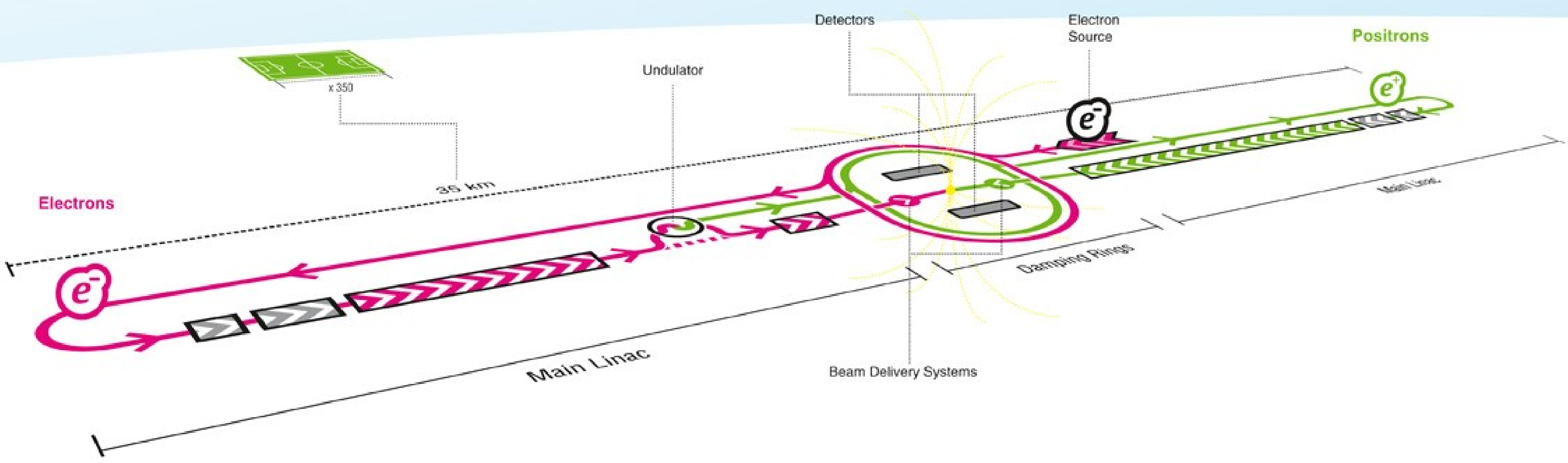
Electron source for Linear Colliders

KURIKI Masao (Hiroshima/KEK)



Contents

- Electron Emission
- Related Physics Process
- Electron Gun
- Electron Source for Linear Colliders
- Laser
- Summary

Electron Emission

Related Physics

Electron Gun

e- Source for LCs

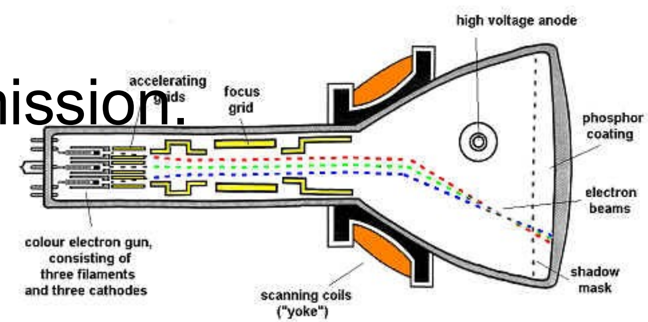
Laser

Summary

Electron Emission

History of Electron Source (1)

- 1883: T. A. Edison discovered *Edison effect ;* electric current was detected between the positively biased electrode and the filament in a bulb.
- 1897: K. F. Brown developed CRT (Cathode Ray Tube).
- 1899:J. J. Thomson identified that Edison effect is caused by the thermal electrons.
- 1905: A. Einstein published the photo-electron hypothesis.
- 1928: J. R. Oppenheimer predicted the field emission.
- 1934: E. Mueller confirmed the field emission.



History (2)

Electron Emission
Related Physics
Electron Gun
e- Source for LCs
Laser
Summary

- 1939: Pierce and Wehnelt invented Pierce Gun, which generates the parallel static electron current with the diode structure.
- 1966: 1st Linac (SLAC) operation with Pierce Gun was started; thermionic DC gun.
- 1970': Polarized electron gun was developed; photo-cathode DC Gun.
- 1980': R&D for RF Gun concept has been started (still now) ; photo-cathode, thermionic RF gun.

Electron Emission (1)

- Thermal electron emission : Electron emission from the heated material (typically 1000 - 3000K).
- Field emission: Emission from the high field gradient surface.
- Photo-electron emission: Emission by photo-electron effect.
- Secondary electron emission: Emission induced by electron absorption.

Electronic States

- Electron Emission
- Related Physics
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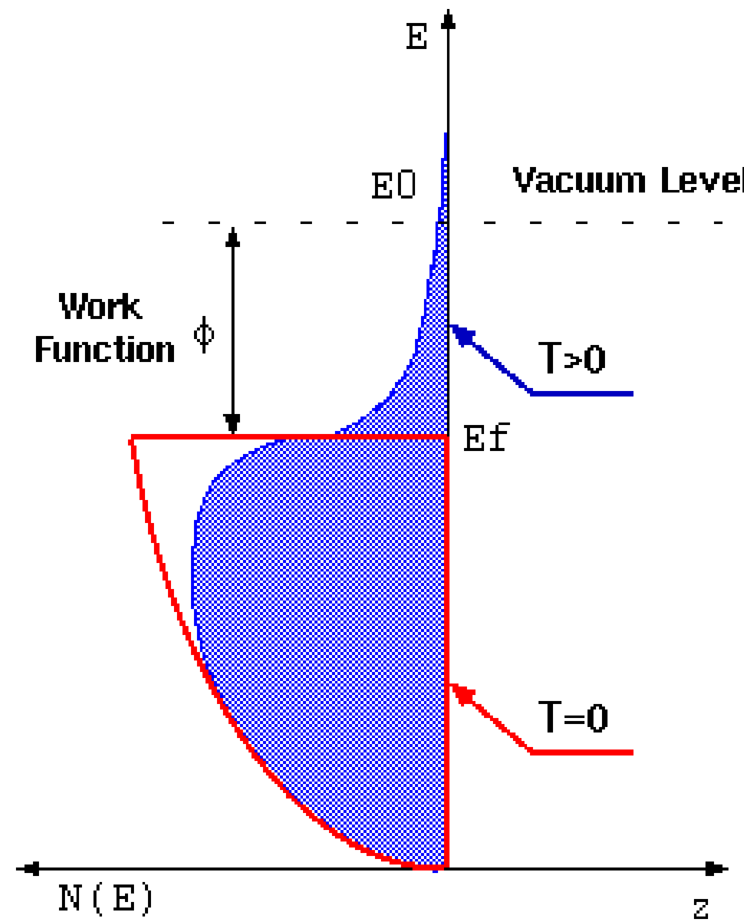
- e- Source for LCs
- Laser
- Summary

- Electrons in a metal are confined in a well potential and distributed according to Fermi-Dirac Distribution.
- T=0: Electrons occupy the energy states up to Fermi-level (Fermi energy, E_f).
- T>0: Electron distribution extends to higher energy state due to the thermal energy.



Electronic States (2)

Electron density in a metal is product of state density D(ε) and distribution function f(ε),

$$n(\epsilon) = D(\epsilon) f(\epsilon) \qquad (1-1)$$

State density in phase space $(x, v_x) - (x+dx, v_x+dv_x)$.... is

$$D(\epsilon) = \frac{2m^2}{h^3} dx\, dy\, dz\, dv_x\, dv_y\, dv_z \qquad (1-2)$$

Distribution function f(ε) is given by Fermi-Dirac function

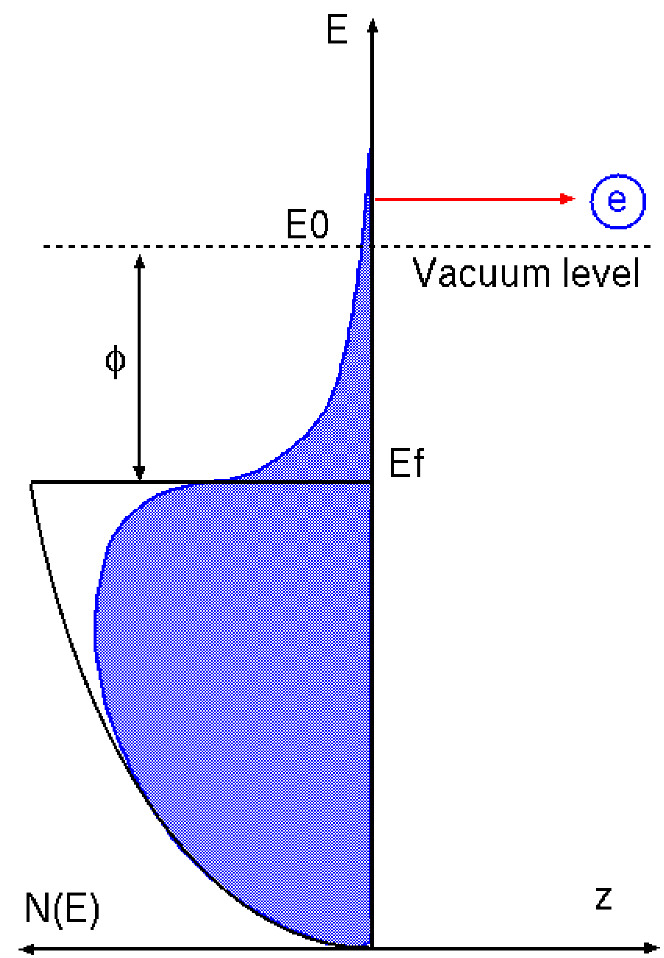
$$f(\epsilon) = \frac{1}{\exp\left(\frac{\epsilon - \mu}{kT}\right) + 1} \qquad (1-3)$$

Number of electron with energy ε<E is

$$N(\epsilon) = \int_0^E f(\epsilon) D(\epsilon) d\epsilon \qquad (1-4)$$

Thermal Electron Emission

- If the temperature is sufficiently high, so that the electrons are distributed up to the vacuum level (E_0), electrons escape out to the outside.
- The gap between the vacuum level and the Fermi energy is Work function, φ, which characterize the thermal emission.



Emission Density (1)

To obtain emission density from cathode, we have to integrate the electron density in a metal with proper conditions. In depth (z-direction), electrons have to be

$$z \leq v_z \Delta t \qquad (1-5)$$

where Δt is time duration. Kinetic energy for z-direction must be more than vacuum potential energy, μ+Φ, then

$$v_z \leq v_{vac} \equiv \sqrt{\frac{2(\mu+\phi)}{m}} \qquad (1-6)$$

Number of electron emitted from the cathode is give by

$$N = \int dx \int dy \int_0^{v_z \Delta t} dz \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z\, f(\epsilon) D(\epsilon) \quad (1-7)$$

Emission Density (2)

- **Electron Emission**
- Related Physics
- Electron Gun
- e- Source for LCs
- Laser
- Summary

By integrating x, y, z and inserting distribution function,

$$N=\Delta x \Delta y \Delta t \frac{2m^3}{h^3} \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z \frac{v_z}{\exp\left(\frac{\epsilon-\mu}{kT}\right)+1} \qquad (1-8)$$

From this equation, emission density per unit time is obtained

$$\sigma \equiv \frac{N}{\Delta x \Delta y \Delta t} = \frac{2m^3}{h^3} \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z \frac{v_z}{\exp\left(\frac{\epsilon-\mu}{kT}\right)+1} \qquad (1-9)$$

Emission Density (4)

Because ε-μ≫kT, the expression is approximated to be

$$\frac{1}{\exp\left(\frac{\epsilon-\mu}{kT}\right)+1} \sim \exp\left(\frac{\mu-\epsilon}{kT}\right) \qquad (1-10)$$

The density is simplified as

$$\sigma = \frac{2m^3}{h^3}\int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z\, v_z \exp\left(\frac{\mu-\epsilon}{kT}\right) \qquad (1-11)$$

Replacing the energy with the velocity,

$$\epsilon = \frac{m}{2}\left(v_x^2+v_y^2+v_z^2\right)$$

$$\sigma = \frac{2m^3}{h^3}\exp\left(\frac{\mu}{kT}\right)\int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{v_{vac}}^{+\infty} dv_z\, v_z \exp\left(\frac{-m\left(v_x^2+v_y^2+v_z^2\right)}{2kT}\right) \qquad (1-12)$$

Emission Density (5)

Integral for v_x and v_y can be performed as

$$\int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \exp\left(\frac{-m\left(v_x^2+v_y^2\right)}{2kT}\right) = \frac{2\pi kT}{m} \qquad (1-13)$$

and for v_z as

$$\int_{v_{vac}}^{+\infty} dv_z\, v_z \exp\left(\frac{-mv_z^2}{2kT}\right) = \frac{kT}{m} \exp\left(\frac{-mv_{vac}^2}{2kT}\right) \qquad (1-14)$$

Integrating those formulae, we obtain

$$\sigma = \frac{4\pi m k^2 T^2}{h^3} \exp\left(-\frac{\phi}{kT}\right) \qquad (1-15)$$

Electric current density J is given by

$$J = \frac{4\pi e m k^2 T^2}{h^3} \exp\left(-\frac{\phi}{kT}\right) \qquad (1-16)$$

Richardson-Dushman Equation

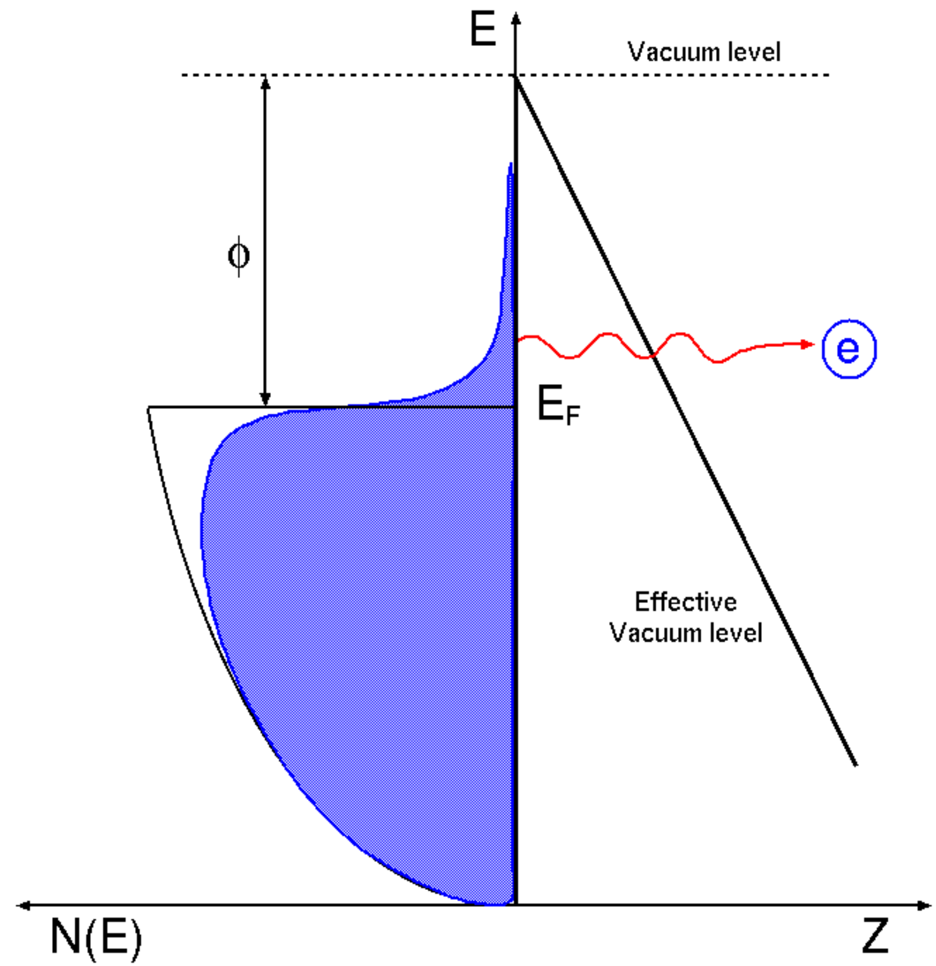
$$J = AT^2 e^{-\frac{\phi}{kT}} \qquad (1-17)$$

$$A = \frac{4\pi emk^2}{h^3} = 1.20\times 10^6 [A/m^2 K^2]$$

A : thermionic emission constant
T: Temperature (K)
k : Boltzmann constant ; 1.38E-23 (J/K)
e : electronic charge
m : electron mass
h : Plank constant ; 6.63E-34 (Js)

Field Emission

- With large surface field, the potential barrier to the outside becomes very thin.
- When the field is more than 1E+8 V/m, the tunnel current becomes significant.
- Because of the emission at the cold temperature, it is called sometimes as cold emission.

