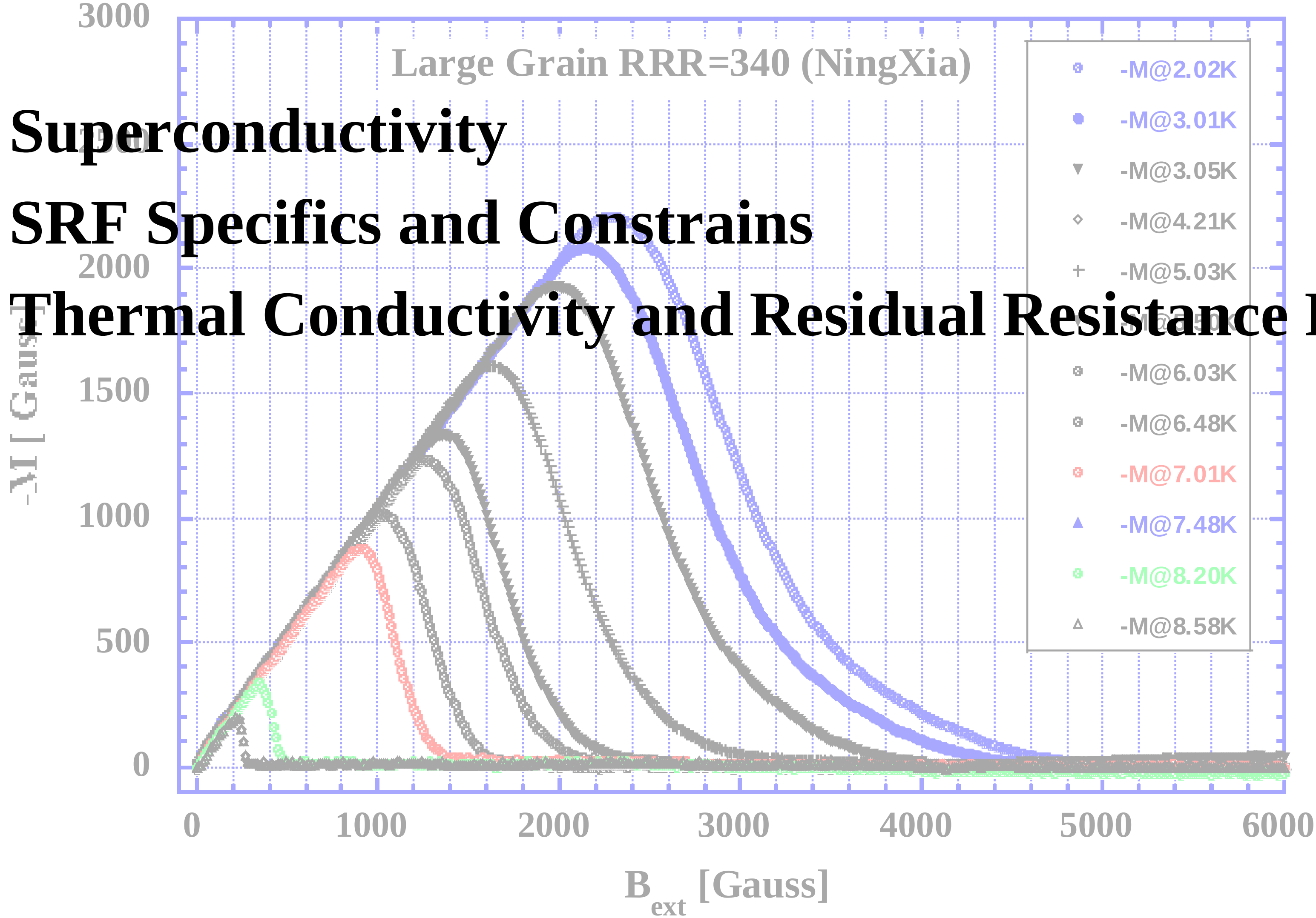
1. Superconductivity Basics for SRF Cavity



1.1 Superconductivity

1.2 SRF Specifics and Constrains

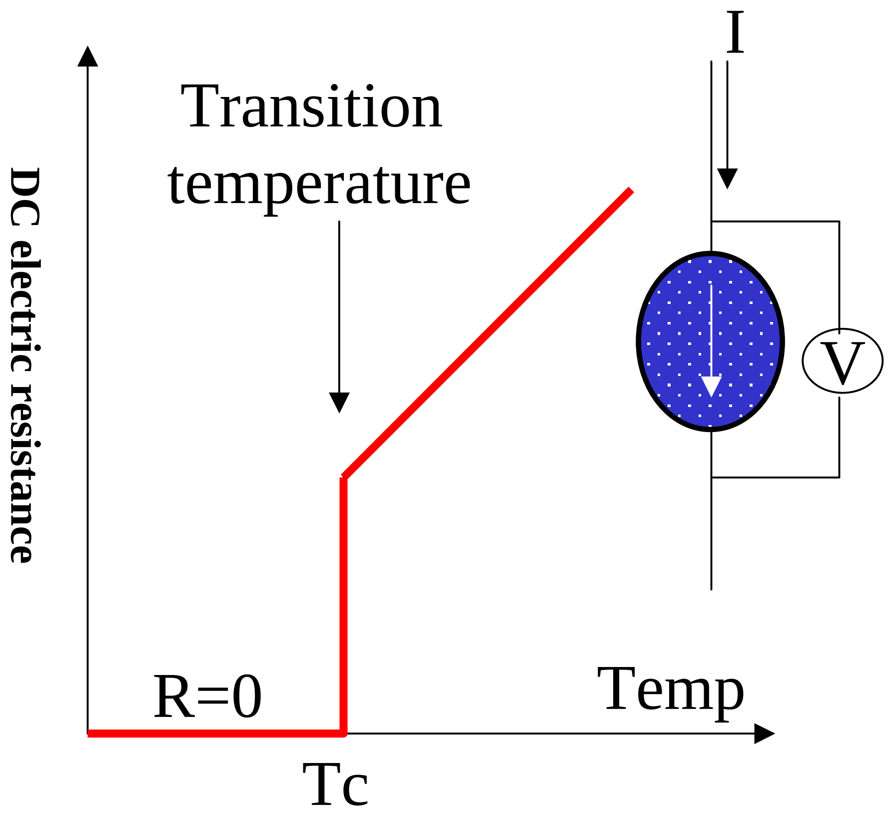
1.3 Thermal Conductivity and Residual Resistance Ratio

Superconductivity

1911 by K.Onnes

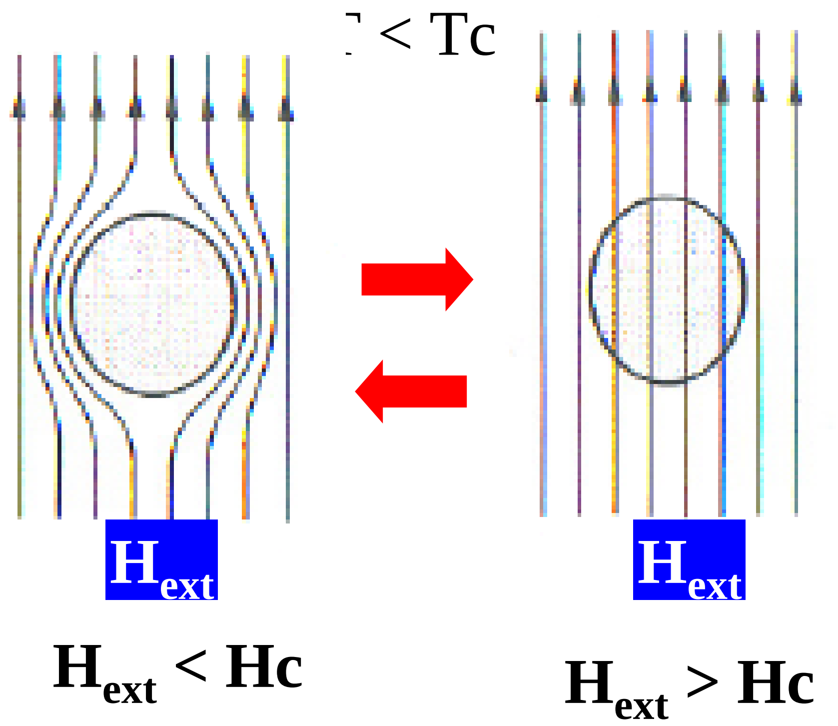
19933 by Meissner and Ochsenfeld (experiment)
1935 Phenomenological theory by F.and H.London

