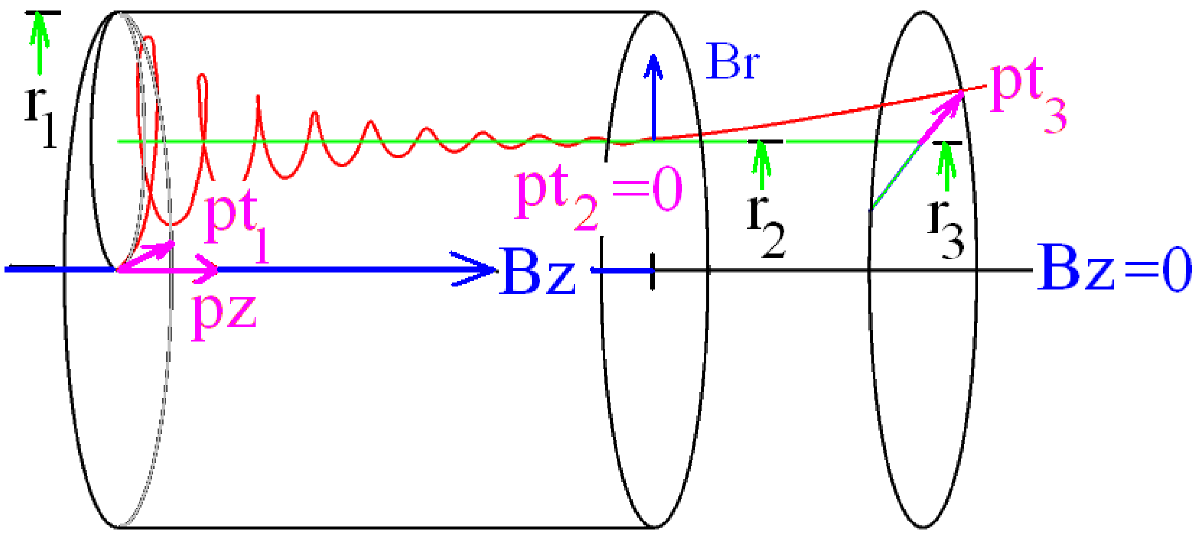
(a) The motion in the Larmor plane will be a helix about the axis with radial amplitude $r_2 = r_1/2$, wavelength 2.5 m, and transverse momentum of pt

(b) When it leaves the magnet the muon will see radial field components that will give it a lateral kick

$$[ptc/e]_3 = \Delta[ptc/e] = +\frac{r_2\ c\ B_z}{2} = \frac{[ptc/e]_1}{2} = 10\ \mathrm{MeV/c}$$

Thus $\epsilon_3 = \frac{\epsilon_1}{2}$ not a very good gain.

(c) The problem arises because the cooling reduces the lab angular momentum inside the solenoid. This causes the canonical angular momentum to grow (eq.23). Periodic reversals of B_z during the cooling avoids a build up of canonical angular momentum



2. Assume a transverse cooling channel with, at the absorber, $\beta_\perp = 0.18$ m, $\beta_v = v/c = 0.85$, and the dimensionless constant defined by eq.35: $C(mat, E) = 45\ 10^{-4}$, and emittance $\epsilon = 2.2$ (pi mm rad). These are the values for a lattice given in Feasibility study 2. The value of $C(mat, E)$ differs somewhat from the value for pure liquid hydrogen ($C \approx 35\ 10^{-4}$) because of the effect of the hydrogen and rf windows.

   a) calculate the theoretical equilibrium emittance.

   b) calculate the final differential rate of cooling $d\epsilon_\perp/\epsilon_\perp$ compared with its ideal value dp/p.

   c) This rate would be improved if the lattice parameter $\beta_\perp$ were lowered. What is the likely problem that will be encountered if this approach were followed?

answers

a) calculate the theoretical equilibrium emittance.

eq.37

$$\epsilon_{x,y}(min) \;=\; \frac{\beta_\perp}{\beta_v}\; C(mat, E)$$

so

$$\epsilon_\perp(\mathrm{min}) \;=\; \frac{45\ 10^{-4}\ 0.18}{0.85} \;=\; 950\ \ (\pi\ mm\ mrad)$$

b) calculate the final differential rate of cooling $d\epsilon_\perp/\epsilon_\perp$ compared with its ideal value dp/p.

eq.38

$$\frac{d\epsilon}{\epsilon}(\mathrm{end}) \;=\; \left(1 - \frac{\epsilon(\mathrm{min})}{\epsilon}\right)\ \frac{dp}{p} \;\approx\; 0.57\ \frac{dp}{p}$$

c) This rate would be improved if the lattice parameter $\beta_\perp$ were lowered. What is the likely problem that will be encountered if this approach were followed?

This rate can be improved if $\frac{\epsilon(\mathrm{min})}{\epsilon}$ is reduced by decreasing ϵ(min), and this can be accomplished by reducing the lattice parameter $\beta_\perp$. The main problem is that if one does this, is that the angular spread of the resulting beam, and thus required angular aperture rises because eq.39:

$$\sigma_\theta \;=\; \sqrt{\frac{\epsilon_\perp}{\beta_\perp\ \beta_v\gamma}}$$

3. In the longitudinal cooling section, we describe a cooling ring that, with emittance exchange in wedges cools all 6 dimensions. Assume $\beta_\perp = 0.4$ m, dispersion at the hydrogen wedge $D = 7$ cm, the length of the wedge on axis $\ell = 28.6$ cm, and the height from the axis to the apex of the wedge $h = \frac{\ell}{2\ \tan(100^o/2)} = 12 \quad \mathrm{cm}.$ Assume that the sum of partition functions $\Sigma J_i \approx 2.0$, $C(mat, E) = 38\ 10^{-4}$, and assume good mixing between x and y. As before, β_v=0.85.

   a) What are the three partition functions in this case?

   b) What is the expected equilibrium transverse emittance?

answer

a) What are the three partition functions in this case?

The longitudinal partition function is thus

$$J_z = \frac{D}{h} = 0.58$$

and since there is good mixing between x and y, $J_x = J_y$

From $\Sigma J_i \approx 2.0$:

$$J_x + J_y + 0.58 = 2.0$$

$$J_x = J_y \approx \frac{2 - 0.58}{2} = 0.71$$

i.e. The wedge angle gave somewhat less cooling (0.58) in the longitudinal direction than in the other two (.71) .

b) What is the expected equilibrium transverse emittance?

The equilibrium emittances, with values of $J_{x,y} \neq 1$ from eq.45

$$\epsilon_\perp(\mathrm{min}) = \frac{C\ \beta_\perp}{J_{x,y}\ \beta_v} = \frac{38\ 10^{-4}\ 0.4}{0.71\ 0.85} = 2.5\ \ (\pi\ mm)$$