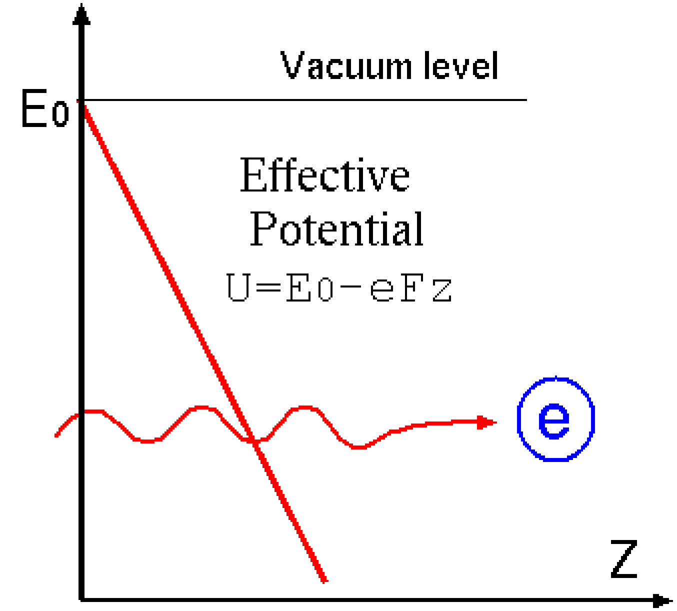


Field Emission (2)

Field emission density is calculated as

$$J = e\int_0^{\infty} n(\epsilon_z)\,P(\epsilon_z)\,d\epsilon_z \qquad (1-18)$$

where P(ε) is transmission probability by tunneling effect. P(ε) is obtained by WKB method



$$P(\epsilon_z, F) = \exp\left[-\int_0^{w}\sqrt{\frac{8\mathrm{m}(2\pi)^2}{h^2}\{U(z)-\epsilon_z\}}\,dz\right]$$

$$= \exp\left[\frac{-8\pi\sqrt{2\mathrm{m}}}{3\mathrm{heF}}(E_0-\epsilon_z)^{3/2}\right] \qquad (1-19)$$

where $w=(E_0-\epsilon_z)/eF$ and effective vacuum potential U is

$$U(z) = E_0 - eFz \qquad (1-20)$$

Field Emission (2)

Electron Emission

Related Physics

Electron Gun

e- Source for LCs

Laser

Summary

By Taylor expansion,

$$(E_0-\epsilon_z)^{3/2}=\left[\phi+(\mu-\epsilon_z)\right]^{3/2}=\phi^{3/2}+\frac{3}{2}\phi^{1/2}(\mu-\epsilon_z) \qquad (1-21)$$

In the low temperature limit, the current density is

$$J(F)=\frac{4\pi em}{h^3}\int_0^\infty d\epsilon_z(\mu-\epsilon_z)\exp\left[-8\pi\frac{\sqrt{2\mathrm{m}}}{3\mathrm{heF}}(E_0-\epsilon_z)\right]$$

$$=\frac{4\pi em}{h^3}\exp\left(\frac{-8\pi\sqrt{2\mathrm{m}}}{3\mathrm{heF}}\phi^{3/2}\right)\int_0^\infty d\acute{\epsilon}\,\acute{\epsilon}\exp\left[-4\pi\frac{\sqrt{2\mathrm{m}}}{heF}\phi^{1/2}\acute{\epsilon}\right] \qquad (1-22)$$

where ε'=ϵ_z-μ. The integral can be performed easily and we get

$$J=\frac{e^3F^2}{8\pi\phi}\exp\left(\frac{-8\pi\sqrt{2\mathrm{m}}}{3\mathrm{heF}}\phi^{3/2}\right) \qquad (1-23)$$

Fowler-Nordheim Formula

- Electron Emission
- Related Physics
- Electron Gun
- e- Source for LCs
- Laser
- Summary

$$J = \frac{e^3 F^2}{8 h \pi \phi} \exp\left(\frac{4\sqrt{2m}}{3 h e F} \phi^{3/2}\right) \qquad (1-24)$$

Introducing field enhancement factor κ , which means local field enhancement by surface condition,

$$J = \frac{e^3 \kappa^2 F^2}{8 h \pi \phi} \exp\left(\frac{4\sqrt{2m}}{3 h e \kappa F} \phi^{3/2}\right) \qquad (1-25)$$

Taking ln(J/F^2) and plotting as a function of 1/F,

$$\ln\left(J/F^2\right) = \ln\left(\frac{e^3 \kappa^2}{8 h \pi \phi}\right) + \left(\frac{4\sqrt{2m}}{3 h e \kappa} \phi^{3/2}\right) \frac{1}{F} \qquad (1-26)$$

The field enhancement factor κ is extracted from the gradient of this plot, Fowler-Nordheim plot.

Field Enhancement Factor

$$\ln\left(J/F^2\right)=\ln\left(\frac{e^3\kappa^2}{8h\pi\phi}\right)+\left(\frac{4\sqrt{2m}}{3he\kappa}\phi^{3/2}\right)\frac{1}{F} \qquad (1-26)$$

- Field enhancement factor is average of local field enhancement by surface condition.
- It is a measure to qualify the flatness of the surface of RF cavity.

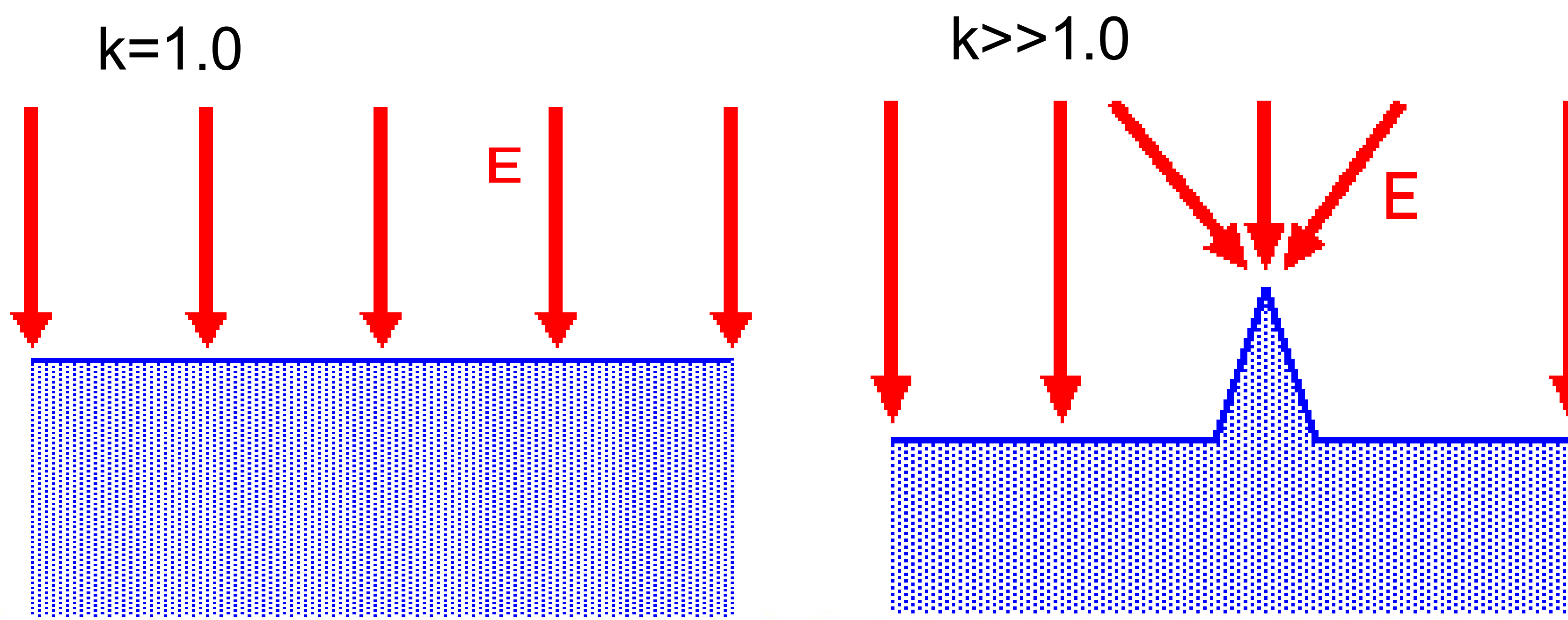
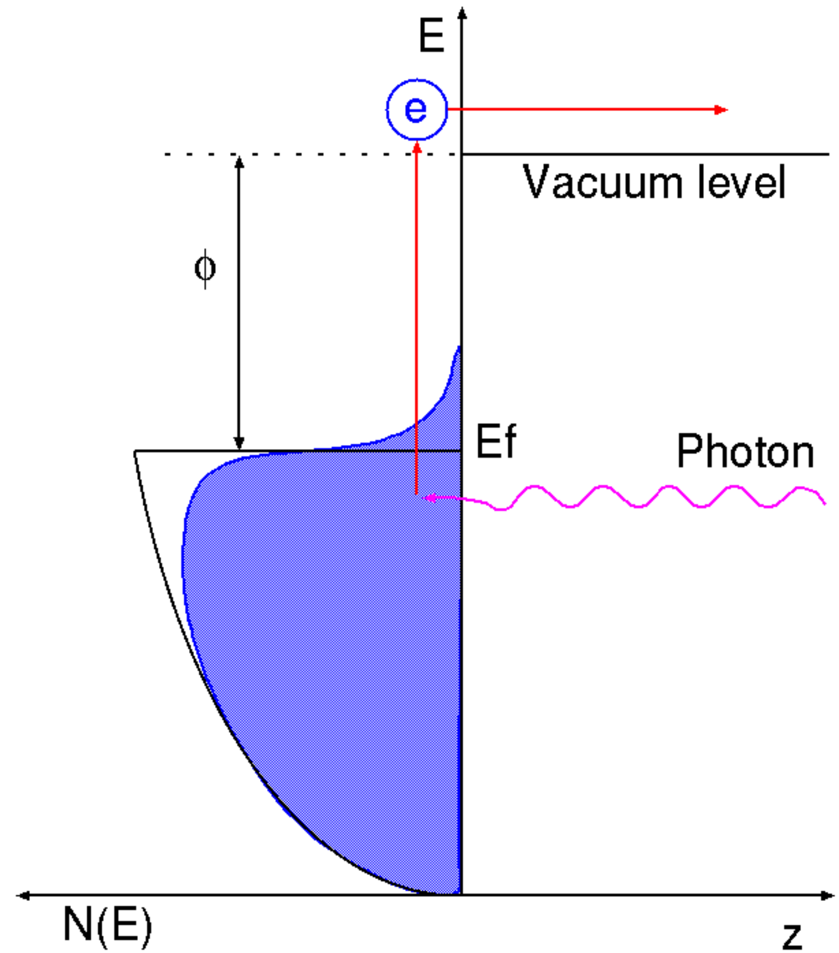


Photo-electron Emission

- Photons excite electrons into higher energy states.
- If the states are higher than the vacuum level, the excited electrons are extracted as the photo-electrons; Photo-electron effect.
- In the lower temperature limit, the electron stay below Fermi-energy and condition for photo-emission is $h\nu \geq \phi$.



Emission Densiy (1)

By similar manipulation with thermionic emission, the photo-electron current density is given by

$$J=\frac{4\pi emkT}{h^3}P\int_{E_0-h\nu}^{\infty}d\epsilon_z \ln\left[1+\exp\frac{(\mu-\epsilon_z)}{kT}\right] \qquad (1-27)$$

where P is transition probability by photon excitation. Note that the lower limit of the integral is E_0-hv instead of E_0.

$$J=\frac{4\pi emkT}{h^3}P\int_{E_0-h\nu}^{\infty}d\epsilon_z \ln\left[1+\exp\frac{(\mu-\epsilon_z)}{kT}\right] \qquad (1-28)$$

In addition, we can not use the approximation in the thermionic case since ε_z can be less than μ . For further manipulation, replacing $y=(\varepsilon_z+h\nu-E_0)/kT$ and $\delta=h(\nu-\nu_0)/kT$,

$$J=\frac{4\pi em k^2 T^2}{h^3}P\int_{0}^{\infty}dy \ln[1+\exp(\delta-y)] \qquad (1-29)$$

Emission Density (2)

Electron Emission
Related Physics
Electron Gun
e- Source for LCs
Laser
Summary

$$J = \frac{4\pi e m k^2 T^2}{h^3} P \int_0^\infty dy \ln\left[1 + e^{\delta - y}\right]$$
$$= \frac{4\pi e m k^2 T^2}{h^3} P f(\delta) \qquad (1-30)$$

When $\delta = h(\nu - \nu_0)/kT < 0$ (photon energy is less than the work function), the formula is expanded as

$$f(\delta) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{e^{n\delta}}{n} \int_0^\infty dy\, e^{-ny}$$
$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{e^{n\delta}}{n^2} \qquad (1-31)$$

since the expansion with small x

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad (1-32)$$

it leads the following formula by substituting $x = e^{\delta - y}$,

$$\ln(1 + e^{\delta - y}) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{e^{n(\delta - y)}}{n} \qquad (1-33)$$

Emission Density (3)

When $\delta = h(\nu-\nu_0)/kT > 0$ (photon energy is more than the work function), the integral is separated to two region,

$$f(\delta) = \int_0^{\delta} dy + \int_{\delta}^{\infty} dy \left[\ln\left(1+e^{\delta-y}\right)\right] \qquad (1-34)$$

for first integral, we replace the variable as $w=\delta-y$

$$\int_0^{\delta} dy \ln\left(1+e^{\delta-y}\right) = \int_0^{\delta} dw \ln\left(1+e^{w}\right)$$

$$= \int_0^{\delta} dw \left\{ w + \ln\left(1+e^{-w}\right) \right\}$$

$$= \left[\frac{w^2}{2}\right]_0^{\delta} + \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2} \left[e^{-nw}\right]_0^{\delta}$$

$$= \frac{\delta^2}{2} + \frac{\pi^2}{12} + \sum_{n=1}^{\infty} (-1)^n \frac{e^{-n\delta}}{n^2} \qquad (1-35)$$

Emission Density (4)

for second integral, changing the variable as w=y-δ

$$\int_{\delta}^{\infty} dy \ln\left(1+e^{\delta-y}\right)=\int_{0}^{\infty} dw \ln\left(1+e^{-w}\right) \qquad (1-36)$$

take the partial integral

$$\int_{0}^{\infty} dw \ln\left(1+e^{-w}\right)=\left[w \ln\left(1+e^{-w}\right)\right]_{0}^{\infty}+\int_{0}^{\infty} dw \frac{w}{1+e^{w}} \qquad (1-37)$$

the first term of rhs is 0 and the second term is

$$\int_{0}^{\infty} dw \frac{w}{1+e^{w}}=\frac{\pi^{2}}{12} \qquad (1-38)$$

Finally, f(δ) is calculated to be sum of these two integrals

$$f(\delta)=\frac{\delta^{2}}{2}+\frac{\pi^{2}}{6}+\sum_{n=1}^{\infty}(-1)^{n} \frac{e^{-n\delta}}{n^{2}} \qquad (1-39)$$

Fowler Equation

$$J = AT^2 P f(\delta) \qquad (1-40)$$

$$A = \frac{4\pi e m k^2}{h^3}$$

$$f(\delta) = \begin{cases} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{e^{n\delta}}{n^2} & \delta < 0 \\ \frac{\delta^2}{2} + \frac{\pi^2}{12} + \sum_{n=1}^{\infty} (-1)^n \frac{e^{-n\delta}}{n^2} & \delta > 0 \end{cases}$$

- Fowler equation gives photo-current spectrum. By taking the spectrum and fit to the curve, work function can be extracted.
- However, the absolute density is hard to estimate with this formula because *P* strongly depends on the surface optical condition.

