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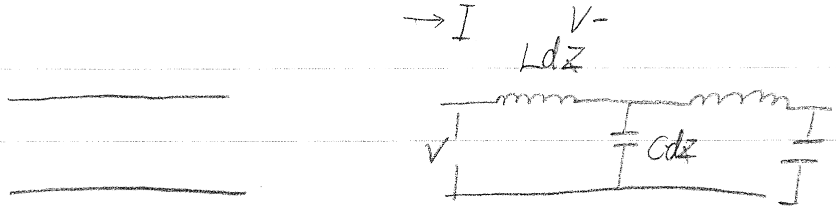
12.

$1{:}00 \sim 4{:}30$ PM

1-①

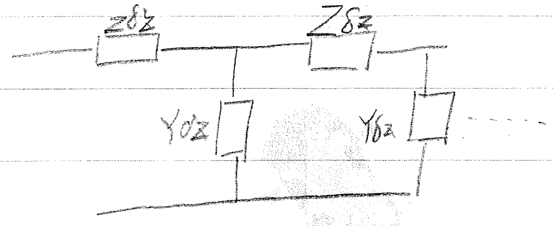
lecture 1

①~⑫



L = inductor / unit length

C = capacity / unit length



$$\frac{di}{dz} = vY \qquad \frac{dv}{dz} = iZ$$

Y - admitance per unit length

$$\frac{d^2 i}{dz^2} = Y\frac{dv}{dz} = YiZ$$

$$i = A e^{\pm\sqrt{YZ}\,z}$$

$$Z = j\omega L \qquad Y = j\omega C \qquad e^{j\omega t}$$

$I = ie^{j\omega t}$

$V = ve^{j\omega t}$

$$i = Ae^{\pm\sqrt{-\omega^2 LC}\,z} = A e^{\pm j\omega\sqrt{LC}\,z}$$

$$V = \frac{1}{Y}\frac{di}{dz} = \frac{1}{Y} \pm j\omega\sqrt{LC}\; i$$

$$= \frac{1}{Y} \pm \sqrt{-\omega LC} = \pm A\frac{1}{Y}\sqrt{YZ}\, e^{\pm\sqrt{YZ}\,z}$$

$$= A\boxed{\mp\sqrt{\frac{L}{C}}}\,e^{\pm j\omega\sqrt{LC}z} = \mp\sqrt{\frac{L}{C}}\, e^{\pm}$$

Characteristic impedance $Z_c = \pm\sqrt{\frac{L}{C}} = \pm\sqrt{\frac{Z}{Y}}$ $\qquad \gamma = \pm\sqrt{YZ}$ propagation constant

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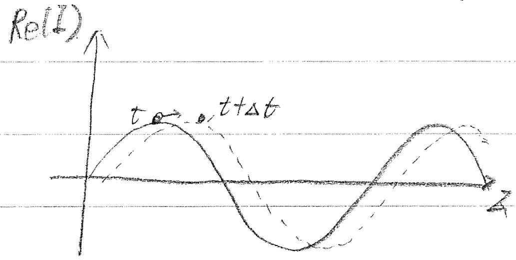
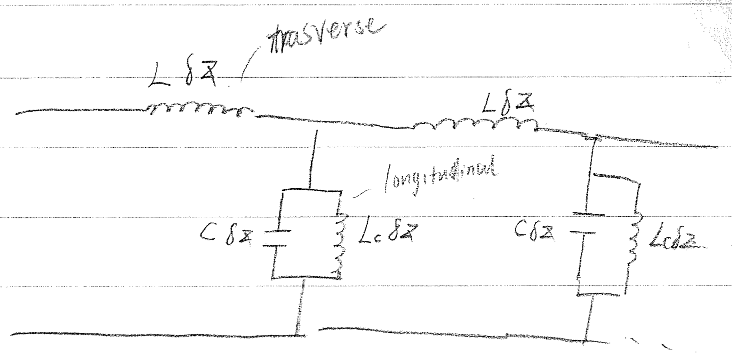
$\frac{v}{i} = Z_c$ independent of $\cancel{z}$ i

$I = A e^{j(\omega t \mp \beta z)}$ where $\beta = \sqrt{LC}$

$z = \frac{\omega t}{\beta}$ $\quad V_{ph} = \frac{\omega}{\beta}$ phase velocity



sign!!



$$\gamma = \pm\sqrt{YZ} = \pm\sqrt{j\omega L\left(j\omega C + \frac{1}{j\omega L_c}\right)}$$

$$= \pm\sqrt{j\omega L\, j\omega C\left[1 - \frac{1}{\omega^2 L_c C}\right]} = \mp j\omega\sqrt{LC\left(1 - \frac{\omega_c^2}{\omega^2}\right)} \quad \text{when } \omega_c^2 = \frac{1}{L_c C}$$

$\omega > \omega_c$ $\quad \gamma$ is imaginary — propagation wave

$\omega < \omega_c$ $\quad \gamma$ is real — exponatial decay

Characteristic Impedence

$$Z_c = \pm\sqrt{\frac{L}{C}} = \pm\sqrt{\frac{Z}{Y}}$$

$$= \pm\sqrt{\frac{j\omega L}{j\omega C + \frac{1}{j\omega L_c}}} = \pm\sqrt{\frac{L}{C}}\cdot\sqrt{\frac{1}{1 - \frac{\omega_c^2}{\omega^2}}}$$

$\omega < \omega_c$ $\quad Z_c$ imaginary
v & i are out of phase
no power flow

$\boxed{\text{TE waveguide}}$

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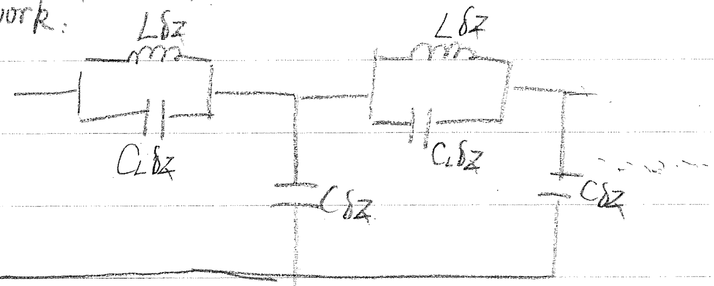
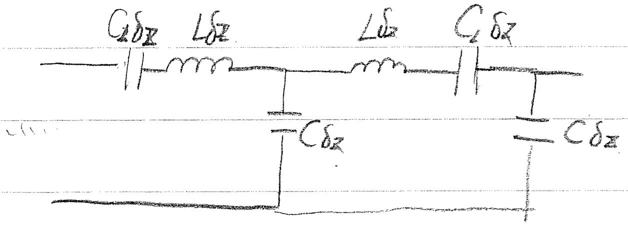
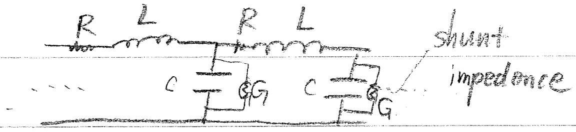
Homework: (extras)



Z_c? γ?

TE or TM



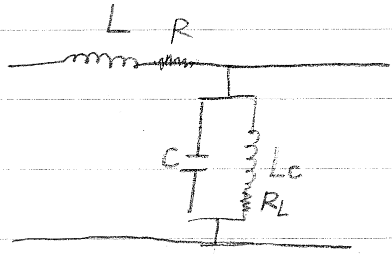


$$\gamma = \pm\sqrt{(j\omega L+R)(j\omega C+G)}$$

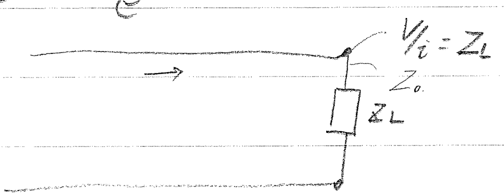
$$= \mp j\omega\sqrt{LC}\sqrt{\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)}$$

$$\simeq \mp j\omega\sqrt{LC}\left(1+\frac{1}{2}\frac{R}{j\omega L}\right)\left(1+\frac{1}{2}\frac{G}{j\omega C}\right)$$

$$\simeq \mp j\omega\sqrt{LC}\left(1-\frac{1}{2}\frac{jR}{\omega L}\right) \qquad G \ll 1$$



towards generator
← $e^{-j\beta z}$ $V^f = A^f e^{-j\beta z}$ $i^f = \frac{A_f}{Z_c} e^{-j\beta z}$



← $e^{j\beta z}$ $V^B = A^B e^{j\beta z}$ $i^B = -\frac{A^B}{Z_c} e^{j\beta z}$

towards load

moving backward direction

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When $Z_c = Z_L$ - line is matched - only forward wave
no backward wave

$$V = V^f + V^B = A^f e^{-j\beta z} + A^B e^{j\beta z}$$

$$I = I^f + I^B = \frac{A^f e^{-j\beta z} - A^B e^{j\beta z}}{Z_C}$$

At the load position

$$\frac{V}{i} = \left(\frac{A^f e^{-j\beta z_0} + A^B e^{j\beta z_0}}{A^f e^{-j\beta z_0} - A^B e^{j\beta z_0}}\right) \cdot Z_c = Z_L$$

$$\Rightarrow \frac{e^{-j\beta z_0} + \Gamma e^{j\beta z_0}}{e^{-j\beta z_0} - \Gamma e^{j\beta z_0}} = \frac{Z_L}{Z_C} = \hat{Z}_L$$

normalized
load impedence

$$\boxed{\Gamma = \frac{A_B}{A_f}} \text{ - reflection coefficient}$$

Assume $Z_0 = 0$ $\Rightarrow$ $\boxed{\frac{1+\Gamma}{1-\Gamma} = \hat{Z}_L}$ $\boxed{\Gamma = \frac{\hat{Z}_L - 1}{\hat{Z}_L + 1}} = \frac{Z_L - Z_C}{Z_L + Z_C}$

only valid at the load

$$= \left.\frac{A_B}{A_f}\right|_{z=0}$$

$$\hat{Z}_L - \Gamma \hat{Z}_L = 1 + \Gamma$$

$$\hat{Z}_L - 1 = \Gamma \hat{Z}_L + \Gamma$$

$$\Gamma = \left.\frac{A_B}{A_f}\right|_{z=0}$$

$$\Gamma'|_l = \frac{e^{-j\beta l} A^B}{e^{j\beta l} A^f} = e^{-2j\beta l}\,\Gamma$$ - move from load to generator

$$e^{2j\beta l}\,\Gamma$$ - move from generator to load

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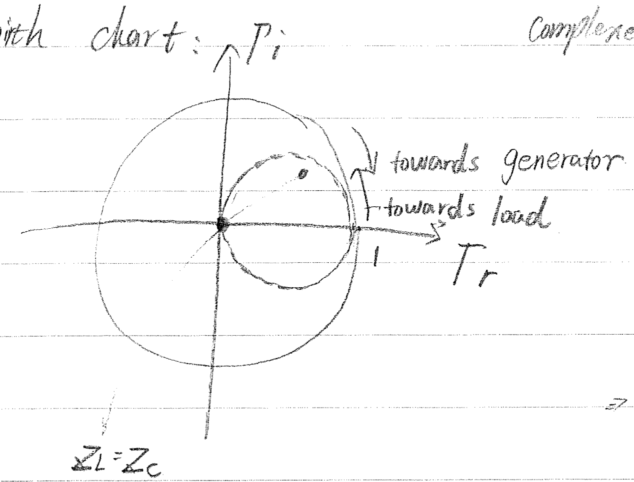
smith chart: complexed plane



$$\frac{1+\Gamma_r+i\Gamma_i}{1-\Gamma_r-i\Gamma_i} = \hat{z}_r + i\hat{z}_i$$

$$\frac{(1+\Gamma_r+i\Gamma_i)(1-\Gamma_r+i\Gamma_i)}{(1-\Gamma_r)^2+\Gamma_i^2} = \hat{z}_r + i\hat{z}_i$$

$$\Rightarrow = \frac{1-\Gamma_r^2 + i\Gamma_i(1-\Gamma_r) + i\Gamma_i(1+\Gamma_r) - \Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$$

$$\Rightarrow \hat{z}_r = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2} \qquad \hat{z}_i = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

$$\hat{z}_r(1-\Gamma_r)^2 + \hat{z}_r\Gamma_i^2 = 1-\Gamma_r^2-\Gamma_i^2$$

$$\hat{z}_r - 2\hat{z}_r\Gamma_r + \hat{z}_r\Gamma_r^2 + \hat{z}_r\Gamma_i^2 = 1-\Gamma_r^2-\Gamma_i^2$$

$$\Rightarrow \Gamma_r^2\left(1+\frac{1}{\hat{z}_r}\right) + \Gamma_i^2 - 2\Gamma_r = 1/\hat{z}_r - 1 - \frac{1}{\hat{z}_r}\Gamma_i^2$$

$$\left(1+\frac{1}{\hat{z}_r}\right)\Gamma_i^2 + \left(\Gamma_r\sqrt{1+\frac{1}{\hat{z}_r}} - \frac{1}{\sqrt{1+\frac{1}{\hat{z}_r}}}\right)^2 - \frac{1}{1+\frac{1}{\hat{z}_r}} = 1/\hat{z}_r - 1$$

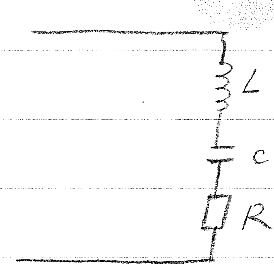
circle
$$\boxed{\Gamma_i^2 + \left(\Gamma_r - \frac{1}{1+\frac{1}{\hat{z}_r}}\right)^2 = \left[(1/\hat{z}_r - 1) + \frac{1}{1+\frac{1}{\hat{z}_r}}\right] \Big/ \left(1+\frac{1}{\hat{z}_r}\right)}$$

if $\hat{z}_r = 1$ $(Z_r = Z_C)$

$$\Gamma_i^2 + \left(\Gamma_r - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2$$

① Alway go through (1, 0) then

② The larger $\hat{z}_r$ the smaller radius



Load impedence $Z_L = j\omega L + \frac{1}{j\omega C} + R$

$$= j\omega_0 L\left(\frac{\omega}{\omega_0} - \frac{1}{\omega\omega_0 LC}\right) + R$$

$$= j\omega_0 L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) + R \qquad \omega_0^2 = \frac{1}{LC}$$

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$$Z_L = R\left[\frac{j\omega_0 L}{R}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) + 1\right]$$

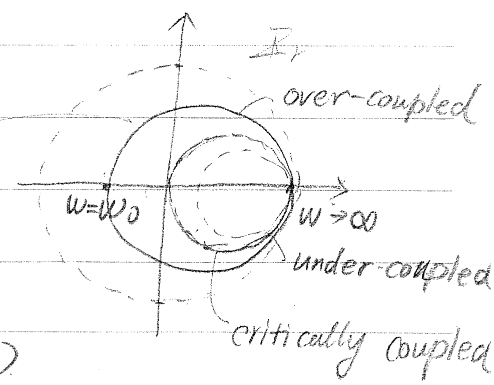
$$= R\left[1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] \qquad Q = \frac{\omega_0 L}{R} \quad \text{quality factor}$$

when $\omega = \omega_0$ $\quad Z_L = R \quad Z_r = R \quad Z_i = 0$

when ω changes $\quad Z_r = R$ doesn't change. only Z_i changes.

$$\Gamma = \frac{\hat{R}\left(1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right) - 1}{\hat{R}\left(1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right) + 1}$$

Q circle



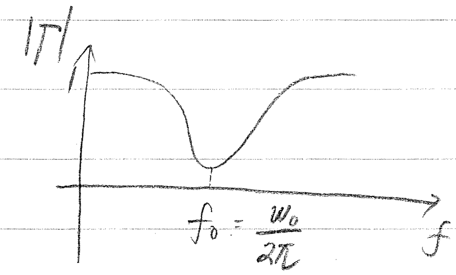
$$\Gamma\Big|_{\omega=\omega_0} = \frac{\hat{R} - 1}{\hat{R} + 1}$$

$\hat{R} < 1$ - over-coupled

$\hat{R} > 1$ - under-coupled

$\hat{R} = 1$ - go through $(0,0)$

critically coupled



From here, you can only get ω_0. cannot determine wheather it's over-coupled undercoupled or critically coupled

Quality factor $\begin{cases} \text{internal} \\ \text{external} \end{cases}$

under coupled $\quad Q_{external} > Q_{internal}$

over-coupled $\quad Q_{external} < Q_{internal}$

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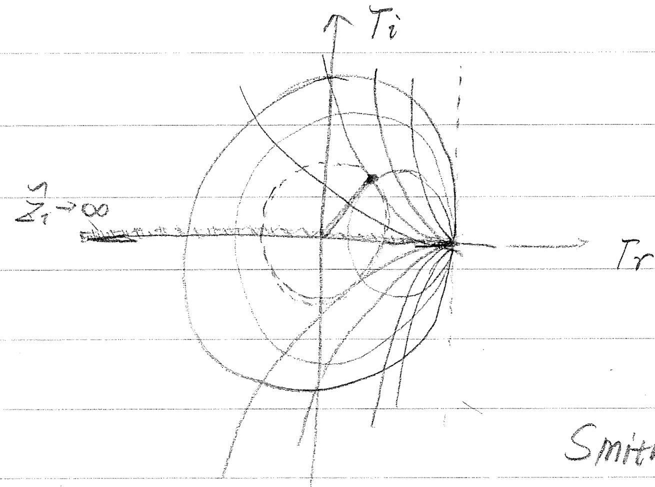
$$\frac{2\Gamma_i}{z_i} = 1-2\Gamma_r+\Gamma_r^2+\Gamma_i^2$$

$$\Gamma_r^2+\Gamma_i^2-\frac{2\Gamma_i}{z_i} = 2\Gamma_r-1$$

$$\left(\Gamma_i-\frac{1}{\hat{z}_i}\right)^2 - \frac{1}{\hat{z}_i^2} + \Gamma_r^2 = 2\Gamma_r-1$$

$$\left(\Gamma_i-\frac{1}{\hat{z}_i}\right)^2 + (\Gamma_r-1)^2 = \frac{1}{\hat{z}_i^2}$$



D $\hat{z}_i \to \infty$

Smith Chart! — $z = \frac{1+\Gamma}{1-\Gamma}$

$$\Gamma' = \Gamma e^{-j2\beta l} = \frac{Z-Z_c}{Z+Z_c} e^{-2j\beta l} = \frac{Z'-Z_c}{Z'+Z_c}$$

$$\frac{\hat{z}-1}{\hat{z}+1} e^{-2j\beta l} = \frac{\hat{z}'-1}{\hat{z}'+1}$$

? $$\hat{z}' = \frac{1+\Gamma'}{1-\Gamma'} = \frac{1+\frac{\hat{z}-1}{\hat{z}+1}e^{-2j\beta l}}{1-\frac{\hat{z}-1}{\hat{z}+1}e^{-2j\beta l}}$$

$$= \frac{\hat{z}+1+(\hat{z}-1)e^{-2j\beta l}}{\hat{z}+1-(\hat{z}-1)e^{-2j\beta l}} = \frac{(\hat{z}+1)e^{j\beta l}+(\hat{z}-1)e^{-j\beta l}}{(\hat{z}+1)e^{j\beta l}-(\hat{z}-1)e^{-j\beta l}}$$

$$= \frac{\hat{z}\cos\beta l + j\sin\beta l}{\hat{z}j\sin\beta l + \cos\beta l}$$

$$\Rightarrow \hat{z}' = \frac{\hat{z}+j\tan\beta l}{1+j\hat{z}\tan\beta l}$$

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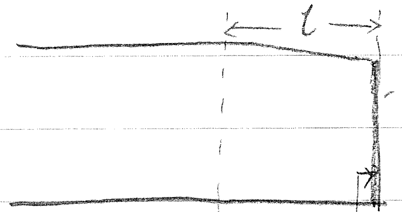
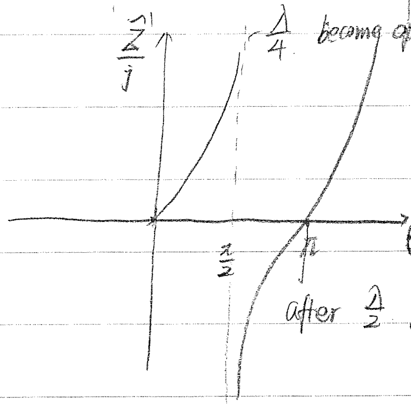


short line $\hat{Z}=0$

$$\hat{Z}' = \frac{j\tan\beta l}{1} = j\tan\beta l$$



$\frac{\lambda}{4}$ becomes open circuit

repeat itself after π.

$$e^{j(\omega t - \beta z)}$$

$\beta l = \frac{2\pi l}{\lambda} = 2\pi \hat{l}$ normalized wavelength $\lambda = \frac{2\pi}{\beta}$

after $\frac{\lambda}{2}$ go back to short circuit

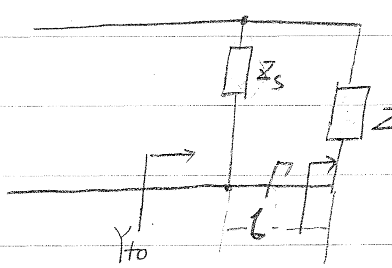
Only has L.C β is constant

generally β is a function of frequency

Γ repeat itself every half-wavelength.

$e^{-2j\beta l}$ from both forward and backward effect.

Matching



$$\Gamma = \frac{\hat{Z}_L - 1}{\hat{Z}_L + 1}$$

$$\hat{Z}' = \frac{\hat{Z}_L + j\tan\beta l}{1 + j\hat{Z}_L\tan\beta l}$$

$$\hat{Y}' = \frac{1 + \hat{Z}_L\tan\beta l}{\hat{Z}_L + j\tan\beta l}$$

$\hat{Y}_{to} = \hat{Y}' + \frac{1}{\hat{Z}_s} = 1$ - complex equation.

$\hat{Y}' = 1 +$ pure imaginary $\frac{1}{\hat{Z}_s}$ cancel imaginary part

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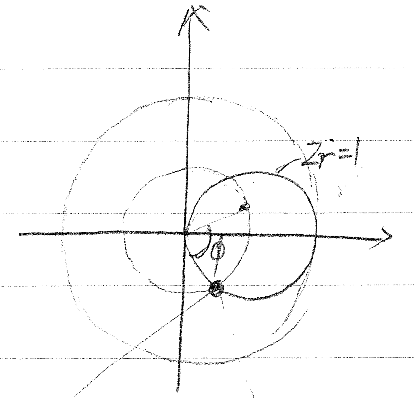


$\phi = \frac{4\pi}{\lambda} \cdot l$

$\hat{Z} = 1 + iY$ Z_s pure imaginary to cancel

$\frac{1}{\hat{Z}} \neq 1 + i\hat{Z}$

$$\Gamma = \frac{1/\hat{Y} - 1}{1/\hat{Y} + 1} = \frac{1 - \hat{Y}}{1 + \hat{Y}}$$

Smith chart can also be done for Y, Z_c ...

