Problems Lecture 3: Static and Dynamic Imperfections

1) A linac that transports a beam with very small vertical and large horizontal emittance suffers from quadrupole vibrations. All quadrupoles have the same jitter amplitude. Unfortunately due to budget limitations only some quadrupoles can be stabilised. How should the quadrupoles be selected for stabilisation? You can assume thin quadrupoles.

2) A transport line consists of a FODO lattice with quadrupole spacing L_0 and focal strength f_0. The line is rebuilt changing the quadrupole spacing to $L = 2L_0$ and the focal strength to $f = 2f_0$. How does the quadrupole jitter tolerance σ_q change?

3) A transport line with no acceleration is built from FODO cells with 90° phase advance. The first two focusing quadrupoles are used as correctors, in the centres of the next two focusing quadrupoles BPMs are located. Write the response matrix for the correctors (thin lens approximation).

Solutions

1) We need to stabilise the beam in the vertical plane since the emittance is much smaller in this plane. The angular deflection by a quadrupole with offset d is

$$\Delta y' = \frac{d}{f}$$

We want to minimise the deflection in the normalised phase space

$$\Delta y'_N = \frac{\Delta y'}{\sqrt{1/(\beta\gamma)}}$$

$$\Rightarrow \Delta y'_N = \frac{\sqrt{\beta\gamma}}{f} d$$

Hence we have to chose the quadrupoles with the largest values

$$\frac{\sqrt{\beta\gamma}}{f}$$

Solutions

2) Since the ratio of focal length to quadrupole spacing remains constant ($f/L = f_0/L_0$) the phase advance per cell remains constant. Therefore the beta-function is now twice the original value $\beta = 2\beta_0$. The multi-pulse emittance growth due the jitter for one quadrupole is

$$\Delta\epsilon \propto \frac{\beta}{f^2}\sigma_q^2$$

The emittance growth per length of the line is given by

$$\frac{\Delta\epsilon}{L} \propto \frac{\beta}{f^2 L}\sigma_q^2$$

$$\rightarrow \frac{\beta}{f^2 L}\sigma_q^2 = \frac{\beta_0}{f_0^2 L_0}\sigma_{q,0}^2$$

from the scaling we calculate

$$\sigma_q = 2\sigma_{q,0}$$

Solutions

3) We can calculate the response matrix R by going into normalised phase space and out again. The kick of moving a quadrupole by δ is

$$\Delta y' = \frac{\delta}{f}$$

in normalised phase space

$$\Delta y'_N = \sqrt{\beta}\frac{\delta}{f}$$

we consider that the phase advance of the first corrector to the first BPM is 180° and two the second 270°. Hence we find at these locations

$$\Delta y_N = 0$$

and

$$\Delta y_N = \sqrt{\beta}\frac{\delta}{f}$$

respectively. We transfor back into normal phase space and find

$$\Delta y = 0$$

and

$$\Delta y = \beta\frac{\delta}{f}$$

Similarly for the second corrector. Hence we find

$$R = \begin{pmatrix} 0 & -\frac{\beta}{f} \\ \frac{\beta}{f} & 0 \end{pmatrix}$$

Problems Lecture 4: Multi-Bunch Effect and Parameter Optimisation

1) In a linac with a very long bunch train the longrange wakefield of each bunch only acts on the next-to-next bunch with $a_2 = 1$.

- Please calculate $A_{1,k}$

Solutions

1) The calculation is performed in the same way as for the wakefield kick on only the next following bunch. We have two trains that do not interact with each other. With $a_2 = 1$, $a_{k \neq 2} = 0$ and the Taylor series for the exponential

$$A = \exp(a) = \sum_{k=0}^{n-1} \frac{a^k}{k!}$$

we find

$$A_{1,2k} = \frac{(ia_2)^k}{k!} \qquad A_{1,2k+1} = 0$$