Quantum Efficiency

▶ Practically, Quantum Efficiency, η, is defined as

$$\eta = \frac{number\ of\ \ photo\, electrons}{number\ of\ \ photons} \qquad (1-41)$$

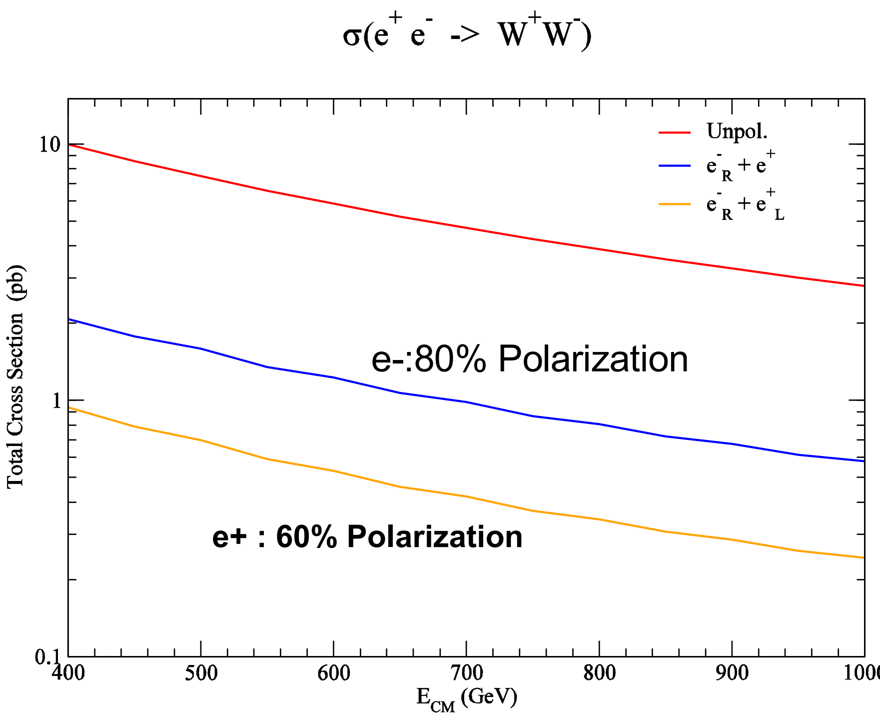
It qualifies the emission density per number of electron. With practical units, it is

$$\eta[\%] = 124\frac{J[nA]}{P[\mu W]\lambda[nm]} \qquad (1-42)$$

Polarized Electron (1)

- Electron has spin ½ and two states.
  - **Right handed electron e_R (spin ½)**
  - **Left handed electron e_L (spin - ½)**
- In e+e- collider, WW-scattering is the biggest background.
- Polarized electron (and also positron) can compensate this background very small.
- Polarization is defined as

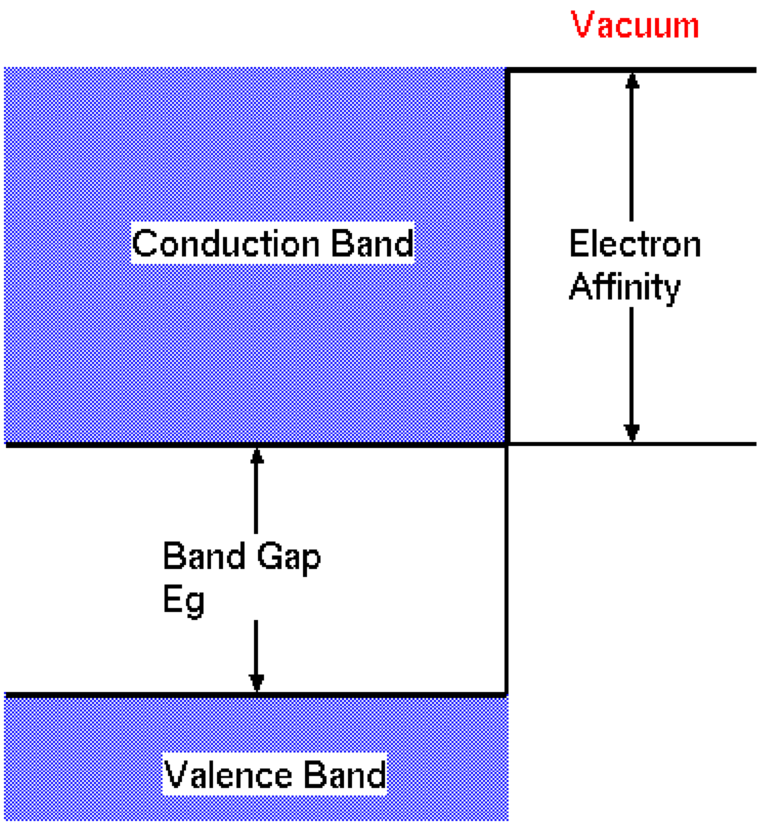
$$P = \frac{N_R - N_L}{N_R + N_L} \qquad (1-43)$$



with GRACE System Developed by Computational Physics Group in KEK

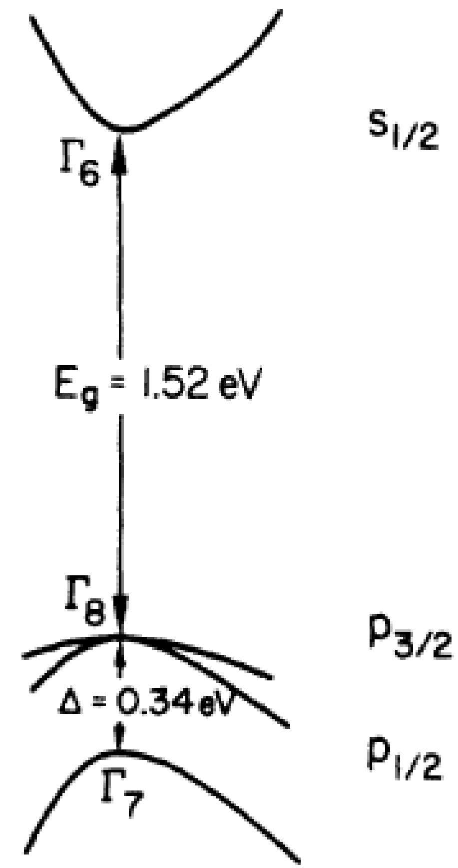
Polarized Electron (2)

- Polarized Electron generation in 3 steps: 1)Excitation, 2)Transportation, 3)Emission
- Electron polarization is made as consequence of selective excitation from the valence band to conduction band.
- Excited electron should be transported without a significant depolarization effect to the surface.
- The polarized electron at the surface should be extracted to the vacuum.



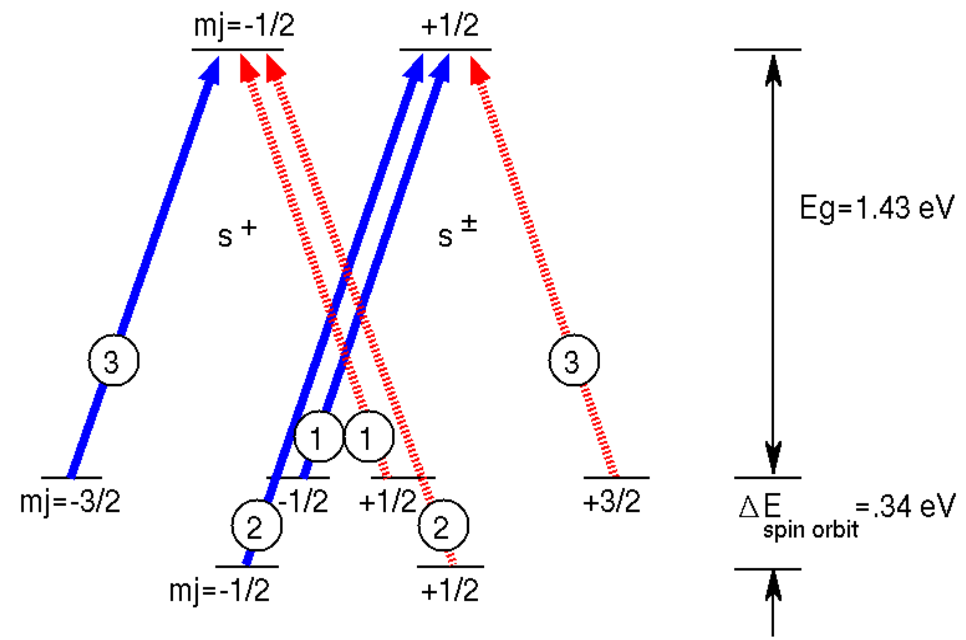
Polarized Electron (3)
Polarization

- Bulk GaAs (Γ point) has two group of degenerated states : J=3/2 (heavy hole) and ½ (light hole).
- The polarized electron is generated by selective transition only from one of the degenerated states: J=3/2 (Heavy hole).
- The selection is made by energy difference between J=3/2 and J=1/2 states.



Polarized Electron (4) Polarization

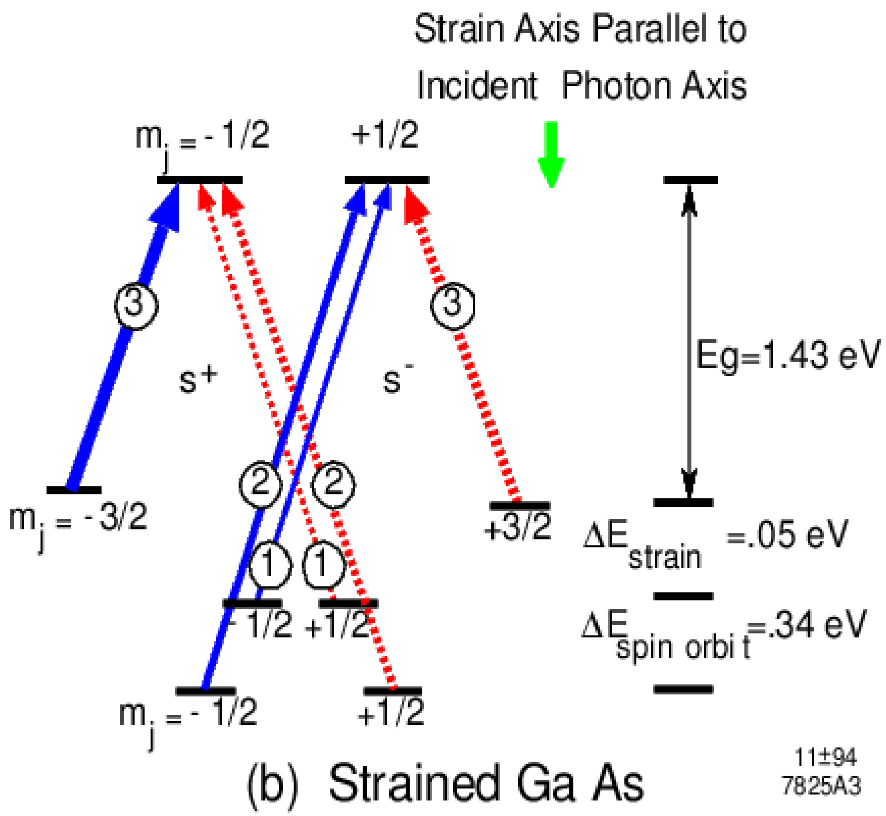
- If the photon is circularly polarized, possible transition is limited to two of four, because the final state is m=+1/2 or -1/2.
- The transition probability by circularity polarized photons($s_z=\pm1$) is described by Clebsh – Gordon co-efficients ($3/2\oplus1$) and the transition occurs 3 times more from m=±3/2.
- Electron polarization becomes 50% (75% m=-1/2,25% m=+1/2)



Unstrained Ga As

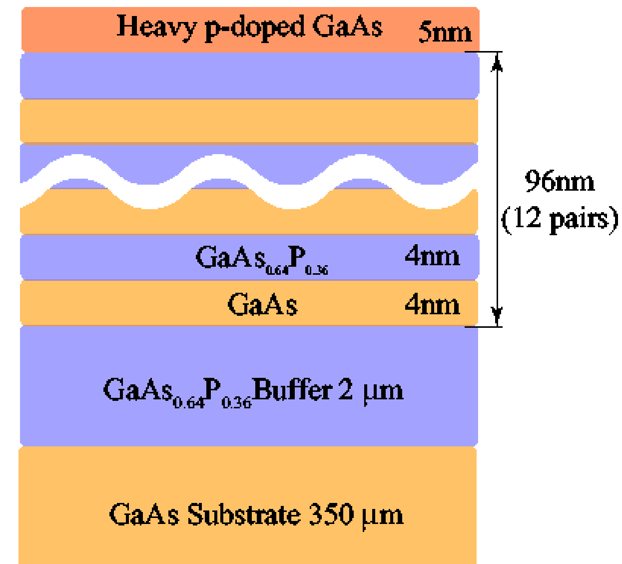
Polarized Electron (5) More than 50% P

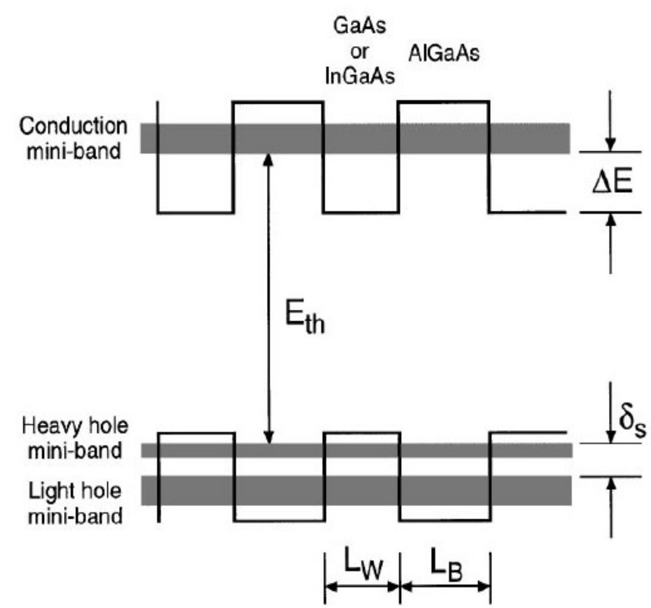
- If the degeneration of heavy hole states is broken, one transition is enhanced and the polarization can be more than 50%.
- Constraint (lattice mismatch) or super-lattice (layer structure with different lattice constant) realize the band split.
- As consequence, 90% polarization is realized.