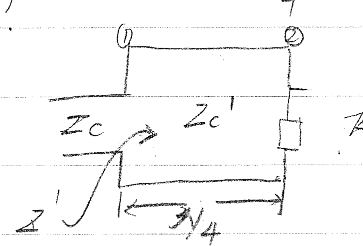
$$Y' = \frac{1 + \hat{Z}_L \tan\beta l}{\hat{Z}_L + j\tan\beta l} = \frac{(1 + \hat{Z}_L \tan\beta l)(\hat{Z}_L^* - j\tan\beta l)}{|\hat{Z}_L|^2 + (\tan\beta l)^2}$$

$$= \frac{\hat{Z}_L^* + |\hat{Z}_L|^2 \tan\beta l - j\tan\beta l - j\hat{Z}_L(\tan\beta l)^2}{|\hat{Z}_L|^2 + (\tan\beta l)^2}$$

$Re(Y') = 1 \Rightarrow$ solve Y' then determine the imaginary cancel Z_s

$$\hat{Z}' = \frac{\hat{Z} + j\tan\beta l}{1 + \hat{Z} j \tan\beta l} \qquad \beta l = \frac{\pi}{2} \qquad \hat{Z} = \frac{Z_L}{Z_C}$$

$\rightarrow \hat{Z}'\Big|_{l = \frac{\lambda}{4}} = \frac{1}{\hat{Z}}$ After each equater of wavelength. $\hat{Z} \rightarrow \frac{1}{\hat{Z}}$



$\hat{Z}'$ at ① $\frac{1}{\hat{Z}}$ at ②

$$\frac{Z'}{Z_c'} = \frac{Z_c'}{Z_L} \Rightarrow Z' = \frac{Z_c'^2}{Z_L} = Z_c$$

match

Quater-Wavelength transform $Z_c' = \sqrt{Z_c Z_L}$

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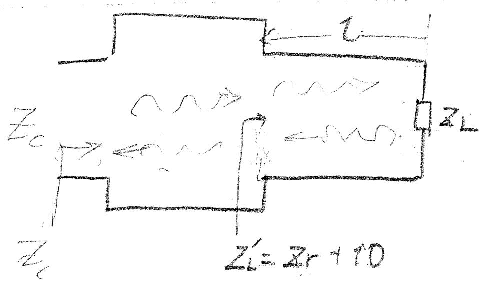
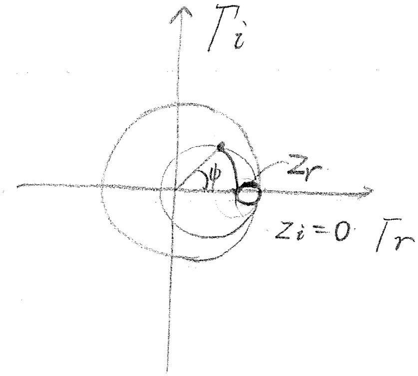
$$Z_L = Z_r + jZ_i$$
$$-jZ_i$$

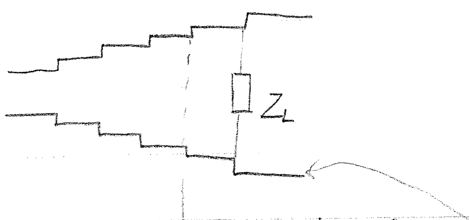




?

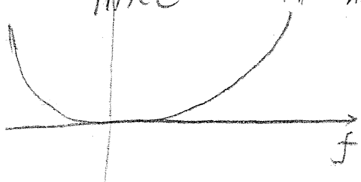
All these happen at particular frequency

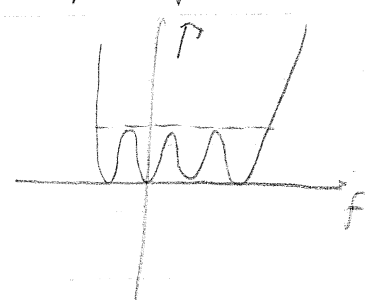
$$\beta = \omega \cdot \sqrt{LC}$$



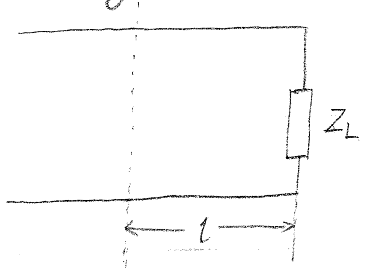
very little reflection, neglect multiple reflection

$\Gamma(f) \in C^N$ — N = number of steps





Standing Wave



$$\Gamma = \frac{Z_L - Z_c}{Z_L + Z_c}$$

$$V = A^f(e^{j\beta l} + \Gamma e^{-j\beta l})$$

$$i = \frac{A^f}{Z_c}(e^{j\beta l} - \Gamma e^{-j\beta l})$$

$$V = A^f e^{j\beta l}(1 + \Gamma e^{-2j\beta l})$$

$$|V| = A^f \left|1 + \Gamma e^{-2j\beta l}\right|$$

$$|V|_{max} = |A^f|(1 + |\Gamma|) \qquad |V|_{min} = |A^f|(1 - |\Gamma|)$$

$$\frac{|V|_{max}}{|V|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \equiv VSWR \text{ (voltage standing wave ratio)}$$

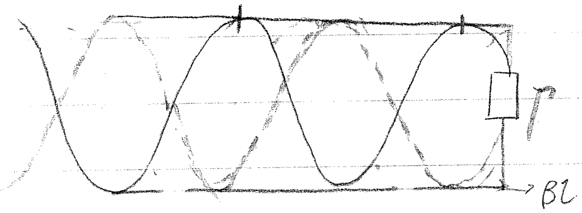
$$|v| = |A^+|\,\left|1 + |\Gamma| e^{j(\phi - 2\beta l)}\right|$$ ϕ angur of Γ at the load

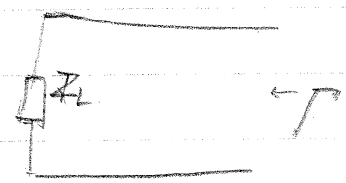
$$\phi - 2\beta l = 2N\pi \qquad l_{max} = \frac{\phi - 2N\pi}{2\beta}$$

$$|i| = \frac{|A^+|}{|Z_c|}\left|1 - |\Gamma| e^{j(\phi - 2\beta l)}\right|$$

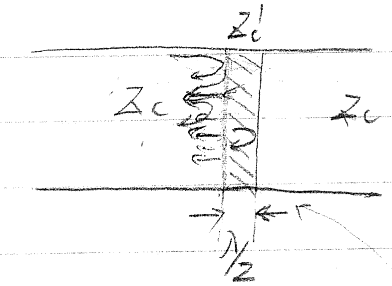
$$\phi - 2\beta l = (2N+1)\pi \qquad l_{max} = \frac{\phi - \pi - 2N\pi}{2\beta}$$

Maximum current & voltage differ by $\frac{\pi}{2\beta} \simeq \frac{\lambda}{4}$





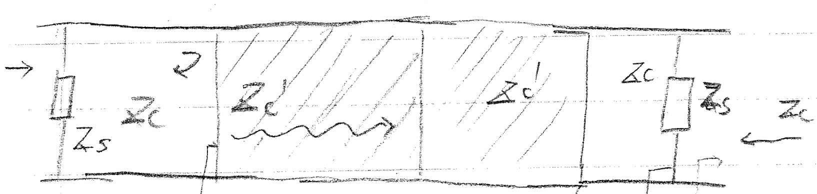
Vacuum Windows



$$Z' = \frac{\hat{Z} + j\tan\beta l}{1 + j\hat{Z}\tan\beta l}$$

put $\beta l = \pi n$ $\qquad Z' = \hat{Z}$

Homwork find out VSWR

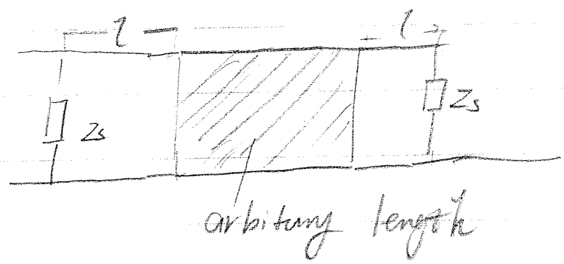


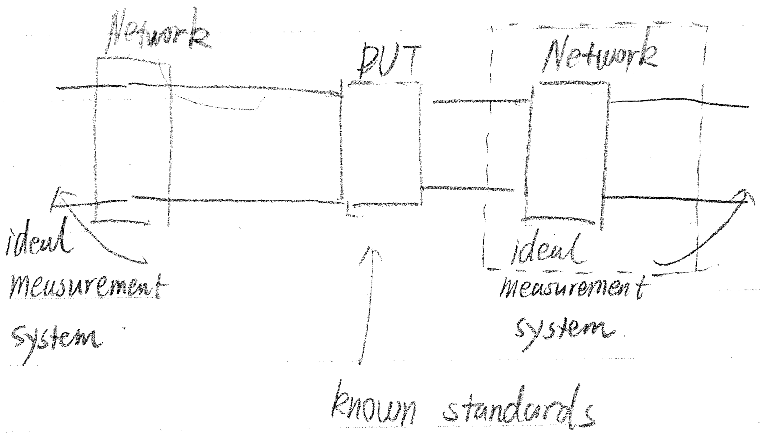
$Z_c'|_l$ $\qquad Z_c'$ $\qquad Z_c$

$\frac{1}{Z_c} + \frac{1}{Z_s}$

$\frac{1}{Z_s} + \frac{1}{Z_c'}\Big|_l$ $\qquad Z_c' = \frac{1}{Z_c} + \frac{1}{Z_s}\Big|_{at\, l}$

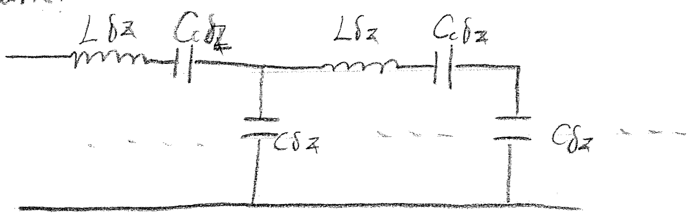
$= \frac{1}{Z_c}$



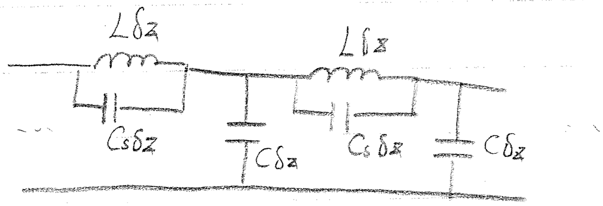


Homework

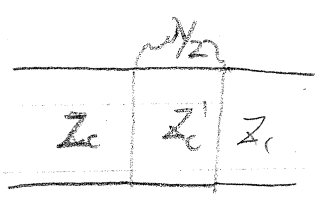
1)



find propagation constant
characteristic impedence

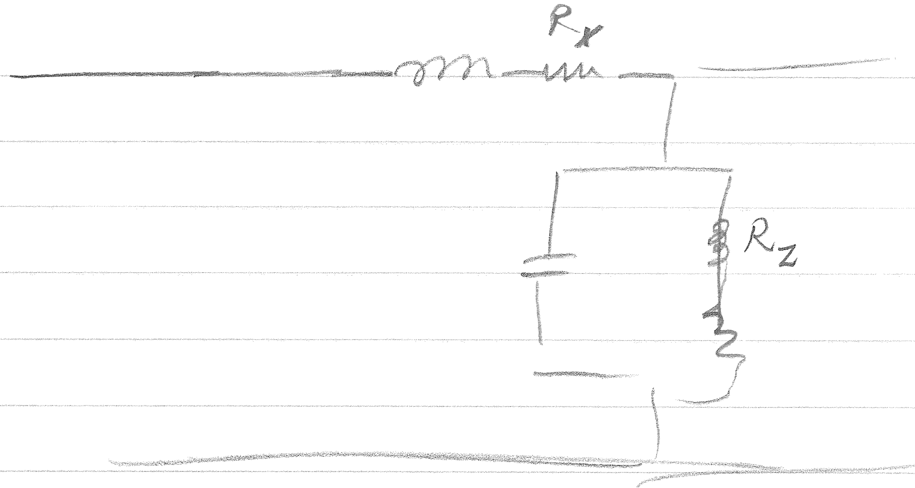


2)



find VSWR

3.10 4.9 4.15 5.3



$$\gamma^2 = (j\omega L + R)\left(j\omega C + \frac{1}{j\omega L_c + R_z}\right)$$

$$= j\omega L\left(1 + \frac{R}{j\omega L}\right) j\omega C\left(1 + \frac{1}{j\omega L_c j\omega C + R_z j\omega C}\right)$$

$$= j\omega L\, j\omega C\left[\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{1}{-\omega^2/\omega_c^2 + jR_z j\omega C}\right)\right]$$

$$= -\omega^2 \sqrt{LC}\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{1}{j\omega L_c j\omega L_c\left[1 + \frac{R_z}{j\omega L_c}\right]}\right)$$

$$= -k_0^2\left(1 + \frac{R_x}{j\omega L}\right)\left(1 + \frac{1}{-(\omega/\omega_c)^2\left(1 + \frac{R_z}{j\omega L_c}\right)}\right)$$

but $$\frac{R}{j\omega L} = \frac{I^2 R}{j\omega L I_a^2} =$$

$$\frac{R_s \int (H_x)^2\, dx}{\frac{b\mu}{4}\int_0^a (H_x)^2\, dx} = \frac{4R}{jb\mu}$$

$$\frac{R_z}{j\omega L_c} = \frac{R_s b H_z l a}{\mu \frac{(H_z)^2}{8}} = \frac{8R_s}{ja\mu}$$

$$\gamma^2 = -k_0^2\left(1 + \frac{j4R_s}{a\mu}\right)\left(1 + \frac{1}{(\omega/\omega_c)\left(1 + \frac{8R_s}{ja\mu}\right)}\right)$$

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lecture 2 13th.

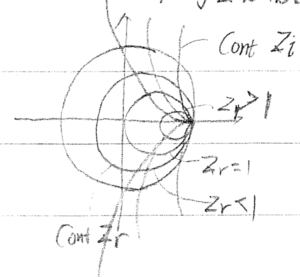
Refresh:

① Uniform along longitudinal direction

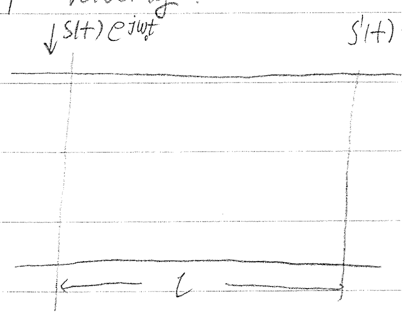
② vary as $e^{j\omega t}$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad \hat{Z} = \frac{1+\Gamma}{1-\Gamma} \qquad \Gamma = \frac{Z-Z_c}{Z+Z_c}$$

$$\hat{Z}' = \frac{\hat{Z}+j\tan\beta l}{1+j\hat{Z}\tan\beta l} \qquad \Gamma' = \Gamma e^{-2j\beta l} \qquad \lambda = \frac{2\pi}{\beta} \qquad V_{ph} = \frac{\omega}{\beta}$$



group velocity.

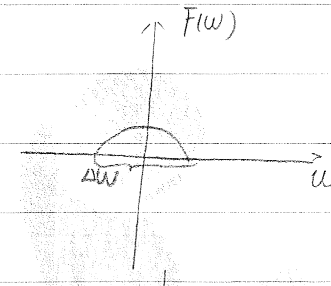


if $\beta(\omega)$ depends on ω, different frequency have different propagation constant

$$F(\omega) = \int_{-\infty}^{+\infty} s(t)e^{-j\omega t}\,dt$$

$$s(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \underbrace{\quad}_{F(\omega)} e^{j\omega t}\,d\omega$$

} fourier transform



$$\hat{F}(\omega) = \int_{-\infty}^{+\infty} s(t)e^{-j\omega t}\, e^{j\omega_0 t}\,dt = \hat{F}(\omega-\omega_0)$$

$$\hat{F}(\omega)e^{-j\beta l}$$

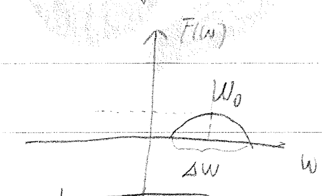
$$\beta(\omega) = \beta_0 + \frac{\partial\beta}{\partial\omega}(\omega-\omega_0)+\cdots \quad \beta_0 = \beta(\omega_0)$$

$$= \hat{F}(\omega-\omega_0)\,e^{-j\beta_0 l}\,e^{-j\beta_0'(\omega-\omega_0)l}$$

$$s'(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{F}(\omega)\,e^{-j\beta l}\,e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{F}(\omega-\omega_0)\,e^{-j\beta_0 l}\,e^{-j\beta_0'(\omega-\omega_0)l}\,e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}e^{-j\beta_0 l}\,e^{j\omega_0 t}\int_{-\infty}^{+\infty}\hat{F}(\omega-\omega_0)\,e^{-j\beta_0'(\omega-\omega_0)l}\,e^{j(\omega-\omega_0)t}\,d\omega$$



$$\boxed{\Delta\omega \ll \omega_0}$$

$$\boxed{s'(t) = s(t-\beta' l)\cdot e^{-j(\beta_0 l-\omega_0 t)}}$$

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1> $e^{-\beta_0 l}$ ~ propagation

2> signal delayed by $\beta_0' l$ $\qquad \beta_0' l = Z$

$1/Z$ = Group velocity $\equiv V_{group} = \frac{1}{\beta'} = \frac{\partial \omega}{\partial \beta} = \frac{1}{\frac{\partial \beta}{\partial \omega}} \neq V_{phase} = \frac{\omega}{\beta}$

Only when $\beta \propto \omega$. $V_{group} = V_{phase}$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{D} = \rho$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \qquad \nabla \cdot \vec{B} = 0$$

$\vec{E}\ \vec{B}\ \vec{H}\ \vec{D} \sim e^{j\omega t}$. $\qquad \frac{\partial}{\partial t} \sim j\omega$.

$$\begin{bmatrix} D \\ \\ B \end{bmatrix} = \begin{bmatrix} \epsilon & & & & & \\ & \epsilon & & & & \\ & & \epsilon & & & \\ & & & u & & \\ & & & & u & \\ & & & & & u \end{bmatrix} \begin{bmatrix} E \\ \\ H \end{bmatrix}$$

diaganol

ϵ u. are not. functions of neither time or space.

$\frac{\partial \epsilon}{\partial t} = 0 \qquad \nabla \epsilon = 0$

$\Rightarrow \quad \nabla \times \vec{E} = -j\omega u \vec{H}$ ① $\qquad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$ ②

$\nabla \times \vec{H} = j\omega\epsilon \vec{E} + \vec{J}$ ③ $\qquad \nabla \cdot \vec{H} = 0$ ④

$\nabla \cdot (\nabla \times \vec{E}) = -j\omega u \nabla \cdot \vec{H} = 0$ $\qquad$ ① $\Rightarrow$ ④ $\qquad$ Charge conservatio

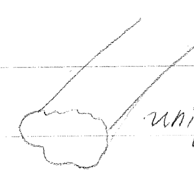
$\nabla \cdot (\nabla \times \vec{H}) = j\omega\epsilon \nabla \cdot \vec{E} + \nabla \cdot \vec{J} = 0 \quad \Rightarrow \quad j\omega\epsilon \frac{\rho}{\epsilon} + \nabla \cdot \vec{J} = 0 \quad \Rightarrow \quad \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$

Continuity equation

$\Rightarrow$ Contininity equation can replace ② in Maxwell's eq.

$\Rightarrow$ Maxwell's eq. $\quad \nabla \times \vec{E} = -j\omega u \vec{H}$

$\nabla \times \vec{H} = j\omega\epsilon \vec{E} + \vec{J}$



uniform in Z direction

No current $\quad \nabla \times \vec{E} = -j\omega u \vec{H}$

$\nabla \times \vec{H} = j\omega\epsilon \vec{E}$

$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -j\omega u \nabla \times \vec{H}$

$\Rightarrow \quad -\nabla^2 \vec{E} = -j\omega u\, j\omega\epsilon \vec{E} = \omega^2 u \epsilon \vec{E}$ $\qquad$ No free space charge!

$\omega^2 \epsilon u = k^2 \qquad \sqrt{\omega^2 \epsilon_0 u_0} = k_0 = \frac{\omega}{C} \qquad C = \frac{1}{\sqrt{u_0 \epsilon_0}}$

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$\nabla^2 \vec{E} + K_0^2 \vec{E} = 0$ Helmhotz Eq.

In Catersial $\nabla^2 E_{x(y,z)} + K_0^2 E_{x(y,z)} = 0$

$\nabla^2 \psi + K_0^2 \psi = 0$

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \equiv \nabla_t^2 + \frac{\partial^2}{\partial z^2}$

$\psi = T(x,y) Z(z)$

$Z(z) \nabla_t^2 T(x,y) + \frac{\partial^2}{\partial z^2} Z(z) T(x,y) + K_0^2 T(x,y) Z(z) = 0$. Seperation of variable.