Zero resistance @ Tc



Perfect diamagnetism < Hc

Miessner effect

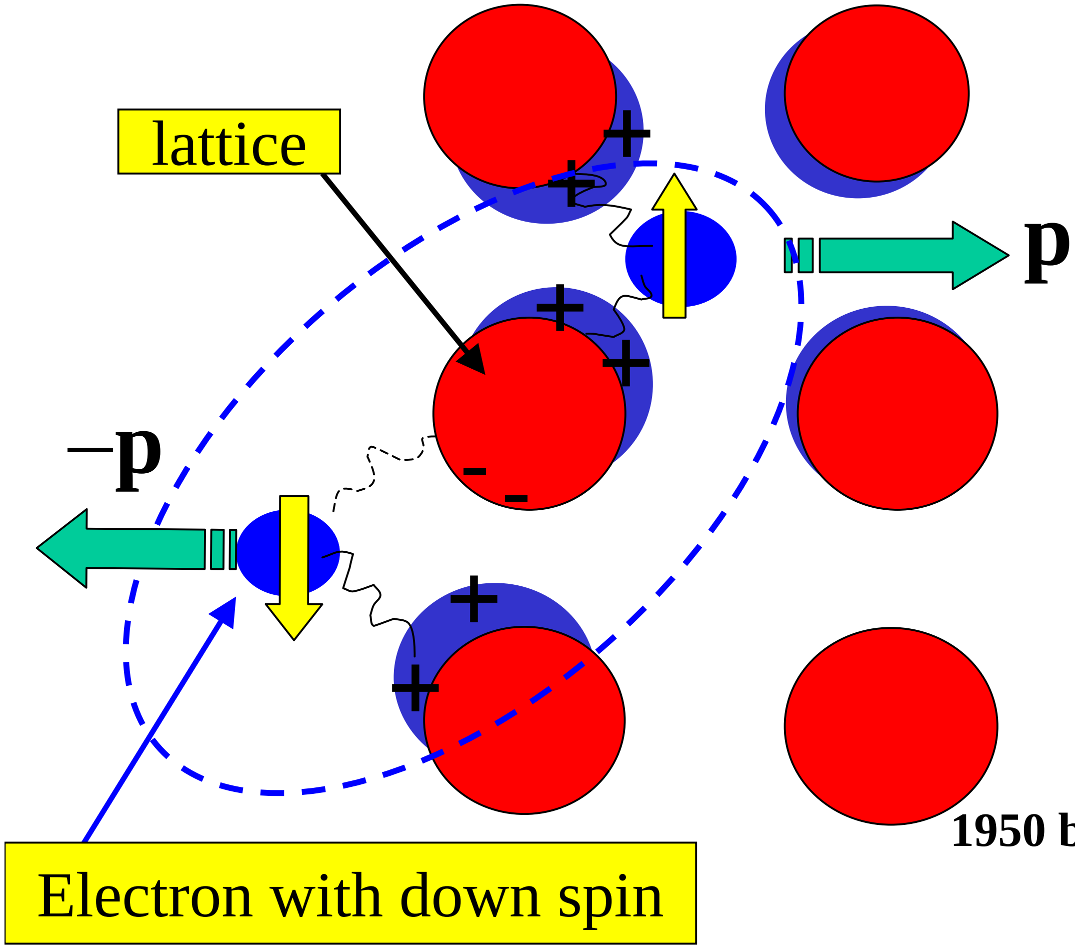


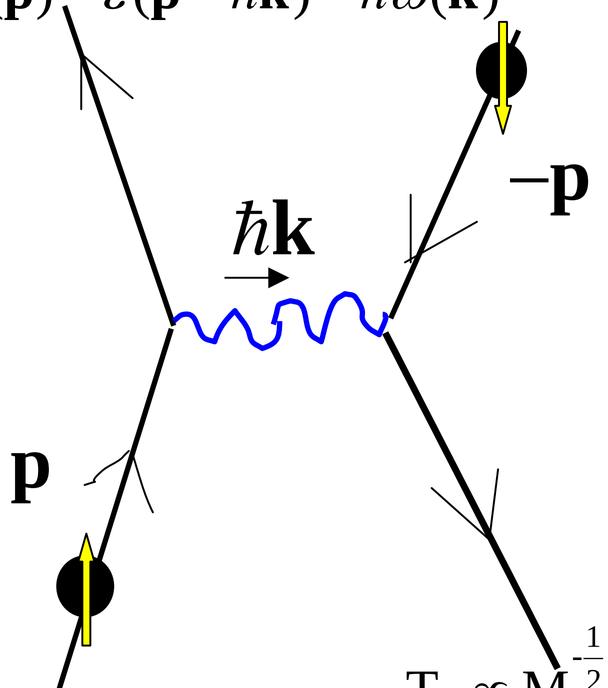
Microscopic Theory

Two electrons having opposite spin and momentum get an attractive interaction through lattice/electron interaction.

Attractive interaction through electron-lattice interaction

$$V = \frac{\left|V_{\mathbf{p}-\hbar\mathbf{k},\mathbf{p}}\right|^2}{\varepsilon(\mathbf{p}) - \varepsilon(\mathbf{p}-\hbar\mathbf{k}) - \hbar\omega(\mathbf{k})}$$





$T_C \propto M^{-\frac{1}{2}}$, $H_C \propto M^{-\frac{1}{2}}$

Isotope effect of Tc
1950 by Reynolds and Maxwell

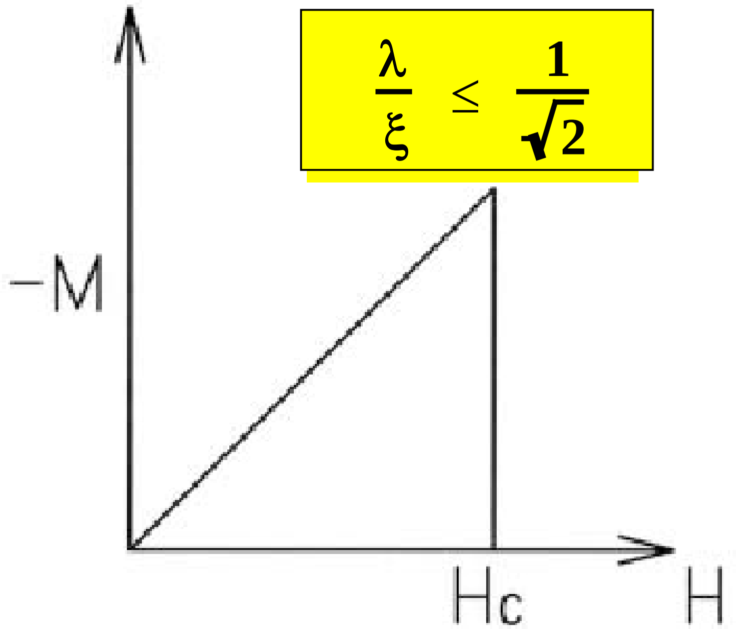
BCS theory
1957 by Bardeen, Cooper, and Schrieffer

Two Types of Superconductor

1937 by Schubnikov (experiment), 1957 Abrikosov (theory)

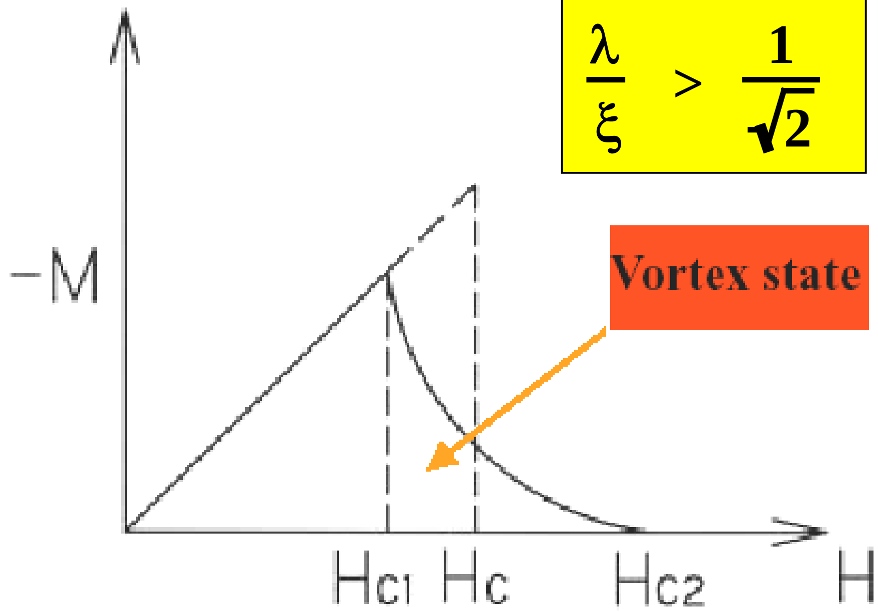
Type-I

$$\frac{\lambda}{\xi} \leq \frac{1}{\sqrt{2}}$$



Type-II

$$\frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$$

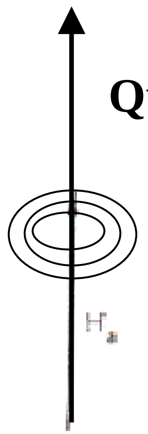


$$G_n - G_s = \frac{1}{2}\mu H_C^2 \equiv \int_0^{H_{c2}} MdH$$

Vortex state

Flux quantization, 1961 by Deaver and Fairbank

Observed by iron powder



Quantized magnetic flux

supercurrent

Vortex state

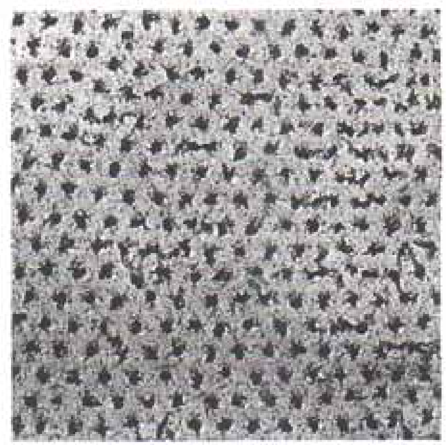
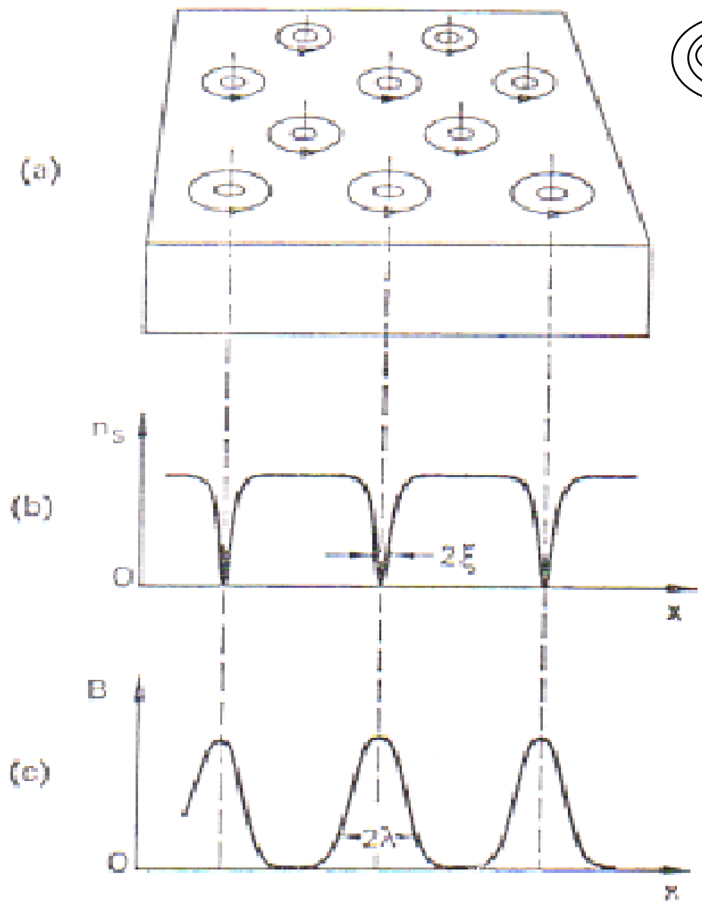


Figure 19 Triangular lattice of fluxoids through top surface of a superconducting cylinder. The points of exit of the flux lines are decorated with fine ferromagnetic particles. The electron microscope image is at a magnification of 8300, by U. Essmann and H. Träuble.