Polarization (6) Super-Lattice

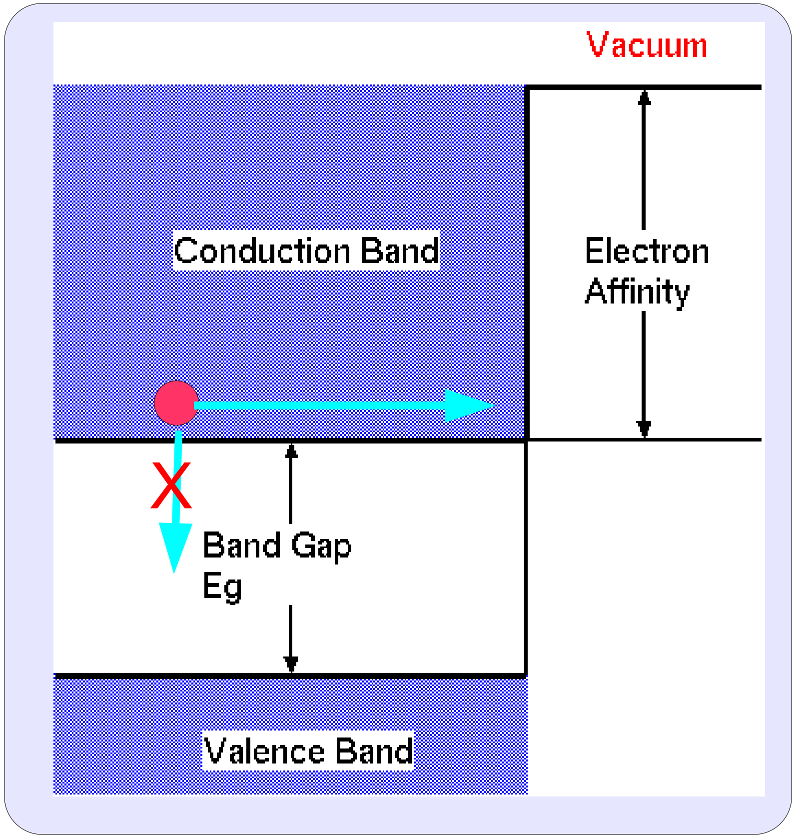
- Super-Lattice structure : sandwiches of GaAsP and GaAs, with different band gap energy.
- The periodic well structure forms mini-bands in conduction band and valence band, respectively.
- As a result, the degeneration in VB is untied and transition can be selective.
- The confinement in the mini-band suppresses the energy spread of the electron beam.



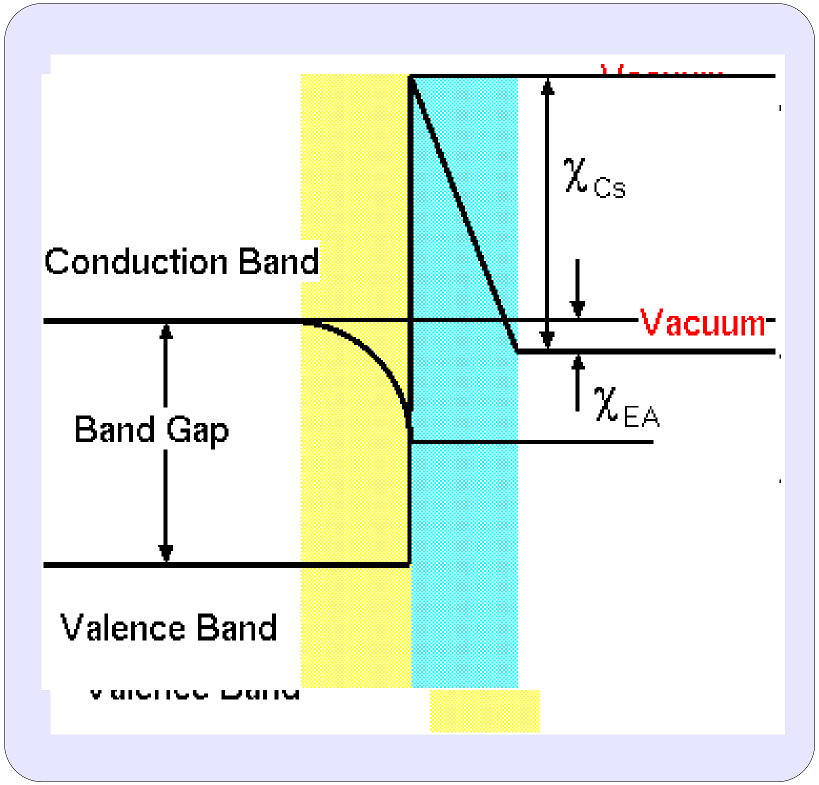


Polarized Electron (7)

- The excited electrons are in near of bottom of CB (in bulk GaAs case) and in conduction mini-band (in SL case), respectively.
- Interaction to electrons in the valence band is compensated because any electrons can not be in the band gap: forbidden band.
- The polarized electron arrives to the surface without any strong interaction to the electrons in VB.



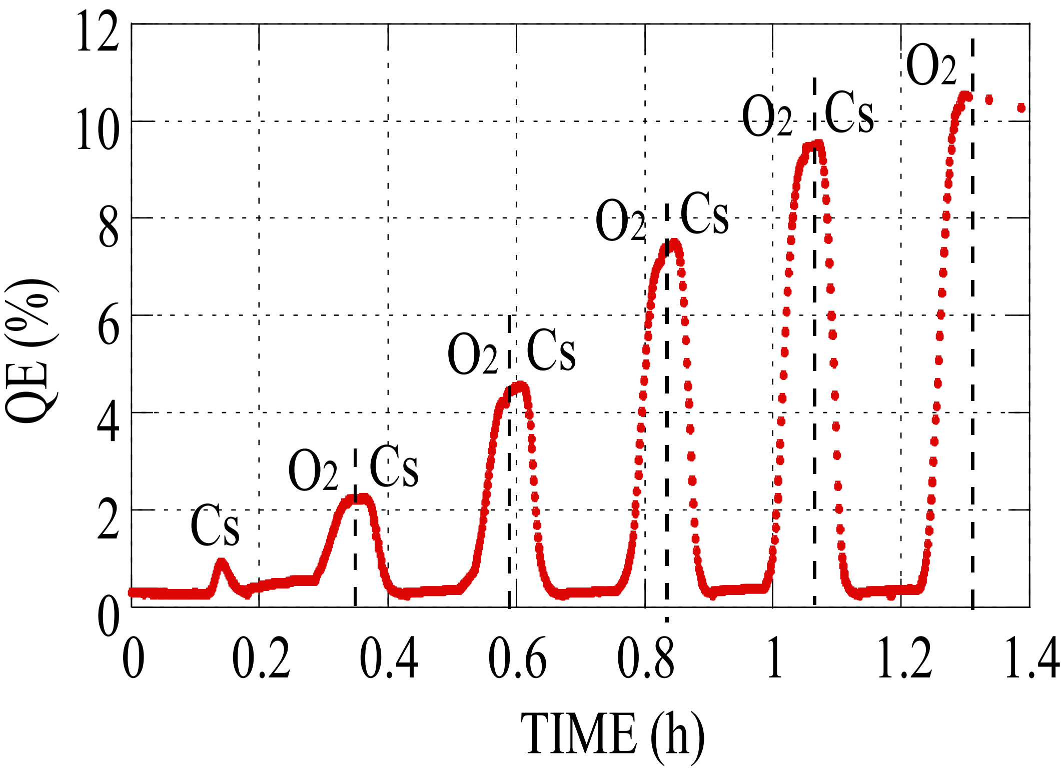
- Nominal material has positive electron affinity; Electrons are confined in the well potential.
- The electron affinity can be negative (NEA surface) by two treatments:
  - Band bending: Zn doping makes hole states, which attract un-paired electrons, resulting potential bending.
  - Dipole layer by Cs and O_2 pulls down the vacuum level.
- Polarized electron in the conduction band can be extracted to the vacuum by the NEA surface.



Polarized Electron (9)

- Electron Emission
- Related Physics
- Electron Gun
- e- Source for LCs
- Laser
- Summary

- NEA surface is made by evaporation of Cs and O_2.
- At first, the surface is conditioned by chemical etching by H_2S0_4 and treatment by HCl-Isopropanol solution followed by heat cleaning.
- Alternating deposition of Cs and O_2 make NEA surface on GaAs.
- The process should be made extremely high vacuum, <5.0E-9Pa.

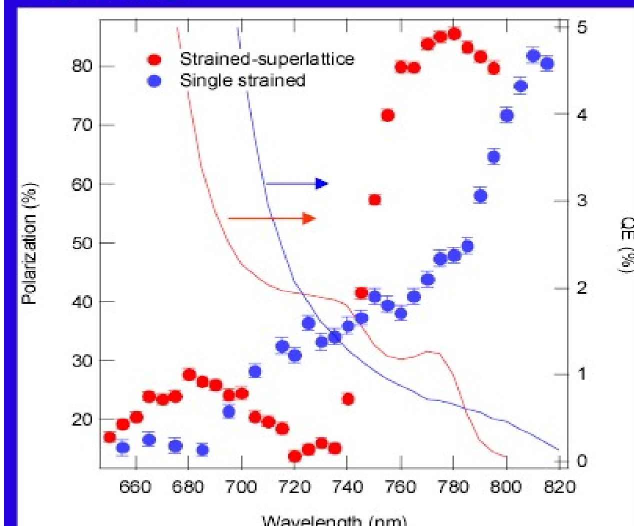


C. Shonaka

Polarized Electron (10)

Performance of GaAs/GaAsP superlattice

SLAC



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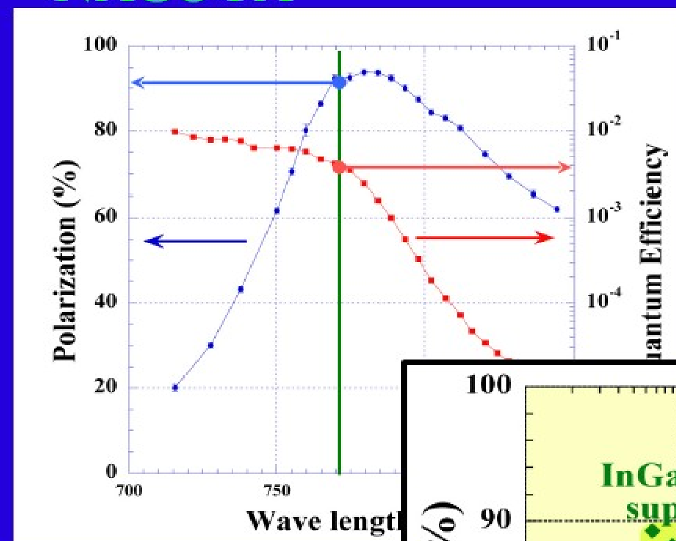

By N. Yamamoto (Nagoya Univ.)

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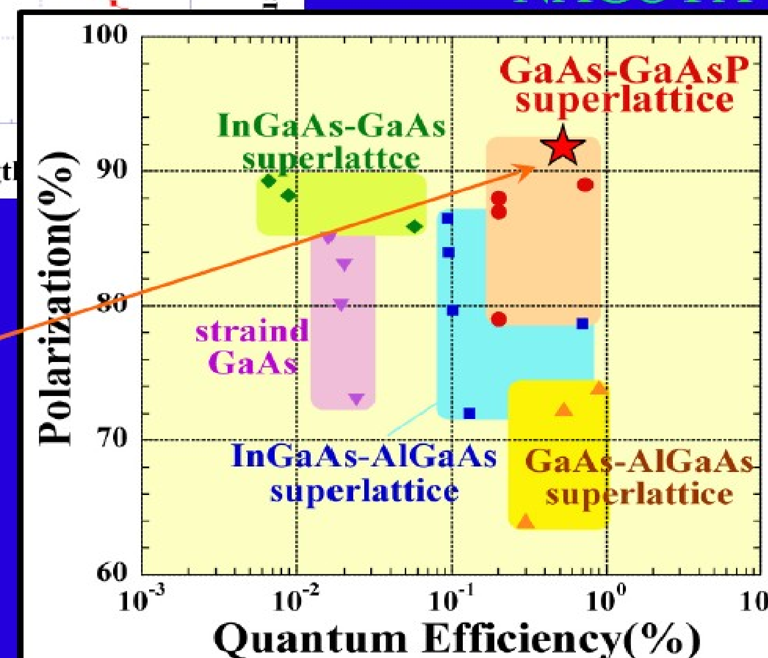


GaAs-GaAsP superlattice shows the best performance !

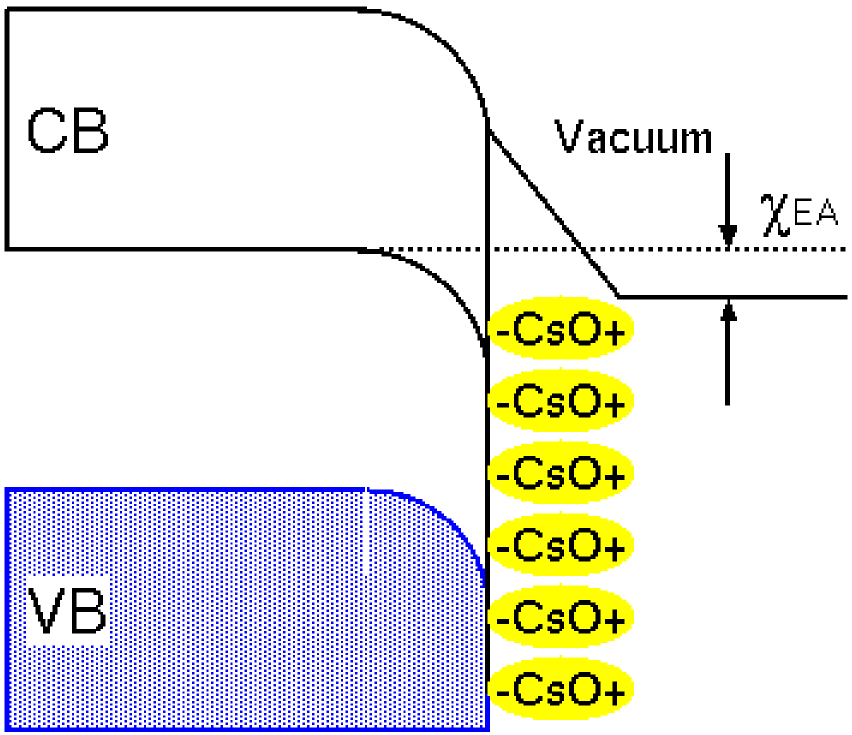
@778nm

Polarization ~ 90%

Q.E. ~ 0.5%

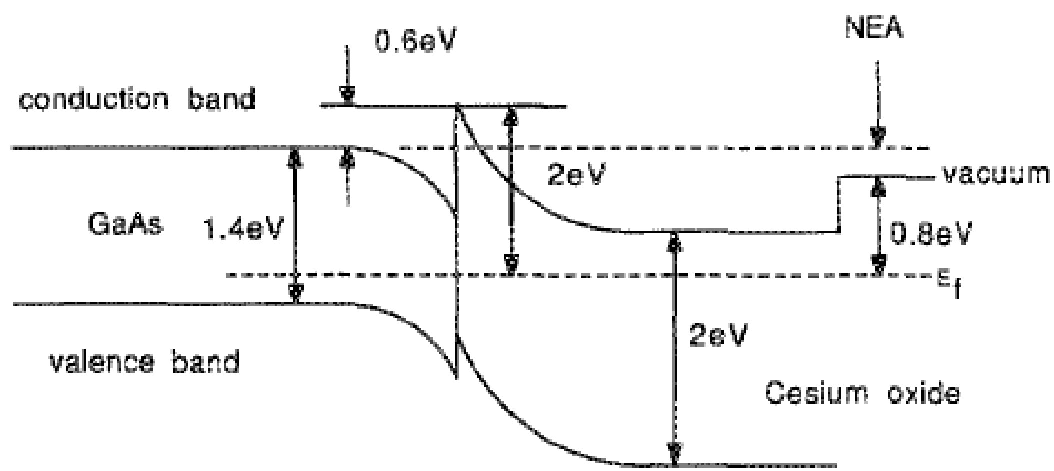
NEA model (1)

- There is no established model for NEA surface. There are two main candidates.
- **Cs-O electric dipole model**
  - Composition of Cs-Ox forms electric dipole on the surface.
  - The vacuum potential is effectively decreased by the dipole potential.