$\frac{\nabla_t^2 T(x,y)}{T(x,y)} + \frac{\frac{\partial^2}{\partial z^2} Z(z)}{Z(z)} + K_0^2 = 0$

$\frac{1}{Z(z)} \frac{\partial^2 Z(z)}{\partial z^2} = -\beta^2$ $\qquad \frac{\nabla_t^2 T(x,y)}{T(x,y)} = -K_t^2$

$-\beta^2 - K_t^2 + K_0^2 = 0 \Rightarrow \boxed{K_0^2 = \beta^2 + K_t^2}$ – dispersion relation

when $K_t = 0$
$K_0 = \beta = \omega\sqrt{u\epsilon}$
$\propto \omega$
when $K_t \neq 0$
$K_0 \not\propto \omega$ V_{group}
$\neq V_{ph}$
disperse

$\frac{\partial^2 Z(z)}{\partial z^2} + Z(z)\beta^2 = 0 \Rightarrow Z(z) = A^f e^{-j\beta z} + A^R e^{j\beta z}$

For a forward wave. $\frac{\partial}{\partial z} \sim -j\beta$

$\nabla \equiv \nabla_t + \frac{\partial}{\partial z} a_z = \nabla_t - j\beta a_z$

$\vec{E} = \vec{E}_t + \vec{E}_z$ $\qquad \vec{H} = \vec{H}_t + \vec{H}_z$

$(\nabla_t - j\beta a_z) \times (\vec{E}_t + \vec{E}_z)$

$\nabla_t \times \vec{E}_t - j\beta a_z \times \vec{E}_t + \nabla_t \times \vec{E}_z = -j\omega u_0 (\vec{H}_t + \vec{H}_z)$

$$\Rightarrow \boxed{\begin{aligned} &\nabla_t \times \vec{E}_t = -j\omega u_0 \vec{H}_z \\ &-j\beta a_z \times \vec{E}_t + \nabla_t \times \vec{E}_z = -j\omega u_0 \vec{H}_t \end{aligned}}$$

$(\nabla_t - j\beta a_z) \times (\vec{H}_t + \vec{H}_z)$

$= \nabla_t \times \vec{H}_t - j\beta a_z \times \vec{H}_t + \nabla_t \times \vec{H}_z = j\omega\epsilon_0 (\vec{E}_t + \vec{E}_z)$

$$\Rightarrow \boxed{\begin{aligned} &\nabla_t \times \vec{H}_t = j\omega\epsilon_0 \vec{E}_z \\ &-j\beta a_z \times \vec{H}_t + \nabla_t \times \vec{H}_z = +j\omega\epsilon_0 \vec{E}_t \end{aligned}}$$

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TEM mode $H_z = 0 \quad E_z = 0$

same with $\omega = 0$ static field

$$\nabla_t \times \vec{E}_t = 0 \qquad -j\beta a_t \times \vec{E}_t = -j\omega u_0 \vec{H}_t$$

$$\nabla_t \times \vec{H}_t = 0 \qquad -j\beta a_t \times \vec{H}_t = +j\omega \varepsilon_0 \vec{E}_t$$

$$\boxed{\nabla_t \times \vec{E}_t = 0 \quad \nabla_t \times \vec{H}_t = 0 \quad \beta = \omega\sqrt{u_0 \varepsilon_0} \quad H_t = \frac{\omega_0 \varepsilon_0}{\beta} a_z \times \vec{E}_t}$$

$$a_t \times (a_t \times \vec{E}_t) = \frac{\omega u_0}{\beta} a_t \times H_t$$

$$\Rightarrow \quad -\vec{E}_t = -\frac{\omega u_0}{\beta} \frac{\omega \varepsilon_0}{\beta} \vec{E}_t$$

$$\frac{\omega_0 \varepsilon_0}{\omega_0 \sqrt{u_0 \varepsilon_0}} = \sqrt{\frac{\varepsilon_0}{u_0}} = \frac{1}{\zeta}$$

$$\Rightarrow \quad \frac{\omega^2}{\beta^2} = \frac{1}{u_0 \varepsilon_0} \quad \Rightarrow \beta = \omega\sqrt{u_0 \varepsilon_0}$$

$\rightarrow \beta^2 = k_0^2$

$\Rightarrow k_t^2 = 0$

free space wave impedence $\approx 120\pi$

$$\nabla_t \times \vec{E}_t = 0 \quad \Rightarrow \quad E_t = -\nabla_t \varphi$$

$$-\nabla_t \times \nabla_t \varphi = 0$$

$$\zeta = \sqrt{\frac{u_0}{\varepsilon_0}} = u_0 c$$

$$= 4\pi \times 10^{-7} \times 2.99792 \times$$

$$= 40\pi \times 2.99792$$

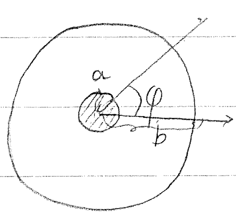
when use ζ may cause a big differenc

solve φ

$$E_t = -\nabla_t \varphi$$

$$H_t = (a \times E_t)\frac{1}{\zeta}$$

$$\nabla_t \cdot E_t = 0 \qquad \nabla_t \cdot (\nabla_t \varphi) = \nabla_t^2 \varphi = 0$$



$$\nabla_t^2 \Phi = 0$$

$$\nabla_t^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \varphi^2}$$

$$= \frac{1}{r}\left(r\frac{\partial^2 \Phi}{\partial r^2} + \frac{\partial \Phi}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \varphi^2}} = 0$$

$$= \frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r}$$

$$\Rightarrow r\frac{\partial \Phi}{\partial r} = \text{Cons}$$

$$\Rightarrow \frac{d\Phi}{dr} = \frac{c}{r} \qquad \Phi = c\ln r + C_1$$

$\Phi = V_1$ at $r = a$ $\qquad V_1 = C\ln a + C_1 \quad \Rightarrow \quad C = \frac{V_1 - V_2}{\ln(a/b)}$

$= V_2$ at $r = b$ $\qquad V_2 = C\ln b + C_1 \qquad C_1 =$

$$\vec{E} = -\nabla_t \phi = -\frac{\partial \phi}{\partial r} = \frac{-C}{r} = \frac{V_2 - V_1}{\ln a/b}\frac{1}{r}\hat{e}_r$$

$$\vec{H} = a_z \times E_t \frac{1}{\zeta} = \frac{1}{\zeta}\frac{V_2 - V_1}{\ln a/b}\frac{1}{r}\hat{e}_\varphi$$

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$V = -\int_a^b E \cdot dl = -V_1 + V_2$

$\vec{J} = \vec{n} \times \vec{H}$

$I = \oint H|_{r=a}\, dl = \frac{2\pi}{3} \frac{V_1 - V_2}{\ln a/b}$

$\Rightarrow \frac{V}{I} = \frac{3}{2\pi} \ln b/a = Z_c$

Capacitance $C = \frac{Q}{V_2 - V_1}$ $q = D|_{at\, r=a} = \epsilon E|_{r=a}$

$Q = 2\pi \cdot a \cdot \epsilon E = 2\pi a \cdot \epsilon \frac{V_1 - V_2}{\ln a/b} \frac{1}{a}$

$\Rightarrow C = \frac{2\pi \epsilon}{\ln b/a}$

$\Rightarrow Z_c = \frac{1}{C v}$ $v = \frac{1}{\sqrt{\mu \epsilon}}$

$L = \frac{\Phi}{I}$

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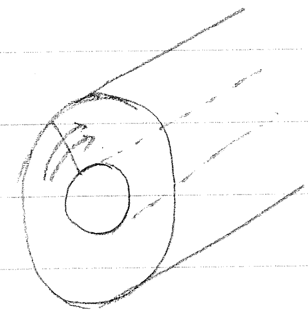
Lecture 3.



$$L = \frac{\Phi}{I} \qquad \Phi = \int u H\,dr \qquad J = n \times \vec{H}$$

$$I = \int_0^{2\pi} J\,a\,d\gamma = \frac{2\pi}{\zeta}\,\frac{V_2 - V_1}{\ln a/b}$$

$$L = \frac{\phi}{I} = \frac{u \ln a/b}{2\pi} \qquad Z_c = v \cdot L = \frac{1}{vC}$$

$$v^2 = \frac{1}{LC} = \frac{1}{u_0 \epsilon_0} \qquad Z_c = \sqrt{\frac{L}{C}}$$

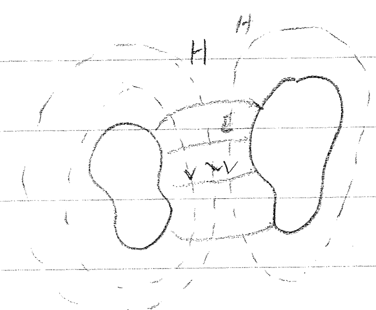
$$\frac{E}{H} \sim e^{i\omega t} \qquad \frac{\partial}{\partial t} \to j\omega$$

$$E_z = H_z = 0 \qquad \nabla_t \times \vec{H}_t = 0 \qquad \nabla_t \times \vec{E}_t = 0 \qquad H_t = 1/\zeta\, a_z \times E_t \qquad \nabla_t \cdot \vec{E}_t = 0$$

$$E_t = -\nabla_t \phi \qquad \nabla^2 \phi = 0$$

Any 2 conductors

$$I = \int H\,ds \qquad V = \int \vec{E} \cdot d\vec{l}$$



$$V = \zeta \int (H_t \times a_z)\,dl$$

$$= \zeta \frac{\Phi}{u} = \zeta \frac{L \cdot I}{u}$$

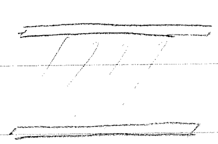
$$\frac{V}{I} = L\frac{\zeta}{u} = \frac{L}{\sqrt{u\zeta}} = L v$$

$$I = \frac{1}{\zeta}\int (a_z \times E_t)\,dl' = \frac{1}{\zeta \epsilon} Q = \frac{1}{\zeta \epsilon} V \cdot C$$

$$\frac{V}{I} = \frac{\zeta \epsilon}{C} = \frac{1}{vC} = Z_c$$

general results for any TEM mode

$$\boxed{\begin{aligned} \frac{1}{v^2} &= LC = u\epsilon \\ Z_c^2 &= \frac{L}{C} = \frac{u}{\epsilon} \end{aligned}}$$



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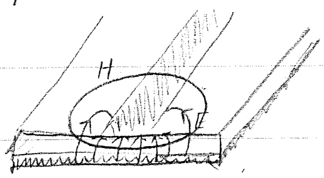
strip line

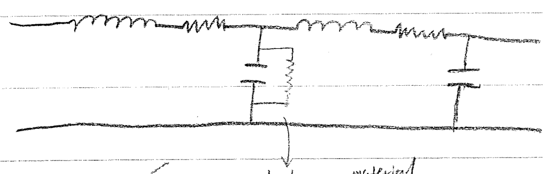


full hybrid mode, very close to TEM

DC capacitance between strip line

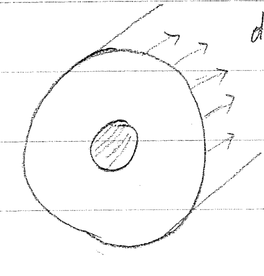
$$C_0 \xrightarrow[\varepsilon]{} C \qquad L = \frac{\mu_0 \varepsilon_0}{C_0}$$

$$Z_c^2 = \frac{\mu_0 \varepsilon_0}{C_0 C} \qquad \frac{1}{v^2} = \frac{\mu_0 \varepsilon_0 C}{C_0}$$



$E_0 \quad H_0$

dielectric material loss in



At high frequency, loss in dielectric material is much less than in conductor

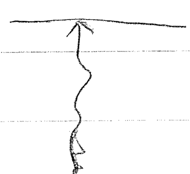
$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} + \vec{J}$$

$$= j\omega\epsilon\vec{E} + \sigma\vec{E}$$

$$= j\omega\epsilon\vec{E}\left(1 + \frac{\sigma}{j\omega\epsilon}\right) = j\omega\tilde{\epsilon}\vec{E}$$

$$\Rightarrow \underbrace{\tilde{\epsilon} = \left(1 + \frac{\sigma}{j\omega\epsilon}\right)\epsilon}_{} \quad \text{complexed dielectric constant}$$

As $\omega \uparrow \quad Im(\tilde{\epsilon}) \to \infty \Rightarrow$ high f, loss in dielectric $\downarrow$



$$\tilde{\epsilon} = \epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right) \qquad \beta = \omega\sqrt{\mu\tilde{\epsilon}} = \omega\sqrt{\mu\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)}$$

when $\sigma \gg 1 \quad \beta = \omega\sqrt{-j\frac{\sigma\mu\epsilon}{\omega\epsilon}} = \left(\frac{\mu\sigma\omega}{2}\right)^{\frac{1}{2}}(1-j)$

when ω is very large $\frac{\sigma}{\omega\epsilon} \ll 1$ metal becomes ~~transparent~~ to lig

$$Z_m = \sqrt{\frac{\mu}{\tilde{\varepsilon}}} = \sqrt{\frac{\mu}{\varepsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)}} = \sqrt{\frac{\mu}{-j\frac{\sigma}{j\omega\epsilon}}} = \sqrt{\frac{\mu\omega}{\sigma}}(1+j)$$

$$e^{-j\beta z} = e^{-j\left(\frac{\mu\sigma\omega}{2}\right)^{\frac{1}{2}}(1-j)z} = e^{-j\left(\frac{\mu\sigma\omega}{2}\right)^{\frac{1}{2}}z}\,\boxed{e^{-\sqrt{\frac{\mu\sigma\omega}{2}}z}}$$

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Skin depth $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$ when E drops to $E\frac{1}{e}$

Energy flux $\int \vec{E}\times\vec{H} = \zeta\int H^2 ds$

$\boxed{\zeta = \left(\frac{\mu\omega}{2\sigma}\right)^{\frac{1}{2}}(1+j)}$ – impedence in the material

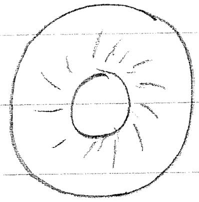
R_s: surface resistance $R_s H^2$

$$\int_V \nabla\cdot(\vec{E}\times\vec{H}^*)\,dv = \oint_S (\vec{E}\times\vec{H}^*)\,d\vec{s}$$

$$= \int (H^*\cdot\nabla\times E - E\cdot\nabla\times H^*)\,dv$$

$$= \int_v \left\{-H^*(j\omega\mu H) - E(j\omega\epsilon E^* + J^*)\right\}dv$$

$$= -j\omega\int(\mu H^* H + \epsilon E E^*)\,dv - \int E\cdot J\,dv = \oint(\vec{E}\times\vec{H}^*)\,ds$$



$P = \frac{1}{2}\int E\times H^* ds =$

time average

$E \& H \propto e^{-j\beta z - \alpha z}$

$P \propto e^{-2\alpha z}$ $\qquad \frac{1}{P}\frac{dP}{dz} = -2\alpha$

$$\alpha = \frac{1}{2}\,\frac{-dP/dz}{P} = \frac{1}{2}\,\frac{\text{Powerloss per unit length}}{\text{power transmitted}}$$

$$P = \frac{\zeta}{2}\int |H|^2 ds = \frac{\zeta}{2}\int_0^{2\pi}\int_a^b \frac{H_0^2}{r^2}\,r\,dr\,d\varphi = \frac{\zeta}{2}\,2\pi\,H_0^2 \ln b/a$$

$$\text{losses}\big|_{r=a} = \frac{1}{2}\int E_z\cdot H_\varphi^*\,ds = \frac{1}{2}\mathrm{Re}(\zeta_m)\int_0^{2\pi} H_\varphi^2\big|_{r=a}\,a\,d\varphi = \left(\frac{\omega\mu}{2\sigma}\right)^{\frac{1}{2}}\times\frac{1}{2}\times 2\pi\cdot\frac{H_0^2}{a}$$

$$\text{losses}\big|_{r=b} = \left(\frac{\omega\mu}{2\sigma}\right)^{\frac{1}{2}}\times\pi\times\frac{H_0^2}{b}$$

$$\alpha = \frac{1}{2}\,\frac{\left(\frac{\omega\mu}{2\sigma}\right)^{\frac{1}{2}}\times\pi\times\left(\frac{H_0^2}{a}+\frac{H_0^2}{b}\right)}{\frac{1}{2}\zeta H_0^2 \ln b/a\; 2\pi} = \frac{1}{2}\,\frac{\left(\frac{\omega\mu}{2\sigma}\right)^{\frac{1}{2}}\left(\frac{1}{a}+\frac{1}{b}\right)}{\zeta \ln b/a}$$

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$R_s = \left(\frac{\omega\mu}{2\sigma}\right)^{\frac{1}{2}} = \left(\frac{\pi f \mu}{\sigma}\right)^{\frac{1}{2}}$ ① $R_s \propto \sigma^{-\frac{1}{2}}$

② high f → high R_s

$\alpha = \frac{1}{2b} \frac{\left(\frac{\omega\mu}{2\sigma}\right)^{\frac{1}{2}}}{\ln x}(x+1)$ $x = b/a$ → find out x to minimize α

Characteristic $Z_c = \frac{\xi}{2\pi} \ln b/a$

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the higher the frequency, the thinner the coaxial cable is — due to higher order modes $\frac{\sqrt{\frac{\mu_0}{\varepsilon_0}}}{2\pi} \ln x_0 = 75$

For either TE or TM mode

$K_0^2 = K_t^2 + \beta^2$ $\nabla_t^2 \Phi + K_t^2 \Phi = 0$

$\Rightarrow \beta = (K_0^2 - K_t^2)$

$= \omega\sqrt{\mu_0\epsilon}\left(1 - \frac{K_t^2}{\omega\sqrt{\mu_0\epsilon}}\right)^{1/2}$

Cut-off frequency $\omega_c = \frac{K_t}{\sqrt{\mu_0\epsilon}}$

$2\pi f_c = \frac{2\pi V}{\lambda_c}$ $f_c = \frac{V}{\lambda_c}$

$K_t = \frac{\omega}{V}$ $\beta = K_0\left(1 - \frac{\omega_c^2}{\omega^2}\right)^{\frac{1}{2}} = K_0\left(1 - \left(\frac{f_c}{f}\right)^2\right)^{\frac{1}{2}}$

TE modes

$H_t = Z_g\, a_z \times E_t$ $Z_{g_{TE}} = \frac{\omega\mu}{\beta} = \frac{\omega\mu}{K_0}\left(1 - \left(\frac{f_c}{f}\right)^2\right)^{-\frac{1}{2}}$ $K_0 = \omega\sqrt{\mu\epsilon}$

$= \sqrt{\frac{\mu}{\varepsilon}}\left(1 - \left(\frac{f_c}{f}\right)^2\right)^{-\frac{1}{2}} = \xi\left(1 - \left(\frac{f_c}{f}\right)^2\right)^{-\frac{1}{2}}$

TM modes

$Z_{g_{TM}} = \frac{\beta}{\omega\epsilon} = \xi\left(1 - \left(\frac{f_c}{f}\right)^2\right)^{\frac{1}{2}}$

At cutoff f of TM $H_t \to 0$

? — — TE $E_t \to 0$

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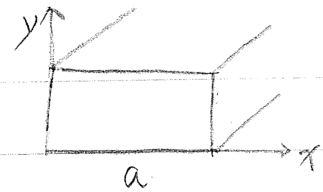
For TE modes

$$\nabla_t^2 H_z + K_t^2 H_z = 0$$



$$H_z = X(x)\,Y(y)$$

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} + K_t^2 XY = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + K_t^2 = 0$$

$$\frac{\partial^2 X}{\partial x^2} = -K_x^2 X \qquad \frac{\partial^2 Y}{\partial y^2} = -K_y^2 Y \qquad K_t^2 = K_x^2 + K_y^2$$

$$X = A\cos(K_x x) + B\sin(K_x x)$$

A B CD determined by B.C

$$Y = C\cos(K_y y) + D\sin(K_y y)$$

$$\begin{cases} \nabla_t \times E_t = -j\omega\mu \vec{H_z} \\ -j\beta a_z \times E_t = -j\omega\mu_0 \vec{H_t} \\ \nabla_t \times H_t = 0 \\ -j\beta a_z \times H_t + \nabla_t \times H_z = j\omega\varepsilon_0 E_t \end{cases}$$

$$\nabla_t \times (\nabla_t \times H_t) = \nabla_t(\nabla_t \cdot \vec{H_t}) - \nabla_t^2 \vec{H_t}$$

$$\nabla \cdot H = 0 \quad \Rightarrow \quad \nabla_t \cdot \vec{H_t} + (-j\beta H_z) = 0$$

$$\Rightarrow \quad -j\beta \nabla_t H_z = -\nabla_t^2 \vec{H_t} \qquad j\beta \nabla_t H_z + K_t^2 \vec{H_t} = 0$$

$$\boxed{H_t = \frac{-j\beta}{K_t^2}\nabla_t H_z} \qquad \boxed{E_t = (-a_z \times H_t)\frac{\omega\mu}{\beta}}$$

B.C $\quad H_z|_{x=0} = A \cdot Y$

$$\frac{\partial H_z}{\partial x} = -AK_x\sin(K_x x) + BK_x\cos K_x x = 0 \quad \text{at } x=0$$

$$\Rightarrow B = 0$$

$$H_z|_{x=a} = A\cos(K_x a)\,Y = 0 \text{ at } x=a \quad \Rightarrow K_x a = m\pi \quad K_x = \frac{m\pi}{a}$$

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In the same way $D=0$ $K_y = \frac{n\pi}{b}$

$$\Rightarrow H_z = A \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y$$

$$H_x = \frac{+j\beta}{K_t^2}\frac{m\pi}{a} A \sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y$$

$$H_y = \frac{j\beta}{K_t^2}\frac{n\pi}{b} A \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y$$

$$E_x = +\frac{j\beta}{K_t^2}\frac{n\pi}{b}\cdot\frac{\omega u}{\beta} A \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y$$

$$E_y = -\frac{j\beta}{K_t^2}\frac{m\pi}{a}\frac{\omega u}{\beta} A \sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

$$K_t^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\beta^2 = K_0^2 - K_t^2 = K_0^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

TE_{mn} – domininat mode – lowest mode above cutoff freqnency.

Cut off f $\quad \beta = 0 \quad \frac{\omega^2}{c^2} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

TE_{10} $(a > b)$ – dominant $\quad m=1 \quad n=0$

$$\begin{cases} H_z = A\cos\frac{\pi x}{a} \\ H_x = \frac{j\beta}{K_t^2}\frac{\pi}{a} A\sin\frac{\pi x}{a} \\ H_y = 0 \\ E_x = 0 \\ E_y = \frac{-j\beta}{K_t^2}\frac{\pi}{a}\frac{\omega u}{\beta} A\sin\frac{\pi x}{a} \end{cases}$$

$$\beta^2 = K_0^2 - \left(\frac{\pi}{a}\right)^2$$

all has factor $e^{j(\omega t - \beta z)}$

$$\frac{2\pi f_c}{c} = \frac{\pi}{a} \Rightarrow f_c = \frac{c}{2a}$$

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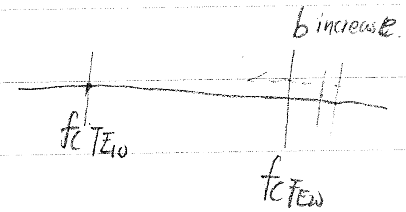
1. H_x E_y in phase. ⇒ power flow only in Z direction.
H_z E_y out of phase

when $a=2b$ TE_{01} and TE_{20} has the same cut-off frequency.
Most standard wave guide choose $a=2b$



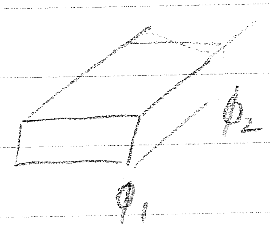
Waveguide
0.9x0.4 inches 1 inche = 2.54cm

$$V_g = \frac{1}{\frac{d\beta}{d\omega}}$$

$f \to \infty$ quasi-opitical
nearly free space.

1) find simple way in F domain to get delay (after find the need calibration procedure for the method)

2) repeat in time domain. verify the answer in 1.



$$\Delta\phi = \phi_2 - \phi_1 = \beta l$$

$$\frac{d\Delta\phi}{d\omega} = \frac{d\beta}{d\omega} \cdot l = \frac{l}{V_g} = \tau$$

delay ✓

1) determine the frequency : dominant frequency.

$$\beta = \frac{\omega}{c}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2} \qquad \frac{1}{\frac{d\omega}{d\beta}} = V_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

2) Choose the right pulse - gaussian minimum - band width for rise & fall ~~time~~ width for gaussian - minimum - wave packet

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Cut off - domain

$$f_{TE10} = \frac{c}{2a} = \frac{29.9792}{2\times 2.54\times 0.9} = 6.55714\ \text{GHz}$$

$$f_{TE20} = 2\times f_{TE} = 13.11428\ \text{GHz}$$

$\Delta f = 65$ ...