ξ : Coherence length
a distance within which the superconducting electron concentration cannot change drastically in a spatially-varying magnetic field

λ_L : London penetration depth

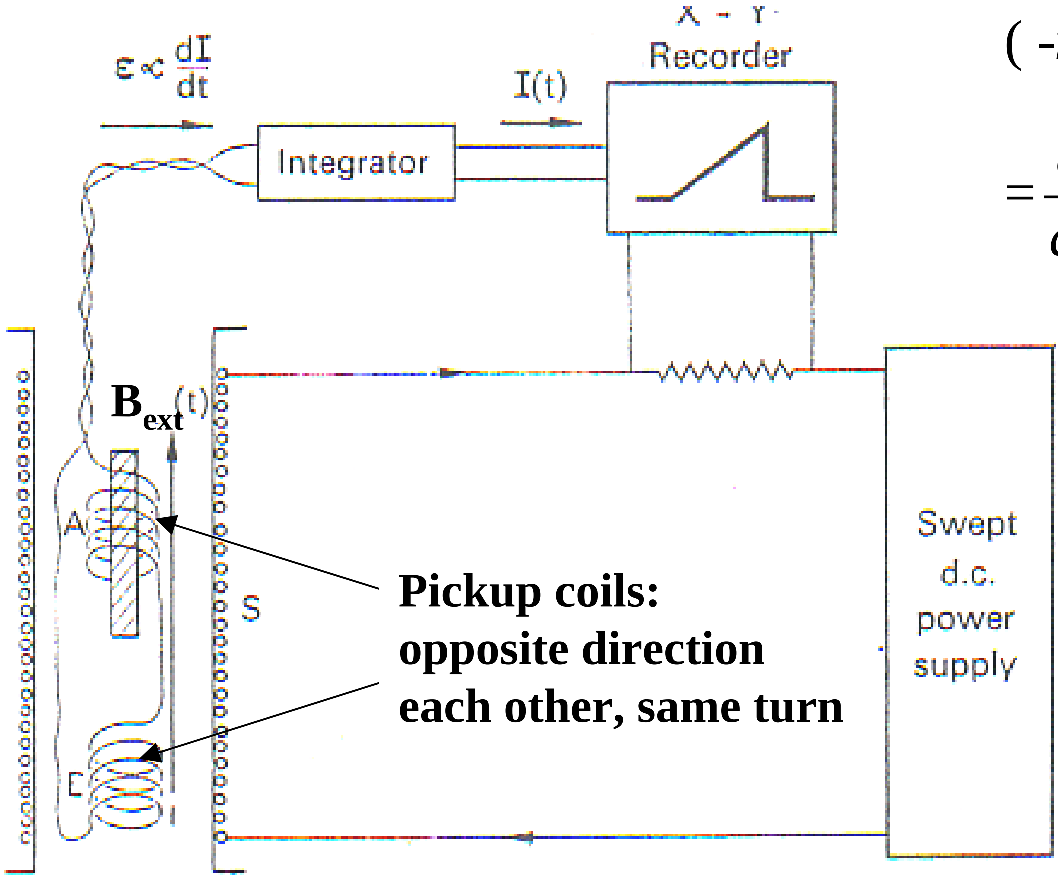
Depth of penetration of the magnetic field

Critical magnetic field measurement

$$V = V_A + V_B = -\frac{d}{dt}\Phi_A + \frac{d}{dt}\Phi_B$$

$$(\ -nS_0\mu\frac{d}{dt}B_{ext} + \frac{d}{dt}M\ ) + nS_0\mu\frac{d}{dt}B_{ext}$$

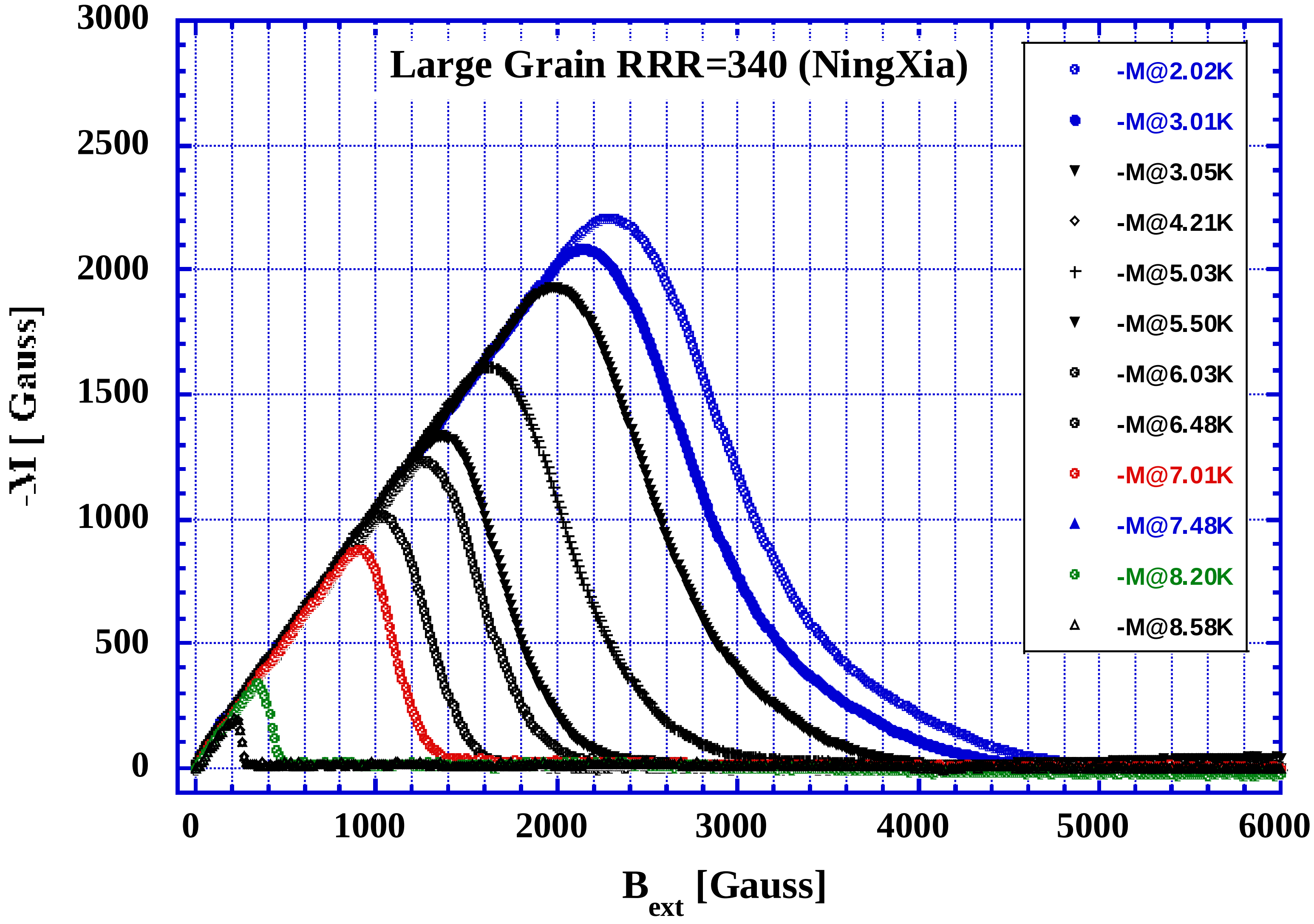
$$= \frac{d}{dt}M$$



$$M = \int_0^t V dt$$

Example of demagnetization curve on Niobium

(NingXia, Large Grain RRR=340)



Abrikosov's Theory for Type-II

$$H_c = \frac{\kappa}{\lambda^2}\frac{\hbar c}{\sqrt{2}e^*} = \frac{\kappa}{\lambda^2}\frac{(hc/2e)}{2\pi\sqrt{2}} = \frac{\phi_0}{2\pi\sqrt{2}\lambda\xi}$$

$$H_{c2} = \sqrt{2}\frac{\lambda}{\xi}\frac{\phi_0}{2\pi\sqrt{2}\lambda\xi} = \frac{\phi_0}{2\pi\xi^2}$$

$$H_{c1} = \frac{\phi_o}{4\pi\lambda^2}\ln(\frac{\lambda}{\xi} + 0.08)$$

$$\phi_0 = hc/2e = 2.0678\times 10^{-7}\,Gauss\cdot cm^2$$

$$= 2.0678\times 10^{-15}\ T\cdot m^2$$

Perturbation theory T ~Tc

$$H_C(T) = H_C(0)\left[1-(T/T_C)^2\right]$$

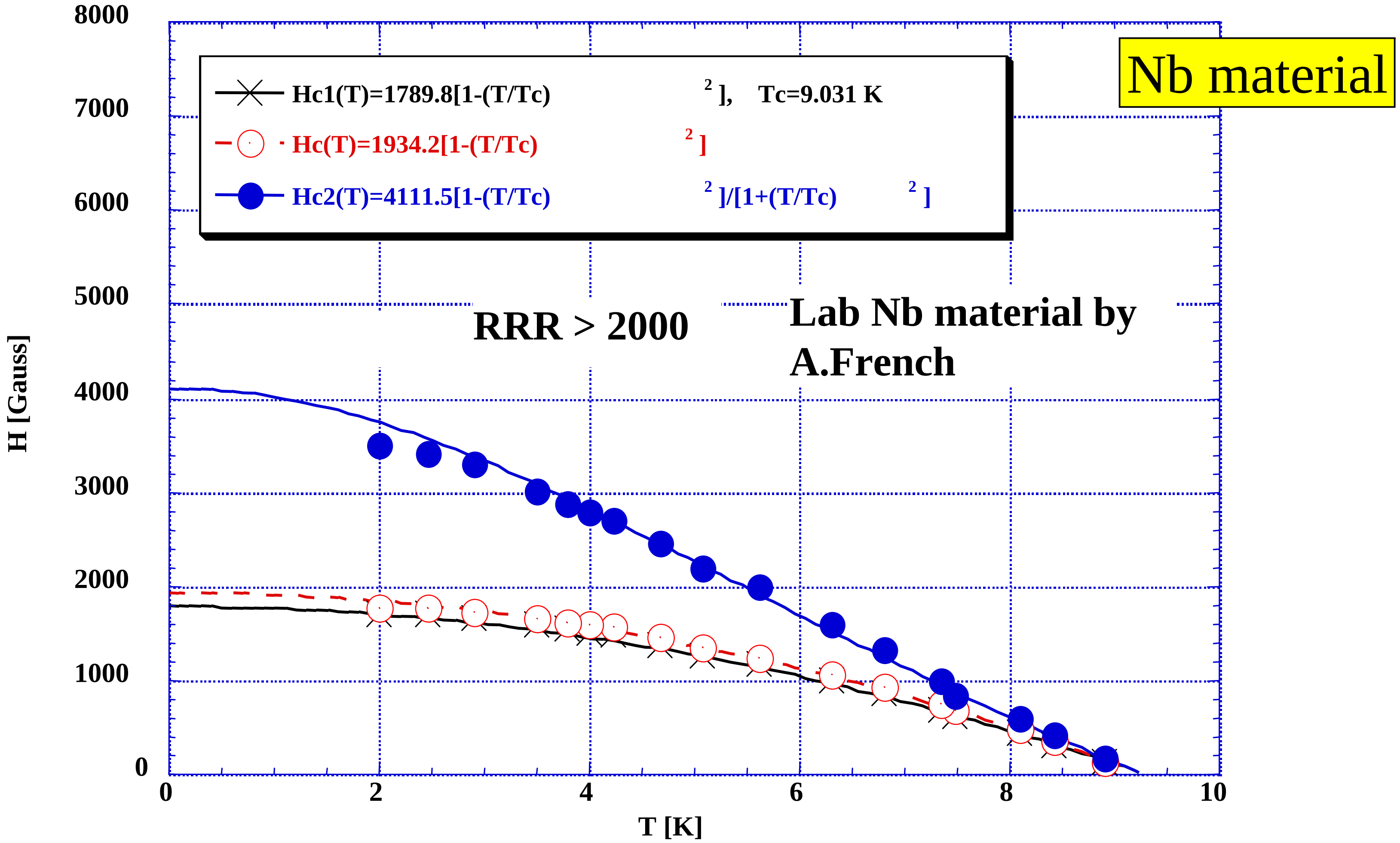
$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1-(T/T_C)^4}}$$

Exercise I.
Show the formulas for ξ, λ by Hc,Hc2.
Get the T-dependences for ξ, H_{C2}, κ, H_C^{RF}.