NEA model (2)

- **Hetero-junction model**
  - III-V semiconductor + Cs_xO_{1-x} hetero-junction is made at the surface of GaAs.
  - Bulk Cs_2O is n-type semi-conductor, φ=0.8eV and electron affinity χ=0.55 eV.
  - In GaAs and Cs_2O hetero-junction, the vacuum level becomes below the conduction band in GaAs.



C. A. Sanford, J. Vac. Sci. Tech. B7(6), 1989

FIG. 1. GaAs/cesium oxide heterojunction band diagram.

Electron Emission

Related Physics

Electron Gun

e- Source for LCs

Laser

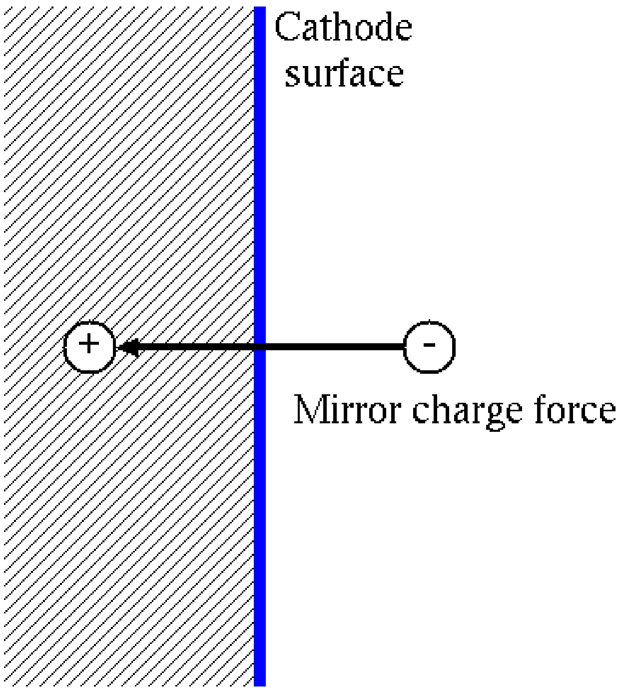
Summary

Related Physics Process

Schottky Effect (1)

Electron near of the cathode feels the force by mirror field

$$F_m(z) = -\frac{1}{4\pi\epsilon}\frac{e^2}{(2z)^2} \quad (2-1)$$



By integrating this field, potential is given by

$$V_m(z) = -\frac{1}{4\pi\epsilon}\int_z^{\infty}\frac{e}{4z'^2}dz' = -\frac{e^2}{16\pi\epsilon z} \quad (2-2)$$

The total vacuum potential is given by

$$V(z) = \phi_0 - \frac{e^2}{16\pi\epsilon z} - eFz \quad (2-3)$$

where F is external field.

Schottky Effect (2)

The potential has a maximum

$$V_{max} = V_0 - \frac{e}{2}\sqrt{\frac{eE}{\pi\epsilon}} \quad (2-4)$$

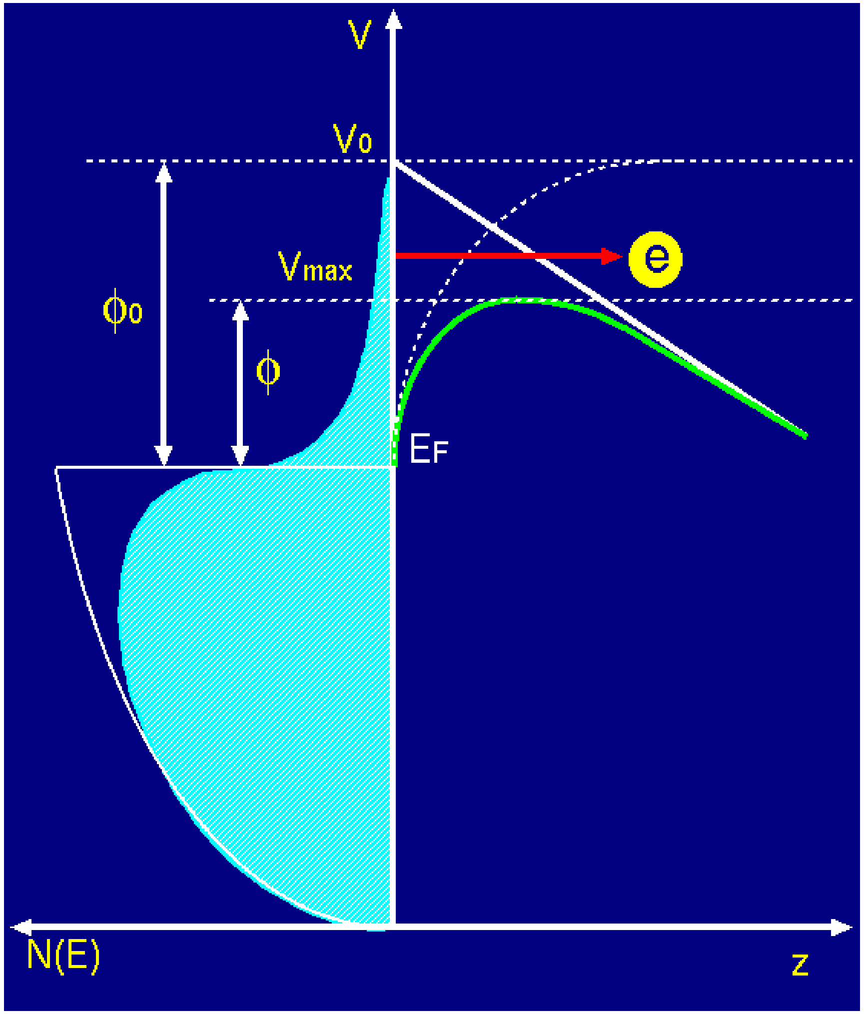
at

$$z_{max} = \frac{1}{4}\sqrt{\frac{e}{\pi\epsilon F}} \quad (2-5)$$

The effective work function is

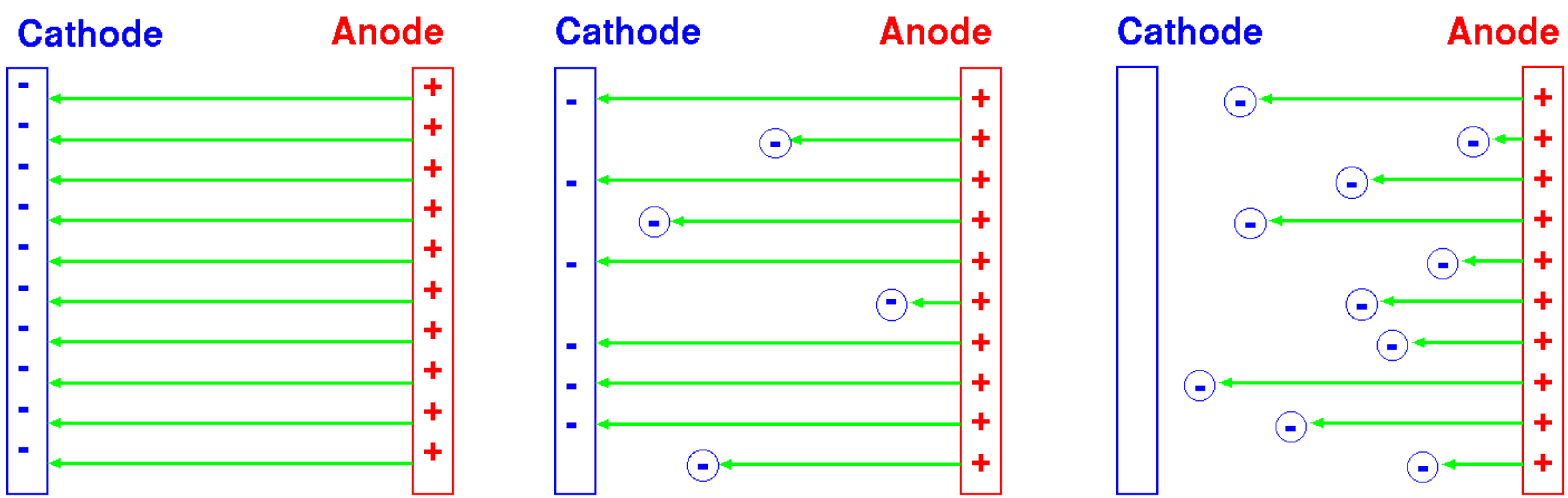
$$\phi(F) = V_{max} - \mu = \phi_0 - e\sqrt{\frac{eF}{4\pi\epsilon}} \quad (2-6)$$

which is lowered by the field.



Space Charge Limit

- Electron terminate the electric flux (remember Gauss's law).
- Electric field is weakened by the space charge.
- At some limit, the field at the cathode surface is disappeared and no electrons extracted further; the space charge limit.



SC Limited Current (1)

We consider the space charge limited current in 1D case. Cathode is V=0, z=0, and anode is V=V_A and z=d. Poisson equation is

$$\frac{d^2V(z)}{dz^2} = -\frac{\rho(z)}{\epsilon_0} \quad (2-7)$$

The current density J is given by the charge density ρ and velocity v,

$$J = -\rho(z)v(z) \quad (2-8)$$

Note that J is a constant because of the static current. According energy conservation,

$$\frac{1}{2}mv(z)^2 = eV(z) \quad (2-9)$$

Integrating these three formulas,

$$\frac{d^2V(z)}{dz^2} = \frac{J}{\epsilon_0}\sqrt{\frac{m}{2e}}V(z)^{-1/2} \quad (2-10)$$

SC Limited Current (2)

Multiplying *2(dV/dz)* and integrating both sides,

$$\left(\frac{dV(z)}{dz}\right)^2 = \frac{4J}{\epsilon_0}\sqrt{\frac{m}{2e}}V(z)^{1/2} \qquad (2-11)$$

Taking square root of both sides and integrate it again,

$$\frac{4}{3}V^{3/4} = \sqrt{\frac{4J}{\epsilon_0}}\sqrt[4]{\frac{m}{2e}}z \qquad (2-12)$$

Extract *J*, we got

$$J = \frac{4\epsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V(z)^{3/2}}{z^2}$$

$$= 2.33\times 10^{-6}\frac{V(z)^{3/2}}{z^2} \qquad (2-13)$$

SC Limited Current (3)

Substituting the anode conditions, the space charge limited current density is obtained as

$$J(V_A, d) = 2.33\times 10^{-6}\frac{V_A^{3/2}}{d^2} \qquad (2-14)$$

In this case, V(z), E(z) , ρ(z) are expressed as a function of z

$$V(z) = V_A\left(\frac{z}{d}\right)^{3/4} \qquad (2-15)$$

$$E(z) = -\frac{dV(z)}{dz} = -\frac{4}{3}\frac{V_A}{d^{4/3}}z^{1/3} \qquad (2-16)$$

$$\rho(z) = -\frac{4\epsilon_0}{9}\frac{V_A}{d^{4/3}}z^{-2/3} \qquad (2-17)$$

Child-Langmuir Law

If the electron source is operated in space charge limited regime, the current is given by C-L law

$$I = 2.33 \times 10^{-6} \frac{S V^{3/2}}{d^2} = P V^{3/2} (A) \qquad (2-18)$$

***V* and *d* : voltage and distance between two electrodes.**

***S* : cathode area**

***P* : perveance defined as;**

$$P = 2.33 \times 10^{-6} \frac{S}{d^2} (A V^{-3/2}) \qquad (2-19)$$

Actual Emission Current

- When the surface field is not sufficiently high, the actual current is determined by the space charge limit.
- When the surface field is sufficiently high, the actual current is determined by that from the cathode.
- Then, the actual emission current form a cathode is

$$I_E = min(I_C, I_{SC}) \qquad (2-20)$$

  - **I_C: Emission current of the fundamental process (thermal emission, etc.)**
  - **I_{SC}:Space charge limit**

Space charge limited flow is lead by assuming a simplified single dimension. It is known that it is realized in 2D or 3D. Let us consider the following 2D case,

The space charge limited flow exists in y<0 region, and y>0 is vacuum (no charge). According to the space charge limited flow, the potential is given by

$$V(z) = V_A \left(\frac{z}{d} \right)^{4/3} \qquad (2-21)$$

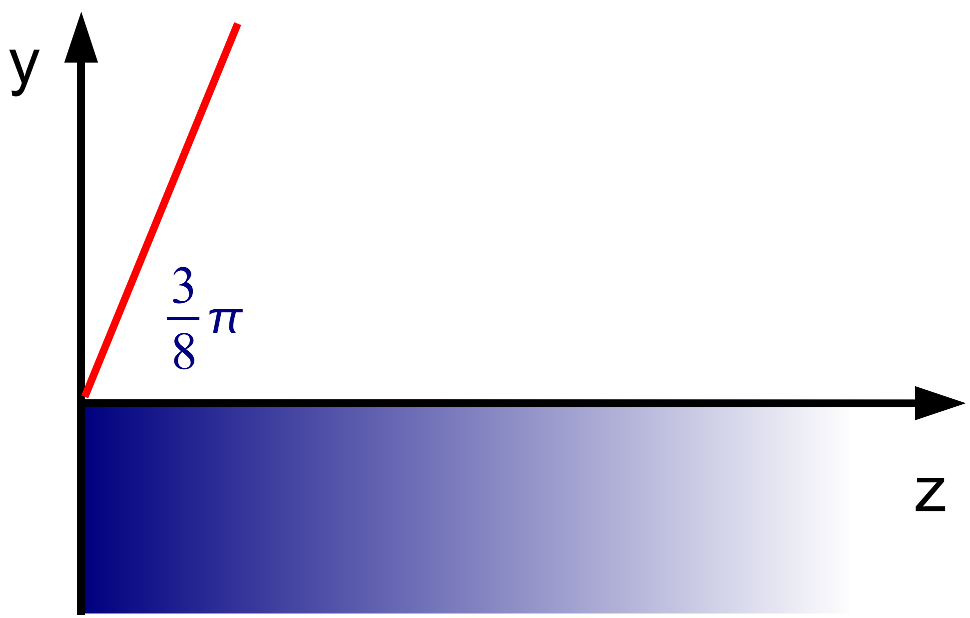
The equation automatically satisfies Poisson equation. For y>0 region, potential V=Φ(z, y) should satisfies Laplace equation. In addition, V=Φ(z, y=0) should be identical to eq. (30). The following solution is derived

$$V(z,y) = V_A \frac{\Re\left[(z+iy)^{4/3}\right]}{d^{4/3}} = V_A (z^2+y^2)^{2/3} \cos\frac{4}{3}\theta \qquad (2-22)$$

For V=0,

$$\cos\frac{4}{3}\theta = 0 \rightarrow \theta = \frac{3}{8}\pi$$

which determines the angle of electrode (Wehnelt).

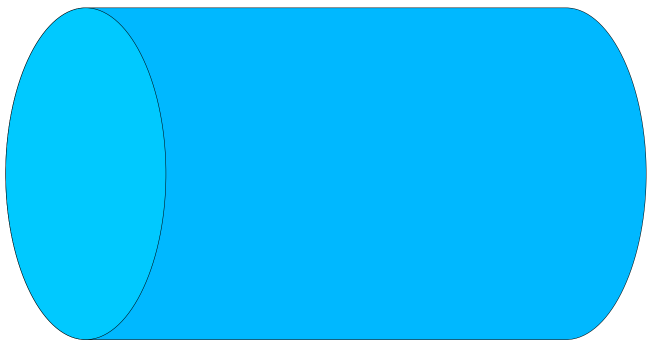


Space Charge Force (1)

Charged particle beam has repulsion force due to the own electronic charge: Space Charge Force. The space charge force causes various beam quality degradations, e.g. bunch lengthening, emittance growth, tune shift, etc. The effect is suppressed by acceleration because it scaled as $1/\gamma^2$.