10% $\sim \Delta f$ 65 MHz - bandwidth

$\Delta\tau\,\Delta f \sim 1$

$S_{12}(W)\cdot S(\omega)$

↓ inverse fourier transform

$$S(t) = A e^{-(t/\tau)^2}$$

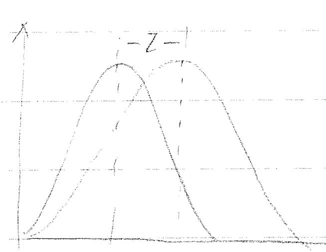
$$\tilde{S}(\omega) = \int_{-\infty}^{\infty} S(t)\, e^{-j\omega t}\, dt$$ fast fourier ~~transfore~~

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{12}(\omega)\,\tilde{S}(\omega-\omega_0)\,e^{j\omega t}\, d\omega$$ ω_0 the center frequency

$$\tilde{Z}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} Z(\omega)\, e^{j\omega t}\, d\omega \qquad \tilde{S}(t) = \int_{-\infty}^{\infty} Z(t)\, S(t-\tau)\, d\tau$$

$$S(\omega) = \int_{-\infty}^{+\infty} s(t)\, e^{-j\omega t}\, dt \qquad \boxed{\tilde{S}(t) = \frac{1}{2\pi}\int Z(\omega)\, S(\omega)\, e^{j\omega t}\, d\omega}$$

$$S(t) = S_0(t)\, e^{j\omega_0 t}$$



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Lecture 4

TM modes

$H_t = (\vec{a}_z \times \vec{E}_t)\dfrac{\omega\epsilon}{\beta}$

$\nabla_t \times \vec{E}_t = 0 \qquad \nabla_t \times (\nabla_t \times \vec{E}_t) = \nabla_t(\nabla_t \cdot \vec{E}_t) - \nabla_t^2 \vec{E}_t = 0$

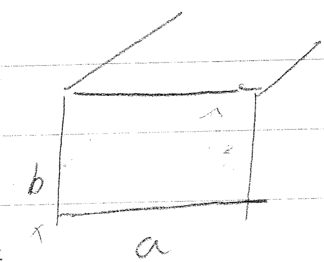
$\nabla \cdot \vec{E} = 0 \qquad \nabla_t \cdot E_t - j\beta E_z = 0$

$\Rightarrow \quad j\beta E_z + k_t^2 E_t = 0$

$$E_t = \frac{-j\beta}{k_t} \nabla_t E_t$$

$$H_t = (a_z \times E_t)\frac{\omega\epsilon}{\beta}$$

$E_z = (A\cos(k_x x) + B\sin(k_x x))(C\cos(k_y y) + D\sin(k_y y))$



$E_z = 0$ at $x=0$ & $x=a$

$y=0$ & $y=b$

$\Rightarrow \quad E_z = A\sin(k_x x)\sin(k_y y) \qquad k_x = \frac{m\pi}{a} \quad k_y = \frac{n\pi}{b}$

$= A\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$

$$E_x = \frac{-j\beta}{k_t}\frac{m\pi}{a} A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$E_y = \frac{-j\beta}{k_t}\frac{n\pi}{b} A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

$$H_x = -\frac{\omega\epsilon}{\beta}\cdot\frac{-j\beta}{k_t}\frac{n\pi}{b} A\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

$$H_y = \frac{\omega\epsilon}{\beta}\frac{-j\beta}{k_t}\frac{m\pi}{a} A\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$\Rightarrow$ No TM_{01} or TM_{10}, the first dominant mode is TM_{11}

$k_t = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

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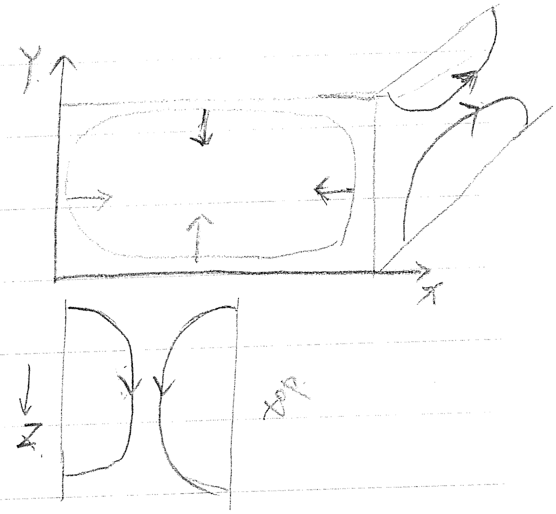
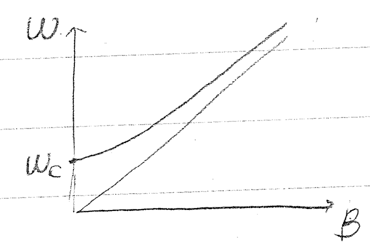


$$\beta = K_0 \left[1 - \left(\frac{\omega_c}{\omega}\right)^2\right]^{\frac{1}{2}}$$

$$v_{ph} = \frac{\omega}{\beta} = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$



$$v_g = \frac{1}{\frac{dB}{d\omega}} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\alpha = \frac{1}{2} \frac{\text{losses / unit length}}{\text{power transmitted}}$$

for: TE_{10} E_y H_z H_x

$$H_z = H_{z0} \cos\left(\frac{\pi x}{a}\right)$$

$$H_x = \frac{+j\beta}{(\pi/a)} H_{z0} \sin\frac{\pi x}{a}$$

$$E_y = -\frac{j\omega\mu}{\pi/a} H_{z0} \sin\frac{\pi x}{a}$$

$$\text{Power} = \frac{1}{2}\int_0^a\int_0^b E_y \times H_x^* \, dx\, dy = \frac{1}{2}\int_0^a\int_0^b H_{z0}^2 \frac{B\omega\mu}{(\pi/a)^2} \sin\frac{\pi x}{a} \sin\frac{\pi x}{a}\, dx\, dy$$

$$= \frac{1}{2} H_{z0}^2 \frac{B\omega\mu}{(\pi/a)^2} \cdot b \cdot \frac{a}{2}$$

$$\text{Losses} = L_z = \boxed{\frac{1}{2}\int_0^a\int_0^b E_y \times H_z^* \, dx\, dy} = \frac{R_s}{2} \times 2\int_0^a H_{z0}^2 \cos^2\left(\frac{\pi x}{a}\right) dx$$

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$$+\frac{R_s}{2}\times 2\int_0^b H_{z0}^2\,dx = R_s H_{z0}^2\left(\frac{a}{2}+b\right)$$

$$L_x = R_s\int_0^a \left(\frac{\beta}{\frac{\pi}{a}}\right)^2 (H_{z0})^2 \sin^2\frac{\pi}{a}x\,dx = \frac{R_s}{2}\left(\frac{\beta}{\frac{\pi}{a}}\right)^2 (H_{z0})^2\cdot a$$

$$\Rightarrow \alpha = \frac{1}{2}\,\frac{L_x+L_z}{\text{Power}}$$

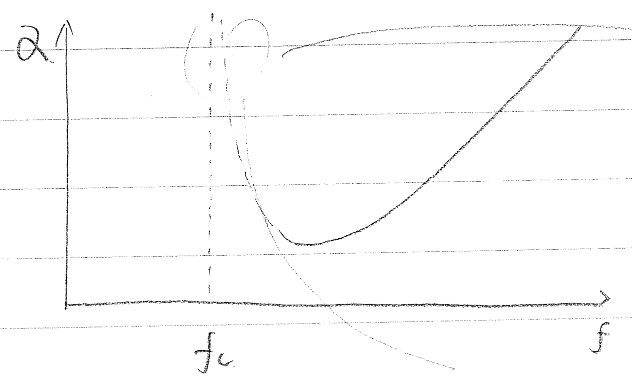
$$= \frac{1}{2}\,\frac{R_s\left(\frac{a}{2}+b + \frac{\beta^2}{2}\left(\frac{a}{\pi}\right)^2\right)}{\frac{ab}{4}\cdot\frac{\beta\omega\mu}{\left(\frac{\pi}{a}\right)^2}}$$

$$= \frac{1}{2}R_s\,\frac{\pi^2\left(\frac{a}{2}+b\right)+\frac{a^2\beta^2}{2}}{\frac{a^3 b}{4}}$$

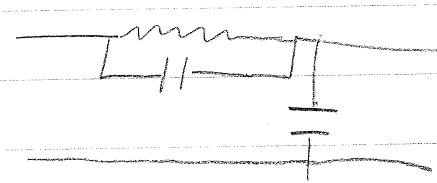
$\omega\mu = Z_g\beta$

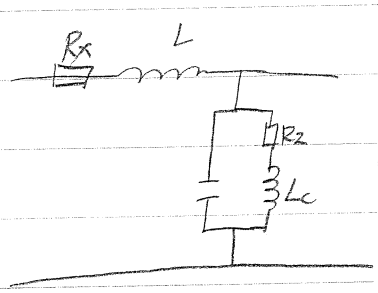
$$= 2\frac{R_s}{Z_g}\,\frac{\left(\frac{a}{2}+b\right)\left(\frac{\pi}{a}\right)^2+\beta^2}{ab\,\beta^2}$$

when $f\to f_c$ $Z_g\to\infty$



Will not see this cut off in reality.





$$\omega_c^2 = \frac{1}{L_c C}$$

$$\beta = \sqrt{LC}\left(1-\left(\frac{f_c}{f}\right)^2\right)^{\frac{1}{2}}$$

$$Z_c = \sqrt{\frac{L}{C}}\left(1-\left(\frac{f_c}{f}\right)^2\right)^{\frac{1}{2}}$$

L - energy stored, transverse magnetic field. (H_x in TE

L_c: longitudinal (H_z

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$\frac{1}{2} L I^2 = \frac{1}{4} u \int H_x^2 \, dA$ I is related to Z_0?

$L_x = \frac{R_s}{2} \times 2 \int H_x^2 \, dx = \frac{1}{2} R_x I^2$

Circuit theory

Loss in x direction.

due the same in z direction

* Calculate α.

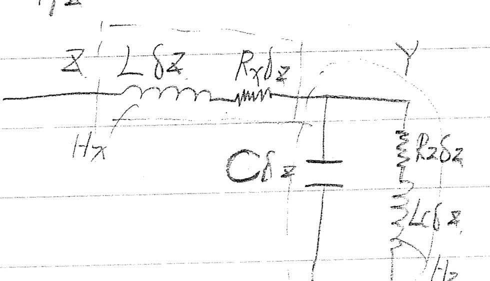
$\frac{\partial H_z}{\partial x}$ at $x=0 \neq 0$
$=a$.

$K_x \neq \frac{\pi}{a}$

Solve maxwell equation by perturbation theory

$K_x = \frac{\pi}{a} + \delta$

$\frac{E_y}{H_z} = R_s(1+j)$



$Z = j\omega L + R_x$

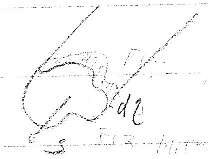
$\gamma = \sqrt{ZY} = \alpha + j\beta$

$Y = j\omega C + \frac{1}{R_z + j\omega L_c}$

α
↓ dimension of waveguide.

$M_1 \rightarrow E_1 \; H_1$

$\int E_m \times H_n \cdot ds = \delta_{mn}$



$M_2 \rightarrow E_2 \; H_2$

$\int_V \nabla \cdot (E_1 \times H_2 + H_1 \times E_2) \, dV$

$= \oint_S E_1 \times H_2 + H_1 \times E_2 \, ds = \frac{\partial}{\partial z} \int_S (\vec{E}_1 \times \vec{H}_2 - \vec{H}_1 \times \vec{E}_2) \, dS \, \Delta z = j(-\beta_1 + \beta_2) \int_S (\vec{E}_1 \times \vec{H}_2 - \vec{H}_1 \times \vec{E}_2$

$\nabla \cdot (E_1 \times H_2 - H_1 \times E_2) = \vec{H}_2 \cdot \nabla \times \vec{E}_1 - \vec{E}_1 \cdot \nabla \times \vec{H}_2 - \vec{E}_2 \cdot \nabla \times \vec{H}_1 + \vec{H}_1 \times \nabla \times \vec{E}_2$

Lorentz's theory

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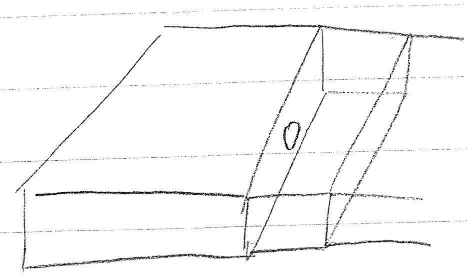
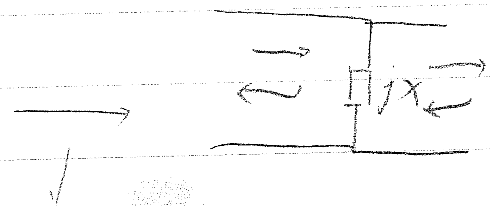
$$= j\omega\mu H_2 H_1 - j\omega\epsilon E_1 E_2 + j\omega\epsilon E_1 E_2 - j\omega\mu H_1 H_2 = 0$$

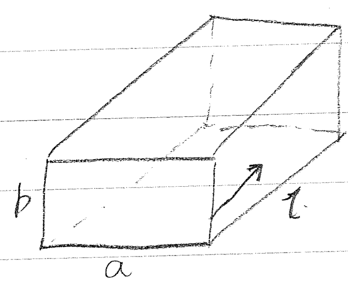
$$\Rightarrow \quad 0 = j(-\beta_1 + \beta_2)\int_S (E_1 \times H_2 - H_1 \times E_2)\, ds$$





modes propagate independently from each other.

each mode in wave guide can be represent by transmission line



$$TM \quad E_z = E_{z0}^{f} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z} + E_{z0}^{B} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{j\beta z}$$

$$E_x = \frac{m\pi}{a}\frac{-1}{k_t^2} \cos\frac{m\pi a}{a} \sin\frac{n\pi y}{b} \left[E_{z0}^{f} e^{-j\beta z}(-j\beta) + E_{z0}^{B} e^{j\beta z}(j\beta) \right]$$

$$E_{z0}^{f} = E_{z0}^{B} = E_z$$

$$\beta = \frac{p\pi}{l} \quad p \in \mathbb{Z}$$

$$= \frac{m\pi}{a}\frac{1}{k_t^2} \cos\frac{m\pi a}{a} \sin\frac{n\pi y}{b}\, 2E_{z0}(-j\beta)(-j\sin\beta z)$$

$$= \frac{-2m\pi}{a}\frac{1}{k_t^2}\left(\frac{m\pi}{a}\right) \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} E_{z0}\, \beta \sin\beta z$$

$$E_y = \frac{2\beta}{k_t^2}\left(\frac{n\pi}{b}\right) \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} E_{z0} \sin\beta z$$

$$H_x = \frac{2j\omega\epsilon}{k_t^2}\left(\frac{n\pi}{b}\right) \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} E_{z0} \cos\beta z.$$

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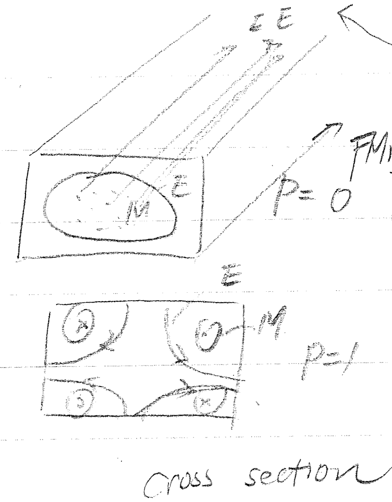
$$H_y = -j\frac{2W\epsilon}{K_t^2}\left(\frac{m\pi}{a}\right)\cdot\cos\frac{m\pi y}{a}\sin\frac{n\pi y}{b}E_{z0}\cos\beta z$$



$$K_0^2 = \beta^2 + K_x^2 + K_y^2$$

$$\left(\frac{\omega}{c}\right)^2 = \left(\frac{p\pi}{l}\right)^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$f = \frac{c}{2}\sqrt{\left(\frac{p}{l}\right)^2+\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$TM^{\square}_{mnp}$

$$\frac{1}{j\omega C} = -j\omega L \Rightarrow \omega^2 = LC$$

$$|E_x|^2 + |E_y|^2 + |E_z|^2 = |H_x|^2 + |H_y|^2$$

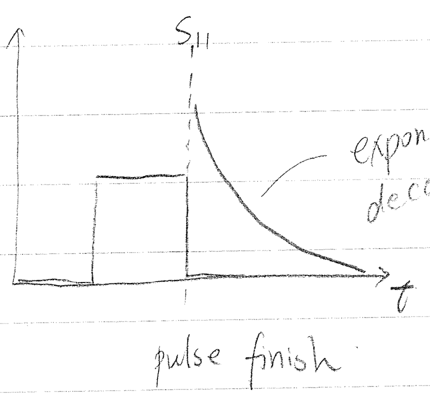
Quality factor Q

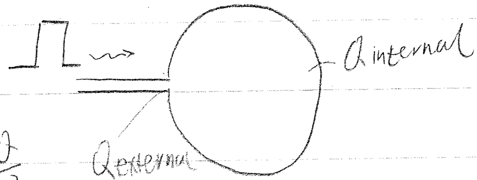
$$Q = \omega\frac{\text{Stored energy } (U)}{P_{losses}}$$ quality factor

$U \propto E^2(t)$ $\qquad P_{losses} \propto E^2(t)$

$$\frac{dU}{dt} = -P_{losses} = -\frac{Q}{\omega}U \Rightarrow U = U_0 e^{-\omega/Q\, t} = U_0 e^{-\frac{t}{\tau}} \qquad \tau = \frac{Q}{\omega}$$

$$U \sim E^2 \rightarrow E \sim e^{-t/2\tau} \qquad \tau_E = \frac{2Q}{\omega}$$





Another way. use Q circle.

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S_{11} - time domain

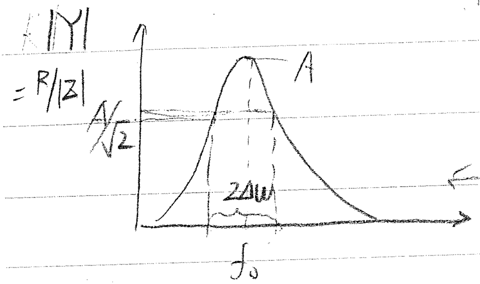
$$Z = R\left(1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right) = R\left[1 + jQ\left(\frac{\omega^2 - \omega_0^2}{\omega\omega_0}\right)\right]$$

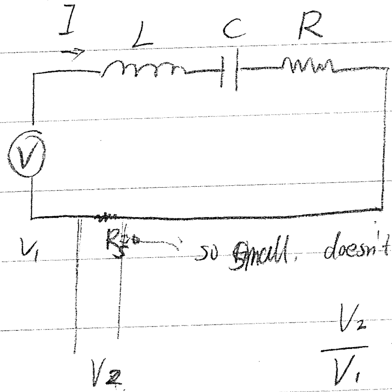
$$= R\left[1 + jQ\,\frac{(\omega-\omega_0)(\omega+\omega_0)}{\omega\omega_0}\right] \simeq R\left[1 + jQ\,\frac{2\Delta\omega}{\omega_0}\right]$$

$$|Z| = R\left[1 + 4Q^2\frac{\Delta\omega^2}{\omega_0^2}\right]^{\frac{1}{2}} \qquad \frac{\Delta\omega^2}{\omega_0^2}\cdot 4Q^2 = 1 \qquad \frac{\Delta\omega}{\omega_0} = \frac{1}{2Q}$$

$$\Rightarrow Q = \frac{\omega_0}{2\Delta\omega}$$

3dB point





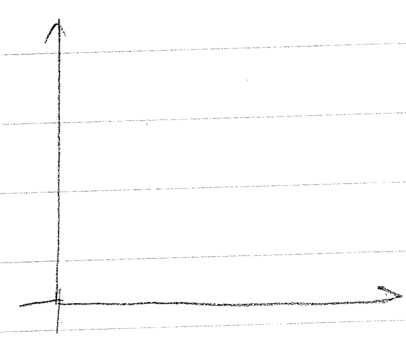
so small. doesn't purturb system

$$\frac{V_2}{V_1} = \frac{IR_s}{V_1}$$

S_{21} is propotional to replace $|Y|$ by S_{21}

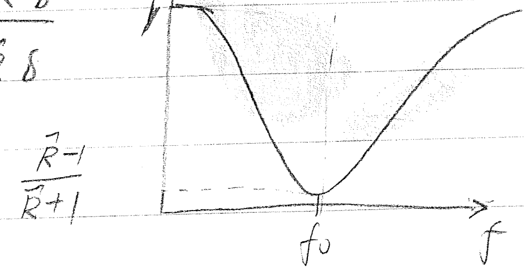
3dB point — Quality factor

transmission measurement



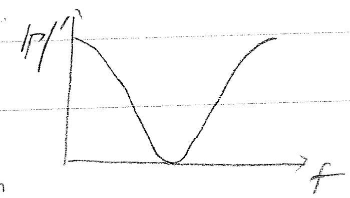
$$\Gamma = \frac{\hat{R}\left[1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] - 1}{\hat{R}\left[1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] + 1} \qquad \delta = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

$$= \frac{\hat{R} - 1 + jQ\hat{R}\delta}{\hat{R} + 1 + jQ\hat{R}\delta}$$



$\hat{R} = 1$ critically coupled

$$\Gamma = \frac{jQ\delta}{2 + jQ\delta}$$



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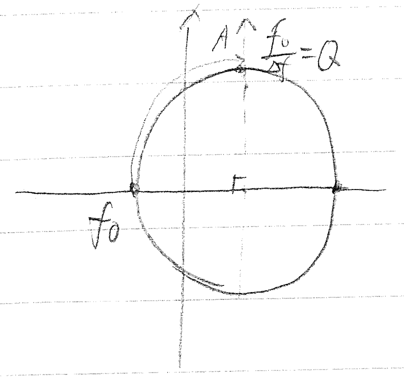
$Q\delta = 1$

$\Gamma = \frac{j}{2+j}$ $\qquad |\Gamma| = \left|\frac{j(2-j)}{5}\right| = \frac{\sqrt{5}}{5}$