Expand for all T range (assumption)

$$\xi(T) = \xi(0)\cdot\sqrt{\frac{1+(T/T_C)^2}{1-(T/T_C)^2}} \qquad \kappa(T) = \frac{\kappa(0)}{1+(T/T_C)^2}$$

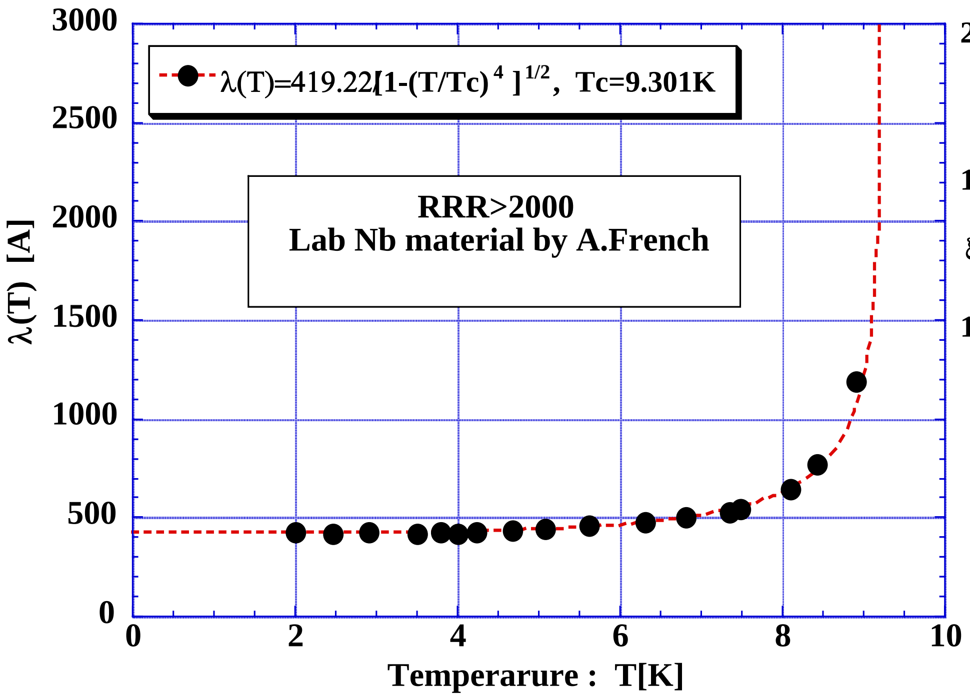
T-dependence of H_{C1}, H_C, H_{C2}

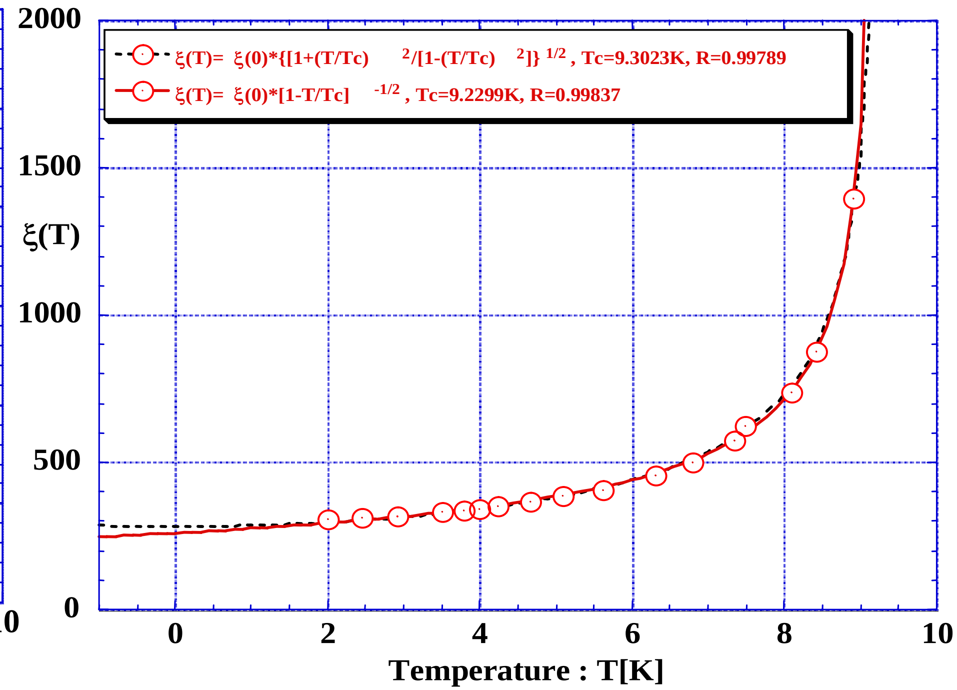


$$H_c(T) = H_c(0)\cdot\left[\, 1-\left(\frac{T}{T_c}\right)^2 \right]$$

T–dependence of λ and ξ

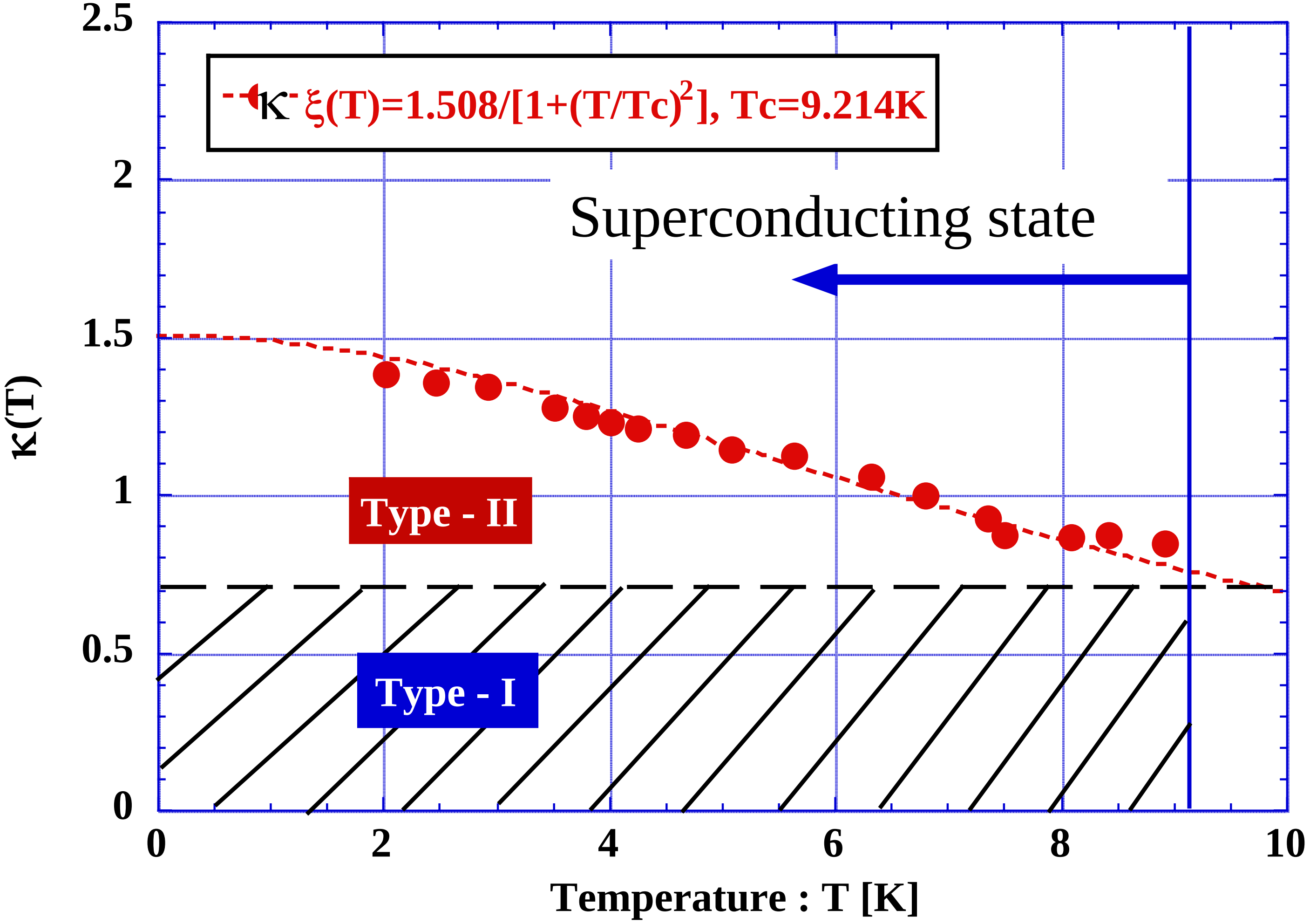
Lab material, RRR>2000





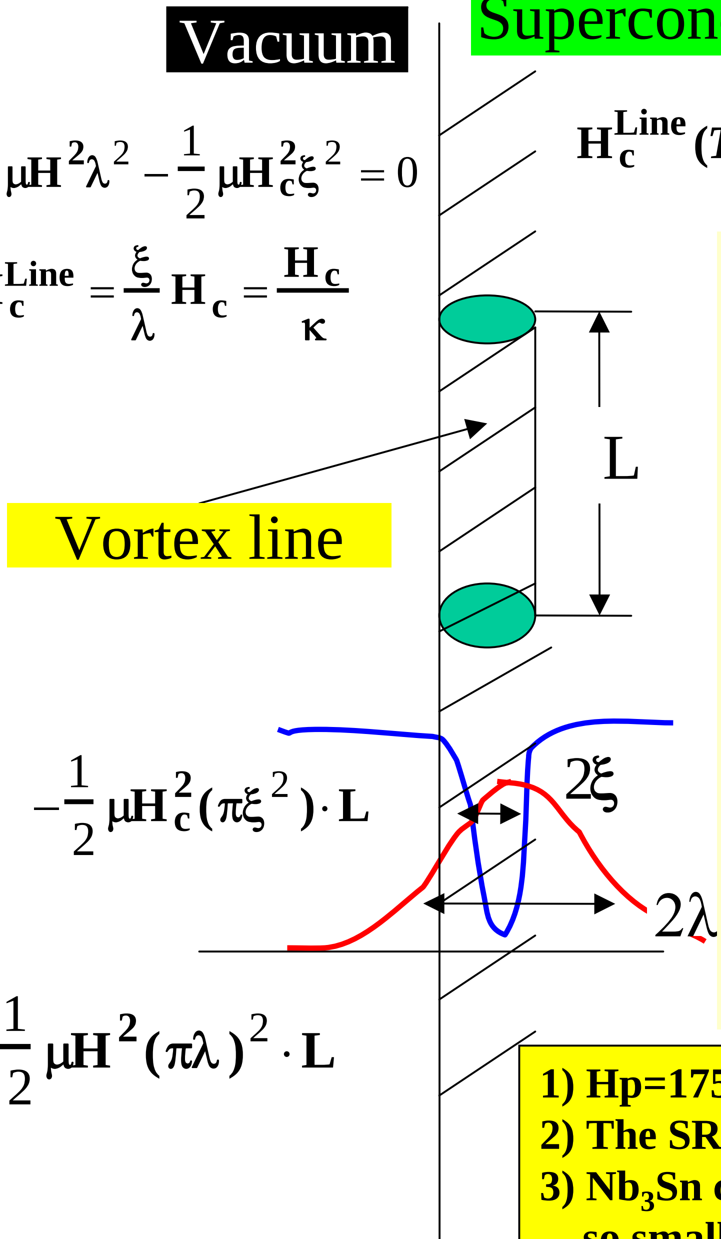
$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1-(T/T_C)^4}}, \qquad \xi(T) = \xi(0)\cdot\sqrt{\frac{1+(T/T_C)^2}{1-(T/T_C)^2}}$$

T-dependence of κ with Lab material



$$\kappa(T) = \frac{\kappa(0)}{1 + (T/T_C)^2}$$

Attempt for RF Field limitation model



Effective field strength

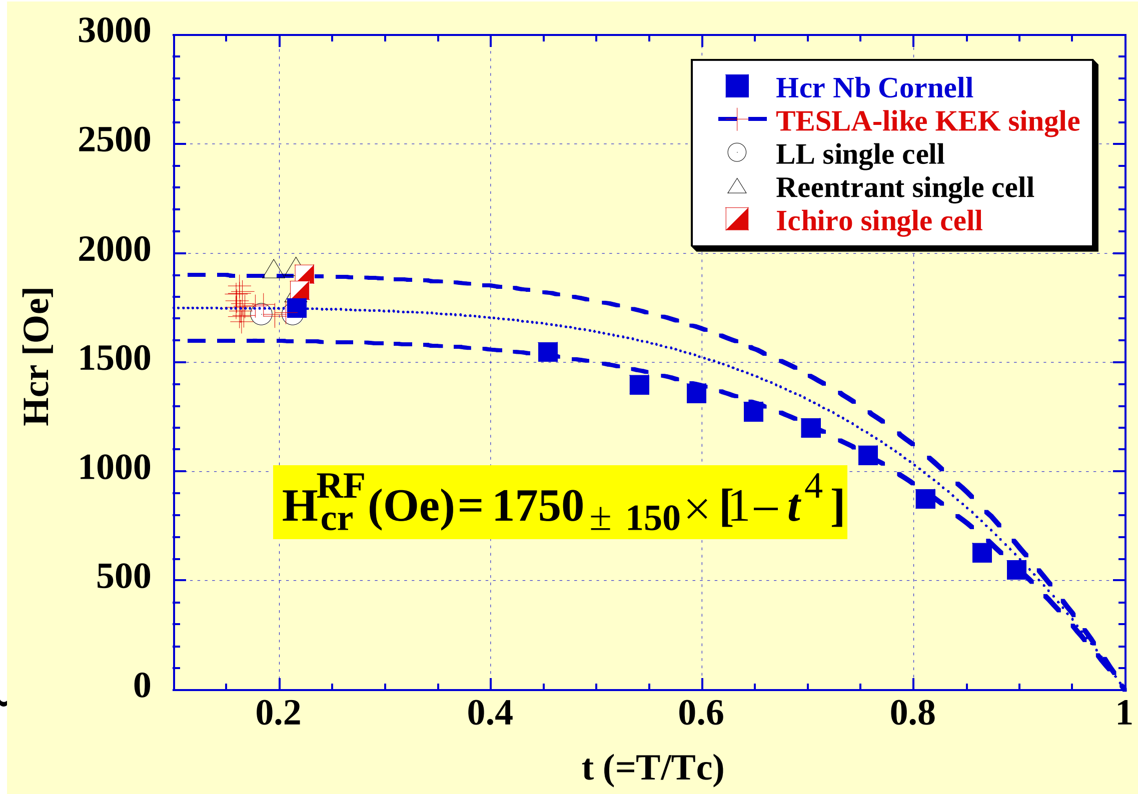
$$H_c^{Line}(T) = \frac{\xi(T)}{\lambda(T)} \cdot \sqrt{2}H_c(T) = \frac{\sqrt{2}H_c(T)}{\kappa(T)} = \sqrt{2}\frac{H_c(0)}{\kappa(0)} \cdot \left[1 - \left(\frac{T}{T_c}\right)^4\right]$$