Consider a cylindrical beam (radius a) with a constant density (N electron per unit length) with infinite length. The space charge force is

$$E_r = \frac{N e}{2\pi a^2 \epsilon_0} r \qquad (2-23)$$



The electron beam is electronic current, which induces magnetic flux density

$$B(r) = \frac{\mu_0}{r} \int_0^r r' J(r') dr' \qquad (2-24)$$

Space Charge Force (2)

Current density is

$$J(r) = \frac{Ne}{\pi a^2} \beta c \qquad (2-25)$$

Then, the magnetic flux is

$$B(r) = \frac{\mu_0 N e \beta c}{2 \pi a^2} r \qquad (2-26)$$

The direction is azimuthal. The force to electron beam itself is

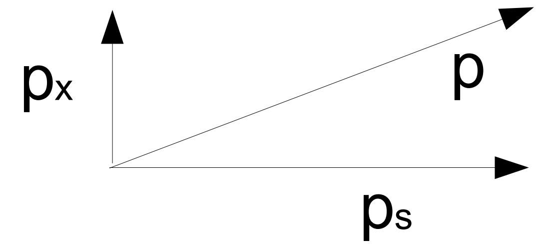
$$\boldsymbol{F} = e\boldsymbol{E} + e\beta c \boldsymbol{B} = \frac{Ne^2 r}{2\pi a^2 \epsilon_0}(1-\beta^2)\vec{e}_r$$
$$= \frac{Ne^2 r}{2\pi a^2 \epsilon_0 \gamma^2}\vec{e}_r \qquad (2-27)$$

which is scaled as $1/\gamma^2$. The space charge force is suppressed greatly by acceleration.

Beam Emittance

Emittance is defined as area in the phase space, where particles occupy. The phase space is defined x and x'=dx/ds

$$x' = \frac{dx}{ds} = \frac{v_x}{v_s} = \frac{p_x}{p_s} \sim \frac{p_x}{p} \qquad (2-28)$$

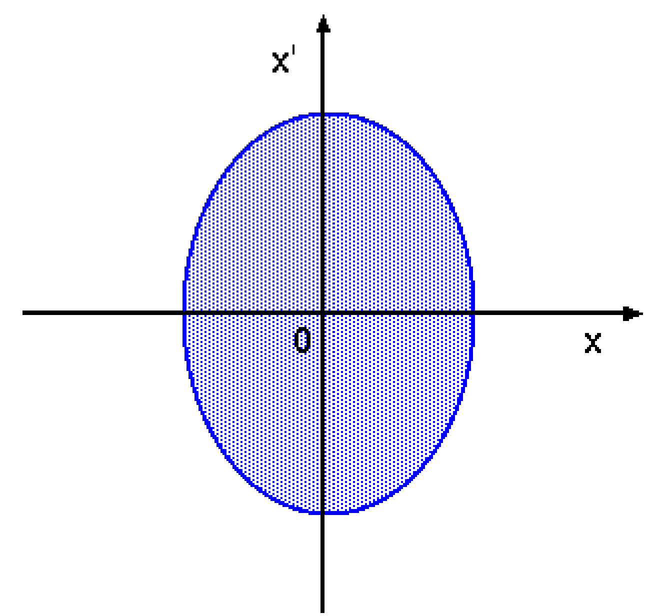


In general, RMS emittance is given as

$$\epsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2} \qquad (2-29)$$

If there is no correlation between x and

$$\epsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle} \qquad (2-30)$$



Normalized emittance

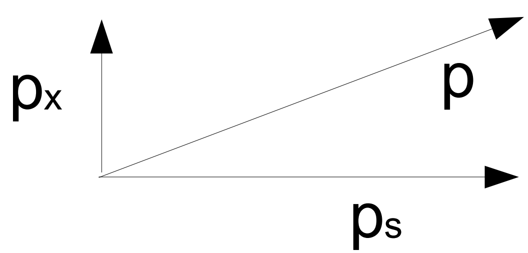
According to Louisville's theorem, 6D volume of the particle distribution in phase space is conserved.

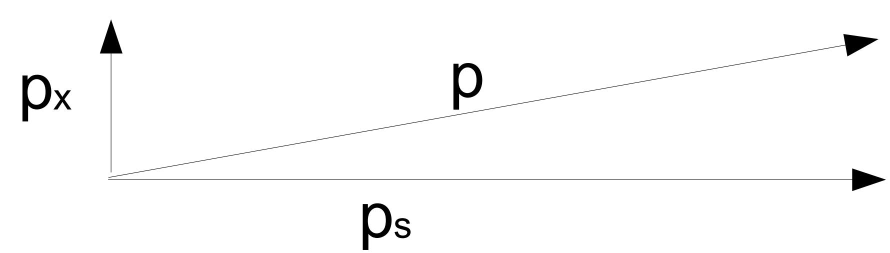
In acceleration, transverse momentum p_x is conserved, but p is scaled as

$$p_s = \gamma \beta mc \qquad (2-31)$$

To avoid the energy dependence ($\gamma\beta$) on the emittance to compare them in different energy, the normalized emittance is defined

$$\epsilon_{nx} = \gamma \beta \epsilon_x \qquad (2-32)$$





Emittance of Beam from Thermionic Cathode (1)

Ignoring reflection at the boundary between vacuum and potential well, thermionic electron emission density is already obtained

$$N=\frac{4\pi m}{h^3}k^2T^2\exp\left(-\frac{\phi}{kT}\right) \qquad (2-33)$$

Total transverse energy of emitted electron is obtained with a similar calculation as

$$E_t=\frac{4\pi m}{h^3}\int_{\mu+\phi}^{\infty}d\epsilon_z\int_0^{\infty}d\epsilon_t\epsilon_t\exp\left(-\frac{\epsilon_z+\epsilon_t-\mu}{kT}\right)$$

$$=\frac{4\pi m}{h^3}k^3T^3\exp\left(-\frac{\phi}{kT}\right) \qquad (2-34)$$

The average transverse energy per electron is

$$\langle\epsilon_t\rangle=\frac{E_t}{N}=kT \qquad (2-35)$$

Emittance of Beam from Thermionic Cathode (2)

The transverse energy is sum of x and y. For each axis,

$$\langle \epsilon_x \rangle = \frac{kT}{2} \qquad (2-36)$$

The transverse emittance is

$$\varepsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle} = \frac{1}{\gamma \beta m c} \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle} \qquad (2-37)$$

Substituting the thermal energy, $\frac{\langle p_x^2 \rangle}{2m} = \langle \epsilon_x \rangle = \frac{kT}{2}$ we get

$$\varepsilon_x = \frac{1}{\gamma \beta} \sqrt{\langle x^2 \rangle \frac{kT}{mc^2}} = \frac{1}{\gamma \beta} \frac{R}{2} \sqrt{\frac{kT}{mc^2}} \qquad (2-38)$$

normalized emittance is

$$\varepsilon_{nx} = \gamma \beta \varepsilon_x = \frac{R}{2} \sqrt{\frac{kT}{mc^2}} \qquad (2-39)$$

Emittance of Beam from Photo-cathode (1)

Transverse emittance of beam from photo-cathode can be evaluated from the average transverse energy, which is expressed as

$$E_t = \frac{4\pi m}{h^3}\int_{\mu+\phi-h\nu}^{\infty} d\epsilon_z \int_0^{\infty} d\epsilon_t \epsilon_t \left[\exp\left(\frac{\epsilon_z+\epsilon_t-\mu}{kT}\right)+1\right]^{-1} \qquad (2-40)$$

It can be calculated analytically only for T=0 case, where F-D function is 0 for $\varepsilon_z + \varepsilon_t > \mu$ and 1 for $\varepsilon_z + \varepsilon_t < \mu$. It becomes

$$E_t = \frac{4\pi m}{h^3}\int_{\mu+\phi-h\nu}^{\mu} d\epsilon_z \int_0^{\mu-\epsilon_z} d\epsilon_t \epsilon_t$$
$$= \frac{4\pi m}{h^3}\frac{(h\nu-\phi)^3}{6} \qquad (2-41)$$

Emittance of Beam from Photo-cathode (2)

Number of emitted electron is calculated similarly,

$$N=\frac{4\pi m}{h^3}\int_{\mu+\phi-h\nu}^{\mu} d\epsilon_z \int_0^{\mu-\epsilon_z} d\epsilon_t$$

$$=\frac{4\pi m}{h^3}\frac{(h\nu-\phi)^2}{2} \qquad (2-42)$$

The average of the transverse energy is

$$\epsilon_t=\frac{E_t}{N}=\frac{h\nu-\phi}{3} \qquad (2-43)$$

Average momentum is obtained

$$\langle p_x^2\rangle=2\mathrm{m}\frac{\epsilon_t}{2}=m\frac{h\nu-\phi}{3} \qquad (2-44)$$

Emittance is

$$\varepsilon_x=\frac{1}{\gamma\beta}\frac{R}{2}\sqrt{\frac{h\nu-\phi}{3\mathrm{mc}^2}} \qquad (2-45)$$

$$\varepsilon_{nx}=\frac{R}{2}\sqrt{\frac{h\nu-\phi}{3\mathrm{mc}^2}} \qquad (2-46)$$

Emittance of Beam from Photo-cathode (3)

In the evaluation, thermal energy is not included at all. If the additional energy is accounted, the transverse energy becomes

$$\epsilon_t = \frac{E_t}{N} = \frac{h\nu - \phi}{3} + kT \qquad (2-47)$$

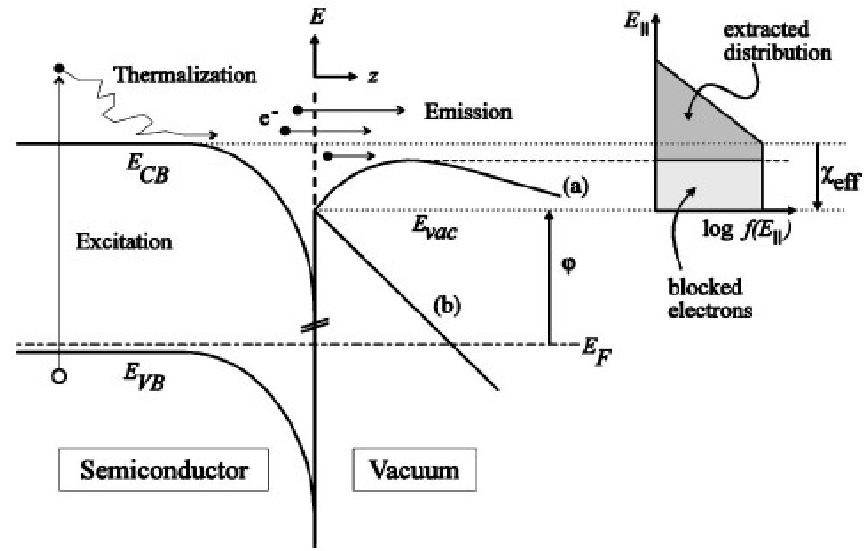
The transverse emittance is extracted as

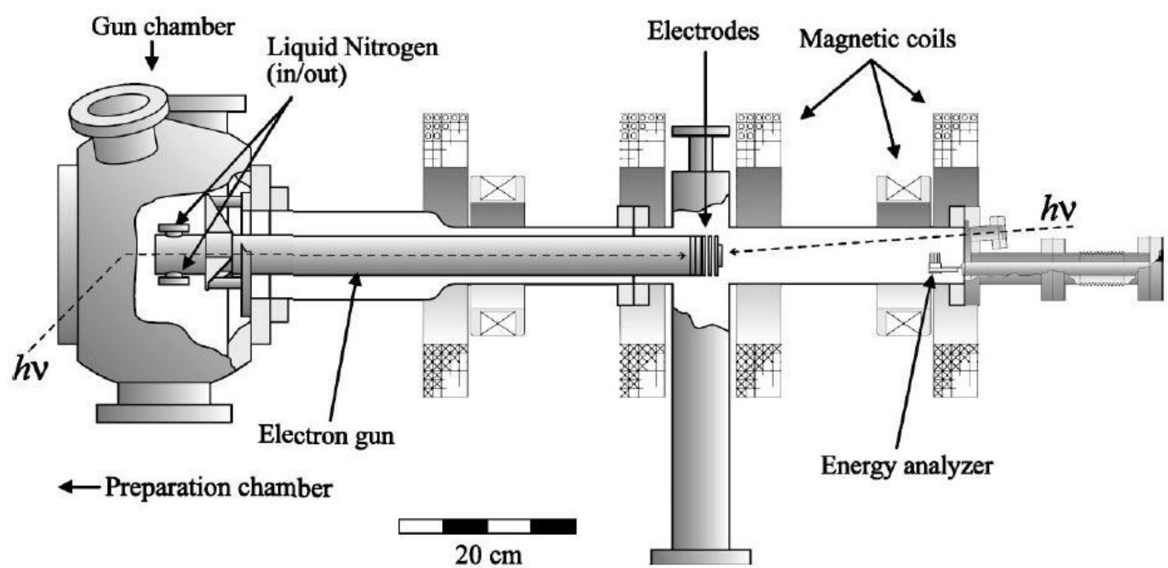
$$\varepsilon_x = \frac{1}{\gamma\beta}\frac{R}{2}\sqrt{\frac{h\nu-\phi}{3\mathrm{mc}^2} + \frac{kT}{mc^2}} \qquad (2-48)$$

$$\varepsilon_{nx} = \frac{R}{2}\sqrt{\frac{h\nu-\phi}{3\mathrm{mc}^2} \frac{kT}{mc^2}} \qquad (2-49)$$

Emittance measurement 1-1

- Energy spread from GaAs photo-cathode is directly measured by blocked electrode.
- Only electrons above the block potential barrier, is observed.
- Cathode is placed longitudinal B field (immerse).





S. Pastuszka, JAP, 88(11), 6788-6800 (2000)

Emittance measurement 1-2

When adiabatic condition

$$\frac{\lambda}{B}\left|\frac{dB}{dz}\right| \leqslant 1 \qquad (2-50)$$

is satisfied, ratio of transverse energy E⊥ and magnetic flux B is an adiabatic constant,

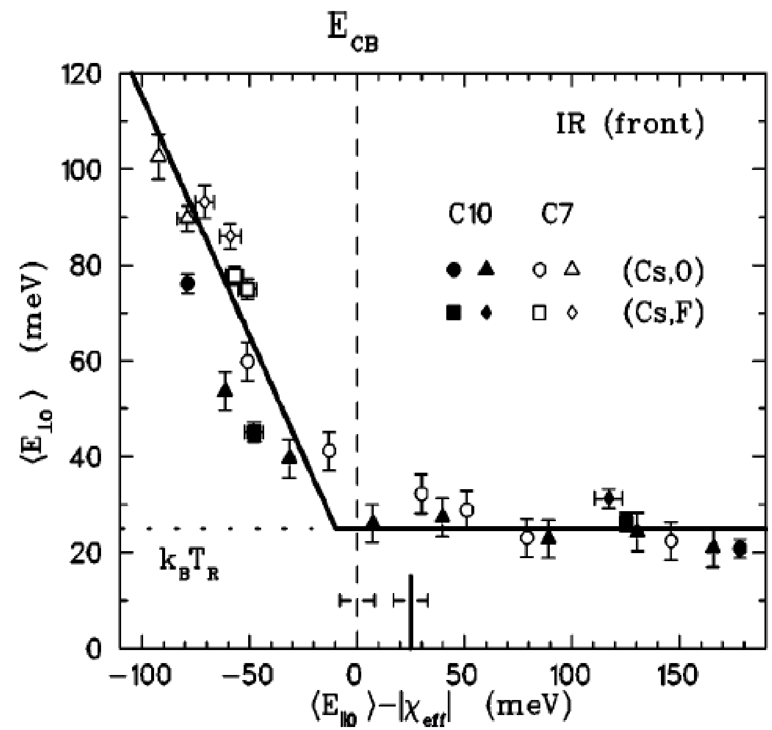
$$\frac{E_\perp}{B} = const \qquad (2-51)$$

From the energy conservation

$$E_{\parallel f} = E_{\parallel i} + \left(1 - \frac{B_f}{B_i}\right) E_{\perp i} \qquad (2-52)$$

The initial transverse energy is obtained as

$$\langle E_{\perp i} \rangle = -\frac{d\langle E_{\parallel f}\rangle}{d\alpha} \qquad (2-53)$$