$\Gamma = \Gamma_r + i\Gamma_i$

$$= \frac{[(\hat{R}-1)+jQ_0\delta\hat{R}][\hat{R}+1-jQ_0\delta\hat{R}]}{(\hat{R}+1)^2 + Q_0^2\delta^2\hat{R}^2}$$

$$= \frac{(\hat{R}-1)(\hat{R}+1) + jQ_0\delta(\hat{R}+1-\hat{R}+1) + Q_0^2\delta^2\hat{R}^2}{(\hat{R}+1)^2 + Q_0^2\delta^2}$$

$$= \frac{\hat{R}^2-1 + jQ_0\delta 2 + Q_0^2\delta^2\hat{R}^2}{(\hat{R}+1)^2 + Q_0^2\delta^2\hat{R}^2}$$

$$\Rightarrow \Gamma_r = \frac{\hat{R}^2-1+Q_0^2\delta^2\hat{R}^2}{(\hat{R}+1)^2+Q_0^2\delta^2\hat{R}^2} \qquad \Gamma_i = \frac{Q_0\delta 2\hat{R}}{(\hat{R}+1)^2+Q_0^2\delta^2\hat{R}^2}$$

when $\delta = 0$ $\quad \Gamma_i = 0 \quad \Gamma_r = \frac{\hat{R}^2-1}{(\hat{R}+1)^2} = \frac{\hat{R}-1}{\hat{R}+1}$

$$\Gamma_i\big|_{f\to\infty}^{f\to 0} \to 0 \quad \Gamma_r \to 1$$

diameter $\quad d = 2r = 1 - \frac{\hat{R}-1}{\hat{R}+1} = \frac{2}{\hat{R}+1} \qquad r = \frac{1}{\hat{R}+1}$

~~At the~~ ~~when~~ $\Gamma_i = \frac{1}{\hat{R}+1} = \frac{Q_0\delta 2\hat{R}}{(\hat{R}+1)^2+Q_0^2\delta^2\hat{R}^2}$

$$\Gamma_R = \frac{1}{2}\left(1+\frac{\hat{R}-1}{\hat{R}+1}\right) = \frac{\hat{R}}{\hat{R}+1} = \frac{\hat{R}^2-1-Q_0^2\delta^2\hat{R}^2}{(\hat{R}+1)^2+Q_0^2\delta^2\hat{R}^2}$$

$$\frac{Q_0}{Q_e} = \frac{I^2R_e}{I^2R} = \frac{R_e}{R} = \frac{1}{\hat{R}} \qquad Q_{total} = \frac{\omega U}{L_i + L_e} = \frac{1}{L_i/\omega U + L_e/\omega U} = \frac{1}{1/Q_0 + 1/Q_e}$$

$$\frac{1}{Q_t} = \frac{1}{Q_0} + \frac{1}{Q_e} = \frac{1}{Q_0}\left(1+\frac{1}{\hat{R}}\right)$$

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$\Rightarrow 1/Q_t = 1/Q_0 + 1/Q_e$

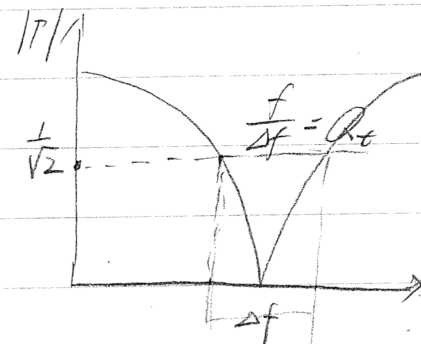
$Q_t = Q_0\left(\frac{\hat R}{1+\hat R}\right) \qquad Q_0 = Q_t \frac{1+\hat R}{\hat R}$

$$\Gamma_r = \frac{(\hat R-1)(\hat R+1) + \hat R^2 \left(\frac{1+\hat R}{\hat R}\right)^2 Q_t^2 \delta^2}{(\hat R+1)^2 + (Q_t \cancel{\hat R}\delta)^2 \left(\frac{1+\hat R}{\cancel{\hat R}}\right)^2}$$

$$= \frac{\hat R^2 - 1 + (1+\hat R)^2 Q_t^2\delta^2}{(\hat R+1)^2 + (1+\hat R)^2 Q_t^2\delta^2} = \frac{\hat R - 1/\hat R+1 + Q_t^2\delta^2}{1+Q_t^2\delta^2} =$$

$$\Gamma_i = \frac{2\hat R Q_t \left(\frac{1+\hat R}{\hat R}\right)\cdot\delta}{(\hat R+1)^2 + \left(Q_t \frac{1+\hat R}{\hat R}\cdot \hat R\delta\right)^2} = \frac{2Q_t(1+\hat R)\delta}{(\hat R+1)^2+(Q_t\delta)^2(1+\hat R)^2}$$

$$= \frac{2Q_t/(1+\hat R)\cdot\delta}{1+(Q_t\delta)^2}$$



for $\Gamma_r = \frac{\hat R}{1+\hat R}$ & $\Gamma_i = \frac{1}{1+\hat R}$

We need $Q_t\delta = 1$

Diametre $= 2/\hat R+1$

Center $= \frac{\hat R}{\hat R+1}$

$\frac{Q_0}{Q_e} = \beta$ - coupling coefficient

$= \frac{1}{\hat R}$

$$\boxed{\begin{array}{l} \text{Diameter} = \frac{2}{\frac{1}{\beta}+1} = \frac{2\beta}{1+\beta} \\ \text{Center} = \frac{\frac{1}{\beta}}{\frac{1}{\beta}+1} = \frac{1}{1+\beta} \end{array}}$$

$\beta > 1$ under couple

$\beta < 1$ over couple

fit the curve of Q circle

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Lecture 5

$Z = R(1 + jQ_0\delta)$

$$Q_0 = Q_t \frac{1+\tilde{R}}{\tilde{R}}$$

Critical coupled Γ can be measured by 3-dB point

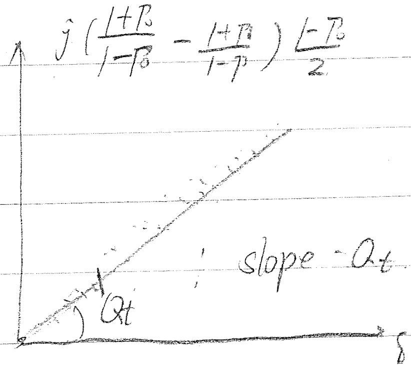
$$Q_t\delta = j\left(\frac{1+\Gamma_0}{1-\Gamma_0} - \frac{1+\Gamma}{1-\Gamma}\right)\frac{1-\Gamma_0}{2}$$

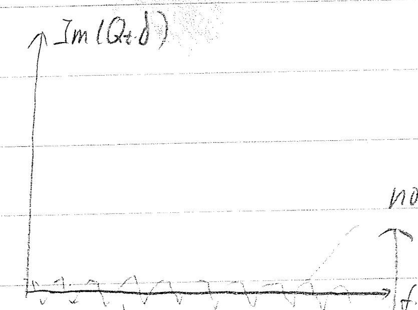
$$\Gamma_0 = \frac{1-\beta}{1+\beta} = \frac{\tilde{R}-1}{\tilde{R}+1}$$

$\Gamma_0 > 0$ undercoupled

$\Gamma_0 < 0$ over coupled

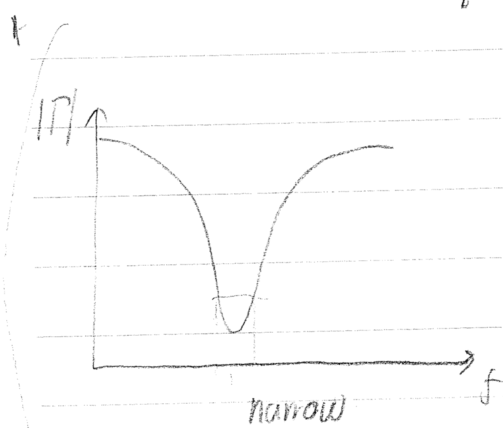
$\Gamma_0 = 0$ critical coupled

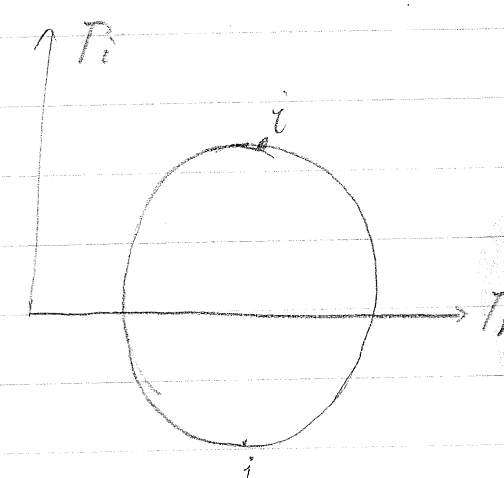


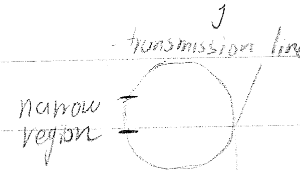


not choosing the proper resonant frequency

average is not zero







fitting $\sum_{i=0}^{N} a_i \delta^i$

increase N until a_i doesn't change any more

$10^{-2} \sim 10^{-3}$ error for Q_t

a_0 is not $0 \to \omega_0$ is not right

① have to decide position

② transmission lin[e] & Other componen[ts]

$\Gamma' = \Gamma F(\omega)$, ne[ed] to let $F(\omega)$ to be c[onstant]
~~constant~~

③ For high Q 2-poi[nt] measurement is good
for low Q. it's hard to

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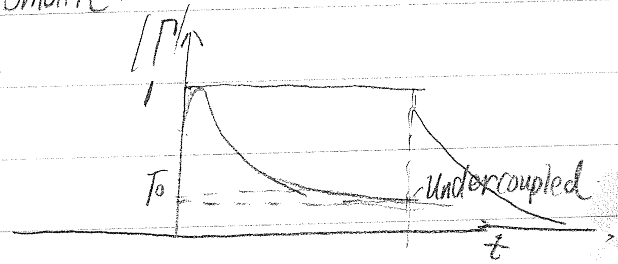
1 – Q_0, Q_e, Q_t and β f_0 (scale & rotate correctly)

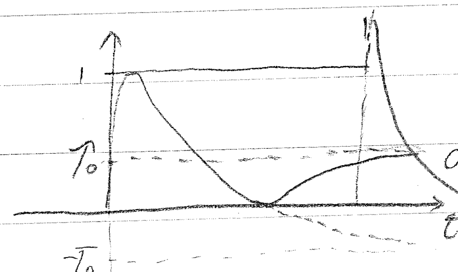
i) measured by Q circle 2-point method.

ii) measured by fitting the data to $Q_t \delta = j\left(\frac{1+\Gamma_0}{1-\Gamma_0} - \frac{1+\Gamma}{1-\Gamma}\right) / 1 - \frac{\Gamma_0}{2}$?

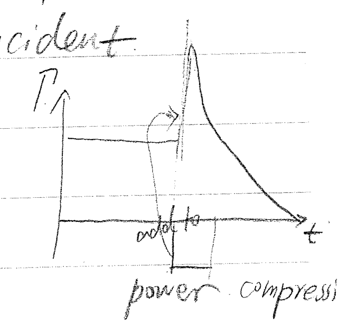
Time domain

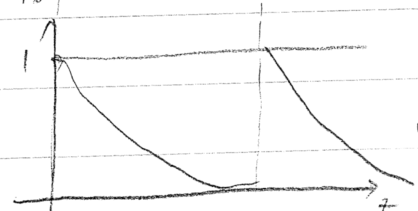


$\tau = \frac{2Q_t}{\omega}$

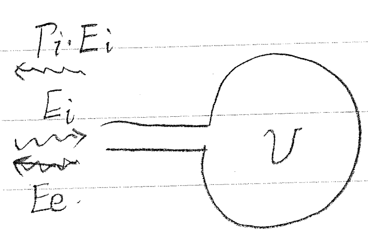


When you shut down the reflected energy, the reflected is larger than the incident





critically coupled



$E_R = E_e + E_i \Gamma_i$

$\Gamma = \frac{E_R}{E_i} = \frac{E_e + E_i\Gamma_i}{E_i} = \frac{E_e}{E_i} + \Gamma_i$

(undercoupled)

$\Gamma_i \approx 1$ $\quad E_e < E_i$ at resonance E_e will have phase $= \pi$

$\Gamma \simeq 1 - \frac{E_e}{E_i} < 1$ & positive

$E_e > E_i$ (overcoupled) at resonance E_e will have phase $= \pi$

$\Gamma = 1 - \frac{E_e}{E_i} < 0$ $\quad |\Gamma| < 1$

$\begin{cases} \Gamma_0 - \beta \\ \text{decay} - Q_t \end{cases}$

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$T(\omega)$ - after ~~rotate~~ expand & rotate

↓ Fourier transform

$T(t)$

$$P_i - P_e - P_{losses} = \frac{dU}{dt}$$

$\tau = \frac{2Q_c}{\omega}$ - cavity filling time

$$K_t^2 + \beta^2 = K_0^2$$

$$\nabla_t^2 \frac{E_z}{H_z} + K_t^2 \frac{E_t}{H_t} = 0$$

TE $\quad H_t = \frac{-j\beta}{K_t^2} \nabla_t H_z$

$$E_t = (-a_z \times H_t) \frac{\omega\mu}{\beta}$$

TM $\quad E_t = \frac{-j\beta}{K_t^2} \nabla_t E_z \qquad H_t = \frac{\omega\varepsilon}{\beta} a_z \times E_t$

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \varphi^2} + K_t^2 \psi = 0$$

$$\psi = f(\varphi) R(r)$$

$$r^2 \frac{\partial^2 R}{\partial r^2} \cancel{f} + \frac{r}{\cancel{r}} \frac{\partial R}{\partial r} f + \frac{1}{\cancel{r^2}} \frac{\partial^2 f}{\partial \varphi^2} R + r^2 K_t^2 f R = 0$$

$$\frac{r^2}{R}\frac{\partial^2 R}{\partial r^2} + \frac{r}{R}\frac{\partial R}{\partial r} + \frac{\partial^2 f}{\partial \varphi^2}\frac{1}{f} + r^2 K_t^2 = 0$$

$$\frac{\partial^2 f}{\partial \varphi^2} = -m^2 f(\varphi) \qquad f = Ae^{jm\varphi} + Be^{-jm\varphi}$$

Circular waveguide

$m \in$ integer

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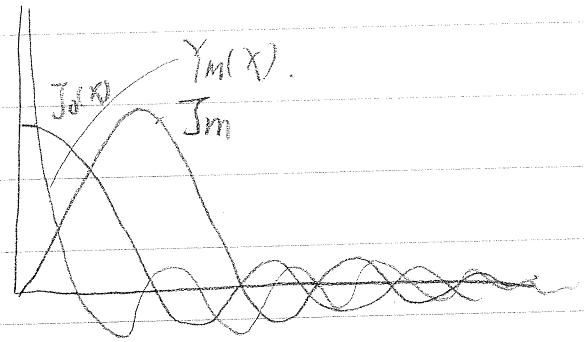
$R(r) = C J_m(K_r r) + D Y_m(K_r r)$

Circular guide is never single mode ↷ ↶ always paired



$J_{-m}(x) = (-1)^m J_m(x)$

$$\boxed{J_m'(x) = \frac{J_{m-1}(x) - J_{m+1}(x)}{2}}$$

$$\boxed{\frac{J_m(x)}{x} = \frac{J_{m-1}(x) + J_{m+1}(x)}{2}}$$

TM mode

$E_z = A J_m(K_t r) e^{jm\varphi}$

$E_r = A \frac{-j\beta}{K_t^2} K_t J_m'(K_t r) e^{jm\varphi}$

$E_\varphi = A \frac{-j\beta}{K_t^2} (jm) \frac{J_m'}{r}(K_t r) e^{jm\varphi}$

$H_t = (a_t \times E_t) \frac{\omega\epsilon}{\beta}$

B.C. $E_\varphi = 0 \big|_{at\ r=a} \Rightarrow J_m'(K_t a) = 0$

$K_t a = X_n \quad J_m(X_n) = 0$

$\Rightarrow K_t = \frac{X_n}{a} \quad X_1 = 2.401$

$TM_{mn} \quad TM_{01}$

TE mode $H_z = A J_m(K_t r) e^{jm\varphi}$

$H_r = A \frac{-j\beta}{K_t^2} K_t J_m'(K_t r) e^{jm\varphi}$

$H_\varphi = A \frac{-j\beta}{K_t^2} \frac{jm}{r} J_m(K_t r) e^{jm\varphi}$

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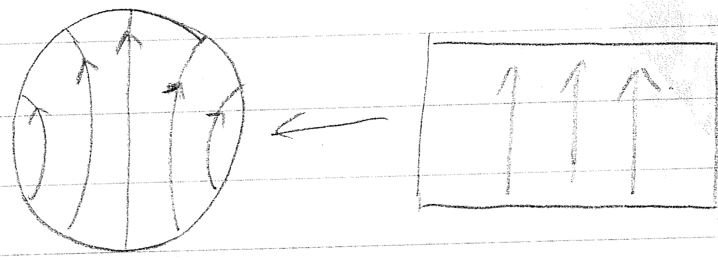
$$E_t = (-a_z \times H_z)\frac{\omega\mu}{\beta}$$

B.C $E_\varphi = 0|_{at\ r=a} \Rightarrow H_r = 0|_{at\ r=a} \Rightarrow J_m'(k_t a) = 0$

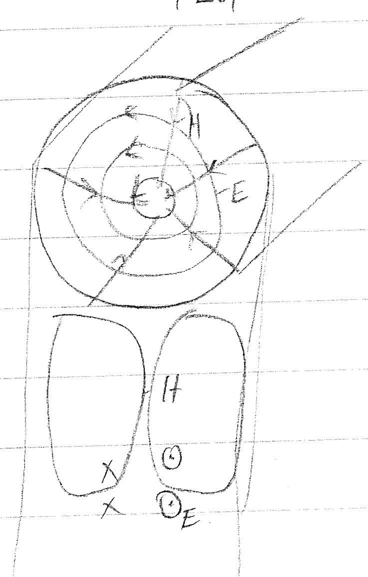
$k_t a = \chi_n'$ $\quad J_m'(\chi_n') = 0 \quad \chi_1' = 1.841$

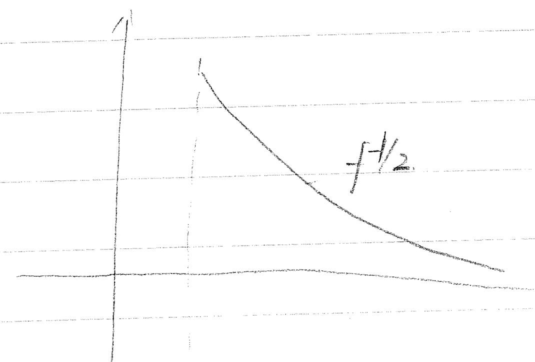
TE_{11} ~~first~~ dominant mode

degenerate with &
(m=1) (m=-1)



TE_{01}





TM_{01} $\quad E_z = A J_0(k_t r)$

$$E_t = E_r = A\frac{-j\beta}{k_t^2}k_t J_0'(k_t r) = A\frac{j\beta}{k_t^2}k_t J_1(k_t r)$$

$$H_\varphi = A\frac{-j\beta}{k_t^2}k_t\frac{\omega\epsilon}{\beta}J_0'(k_t r) = A\frac{j\beta}{k_t}\frac{\omega\epsilon}{\beta}J_1(k_t r)$$

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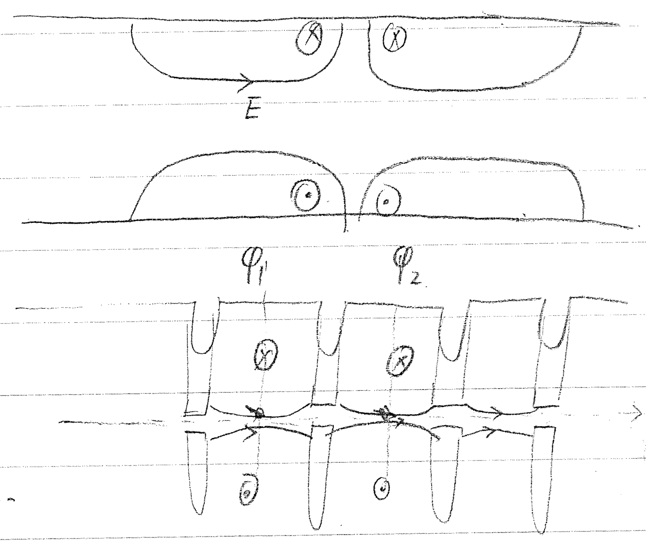
by itself can not accelerate



$Ka = x_1^0 = 2.405$

$K_t = \frac{2.405}{a}$

$\beta = 1 \quad K_t = K_0$

$\frac{2\pi f}{c} = \frac{2.405}{a}$

$f = 11.424\,GHz$

diameter of
phase advance

Wave travel with speed of light. $\beta = K_0 \quad K_t = 0$ $\quad k_0 = \frac{\omega}{c}$

$\Rightarrow E_z = A$

$E_r = A j K_0 r \frac{1}{2}$ $\quad \beta = \frac{\omega}{v = c}$

$H_\varphi = A j\omega\varepsilon \frac{r}{2}$

1> E_z doesn't depends on r. $\quad E/H_\varphi = \zeta$

$F_r = e(E_r - cB_\varphi) = 0$

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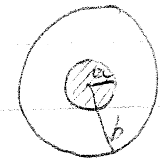
Lecture 6

TE fields



$H_z = [A\, J_m(k_t r) + B\, Y_m(k_t r)]\, e^{jm\varphi}$

$E_\varphi \propto H_r$

$$H_r = -\frac{j\beta}{k_t}\left(A J_m'(k_t r) + B Y_m'(k_t r)\right) e^{jm\varphi}$$

H_r at $(r=a \quad r=b) = 0$

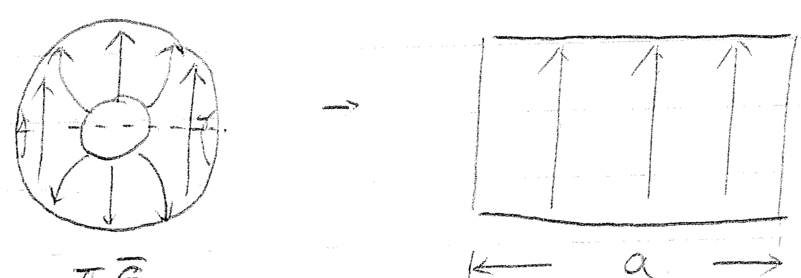
$$H_r = -\frac{j\beta}{k_t} A'\left[Y_m'(k_t a) J_m'(k_t r) - J_m'(k_t a) Y_m'(k_t r)\right] e^{jm\varphi}$$

by solving for k_t from

$$Y_m'(k_t a) J_m'(k_t b) - J_m'(k_t a) Y_m'(k_t b) = 0$$

$$H_z = A'\left[Y_m'(k_t a) J_m(k_t r) - J_m'(k_t a) Y_m(k_t r)\right]$$

foundemental TE mode

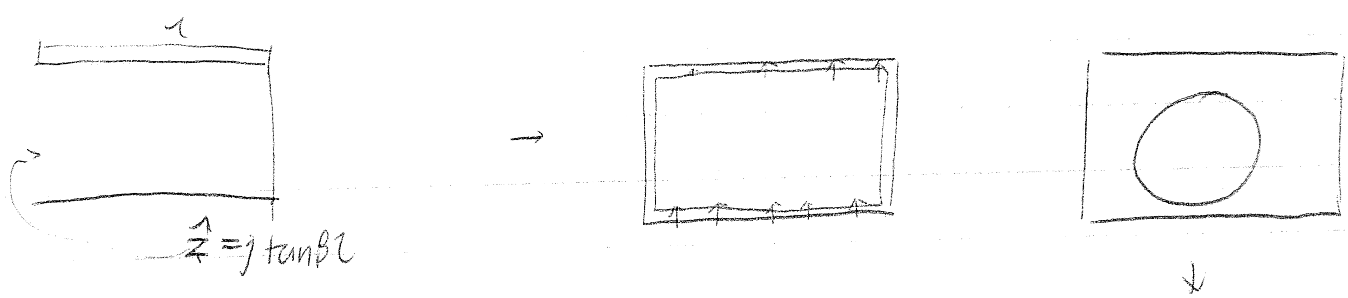


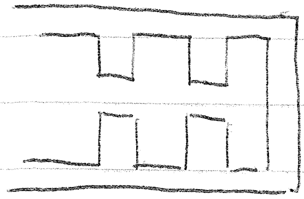
$\pi \bar{c}$

$\bar{c} \propto \frac{b+a}{2}$

$f_c = \frac{c}{2\bar{c}}$

check when $a \approx b$.