$$\frac{1}{2}\mu H^2\lambda^2 - \frac{1}{2}\mu H_c^2\xi^2 = 0$$

$$H_c^{Line} = \frac{\xi}{\lambda}H_c = \frac{H_c}{\kappa}$$

$$-\frac{1}{2}\mu H_c^2(\pi\xi^2)\cdot L$$

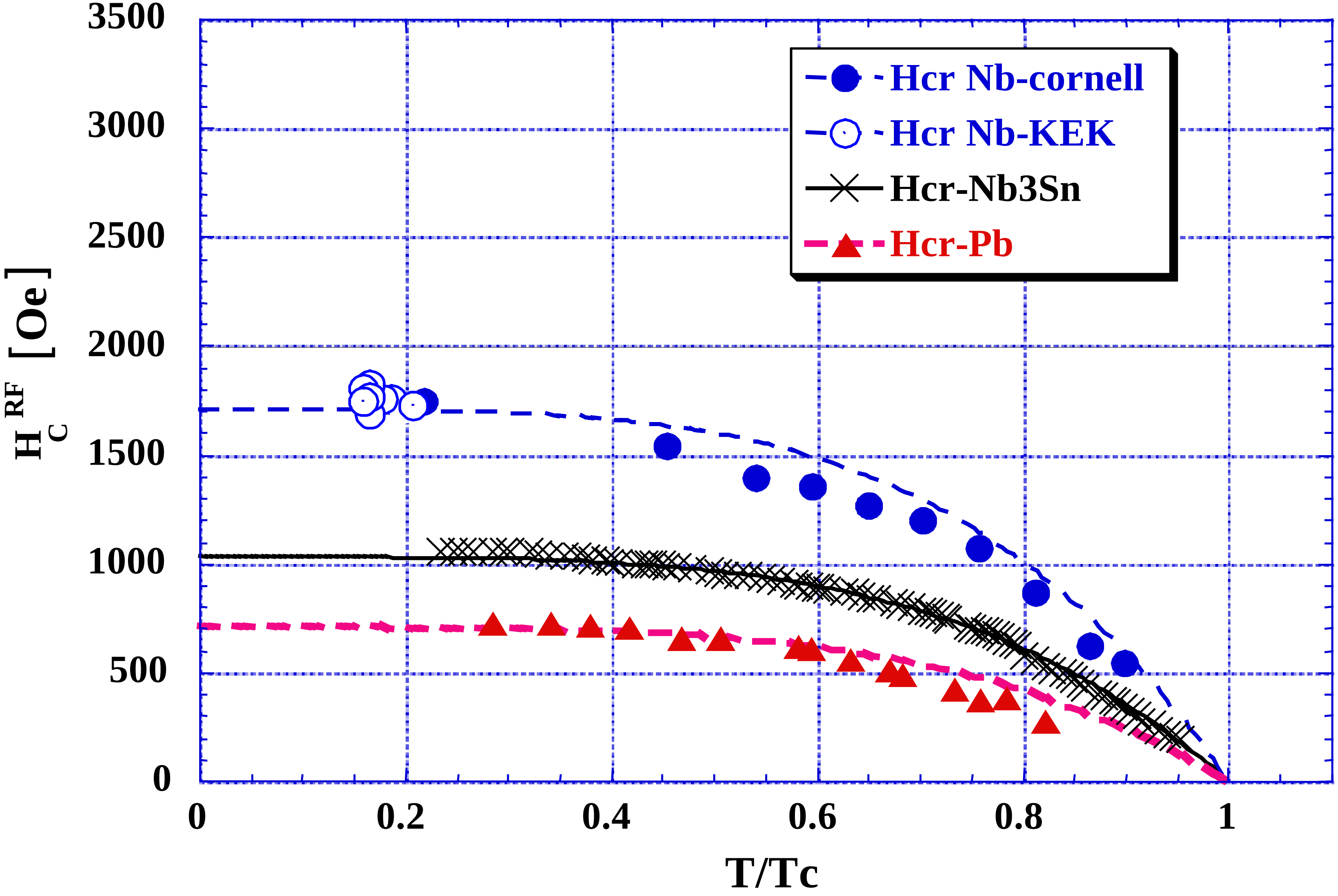
$$\frac{1}{2}\mu H^2(\pi\lambda)^2\cdot L$$



1) Hp=1750±100 Oe with Nb cavity ⟶ Eacc ~ 40MV/m
2) The SRF technology is meeting the theoretical limit.
3) Nb_3Sn cavity has a very larger k(0), therefore the critical field is so small.

Checking of the model for other materials

The model looks good for the type-II material cavity !



What material is best for SRF cavity?

Material point of view:

- Smaller heat loading for refrigerator → Higher T_C
- High gradient

  $H_{RF} > H_C^{RF}$, then normal conducting

  $$H_c^{RF} = \sqrt{2} \cdot \frac{H_c}{\kappa}, \ \kappa : \text{G - L parameter}$$

  The material with higher Hc and smaller κ-value

This is very much different from superconducting magnet

If Hc is high enough, Type-I material is better because of the smaller κ-value.