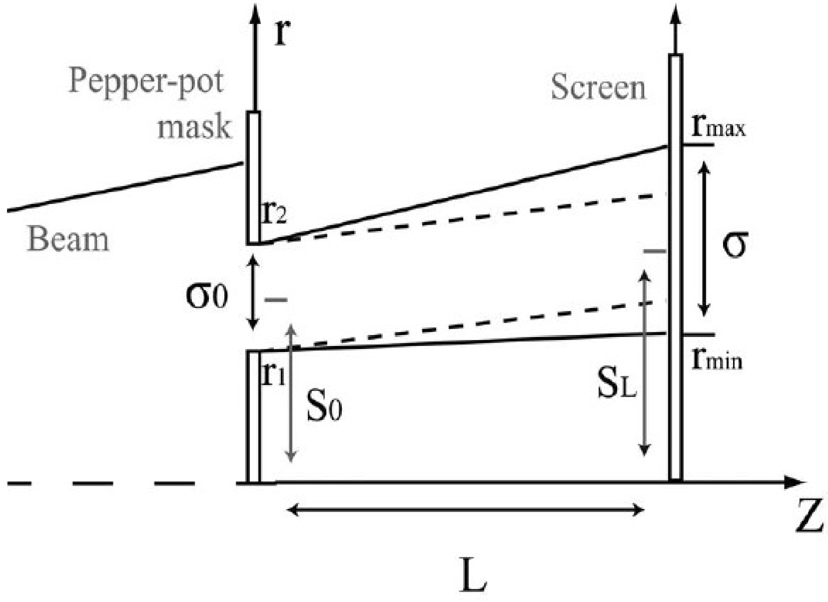


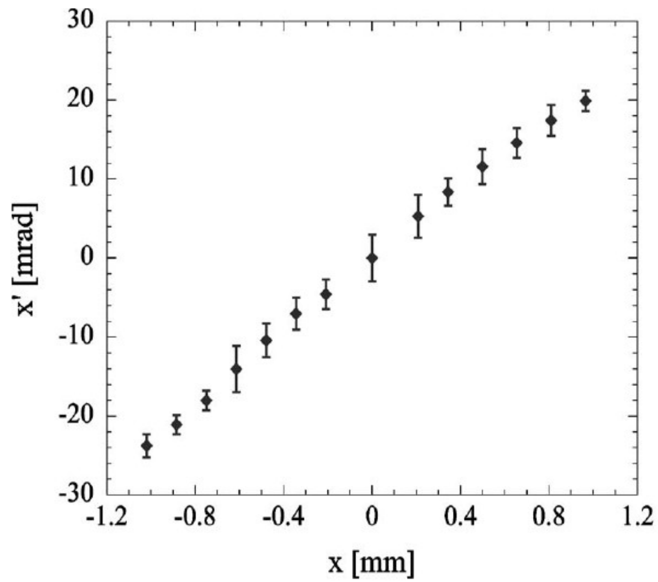
$E_{\parallel i}$ = 25 meV is confirmed.

S. Pastuszka, JAP, 88(11), 6788-6800 (2000)

Emittance measurement 2-1

- Beam emittance from SL GaAs photocathode is measured by pepper-pot method.
- The beam image passing small holes (pepper-pot) are observed.
- The phase-space distribution is reconstructed from the image.

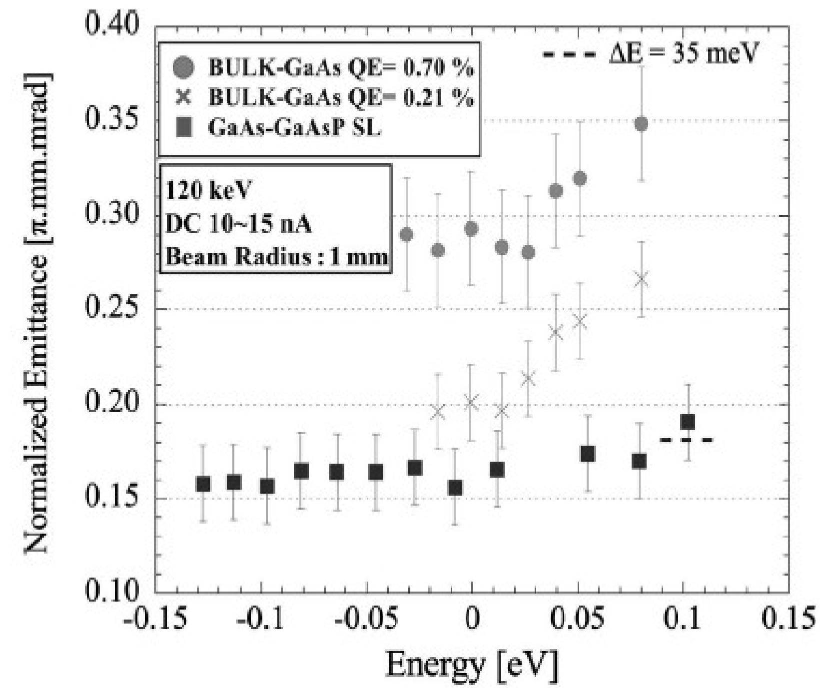


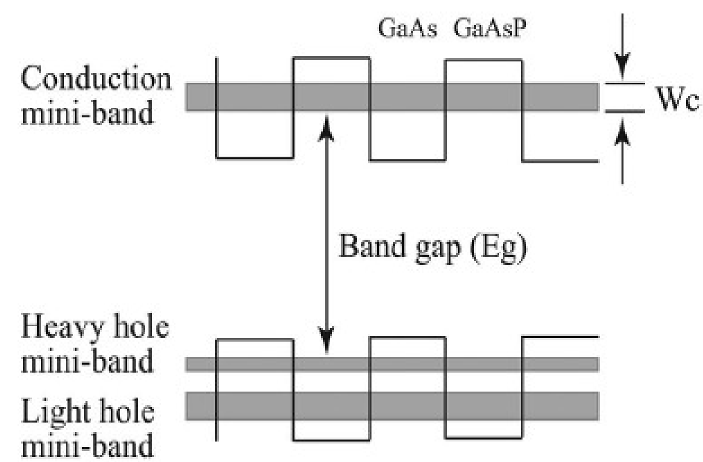


N. Yamamoto, JAP(102) 024904(2007)

Emittance measurement 2-2

- Emittance is measure as a function of laser wave length.
- Comparing Super-lattice GaAs and bulk GaAs, SL has smaller emittance, especially for shorter wave length.
- It can be considered due to confinement of the excited electrons in the conduction mini-band.
- ε_x~0.16 mm.mrad is confirmed.

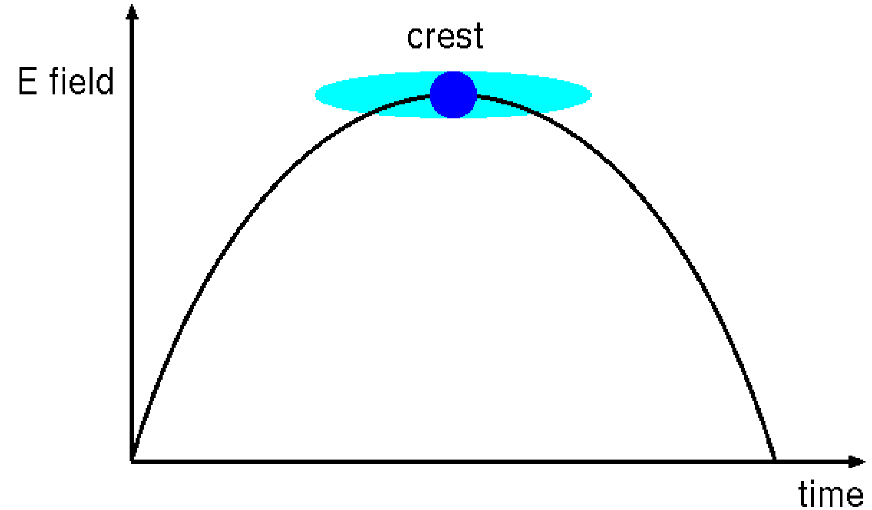




N. Yamamoto, JAP(102) 024904(2007)

Bunch Compression (1)

- In any accelerator with RF field, the beam should be concentrated in a short period of longitudinal space for small energy spread;
  - **$E=E_0\cos(\omega t-ks)$, wt-ks=0 for efficient acceleration.**
- Bunch compressor and buncher shorten the bunch length down to an adequate size for acceleration.



Bunch Compression (2)

- Bunching after the source
  - **Particle source can generate only long bunch or continuous beam.**
  - **Magnetic Bunching**
- Bunching after the storage ring
  - **Long bunch length tend to reduce beam instabilities in DR.**
  - **Thus, the bunch length is compressed after DR.**
- There are two ways for bunch compression:
  - **Velocity Bunching**
  - **Magnetic Bunching**

Velocity Bunching (1)

- Bunch compression is performed by velocity modulation within a bunch;
  - **Bunch head is decelerated.**
  - **Bunch tail is accelerated.**
- Velocity is modulated by this energy modulation according to

$$c\beta = c\sqrt{1-\frac{1}{\gamma^2}} \qquad (2-54)$$

- Velocity is saturated to c at $\gamma>>1$. Then, it works only for low energy particle ($\beta<1$).
  - **Bunch compression at the injector.**

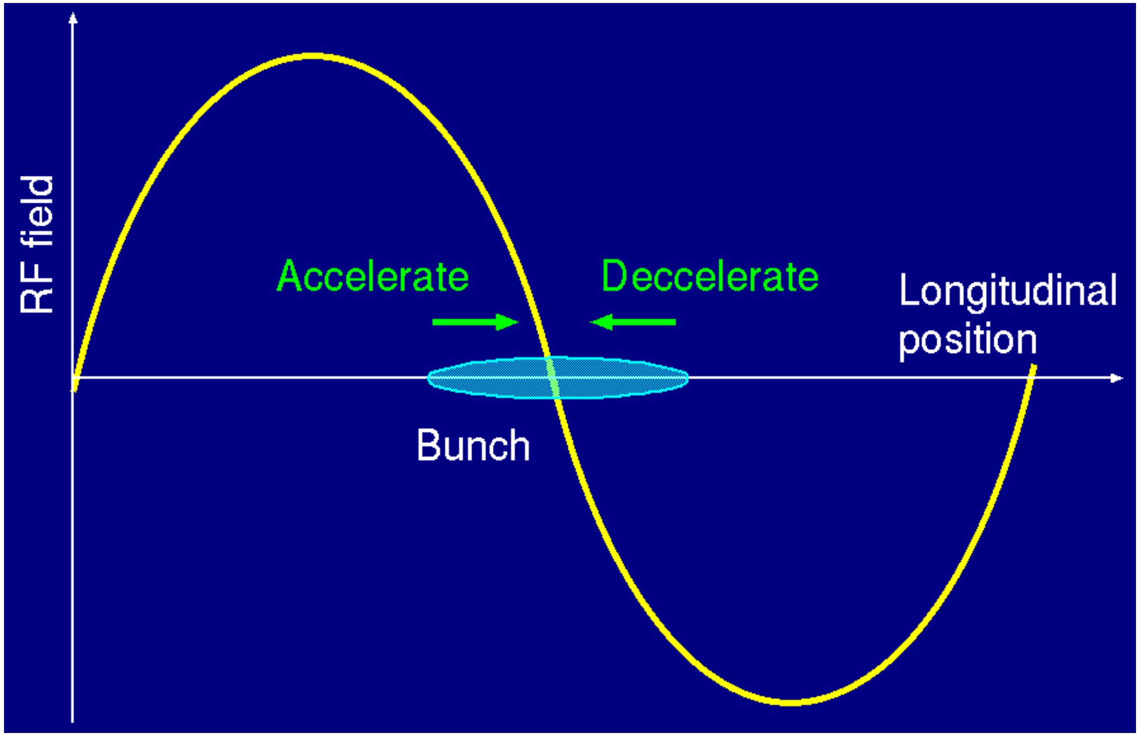
Velocity Bunching (2)

RF cavity voltage varies as a function of relative position (*t*) in a bunch

$$V = -V_0 \sin(\omega t) \qquad (2-55)$$

In linear approximation, energy modulation by the RF field depends on *t* as

$$\frac{dE}{E_0} \sim \frac{-eV_0}{E_0}\omega t \qquad (2-56)$$



► Time to travel distance L is

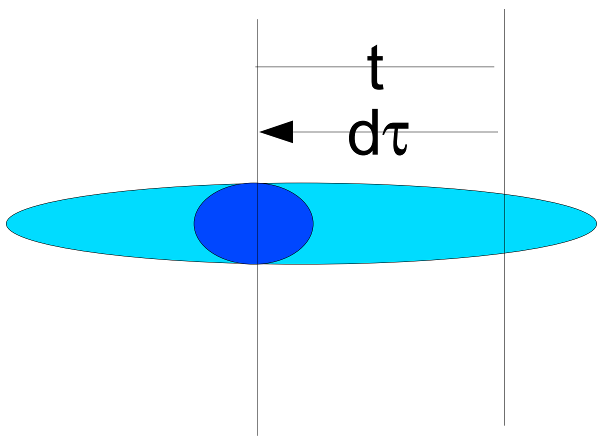
$$\tau = \frac{L}{c\beta} \qquad (2-57)$$

► Relative particle position in a bunch to the bunch center is modulated as

$$d\tau = -\frac{L}{c\gamma^2\beta^3}\frac{dE}{E} \sim -\frac{L}{c\gamma^2\beta^3}\frac{eV_0\omega}{E}t \qquad (2-58)$$

► If dτ equals to -t, all particles are gathered at the bunch center, bunched.

► The bunch compressor has a sub effect, that the bunch timing is less sensitive to it before the bunch compression, because all electrons concentrate at t=0 position.



Magnetic Bunching (1)

- Bunch compression is performed by energy modulation with dispersive path length difference.
  - **Chicane, Wiggler, Arc, etc.**
- A path length difference by a dispersive section, Δz is

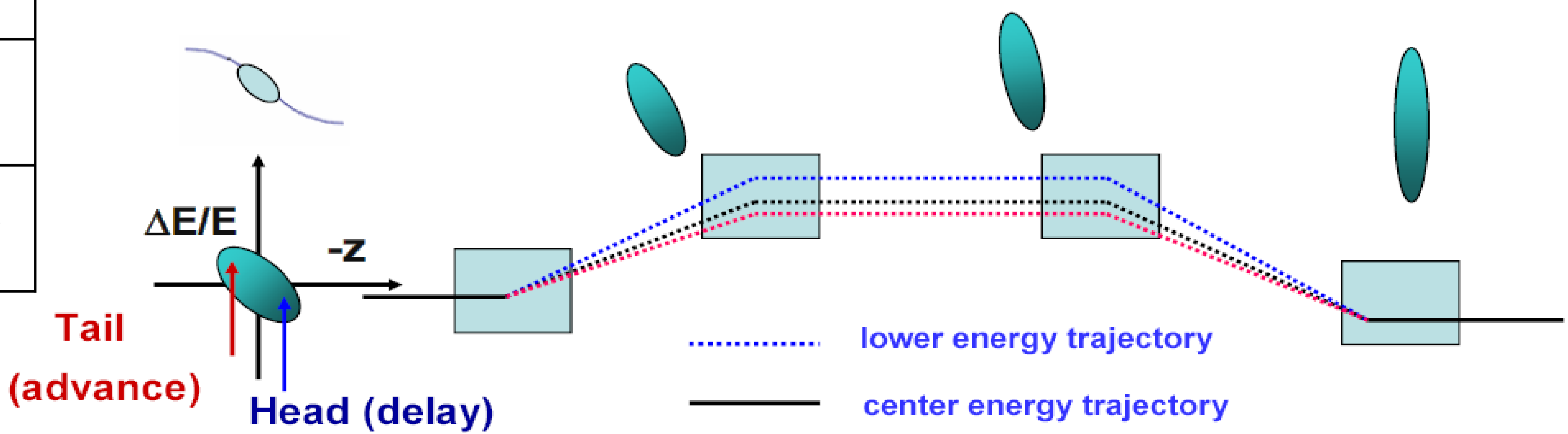
$$\Delta z = \eta_l \frac{\Delta E}{E} \qquad (2-59)$$

where η_l is (longitudinal) dispersion and $\Delta E/E$ is relative energy deviation.