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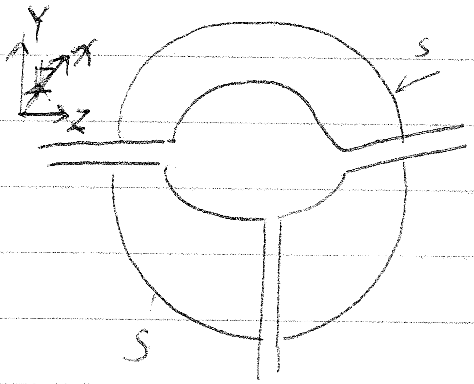


sheild ~~e~~xcept ports

$$\nabla\times\vec{H} = j\omega\epsilon\vec{E} + \sigma\vec{E}\ 0.$$
$$\nabla\times\vec{E} + j\omega\mu\vec{H} = 0$$
$$\nabla\cdot\epsilon\vec{E} = \rho$$
$$\nabla\cdot\mu\vec{H} = 0$$

$$\nabla\cdot(\vec{E}\times\vec{H}^*) = \vec{H}^*\cdot\nabla\times\vec{E} - \vec{E}\cdot\vec{\nabla}\times\vec{H}^*$$
$$= -\vec{H}^*\cdot(j\omega\mu\vec{H}) - \vec{E}\cdot(j\omega\epsilon\vec{E}^* + \sigma\vec{E}^*)$$
$$\Downarrow$$

$$\int_S \vec{E}\times\vec{H}^* = -j\omega\int_V \mu\vec{H}^*\cdot\vec{H}\,dv + j\omega\int_V \epsilon\vec{E}^*\cdot\vec{E}\,dv - \int\sigma\vec{E}^*\cdot\vec{E}\,dv$$

magnetic field stored — electrical fired store — consumed energy

$E_x = e f_x(x,y)$, $H_x = i g_x(x,y)$

$E_y = e f_y(x,y)$, $H_y = i g_y(x,y)$

~~as H~~ at the beginning, E & H ~~they~~ are off by a factor β but β is changing as the wave propergates

$$\int(f_x g_y - f_y g_x)\,dx\,dy = -1$$

$$\Rightarrow \int_S \vec{E}\times\vec{H}^* = -e\,i^*$$

$$e\,i^* = 4j\omega(W_H - W_p) + 2P$$

$W_H = \frac{1}{4}\int \mu H\cdot H^*\,dv$ $\quad W_p = \frac{1}{4}\int \epsilon E\cdot E^*\,dv$ $\quad P = \frac{1}{2}\int \sigma E\cdot E^*\,dv$

define $\frac{e}{i} = Z(\omega) = \frac{1}{Y(\omega)}$ for each particular port

$$Z = \frac{2j\omega(W_H - W_E) + 2P}{\frac{1}{2} i i^*} \qquad \Rightarrow \boxed{Z(\omega) = Z^*(-\omega)}$$

$$Y = \frac{-2j\omega(W_H - W_E) + 2P}{\frac{1}{2} e e^*} \qquad Y(\omega) = Y^*(-\omega)$$

Real part - loss energy

Im part - stored energy

reactance part increase as ω goes away from ω_0

~~w change~~ P

incident E_i H_i reflected E_r H_r

$\downarrow$ a $\;\; \downarrow$ a $\qquad \downarrow$ b $\;\; \downarrow$ b

$$\Gamma = \frac{b}{a}$$

$$e = g(a+b) = ga(1+\Gamma) \qquad i = \frac{1}{g}(a-b) = \frac{1}{g}a(1-\Gamma)$$

$$e \cdot i^* = a \cdot a^*(1+\Gamma)(1-\Gamma^*) = \frac{2j\omega(W_H - W_E) + P}{\frac{1}{2} a \cdot a^*}$$

$$\Rightarrow \quad 1 - \Gamma\Gamma^* = \frac{P}{\frac{1}{2} a \cdot a^*}$$

$$\frac{1}{2}(\Gamma - \Gamma^*) = j\,Im(\Gamma) = \frac{j\omega(W_H - W_E)}{\frac{1}{2} a a^*}$$

$$\boxed{\Gamma \cdot \Gamma^* \leq 1} \qquad \text{if } P = 0 \quad \Gamma^*\Gamma = 1$$

$$\int_S E \times H^* ds = -\sum_m e_n i_n^*$$

$$\sum_n e_n i_n^* = 4j\omega(W_H - W_E) + 2P$$

$$e_p = \sum_q Z_{pq}\, i_q \qquad Z = \begin{pmatrix} Z_{11} & Z_{12} & \cdots \\ Z_{21} & Z_{22} & \cdots \\ \vdots & \vdots & Z_{NN} \end{pmatrix} \qquad i = \begin{pmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{pmatrix} \qquad e = \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{pmatrix}$$

presentation of the junction   Current vector   voltage vector

$e = Z \cdot i \qquad i = Y e$

$\Rightarrow \quad e = Z \cdot Y e \Rightarrow \boxed{ZY = 1 \quad \Rightarrow \quad \cancel{Z} = Y = Z^{-1}}$

$$\nabla \times H^{(1)} - (j\omega\epsilon + \sigma) E^{(1)} = 0$$
$$\nabla \times E^{(1)} + j\omega\mu H^{(1)} = 0$$
$$\nabla \times H^{(2)} - (j\omega\epsilon + \sigma) E^{(2)} = 0$$
$$\nabla \times E^{(2)} + j\omega\mu H^{(2)} = 0$$

$$\nabla \cdot (E^{(1)} \times H^{(2)} - E^{(2)} \times H^{(1)}) = 0$$

$$\cancel{\sum e_p^{(1)} i_p^{(2)} - e_p^{(2)} i_p^{(1)}}$$

$$= \int_S (E^1 \times H^2 - E^2 \times H^1)\, ds$$

$$= \sum e_p^{(1)} i_p^{(2)} - e_p^{(2)} i_p^{(1)}$$

$e_p^{(1)} = 0, \quad p \neq 1$

$e_p^{(2)} = 0, \quad p \neq 2$

$$e_1^{(1)} i_1^{(2)} - e_2^{(2)} i_2^{(1)} = 0$$

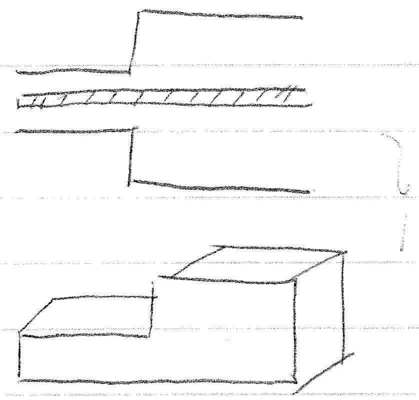
$\Rightarrow \quad i_1^{(2)} = Y_{12}\, e_2^{(2)}$

2 - due to source at 1 $\quad \rightarrow \quad Y_{12} = Y_{21}$.

$i_2^{(1)} = Y_{21}\, e_1^{(1)}$

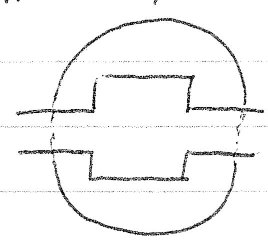
$\boxed{Y_{mn} = Y_{nm}} \qquad \boxed{Z_{mn} = Z_{nm}}$ reciprocal condition

Condition: μ, ϵ, σ are reciprocal
are not affected by Maxwell's equation's solution.



geometry is not symmetric, but Y, Z matrix are symmetric.

Ex $\sum_n i_n^* e_m = \sum_n \sum_m i_n^* Z_{nm} i_m = 4j\omega (W_H - W_E) + 2P$



For 2 ports

~~$i_1^* i_1$~~ $Z_{nm} = R_{nm} + jX_{nm}$

$i_1^* i_1 R_{11} + (i_2^* i_1 + i_2 i_1^*) R_{12} + i_2^* i_2 R_{22} = 2P \geq 0$ real part

let $i_1 = 0$ (put port 1 open) $\Rightarrow R_{11} \geq 0$

In the same way $R_{22} \geq 0$

~~by choosing~~ $(i_1/i_2)^2 R_{11} + 2(i_1/i_2) R_{12} + R_{22} \geq 0$ (1)

$\Rightarrow X^2 R_{11} + 2XR_{12} + R_{22} = 0$ — no real solution to ensure (1)

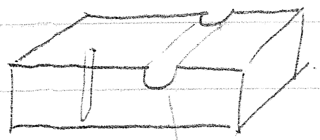
$\Rightarrow R_{12}^2 - R_{11}R_{22} \leq 0$ always true

$$\begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix} \geq 0$$

$R_{pp} \geq 0$

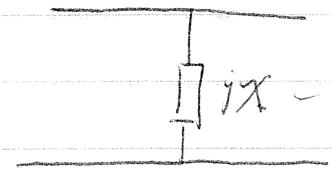
$\det |R_{nm}| \geq 0$ equally: $|Re(Z)| \geq 0$

Lossless junction



bump in waveguide

→



jX – no pure imaginary

~~pure im~~ – no loss

$$Re\left[i_K i_K^* Z_{KK} + (i_m i_K^* + i_K i_m^*) Z_{Km} + i_m i_m^* Z_{mm}\right] = 0 \quad - \; i_p = 0;\ p \neq k \,\&\, m$$

$Re[Z_{KK}] = 0$ all currents are 0 except for one port.

$\Rightarrow \; Re(Z_{Km}) = 0$

$\Rightarrow$ For Lossless junction. Z & Y are pure imaginary.

Scattering Matrix

$e_n = g_n(a_n + b_n) \qquad i_n = 1/g_n\,(a_n - b_n)$ Constant presentation.

for $b_n = 0$ (only forward wave)

$e_n = g_n a_n \qquad i_n = \frac{1}{g_n} a_n$

$\Rightarrow \; \frac{e_n}{i_n} = g_n^2 = Z_c^{(n)} \qquad g_n = \sqrt{Z_c^{(n)}}$ ~ characteristic impedence of $\boxed{\text{wave guide}}$

presentation of field of waveg

let $g_n = 1$.

$e_n = a_n + b_n \qquad i_n = a_n - b_n$.

$a_n = \frac{1}{2}(e_n + i_n) \qquad b_n = \frac{1}{2}(e_n - i_n)$

$$a_n = \frac{1}{2}\left(\sum_m (Z_{nm} + \delta_{nm})\, i_m\right)$$

$$b_n = \frac{1}{2}\left(\sum_m (Z_{nm} - \delta_{nm})\, i_m\right)$$

In the matrix form

$$a_n = \frac{1}{2}(Z+I)i \qquad a = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

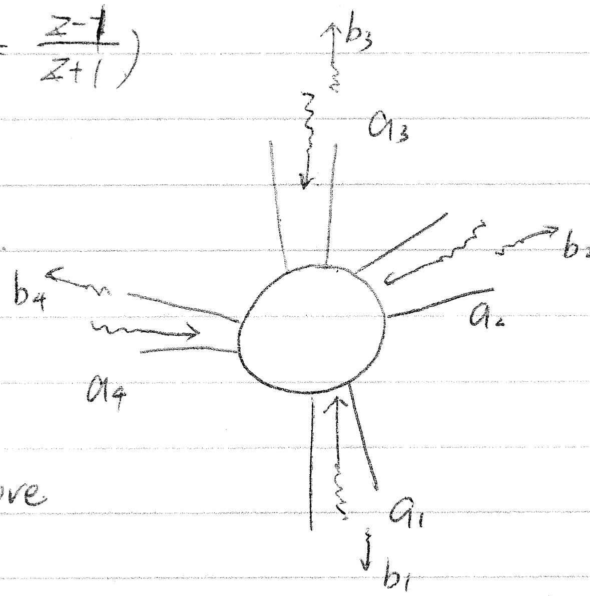
$$b = \frac{1}{2}(Z-I)i$$

$$i = 2(Z+I)^{-1}a$$

$$b = \underbrace{(Z-I)(Z+I)^{-1}}_{S}a \qquad \left(\Gamma = \frac{Z-1}{Z+1}\right)$$

S - scattering matrix

$$S = (Z-I)(Z+I)^{-1} \qquad \boxed{b = Sa}$$



Scattering relates "what comes in" & "what comes out".

In the same way easy to prove

$$S = \frac{1-Y}{1+Y}$$

$$Z-I = G \qquad Z+I = H \qquad S = GH^{-1} \qquad GH = HG$$

$$\cancel{H^{-1}GHH^{-1} = H^{-1}GS\ H^{-1}\ S}$$

$$H^{-1}GHH^{-1} = H^{-1}HGH^{-1}$$

$$\Rightarrow H^{-1}G = GH^{-1} = S$$

$$S^T = (H^{-1})^T G^T = \cancel{H^T G}\ H^{-1}G$$

$$\Rightarrow \boxed{S = S^T} \qquad S \text{ is symmetric}$$

i^* e

$$\Sigma(a_n^* - b_n^*)(a_n + b_n) = 4j\omega(W_H - W_E) + 2P$$

$$\Sigma(a_n^* a_n - b_n^* b_n) = 2P$$

$$\Sigma(a_n^* b_n - b_n^* a_n) = 4j\omega(W_H - W_E)$$

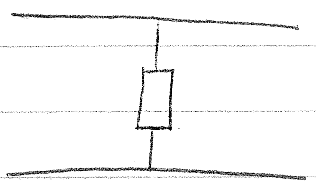
$$a^{*T}(I - S^{*}S) = 2P$$

$$a^{*T}(S - S^{*})a = 4j\omega(W_H - W_E)$$

$$|I - S^{*}S| \geq 0$$

for a lossless junction: $S^{*} \cdot S = I$

$\boxed{\begin{matrix} S^{T} = S \\ S^{*} = S^{-1} \end{matrix}}$ symmetric / unitary



For a 2-port network

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$ lossless

$$\overset{\text{Symmetric}}{=} \begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix}$$

$|S_{11}|^2 + |S_{12}|^2 = 1$ $\quad |S_{22}|^2 + |S_{12}|^2 = 1$

$S_{11} S_{12}^{*} + S_{12} S_{21}^{*} = 0$

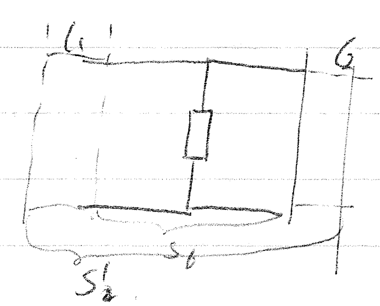
$$= \begin{pmatrix} \cos\theta e^{j\varphi} & \pm j \sin\theta e^{j(\varphi+\delta)} \\ \pm j \sin\theta e^{j(\varphi+\delta)} & \cos\theta e^{j(\varphi+2\delta)} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta e^{j(\varphi-\delta)} & \pm j \sin\theta e^{j\varphi} \\ \pm j \sin\theta e^{j\varphi} & \cos\theta e^{j(\varphi+\delta)} \end{pmatrix}$$ reduce ~~the~~ to 3 unknou

$|S_{11}| = |S_{22}|$ but they can have diffrent phase

$S' = P \cdot S P$ $\quad P = \begin{pmatrix} e^{-j\beta_1 l_1} & 0 \\ 0 & e^{-j\beta_2 l_2} \end{pmatrix}$

$\rightarrow$ De Embiding of scattering matrix.



generally

$$P = \begin{pmatrix} e^{-j\beta_1 l_1} & & 0 \\ & e^{-j\beta_2 l_2} & \\ & & \ddots \\ 0 & & e^{-j\beta_N l_N} \end{pmatrix}$$

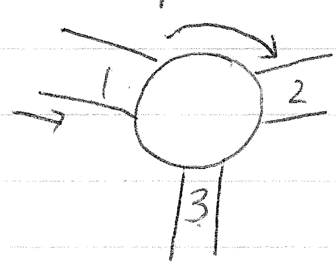
by choosing position of reference plane. we can adjust $\varphi-\delta$ & φ- to make $S_{11} = S_{22}$

Homework:

For a ~~lossless~~ 3-port network.

1. Assume S is lossless $S^{+} \cdot S = 1$
2. Assume S is matched (all diagonal term is 0).
3. Assume S is non-reciprocal ($S_{ij} \neq S_{ji}$).

Prove ~~that~~ S ~~represent~~ a circulator



$1 \to 2$

$2 \to 3$

$3 \to 1$.

Homework.

1. delay using $\frac{d\varphi}{d\omega}$ in a short wave guide ✓
2. " using time domain in a short wave guide ✓
3. time response of a cavity (and get Q_0 Q_e. β. ✓
4. Q_t Q_0 β from Q circle.
   - two points $\begin{cases} \text{calibrated} \\ \text{uncalibrated} \end{cases}$ ✓
   - from fitting the data near resonancie frequency.
     fitting the Q circle ✓

5. Make a ~~cls~~ clibration kit for one port calibration.

- → ~~measurements~~ load
- → measure short circuit
- → measure shifted short circuit
- → measure a cavity (Q circle with your own calibration & TRL)

6. Calculate a position for dent, to get a critically coupled cavity.

   measure the $\underbrace{\text{response}}_{\text{time}}$ of the cavity

* make a over coupled cavity by making extra dent.

  measure response in time domain

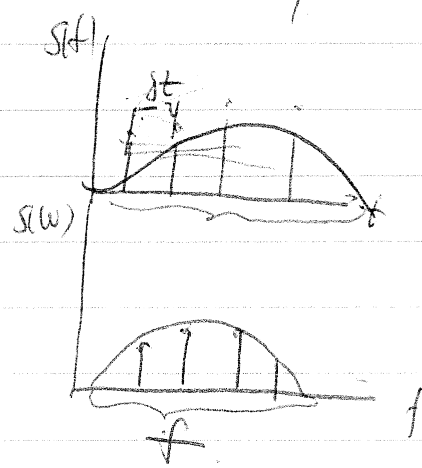
7. Get the ~~sphere~~ phase advance and field ~~prof~~ profile in the disturbed $\frac{2\pi}{3}$ structure

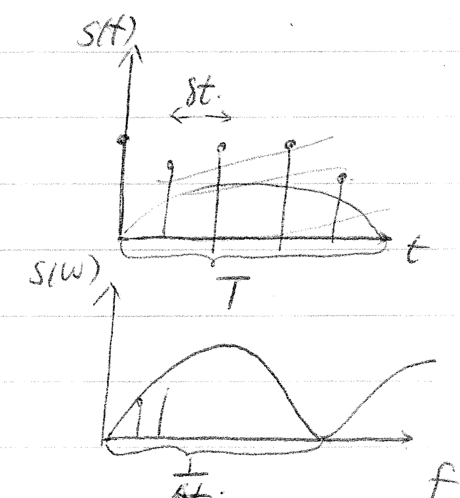
* - measure the ~~v~~ group velocity of the structure

Signal $S(t)$. $S(t-mT) = S(t)$ periodic in time $m \in \mathbb{Z}$

$$S(t) = \sum_n a_n \cos \omega_n t + b_n \sin \omega_n t$$

$$\omega_n = \frac{2\pi n}{T}$$

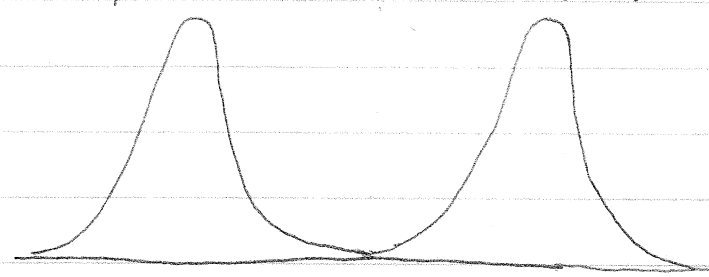


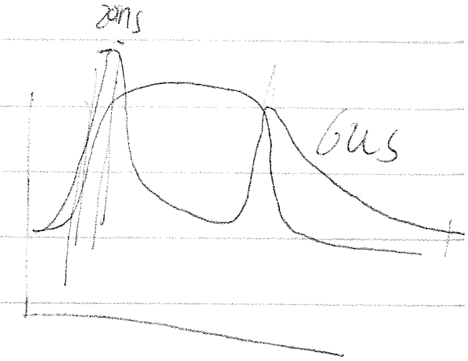


Frequency Band : outside band, assume it's ~~cont~~ repeated
distrete in frequency domain

↓

periodic in time domain





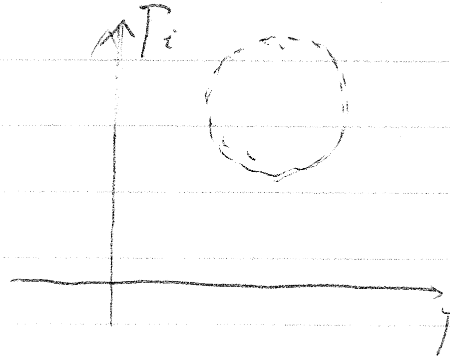
$Q = \cancel{10}\,\cancel{3\times10^3}\,30\times10^3$ $\qquad \frac{2Q}{\omega} = \cancel{0.838}\,6\times10^{-6}s$

$T \geq 6us$

$\Delta f \leq \frac{1}{6\times10^{-6}} = 1.5\times10^5\ Hz \quad \cancel{150} \times 1600 = 2.4\times10^8\ Hz = 240MHz$