- Good formability

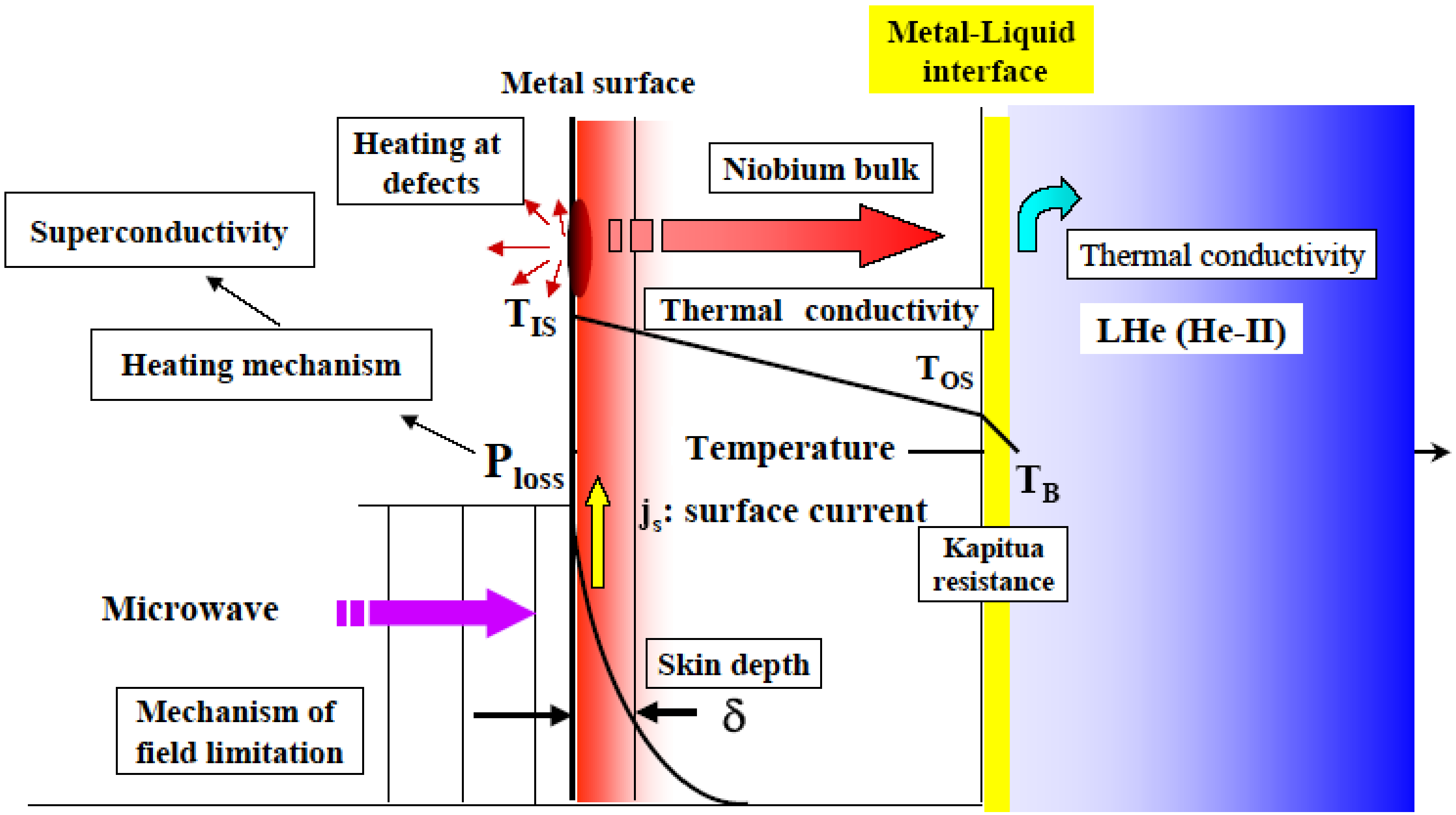
Materials	Tc [K]	Hc, Hc1 [Gauss]	Type	Fabrication
Pb	7.2	803	I	Electroplating
Nb	9.25	1900, 1700	II	Deep drawing, film
Nb_3Sn	18.2	5350, 300	II	Film
MgB_2	39	4290, 300	II	Film

Niobium has higher Tc, Hc and enough formability.

Now, niobium is widely used for RF sc cavity production.

1.2 SRF Specifics and Constrains

What happens when microwave is input a cavity?



Surface resistance **is very very small.**

➡ Cavity performance strongly depends on the surface.

Thermal conductivity @ superconducting state is very small.

➡ *High thermal conductivity* is very important.

Surface resistance of normal conducting Case

Maxwell Equations for conductor (ε, μ, ρ = 0)

$$\nabla \cdot \vec{B} = 0,\ \nabla \times \vec{E} + \mu \frac{\partial \vec{H}}{\partial t} = 0$$

$$\nabla \cdot \vec{D} = 0,\ \nabla \times \vec{H} - \varepsilon \frac{\partial \vec{E}}{\partial t} - \sigma \vec{E} = 0$$

$$\vec{J} = \sigma \vec{E} \quad \text{(Ohm's Law)}$$

$$\vec{E}(\vec{x},t) = \vec{E}_\ell(\vec{x},t) + \vec{E}_t(\vec{x},t),$$

$$\vec{H}(\vec{x},t) = \vec{H}_\ell(\vec{x},t) + \vec{H}_t(\vec{x},t)$$

From Maxwell Equation,

$$\frac{\partial \vec{H}_\ell}{\partial t} = 0, \quad \vec{E}_\ell(x,t) = \vec{E}_\ell(0) \cdot e^{-\frac{\sigma t}{\varepsilon}}$$

For the transvers,

$$\text{Plane wave : } \vec{E}_t(\vec{x},t) = \vec{E}_t(0) \cdot \exp(i\vec{k} \cdot \vec{x} - \omega t)$$

$$\vec{H}_t(x,t) = \frac{1}{\mu\omega}[\vec{k} \times \vec{E}_t(\vec{x},t)],$$

$$[\ k^2 - (\varepsilon\mu\omega^2 + i\mu\omega\sigma)\ ]\begin{Bmatrix} \vec{E}_t(\vec{x},t) \\ \vec{H}_t(\vec{x},t) \end{Bmatrix} = 0$$

Normal Conducting Case, continued

$$k^2 - (\varepsilon\mu\omega^2 + i\mu\omega\sigma) = 0,$$

$$k = \alpha + i\beta,$$

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \sqrt{\varepsilon\mu} \cdot \omega \left[\frac{\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} \pm 1}{2} \right]^{\frac{1}{2}}$$

For good electric conductor

$$\frac{\sigma}{\omega\varepsilon} >> 1$$

$$k \approx (1+i)\sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\delta = \frac{1}{\beta} = \sqrt{\frac{2}{\mu\sigma\omega}}$$: Skin depth

Surface Impedance for normal conducting case

$$Z \equiv R_s + iX_s \equiv \left.\frac{E_t}{H_t}\right|_{Surface} = \frac{\mu\omega}{k}$$

Exercise II.
Get the formula of Rs
for good electric conductor

$$\mathbf{R_s} = \sqrt{\frac{\mu\omega}{2\sigma}} = \frac{1}{\sigma}\sqrt{\frac{\mu\sigma\omega}{2}} = \frac{1}{\sigma\delta}$$

$$P_{loss} = \frac{1}{2} R_S \cdot \int_S H_S^2 dS$$

Surface resistance in superconductor (Two Fluid model)

General equation: $m\frac{\partial \mathbf{v}}{\partial t} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - m\nu\,\mathbf{v}$

Two-fluid model by Gorter and Casimir in 1933

$\mathbf{J} = \mathbf{J}_s + \mathbf{J}_\mathbf{n}$, $\mathbf{J}_s = n_s q_s \mathbf{v}$, $\mathbf{J}_\mathbf{n} = n_n q_n \mathbf{v}$

Maxwell equation: neglecting the Lorentz term, $\mathbf{v}\times\mathbf{B} \ll 1$

$$m_s \frac{\partial \mathbf{v}_s}{\partial t} = q_s \mathbf{E} \ , \ \ m_s = 2m_e \ , \ q_s = -2e$$

$$m_e \frac{\partial \mathbf{v}_n}{\partial t} = q_n \mathbf{E} - m_e \nu\,\mathbf{v}_n \ , \quad q_n = -e$$

$$\mathbf{E} = \mathbf{E}_0 e^{i\omega t} \Rightarrow \mathbf{J}_s = \frac{n_s q_s^2}{i\omega m_s}\mathbf{E} \ , \ \ \mathbf{J}_n = \frac{n_n q_n^2}{i(\omega - i\nu)m_e}\mathbf{E}$$

$$\mathbf{J} = \left(\frac{n_s q_s^2}{i\omega m_s} + \frac{n_n e^2}{i(\omega - i\nu)m_e} \right)\mathbf{E}$$

$$\nu \gg \omega \ \Rightarrow \ \mathbf{J} = \left(\frac{n_n e^2}{\nu m_e} - i\,\frac{n_s q_s^2}{\omega m_s} \right)\mathbf{E} = \sigma E, \quad \sigma = \sigma_\mathrm{n} - i\sigma_s \ \Rightarrow \mathrm{R}_\mathrm{s} = \sqrt{\frac{\mu\omega}{2\sigma}}$$

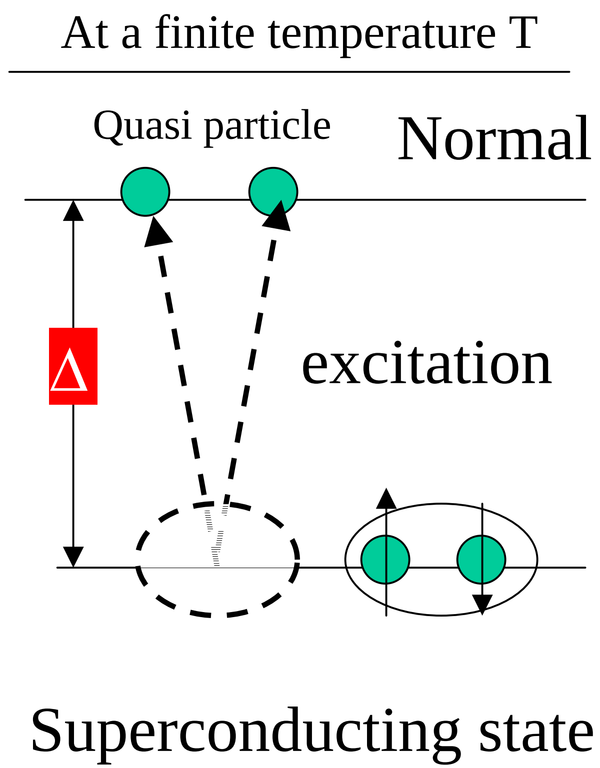
Surface resistance in superconductor

$$\sigma_n = \frac{n_n \cdot e^2 \cdot l}{m \cdot v_F} = \frac{e^2 \cdot l}{m \cdot v_F} \cdot n_s(T=0) \cdot e^{-\frac{\Delta}{k_B \cdot T}}$$

Exercise III.
Get this formula.

$$R_S = \frac{1}{2} \cdot (2\pi)^2 \cdot \mu^2 \cdot f^2 \cdot \lambda_L^3 \cdot l \cdot \frac{n_s(0)}{m v_F} \cdot e^{-\frac{\Delta}{k_B T}}$$