- It works well for any energy particle because the measure is the relative energy deviation.

Magnetic Bunching (2)

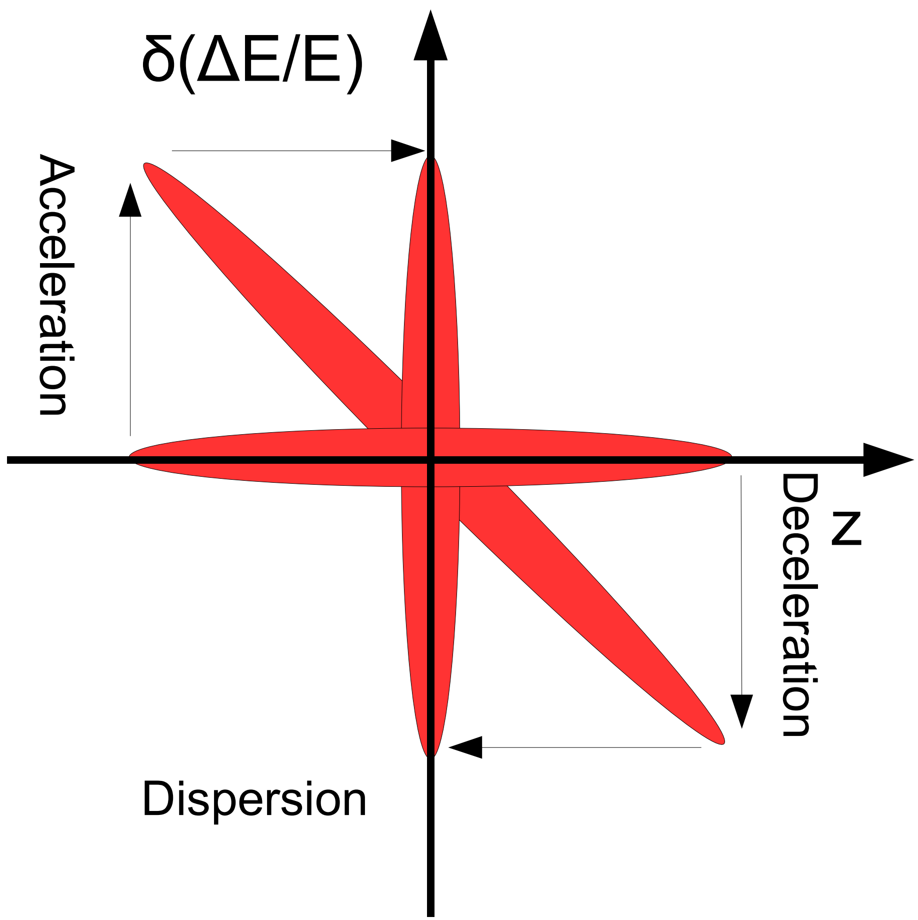
- Energy Modulation : RF cavity.
- Dispersive section : Chicane, Wiggler, Bend,..
  - E.g. four bending magnets compose a chicane
- Bunch head (tail) travels longer (shorter) path and bunch length becomes shorter.



By E.S. Kim

Common formalism (1)

- For both bunching method, it can be formalized with transfer matrix in linear approximation.
- Energy modulation is made by RF (acc- and deceleration).
- Simple drift causes the rotation in phase space in case of velocity bunching.
- Drift through a dispersive section rotates the beam in case of magnetic bunching.
- For both cases, the bunch rotates 90 deg.



Common formalism (2)

- Example of R-matrices
  - Drift space

$$\begin{bmatrix} z(s) \\ \delta(s) \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix} \quad (2-60)$$

$$R_{56} = -\frac{L}{\gamma^2 \beta^2} \quad (2-61)$$

  - Dispersive area

$$\begin{bmatrix} z(s) \\ \delta(s) \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix} \quad (2-62)$$

$$R_{56} = \eta_l = \int ds \frac{\eta}{\rho} \quad (2-63)$$

  - Energy modulation

$$\begin{bmatrix} z(s) \\ \delta(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix} \quad (2-64)$$

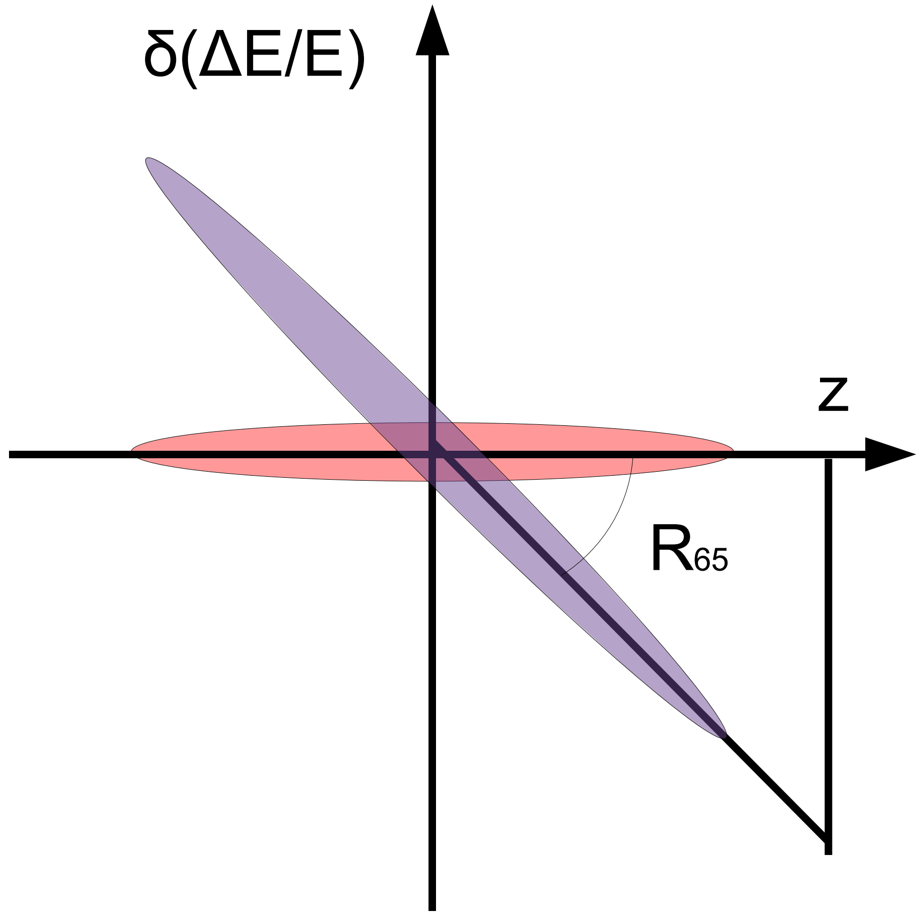
Common formalism (3)

Energy modulation by RF (acc- and deceleration)

$$\begin{bmatrix} z(s_1) \\ \delta(s_1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(0) \\ \delta(0) \end{bmatrix}$$

If the beam is on zero-cross

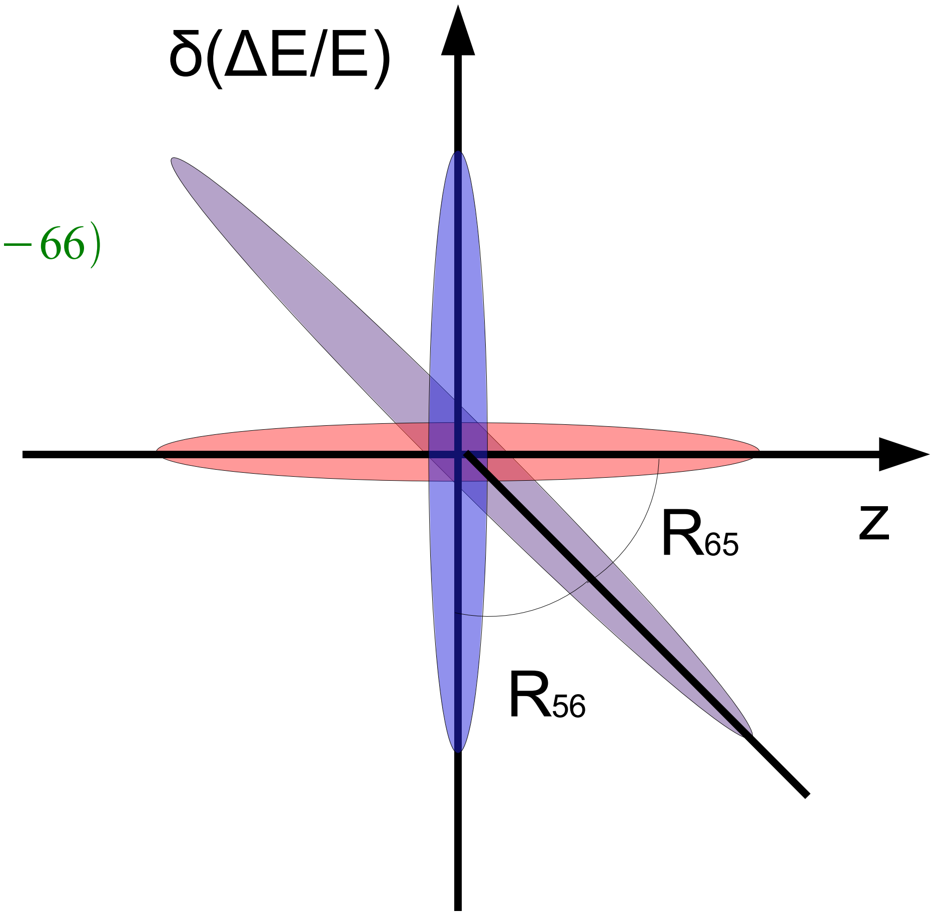
$$R_{65} = \frac{\sigma}{z} = \frac{1}{z}\frac{\Delta E}{E} \sim \frac{eV_0}{E}\frac{\omega}{\beta c} \qquad (2-65)$$



Common formalism (4)

Total Transfer Matrix of BC section.

$$\begin{bmatrix} z(s_2) \\ \delta(s_2) \end{bmatrix} = \begin{bmatrix} 1 & R_{56} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix} = \begin{bmatrix} 1+R_{56}R_{65} & R_{56} \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix} \qquad (2-66)$$

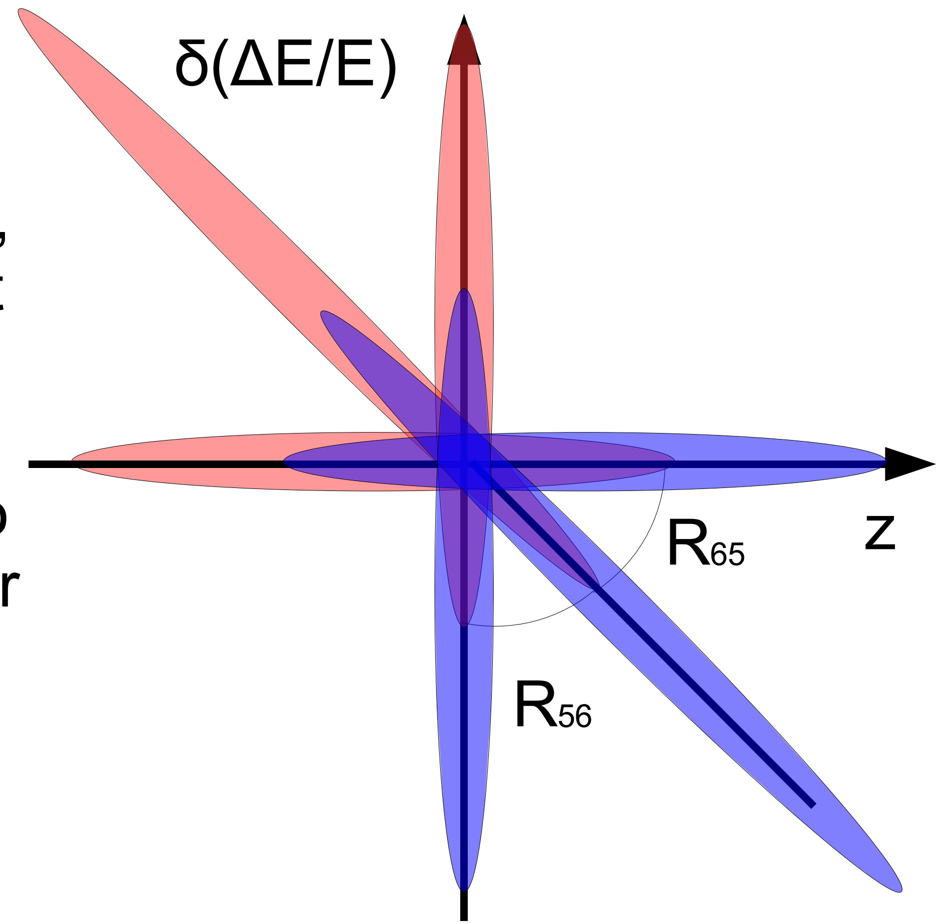


If $1+R_{56}R_{65}=0$, the phase space distribution rotate π/2 and the bunch length is minimized. For velocity bunching, the condition for perfect bunching is

$$1+R_{56}R_{65}=1-\frac{L}{\gamma^2\beta^2}\frac{eV_0}{E}\frac{\omega}{\beta c}=0 \qquad (2-67)$$

$$\begin{bmatrix} z(s_2) \\ \delta(s_2) \end{bmatrix} = \begin{bmatrix} 0 & R_{56} \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} z(s_0) \\ \delta(s_0) \end{bmatrix} \quad (2-68)$$

- After the bunch compressor section, the phase in the linac, $z(s_2)$, depends only on $\delta(s_0)$; It is insensitive to the phase fluctuation or drift.
- This is a good mechanism to stabilize the bunch phase prior to acceleration in Main Linac.



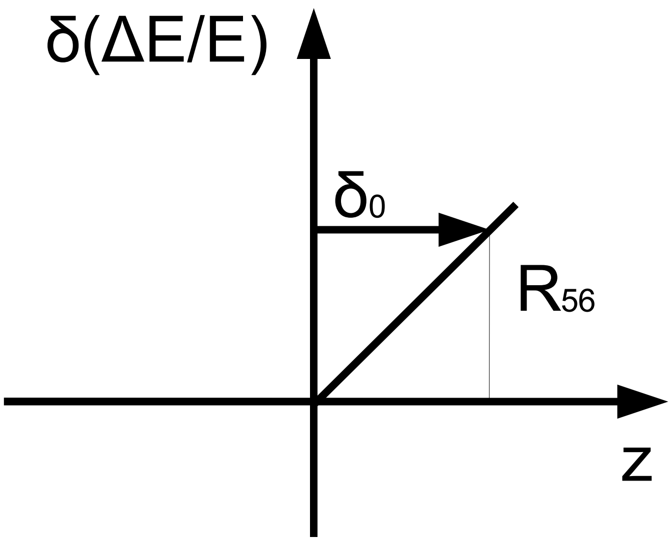
Common formalism (6)

- Final bunch length after an optimized BC section ($1+R_{56}R_{65}=0$) is determined by the initial energy spread as;

$$z_2 = R_{56}\,\delta_0 \qquad (2-69)$$

- It can be understood by considering the transport of a reference point.

$$\begin{bmatrix} R_{56}\delta_0 \\ \delta_0 \end{bmatrix} = \begin{bmatrix} 0 & R_{56} \\ R_{65} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \delta_0 \end{bmatrix} \qquad (2-70)$$



- The actual bunch length is also limited by non-linearity on RF modulation and transfer matrix. To prevent such effects, several compressor sections are employed.