$\delta t = \frac{1}{240}\times10^{-6} = 4.2\times10^{-9}s \simeq 4ns$

Choose Causses ~ 20 ns   keep five points at rise time



$$(\Gamma_i - y)^2 + (\Gamma_r - x)^2 = R^2$$

$$\Gamma_i^2 + y^2 - 2\Gamma_i y + \Gamma_r^2 - 2\Gamma_r x + x^2 = R^2$$

$$\underbrace{\Gamma_i^2 + \Gamma_r^2}_{|\Gamma|^2} - 2\Gamma_i y - 2\Gamma_r x = R^2 - x^2 - y^2$$

$$\begin{pmatrix} -2T_x & -2T_y & -1 \\ & & \\ & & \end{pmatrix} \begin{pmatrix} X \\ Y \\ A \end{pmatrix} = \begin{pmatrix} |T|^2 \\ \vdots \\ \vdots \end{pmatrix}$$

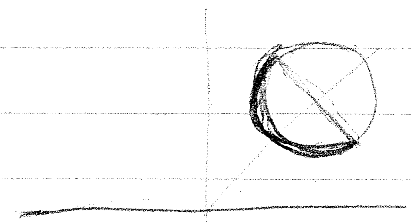
T

$$T \begin{pmatrix} X \\ Y \\ A \end{pmatrix} = G$$

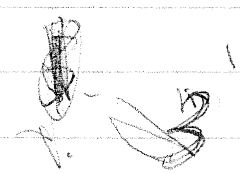
$$T^T T \begin{pmatrix} X \\ Y \\ A \end{pmatrix} = T^T G$$

$$\begin{pmatrix} X \\ Y \\ A \end{pmatrix} = (T^T T)^{-1} T G$$

G



Calibration – $\boxed{\text{essential}}$



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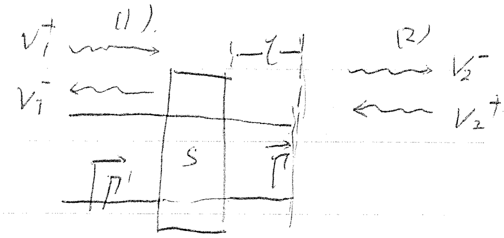
Lecture 7

Two-port Networks



$$S = \begin{pmatrix} \cos\theta\, e^{i(\varphi+\delta)} & \pm j\sin\theta\, e^{i\varphi} \\ \pm j\sin\theta\, e^{i\varphi} & \cos\theta\, e^{i(\varphi-\delta)} \end{pmatrix}$$

No way to do $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

$V^- = S V^+$

$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix} \qquad \Gamma = \frac{V_2^+}{V_2^-}$$

$$= \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} 1 \\ \Gamma V_2^- \end{pmatrix} \qquad V_1^+ = 1$$

$$V_1^- = S_{11} + S_{12}\Gamma V_2^-$$
$$V_2^- = S_{21} + S_{22}\Gamma V_2^-$$

$$\Rightarrow V_2^- = \frac{S_{21}}{1 - S_{22}\Gamma}$$

$$V_1^- = S_{11} + S_{12}\Gamma \frac{S_{21}}{1 - S_{22}\Gamma} = \frac{S_{11} - S_{11}S_{22}\Gamma + S_{12}\Gamma S_{21}}{1 - S_{22}\Gamma} = \Gamma'$$

For matching $\Gamma' = 0$.

$$\cancel{S_{11} =} \Rightarrow \Gamma = \frac{S_{11}}{S_{11}S_{22} - S_{12}S_{21}} = \frac{\cos\theta\, e^{j(\varphi+\delta)}}{\cos^2\theta\, e^{2j\varphi} + \sin^2\theta\, e^{2j\varphi}}$$

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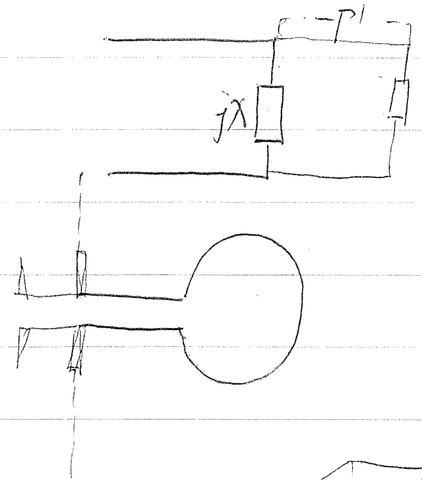
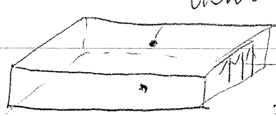
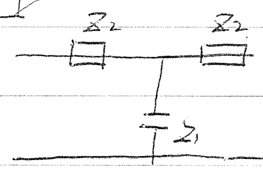
$$\Rightarrow \Gamma = \frac{\cos\theta\, e^{j\delta}}{e^{j\varphi}} = \cos\theta\, e^{j(\delta-\varphi)} = S_{22}^{*}$$



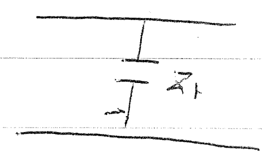
Waveguide



dent ~~of wa~~ - adding capacitance to wave guide



Now only need



$Z =$ $\quad e = Z\cdot i$

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

For open circuit. $\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$Z_{11} = \frac{1}{j\omega C} \qquad Z_{12} = \frac{1}{j\omega C}$$

$$Z_0 = \begin{pmatrix} \frac{1}{j\omega C} & \frac{1}{j\omega C} \\ \frac{1}{j\omega C} & \frac{1}{j\omega C} \end{pmatrix} \Rightarrow S = (Z-I)(Z+I)^{-1}$$

Or: $Y = j\omega C + \frac{1}{\cancel{Y_c}} Y_c$

$$\Gamma = \frac{Y_0}{Y} = \frac{1 - \hat{Y}}{1+\hat{Y}} = \frac{1 - (j\omega C + Y_c)/Y_c}{1 + (j\omega C + Y_c)/Y_c} = \frac{-j\omega\hat{C}}{2 + j\omega\hat{C}} = S_{11}$$

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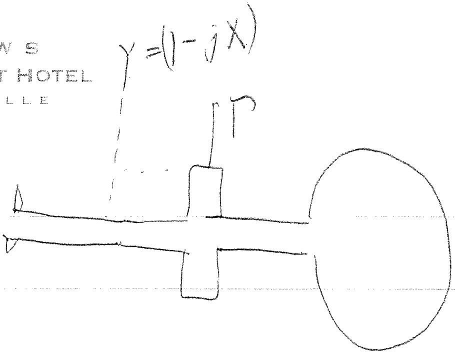
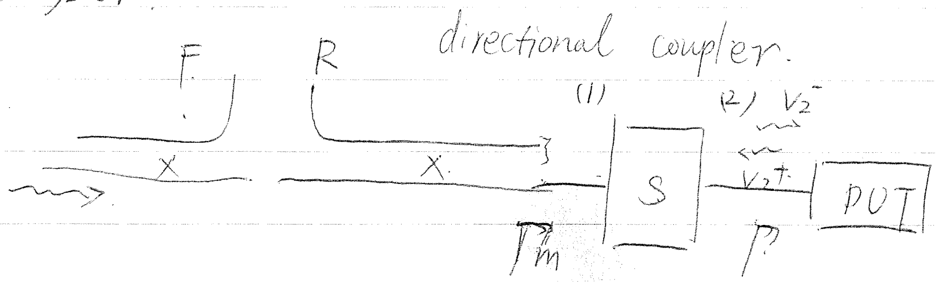


Network Analyzer:

1 port.



$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ = \Gamma V_2^- \end{pmatrix}$$

$$V_2^- = S_{21} V_1^+ + S_{22} \Gamma V_2^-$$

$$\Rightarrow V_2^- = \frac{S_{21} V_1^+}{1 - S_{22}\Gamma}$$

$$V_1^- = S_{11} V_1^+ + S_{12} \Gamma V_2^- = S_{11} V_1^+ + S_{12} \Gamma \frac{S_{21} V_1^+}{1 - S_{22}\Gamma}$$

$$= \frac{S_{11} - S_{11} S_{22} \Gamma + S_{12} S_{21} \Gamma}{1 - S_{22}\Gamma} \cdot V_1^+$$

$$\cancel{\Rightarrow \Gamma_m = \frac{V_1^+}{V_1^-} = \frac{1 - S_{22}\Gamma}{S_{11} - S_{11} S_{22} \Gamma + S_{21} S_{12} \Gamma}}$$

$$\Gamma_m = \frac{V_1^-}{V_1^+} = S_{11} + \left(\frac{S_{12} S_{21} \Gamma}{1 - S_{22}\Gamma} \right)$$

$$\frac{\Gamma_m - S_{11}}{S_{12} S_{21}} = \frac{\Gamma}{1 - S_{22}\Gamma}$$

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$$\frac{S_{12}S_{21}}{\Gamma_m - S_{11}} = \frac{1}{\Gamma} - S_{22}$$

$$\frac{S_{12}S_{21}}{\Gamma_m - S_{11}} + S_{22} = \frac{1}{\Gamma}$$

$$\frac{S_{12}S_{21} + S_{22}\Gamma_m - S_{11}S_{22}}{\Gamma_m - S_{11}} = \frac{1}{\Gamma}$$

$$\Rightarrow \quad \Gamma = \frac{\Gamma_m - S_{11}}{\underbrace{[S_{12}S_{21} - S_{11}S_{22}]}_{\chi} + S_{22}\Gamma_m}$$

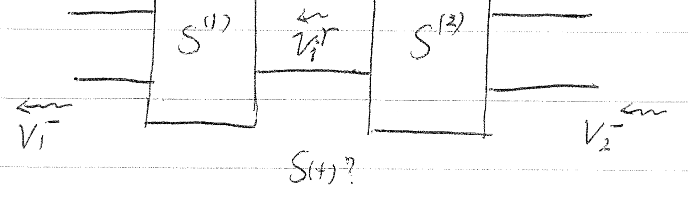
need to figure out. S_{11} S_{22} χ. need 3 times Calibration

For short circuit. $Z = 0$. $\Gamma = -1$.

$$\begin{cases} -1 = \dfrac{\Gamma_m^{(1)} - S_{11}}{\chi + S_{22}\Gamma_m^{(1)}} & \text{short circuit} \\ e^{+j\varphi} = \dfrac{\Gamma_m^{(2)} - S_{11}}{\chi + S_{22}\Gamma_m^{(2)}} & \text{wave guide?} \\ 0 = \Gamma_m^{(3)} - S_{11}. & \text{load.} \end{cases}$$

solve S_{22} S_{11} & χ

done at each particular frequency.



$\cancel{V_1^+ = V_i^r S_{11} + V_i^l S_{12}}$

$\cancel{V_1^- = S_{21}V_i^l + S_{22}V_i^l}$

$\cancel{V_1^- = V \qquad + S_{12}^{(1)} V_i^R}$

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$$V_1^- = S_{11}^{(1)} V_1^+ + S_{12}^{(1)} V_1^R \qquad V_2^- = S_{21}^{(2)} V_2^+ + S_{21}^{(2)} V_1^t$$

$$V_1^t = V_1^R S_{22}^{(1)} + V_1^+ S_{21}^{(1)} \qquad V_1^R = V_1^t S_{11}^{(2)} + V_2^+ S_{12}^{(2)}$$

$$V_1^t = [V_1^t S_{11}^{(2)} + V_2^+ S_{12}^{(2)}]\, S_{22}^{(1)} + V_1^+ S_{21}^{(1)}$$

$$V_1^t (1 - S_{11}^{(2)} S_{22}^{(1)}) = V_2^+ S_{12}^{(2)} S_{22}^{(1)} + V_1^+ S_{21}^{(1)}$$

$$\Rightarrow \quad V_1^t = \frac{V_2^+ S_{12}^{(2)} S_{22}^{(1)} + V_1^+ S_{21}^{(1)}}{1 - S_{11}^{(2)} S_{22}^{(1)}}$$

$$V_2^- = S_{21}^{(2)} V_2^+ + S_{21}^{(2)} \cdot \frac{V_2^+ S_{12}^{(2)} S_{22}^{(1)} + V_1^+ S_{21}^{(1)}}{1 - S_{11}^{(2)} S_{22}^{(1)}}$$

$$= \left(S_{21}^{(2)} + \frac{S_{21}^{(2)} S_{12}^{(2)} S_{22}^{(1)}}{1 - S_{11}^{(2)} S_{22}^{(1)}}\right) V_2^+ + \frac{S_{21}^{(2)} S_{21}^{(1)}}{1 - S_{11}^{(2)} S_{22}^{(1)}} V_1^+$$

$$= \frac{S_{21}^{(2)} - S_{21}^{(2)} S_{11}^{(2)} S_{22}^{(1)} + S_{21}^{(2)} S_{12}^{(2)} S_{22}^{(1)}}{1 - S_{11}^{(2)} S_{22}^{(1)}} V_2^+ + \frac{S_{21}^{(2)} S_{21}^{(1)}}{1 - S_{11}^{(2)} S_{22}^{(1)}} V_1^+$$

$$= S_{22}^t V_2^+ + S_{21}^t V_1^+$$

$$S_{22}^t = S_{21}^{(2)} + \frac{S_{21}^{(2)} S_{12}^{(2)} S_{22}^{(1)}}{1 - S_{11}^{(2)} S_{22}^{(1)}} \qquad S_{21}^t = \frac{S_{21}^{(2)} S_{21}^{(1)}}{1 - S_{11}^{(2)} S_{22}^{(1)}}$$

$$V_1^R = (V_1^R S_{22}^{(1)} + V_1^+ S_{21}^{(1)})\, S_{11}^{(2)} + V_2^+ S_{12}^{(2)}$$

$$V_1^R (1 - S_{22}^{(1)} S_{11}^{(2)}) = V_1^+ S_{21}^{(1)} S_{11}^{(2)} + V_2^+ S_{12}^{(2)}$$

$$\Rightarrow \quad V_1^R = \frac{V_1^+ S_{21}^{(1)} S_{11}^{(2)} + V_2^+ S_{12}^{(2)}}{1 - S_{22}^{(1)} S_{11}^{(2)}}$$

$$\Rightarrow \quad V_1^- = S_{11}^{(1)} V_1^+ + S_{12}^{(1)} \cdot \frac{V_1^+ S_{21}^{(1)} S_{11}^{(2)} + V_2^+ S_{12}^{(2)}}{1 - S_{22}^{(1)} S_{11}^{(2)}}$$

$$= \left(S_{11}^{(1)} + \frac{S_{12}^{(1)} S_{21}^{(1)} S_{11}^{(2)}}{1 - S_{22}^{(1)} S_{11}^{(2)}}\right) V_1^+ + \frac{S_{12}^{(1)} S_{12}^{(2)}}{1 - S^{(1)} S^{(2)}}$$

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$$= S_{11}^t V_1^+ + S_{12}^t V_2^+$$

$$S_{11}^t = S_{11}^{(1)} + \frac{S_{12}^{(1)} S_{21}^{(1)} S_{11}^{(2)}}{1 - S_{22}^{(1)} S_{11}^{(2)}}$$

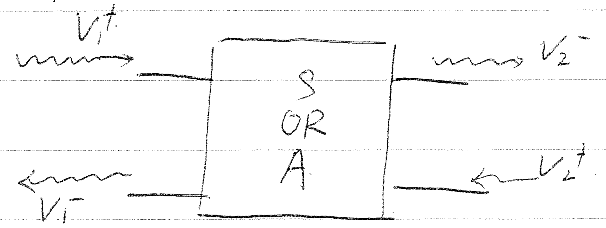
$$S_{12}^t = \frac{S_{12}^{(1)} S_{12}^{(2)}}{1 - S_{22}^{(1)} S_{11}^{(2)}}$$

① resonance condition $1 - S_{22}^{(1)} S_{11}^{(2)}$

② In transfer matrix representation. $T^{(t)} = T^{(1)} T^{(2)}$
T representation & S representation ~~oth~~ can transfer between each other.

③ Assumssion: one single mode.
For multiple mode. $S_{11}^{(1)(2)}$ $S_{12}^{(1)(2)}$ $S_{21}^{(1)(2)}$ $S_{22}^{(1)(2)}$ are matrix

Transfer matrix:



$$\begin{pmatrix} V_1^+ \\ V_1^- \end{pmatrix} = A \begin{pmatrix} V_2^- \\ V_2^+ \end{pmatrix}$$

$$V_1^+ = A_{11} V_2^- + A_{12} V_2^+$$
$$V_1^- = A_{21} V_2^- + A_{22} V_2^+$$

~~$V_1^+ = S_{11} V_2^+ + S_{12} V_2^-$~~

$$\begin{cases} V_1^- = S_{11} V_1^+ + S_{12} V_2^+ \\ V_2^- = S_{22} V_2^+ + S_{21} V_1^+ \end{cases}$$

$$V_1^+ = \frac{V_2^- - S_{22} V_2^+}{S_{21}}$$

$$V_1^- = S_{11} \cdot \frac{V_2^- - S_{22} V_2^+}{S_{21}} + S_{12} V_2^+$$

$$= \left(S_{12} - \frac{S_{11} S_{22}}{S_{21}} \right) V_2^+ + \frac{S_{11}}{S_{22}} V_2^-$$

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$\Rightarrow A_{21} = S_{11}/S_{21}$ $\qquad A_{22} = -\dfrac{S_{11}S_{21}}{S_{21}} + S_{12}$

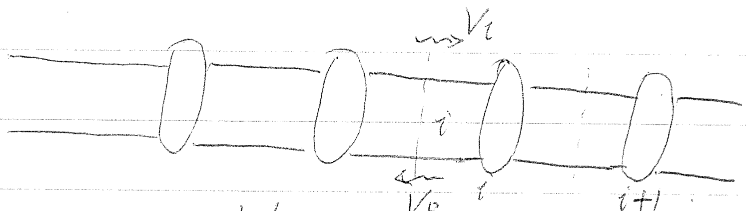
In the same way

$\Rightarrow A_{11} = \dfrac{1}{S_{21}}$ $\qquad A_{12} = -\dfrac{S_{22}}{S_{21}}$

$$\Rightarrow A = \begin{pmatrix} 1/S_{21} & -S_{22}/S_{21} \\ \dfrac{S_{11}}{S_{22}} & \dfrac{S_{21}S_{12} - S_{11}S_{22}}{S_{21}} \end{pmatrix} \qquad A^{(t)} = A^{(1)}S^{(2)}$$

No stable algorithm: S_{21} can be very small

periodic structure



$V_i = \begin{pmatrix} V_i^I \\ V_i^R \end{pmatrix}$ $\qquad V_{i+1} = \begin{pmatrix} V_{i+1}^I \\ V_{i+1}^R \end{pmatrix}$

$V_{i+1} = V_i e^{r}$

~~$V_{i+1} = V_i$~~ $\qquad V_i = \boxed{V_{i+1} A = e^{r} V_{i+1}}$ eigenvalue equation

eigenvalue of A is ~~the~~ ~~pro~~ e^{r}. $-r$. propagation constant.

if $S_{12} = S_{21}$, then if e^{r} is a solution. e^{-r} must be another solution.

$e^{r} = e^{j\beta}$ $\qquad e^{j(\beta + 2\pi i)}$ is also a solution $i \in \mathbb{Z}$.

β is a solution, ~~that~~ then $\beta + 2n\pi$ $n \in \mathbb{Z}$ also ~~so~~ so solution.

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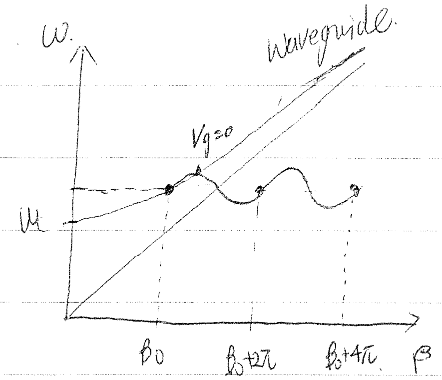
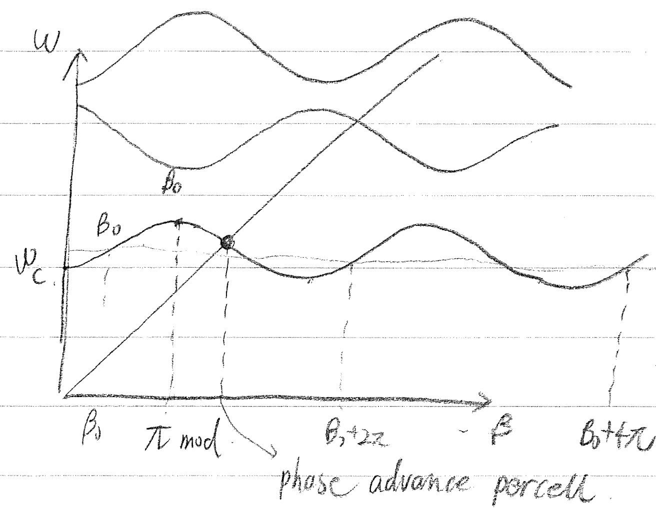
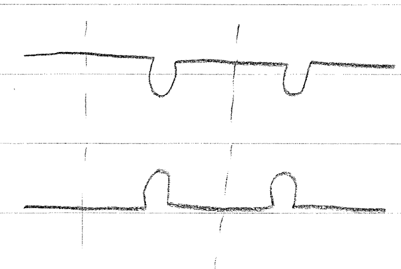


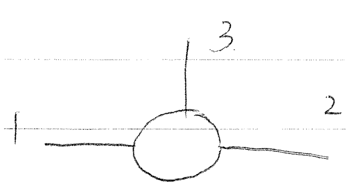
$w(\beta)$ should be periodic

$V_g = 0$ - π mode





3-port Network



$$\begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \xrightarrow[\text{passive}]{\text{reciprocal}} \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{pmatrix}$$

not necessary symmetric

$$\begin{pmatrix} 0 & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & 0 \end{pmatrix}$$

$$\left.\begin{aligned} |S_{12}|^2 + |S_{13}|^2 &= 1 \\ |S_{23}|^2 &= |S_{13}|^2 \\ S_{12} S_{13}^* &= 0 \\ S_{13} S_{23}^* &= 0 \end{aligned}\right\} \text{from unitary}$$

Im possible let
$S_{11} = 0$ & $S_{22} = 0$ & $S_{33} =$

then

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Theory 1: Impossible to match $|S_{12}|^2 + |S_{23}|^2 = 1$
all ports of 3-port lossless reciprocal network

$$\begin{pmatrix} V_1^- \\ V_2^- \\ V_3^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ V_3^- e^{j\phi} \end{pmatrix}$$

choke

$$V_3^- = S_{13} + S_{33} V_3^- e^{j\phi}$$

$$V_3^- = \frac{S_{13}}{1 - S_{33} e^{j\phi}}$$

$$V_1^- = S_{11} + S_{13} V_3^- e^{j\phi} = S_{11} + \frac{S_{13} S_{13} e^{j\phi}}{1 - S_{33} e^{j\phi}}$$

$$V_2^- = S_{12} + S_{23} V_3^- e^{j\phi} = S_{12} + \frac{S_{23} S_{13} e^{j\phi}}{1 - S_{33} e^{j\phi}}$$

If want $V_2^- = 0 \Rightarrow S_{12} + \dfrac{S_{23} S_{13} e^{j\phi}}{1 - S_{33} e^{j\phi}} = 0$

$$\Rightarrow S_{12} - S_{12} S_{33} e^{j\phi} + S_{23} S_{13} e^{j\phi} = 0$$

$$\Rightarrow e^{j\phi} = \frac{S_{12}}{S_{12} S_{33} - S_{23} S_{13}}$$

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Lecture 8. Tianhuan Luo

$$V_2^- = \frac{S_{12} + (-S_{12}S_{33} + S_{13}S_{23})e^{j\varphi}}{1 - S_{33}e^{j\varphi}}$$

$$V_2^- = 0 = S_{12} - S_{12}\left(\frac{-S_{13}S_{23}}{S_{12}} + S_{33}\right)e^{j\varphi} = 0$$

$$\left|\frac{-S_{13}S_{23}}{S_{12}} - S_{33}\right| = 1$$

Adding loss to port 1 $\frac{S_{13}}{S_{12}}$ don't ch

Adding loss to por 2 $\frac{S_{23}}{S_{13}}$ don't ch

$$P\begin{pmatrix} \frac{-\cos\theta + e^{j\varphi}}{2} & \frac{-\cos\theta - e^{j\varphi}}{2} & \frac{\sin\theta}{\sqrt{2}} \\ \frac{-\cos\theta - e^{j\varphi}}{2} & \frac{-\cos\theta + e^{j\varphi}}{2} & \frac{\sin\theta}{\sqrt{2}} \\ \frac{\sin\theta}{\sqrt{2}} & \frac{\sin\theta}{\sqrt{2}} & \cos\theta \end{pmatrix} P$$

$2 + 3 = 5$ free degree
from P

$$\Rightarrow \left|\frac{-\frac{\sin\theta}{\sqrt{2}} \cdot \frac{\cancel{(-\cos\theta + e^{j\varphi})}\sin\theta}{\sqrt{2}}}{\frac{-\cos\theta - e^{j\varphi}}{2}} - \cos\theta\right| = 1$$

$$\left|\frac{\sin\theta\,\cancel{(-\cos\theta + e^{j\varphi})}^{\sin\theta}}{\cos\theta + e^{j\varphi}} - \cos\theta\right| = 1$$

$$-\sin\theta\cos\theta + e^{j\varphi}$$

Theore 2

three port network can couple 1 & 2 ~~any~~, g by adjust φ.

can give any magnitude V_2^- you want

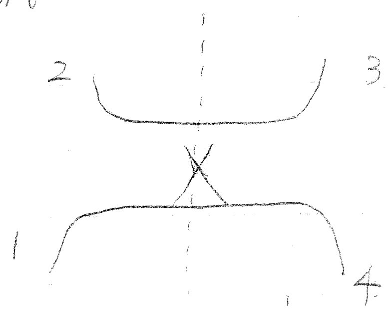
use as a control element

Theore 3

Circulator - (Homework).