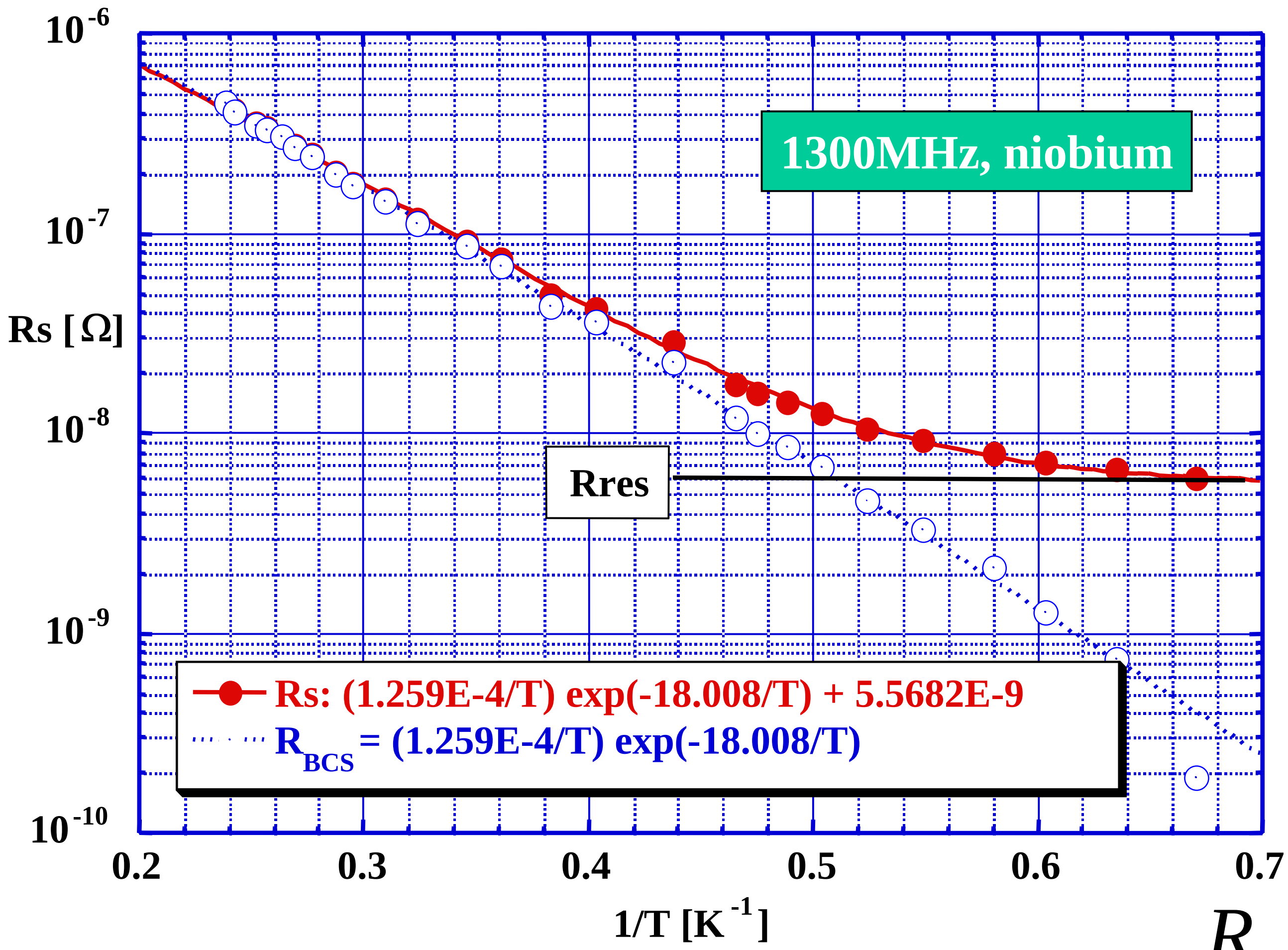
$$= A \cdot f^2 \cdot e^{-\frac{\Delta}{k_B T}}$$



BCS Theory

$$R_S^{BCS}(T,\omega) = A(\lambda, \xi, \ell, T_c) \cdot \frac{f^2}{T} \cdot \exp(-\frac{\Delta}{k_B T})$$

"Very Small" Surface Resistance in SRF Cavity



$$\frac{\Delta}{k_B} = 18.008 \Rightarrow \frac{2\Delta}{k_B T_c} = \frac{2 \cdot 18.008}{9.25} = 3.89$$

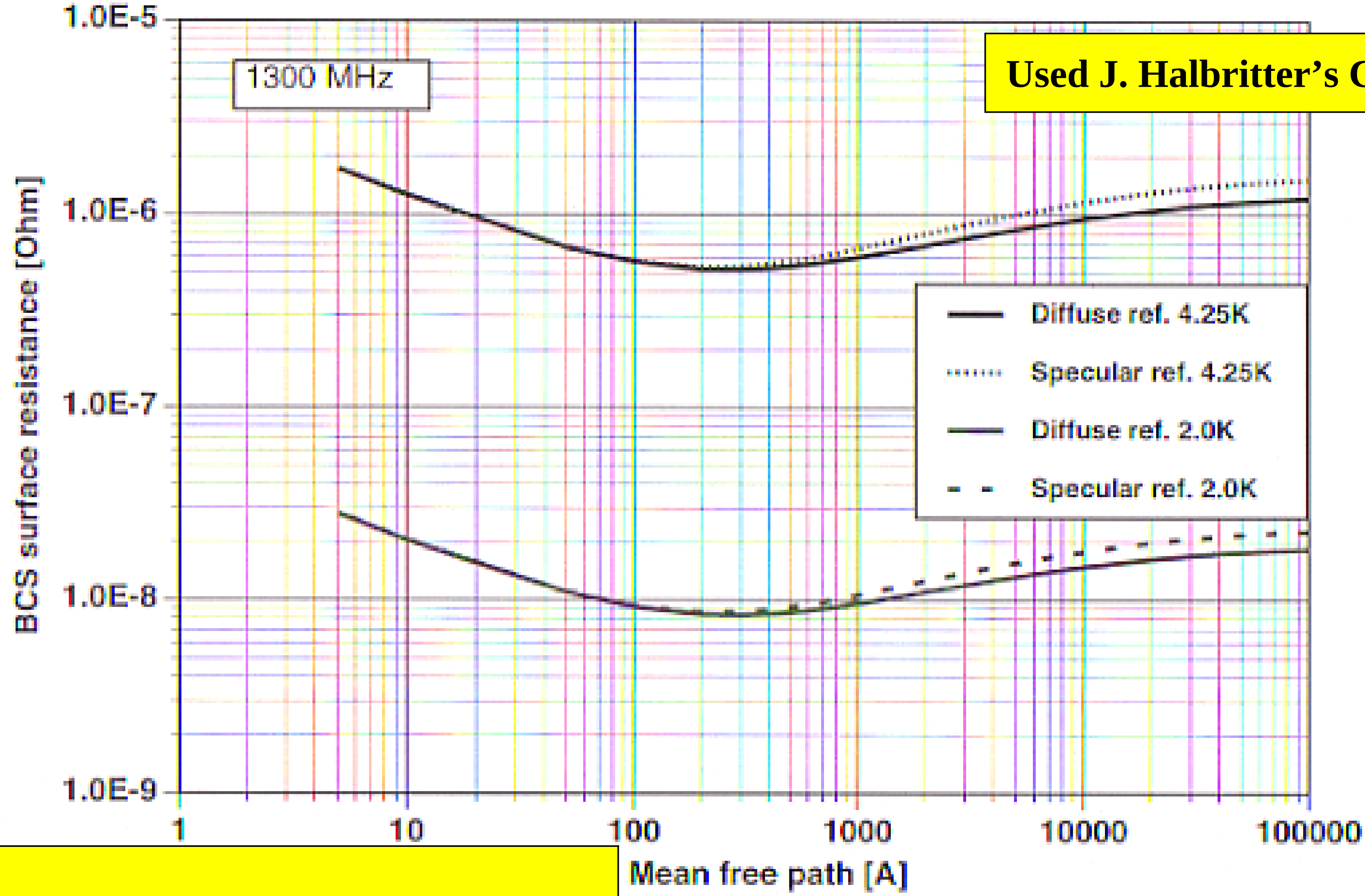
$$\frac{2\Delta}{k_B T_c} = 3.52 \textbf{ (BCS theory)}$$

Residual surface resistance depends on residual magnetic field, surface contamination, and so on.

$$R_S(T) = R_S^{BCS}(T) + R_{res}$$

R_{BCS} ~ 8nΩ,
which is smaller a factor of 10^{-6} than normal conducting!
The performance strongly depends on the surface!

BCS Surface Resistance Calculation for 1.3 GHz niobium cavity at 4.25 and 2K

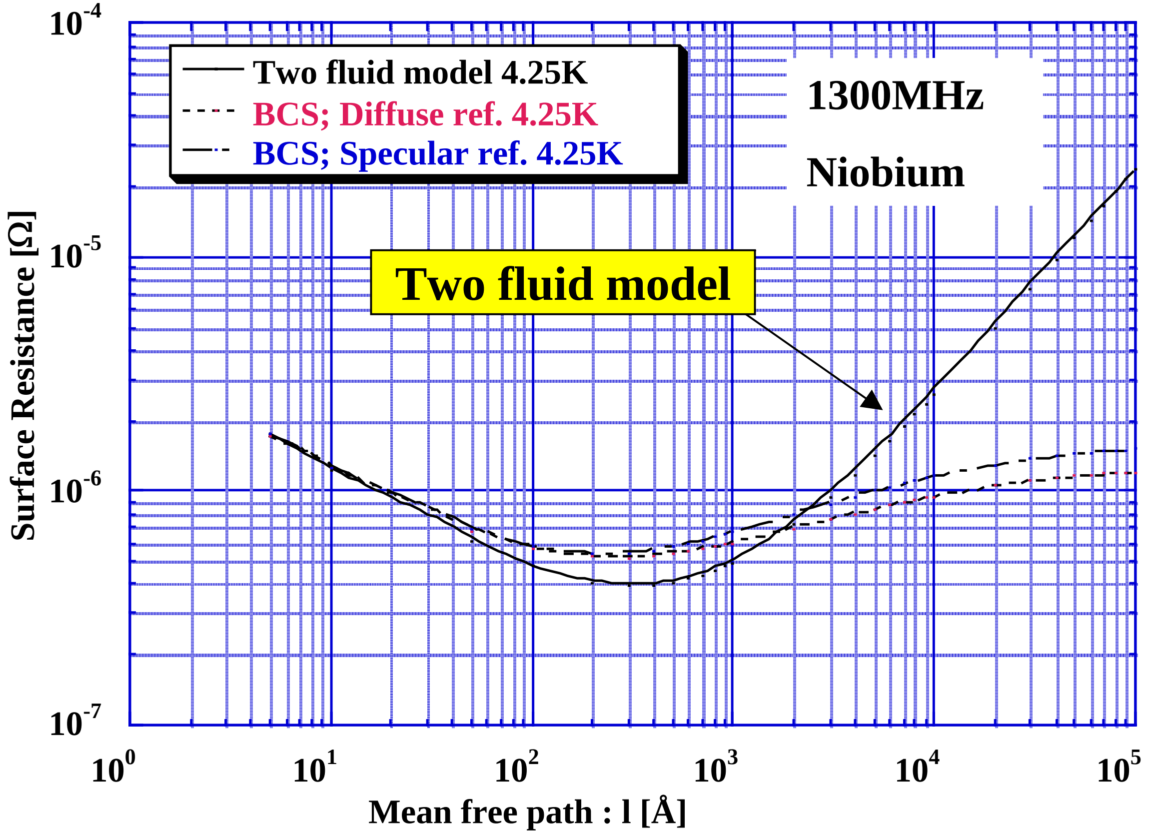


Used J. Halbritter's Code

R_{BCS} ~ 8nΩ @ 2K, 1300MHz

ℓ(mean free path) $\propto$ *RRR*

R_S minimum around mean free path $\ell \approx 300$ Å



Strange behavior on mean free path(ℓ)