8 - 2

4 - port



Directional coupler

$S_{12} = 0 = S_{21}$

$S_{34} = S_{43} = 0$

$$S = \begin{pmatrix} S_{11} & 0 & S_{13} & S_{14} \\ 0 & S_{22} & S_{23} & \cancel{0}\, S_{24} \\ S_{13} & S_{23} & S_{33} & \cancel{S_{34}}\, 0 \\ S_{14} & \cancel{0}\, S_{24} & \cancel{S_{34}}\, 0 & S_{44} \end{pmatrix}$$

from symmetry $S_{23} = S_{14}$

$S_{33} = S_{22}$

$S_{44} = S_{11}$

$S_{13} = S_{24}$

$$S = \begin{pmatrix} S_{11} & 0 & S_{13} & S_{14} \\ 0 & S_{22} & S_{14} & S_{13} \\ S_{13} & S_{14} & S_{22} & 0 \\ S_{14} & S_{13} & 0 & S_{11} \end{pmatrix}$$

$\Rightarrow$ $S_{13} S_{14}^* + S_{14} S_{13}^* = 0$

$\Rightarrow Re(S_{13} S_{14}^*) = 0$

$S_{22} S_{14}^* + S_{22}^* S_{14} = 0$

$\Rightarrow Re(S_{22} S_{14}^*) = 0$

also have $Re(S_{11} S_{14}^*) =$

$Re(S_{22} S_{13}^*) =$

$|S_{11}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$

$|S_{22}|^2 + |S_{14}|^2 + |S_{13}|^2 = 0$

$\Rightarrow |S_{11}| = |S_{22}| \qquad S_{22} = S_{11} e^{j\varphi}$

~~$S_{13} S_{14}^* = |S_{14}| e^{j\varphi} \cdot |S_{13}|$~~

$Re[S_{13} S_{14}^*] = 0 \Rightarrow S_{13}$, S_{14} differ by factor j. or at least one of them is 0.

if $S_{13} = 0$ or S_{14} then it is a 2-port.

only when $S_{11} = 0$ ~~$S_{14} = 0$~~ it's a 4-port network

Theory 1: ~~S~~ 4 port network if it's matched, it's a directional coupler ~~coup~~

$$S = \begin{pmatrix} 0 & 0 & \cos\theta & j\sin\theta \\ 0 & 0 & j\sin\theta & \cos\theta \\ \cos\theta & j\sin\theta & 0 & 0 \\ j\sin\theta & \cos\theta & 0 & 0 \end{pmatrix}$$

~~Theory 2~~ when $\theta = 45°$ - 3dB ~~hbr~~ hybrd. / Magic Tee

8 - ③

$$S=\begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & j\frac{1}{\sqrt{2}} \\ 0 & 0 & j\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & j\frac{1}{\sqrt{2}} & 0 & 0 \\ j\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

One port cannot couple other two ports with the same phase

90° hybryd

0° hybryd

Put short circuit at port 3 & 4

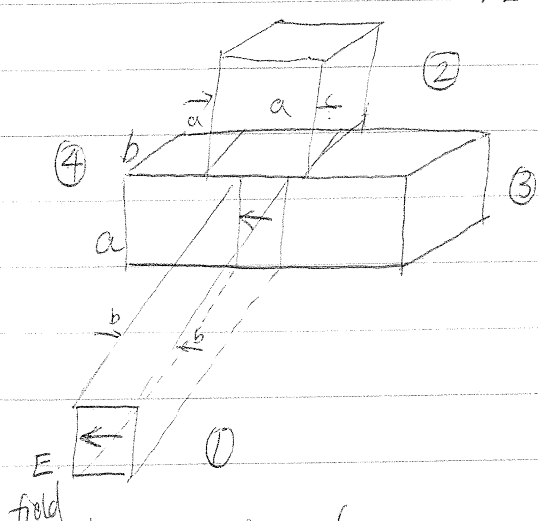
$$S\begin{pmatrix} 1 \\ 0 \\ V_3^- e^{j\varphi} \\ V_4^- e^{j\varphi'} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(V_3^- e^{j\varphi} + jV_4^- e^{j\varphi'}) \\ \frac{1}{\sqrt{2}}(jV_3^- e^{j\varphi} + V_4^- e^{j\varphi'}) \\ \frac{1}{\sqrt{2}} \\ j\frac{1}{\sqrt{2}} \end{pmatrix}$$

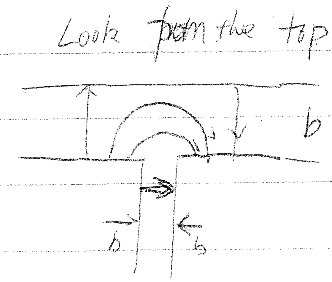
$$V_1^- = \frac{1}{\sqrt{2}}(V_3^- e^{j\varphi} + jV_4^- e^{j\varphi'}) = \frac{1}{2}(e^{j\varphi} - e^{j\varphi'})$$

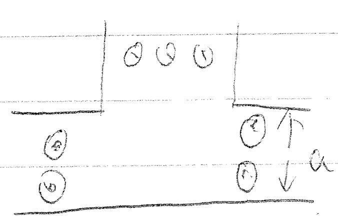
$$V_2^- = \frac{1}{\sqrt{2}}(jV_3^- e^{j\varphi} + V_4^- e^{j\varphi'}) = \cancel{\frac{1}{\sqrt{2}}}(\frac{1}{2}(je^{j\varphi} + je^{j\varphi'})$$

if $\varphi = \varphi'$. $V_1^- = 0$

$V_2^- = je^{j\varphi}$







from front

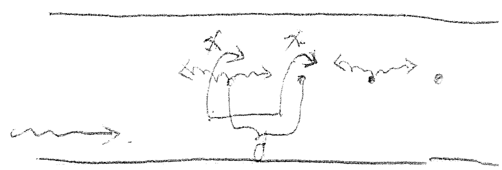
if the same small purterbation is given at ① & ②, ③ & ④ are still not coupled

8 - 4

difference between 3 & 4

put short line, at 1 & 2 adjust position of them ~~the change them~~ simultaneously will change the phase change the relative position will change the coupling between 3 & 4

E-H tuner



$xe^{j\varphi}$ $e^{j(\varphi+Bd)}$ $\cdots$ $e^{j(\varphi+NBd)}$

$$C = \cancel{e^{j\varphi} + e^{j(\varphi+Bd)} + e^{j(\varphi+NBd)}} \quad \sum_i e^{j(\varphi+NBd)}$$

$$= Nxe^{jNBd}$$

$$|C| = Nx$$

$$B = x[\underset{N=1}{e^{j\varphi}} + e^{j(\varphi+Bd)} + e^{j(\varphi+NBd)} + \cdots]$$

$$= \sum_{i=0} xe^{j(\varphi+i2Bd)}$$

$$= (1+e^{j2Bd})^M \qquad 2Bd = 2N\pi + \pi$$

~~B=~~ Maximum reflect directional coupled

x is small

two way to make ~~critically~~ directive coupled

1. Magic Tee
2. many small holes

$$S = \frac{I-Y}{I+Y} \Rightarrow \cancel{S} \quad S + SY = I - Y$$

$$SY + Y = I - S$$

$$Y = \frac{I-S}{I+S}$$

For magic Tee

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -j & 1 \\ 0 & 0 & 1 & -j \\ -j & 1 & 0 & 0 \\ 1 & -j & 0 & 0 \end{pmatrix}$$

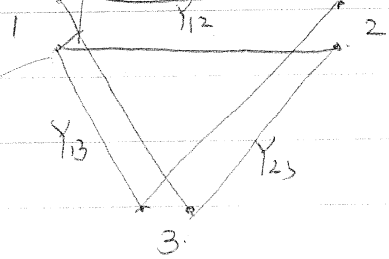
$$Y = \begin{pmatrix} 1 & 0 & +j & -1 \\ 0 & 1 & -1 & +j \\ +j & -1 & 1 & 0 \\ -1 & +j & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -j & 1 \\ 0 & 1 & 1 & -j \\ -j & 1 & 1 & 0 \\ 1 & \cancel{+j} & 0 & 1 \end{pmatrix}^{-1}\cdot\left(-\frac{1}{2}\right)$$

$$= j\begin{bmatrix} 0 & 1 & 0 & +\sqrt{2} \\ 1 & 0 & +\sqrt{2} & 0 \\ 0 & +\sqrt{2} & 0 & 1 \\ +\sqrt{2} & 0 & 1 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} Y_{11} & 0 & \\ 0 & 0 & 0 \\ 0 & 0 & \end{bmatrix} + \begin{bmatrix} 0 & Y_{12} & 0 & 0 \\ Y_{21} & 0 & 0 & \\ 0 & 0 & & \end{bmatrix} + \begin{bmatrix} 0 & 0 & & \\ 0 & Y_{21} & 0 & \\ & 0 & 0 & \end{bmatrix} + \cdots$$

adding impedence at certain position

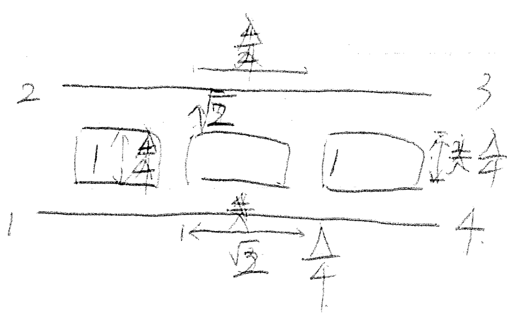
$$i = YU$$



$\cancel{Y = Y_0}$

$Y = -jY_0 \cot \beta l$

5# S matrix → Y matrix → network



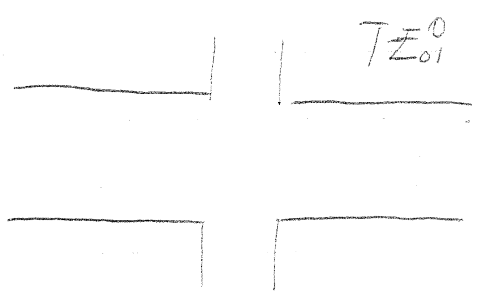
plus + $\frac{\lambda}{4}$

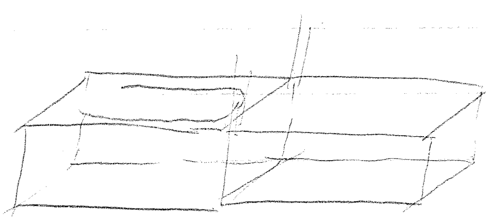
minus − $\frac{\lambda 3}{4}$

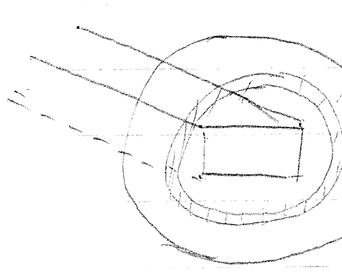
$$i_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3$$

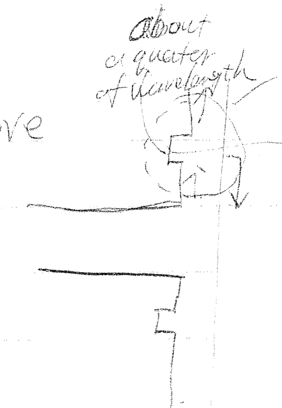
$$i_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{33}V_3$$

choke









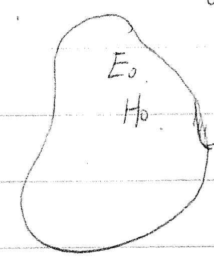
Magic Tee.

choose proper short line, to get totally reflection. back to wave guide.

8- 7

Purtabation theory:

Cavity ω_0

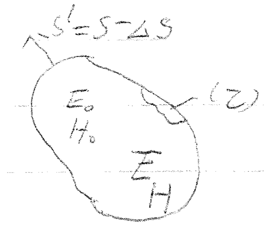


a dent with volume τ

$E . H . \omega .$

Lecture 9

$$\int_{V'} \nabla\cdot(E_0^*\times H^* + E\times H_0^*) = \int_{S'} (E_0\times H^* - E\times H_0^*)\,ds$$



$$\int_{V'} H_0^* \nabla\times E_0^* - E_0^*\cdot\nabla\times H^* + H_0\cdot\nabla\times E - E\cdot\nabla\times H_0^*$$

$$= \int\left(-j\omega_0 u H^*\cdot H_0 - j\omega\epsilon E_0 E^* + j\omega u H_0^*\cdot H + (E\cdot E_0/j\omega_0\epsilon)\,dv\right)$$

$$= j(\omega - \omega_0)\int_{V'} u H^* H_0 - \epsilon E_0\cdot E^*\,dv = \int_{S'} -E_0^*\times H_0$$

E is 0 everywhere on the surface

$$\int_{V'}\nabla\cdot(\vec E_0^*\times H + E\times\vec H_0^*) = \int_{S'}(\vec E_0^*\times H + E\times H_0^*)\,ds$$

$$\int_{V'}(H\,\nabla\times\vec E_0^* - \vec E_0^*\,\nabla\times H + \vec H_0^*\,\nabla\times E - \vec E\cdot\nabla\times\vec H_0^*)$$

$$= \int_{V'} H\, j\omega_0 u H_0^* - \vec E_0^*\, j\omega\epsilon E + j\omega u \vec H^*\cdot H - j\omega_0 u E\cdot E_0^*$$

$$= j(\omega_0-\omega)\int_{V'}(u H\cdot H_0^* + \epsilon E_0^* E)\,dv = \int_{S'} E_0^*\times H\cdot d\vec s$$

$$= \int_S E_0^*\times H\,d\vec s - \int_{\Delta S} E_0^*\times H\,d\vec s$$

$$\Rightarrow \omega_0-\omega = j\frac{\int_{\Delta S}E_0^*\times H}{\int_V(uH\cdot H_0^* + \epsilon E_0^*\cdot E)\,dV}$$

$$= j\frac{\int_{\Delta V}\nabla\cdot(\vec E_0^*\times H)\,dv}{\int_V(uH\cdot H_0^*+\epsilon E_0^*\cdot E)\,dv}$$

$$= j\frac{\int_{\Delta V} H(j\omega_0 u H_0^*) - E_0^* j\omega\epsilon E}{\int_V u H\cdot H_0^* + \epsilon' E_0^* E\,dv}$$

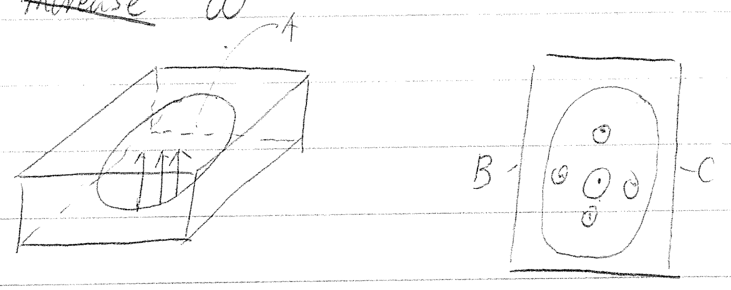
$$= -\frac{\int_{\Delta V} u H H_0^*\,\omega_0 - E_0^*\epsilon E\,\omega}{\int_V u H\cdot H_0^* + \epsilon E E_0^*}$$

$$\cancel{\frac{W_0 - W}{}} \quad \frac{W - W_0}{W_0} = \frac{\int_{\Delta V} (\mu H H_0^* - \epsilon E_0^* E) dV}{\int_V \mu H_0 H_0^* + \epsilon E_0^* E \, dV} = \cancel{\frac{\Delta W_H - \Delta W_E}{W_H + W_E}}$$

$$\boxed{\frac{W - W_0}{W_0} = \frac{\Delta V_H - \Delta V_E}{V_H + V_E}}$$

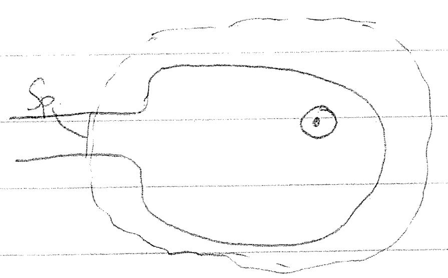
1) squeesz a cavity ~~at~~ where there is E field by no B field
will ~~increase~~ decrease ω

TE_{101}.



squeeze in the middle A $\quad W \downarrow$

squeeze at the side BC $\quad W \uparrow$



$P = E_0 \times H - E \times H_0$

$\int_S P \, ds = \int_V \nabla \cdot P \, dV$

$\int_{Sp} \vec{P} \cdot d\vec{s} = \int_V \vec{\nabla} \cdot \vec{P} \, dv$

$E_0 = E_{i0} + E_{r0} = E_{i0} (1 + \Gamma_0)$

$H_0 = H_{i0} \mp E_{r0} = H_{i0} (1 - \Gamma_0)$

$E_{i0}(1+\Gamma_0) H_i (1-\Gamma) - E_i (1+\Gamma) H_{i0} (1-\Gamma_0) = (\Gamma - \Gamma_0)(E_i H_{i0} + E_{i0} H_0^*)$

$(T-T_0)\iint E_i H_{i0} + E_i H_i \, dS_p = 2$

$\nabla\cdot P = \nabla\cdot(E_0\times H - E\times H_0)$

$= H_0\cdot\nabla\times E_0 - E_0\cdot\nabla\times H - H_0\cdot\nabla\times E + E\cdot\nabla\times H_0$

$= H\cdot H_0(-j\omega\mu_0) \; \overline{\oplus} E_0\cdot E(j\omega\epsilon) - H_0\cdot H(-j\omega\mu) + E\cdot E_0(j\omega\epsilon_0) - E_0 J$ current induced by bead

$= +j\omega(\underbrace{\cancel{\mu_0 - \mu}}_{\mu-\mu_0}) H H_0 - j\omega(\epsilon \underbrace{-}_{+\frac{J}{j\omega E}} \epsilon_0) E_0 E$

$\Rightarrow 2(T-T_0)P = \int_v \left[j\omega(\mu-\mu_0) H H_0 - j\omega\left(\epsilon + \frac{J}{j\epsilon E\omega} - \epsilon_0\right) E_0 E \right] dV$

$= \int_{\Delta v} j\omega(\mu-\mu_0) P_m H_0^2 - j\omega\left(\epsilon + \frac{J}{j\epsilon E\omega} - \epsilon_0\right) P_E E_0^2$

$\Rightarrow 2(T-T_0)P_i = j\omega(\alpha_m H_0^2 - \alpha_e N E_0^2)$

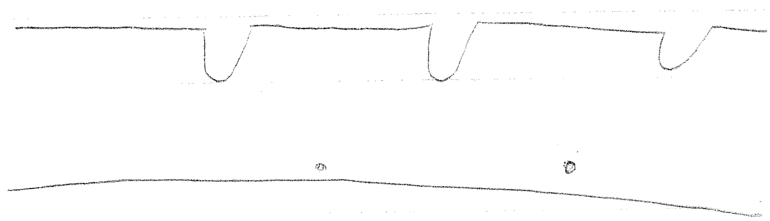
① by choosing material, we might control it to be only H_0^2 or E_0^2 effect.

Eg. For ~~metal~~ small metal pieces like Cu only E_0^2 effect

② When $T_0 = 0$ $\quad T \propto E_0^2$

"usually took T_0 out numerically like 'substracting'"

③



$T - T_0 \propto \alpha_e E_0^2$

$\Rightarrow T - T_0$

$\frac{29.9792}{11.424} \cdot \frac{1}{3}$

$$\frac{\omega - \omega_0}{\omega} = \frac{\Delta V_H - \Delta V_E}{V_H + V_E}$$

$$(\Gamma - \Gamma_0) \simeq \partial_m H^2 - \partial_E E^2$$

becareful of frequency : the character of material change over different ~~frequency~~ frequencies

further reading:

~~Harmonical~~

1) Time-Harmonic Electromagnetic field.
R. F. Harrington.

2) Electro magnetic waveguide Theory and application.
S. F. Mahmand

3) Microwave Engineering . David Pozar.

4) Rad Lab series ~~Vol~~ Volam 8
Dikee, Monteyomary and Purcell.

5) Charles W. steele IEEE Transactions on microwave theory and tech
Vol MIT-14. NO. 2.

Digital processing.

6) Slater. Microwave Electronics