R_s minimum

@ ℓ~300 Å

London penetration depth λ : $\lambda(\ell) = \lambda_{\ell=\infty} \cdot \sqrt{1 + \frac{\xi_o}{\ell}}$,

$R_S(TF\ model) \propto (1 + \frac{\xi_o}{\ell})^{\frac{3}{2}} \cdot \ell$,

$\ell << 1,\ R_S \to \frac{\xi_o^{\frac{3}{2}}}{\sqrt{\ell}}$,

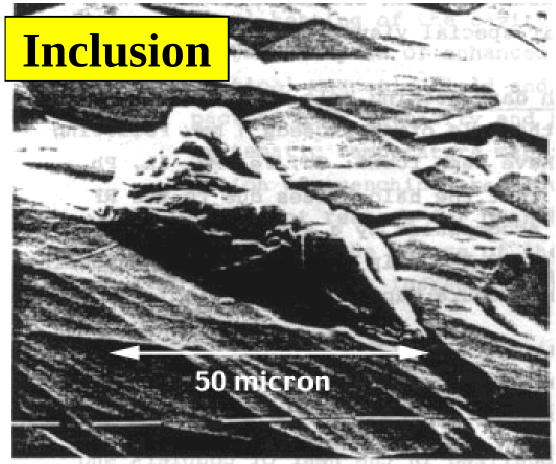
$\ell >> 1,\ R_s \to \ell$

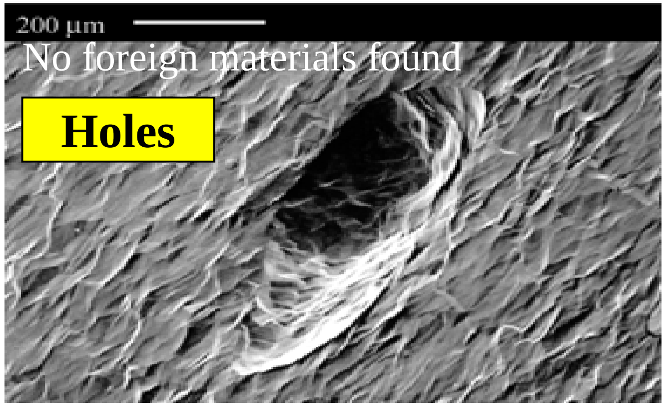
1.3 Thermal Conductivity

and

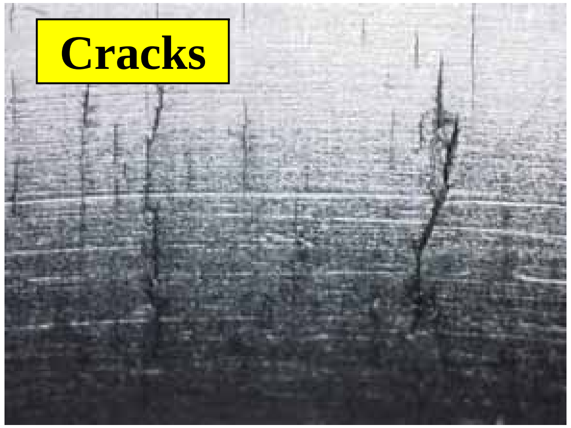
Residual Resistance Ratio (RRR)

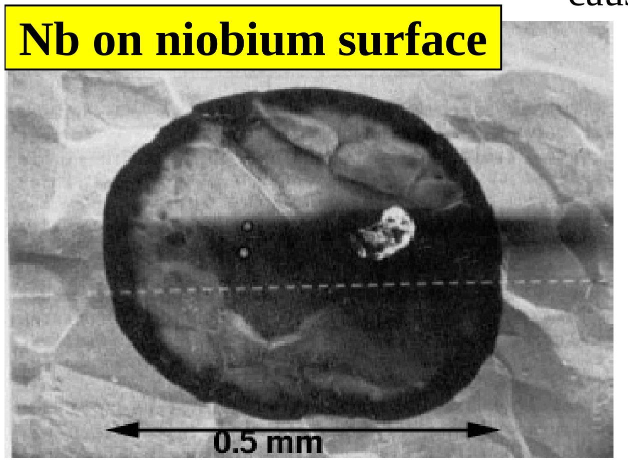
Surface defects on the SRF cavities

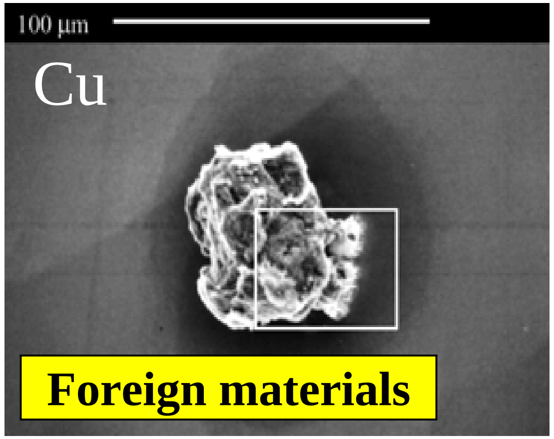


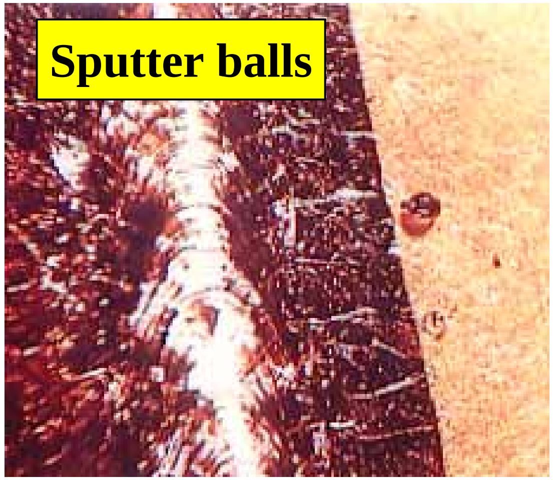


Surface defects, holes can also cause TB

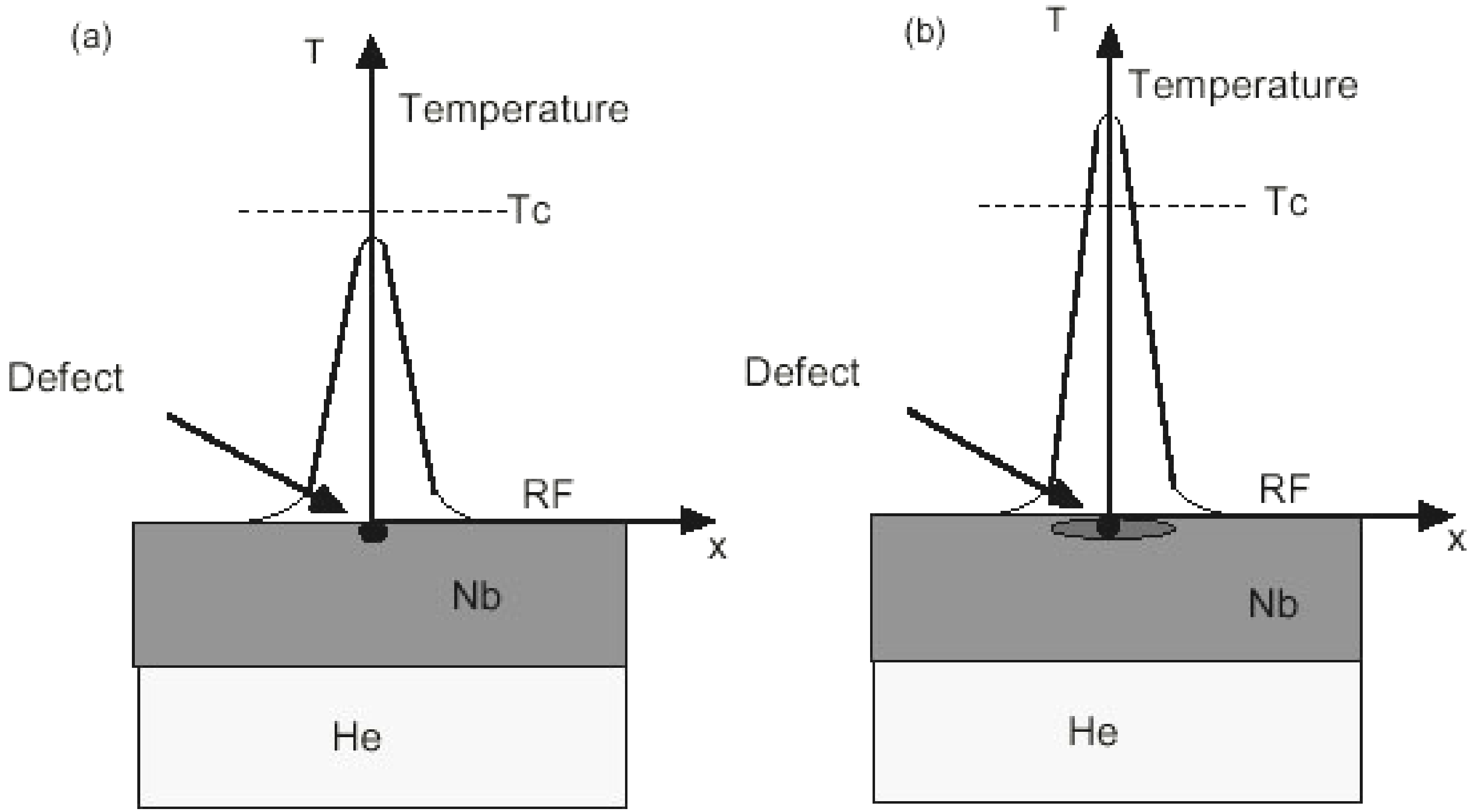








Breakdown at the surface defects :"Thermal Instability"



Thermal conductivity

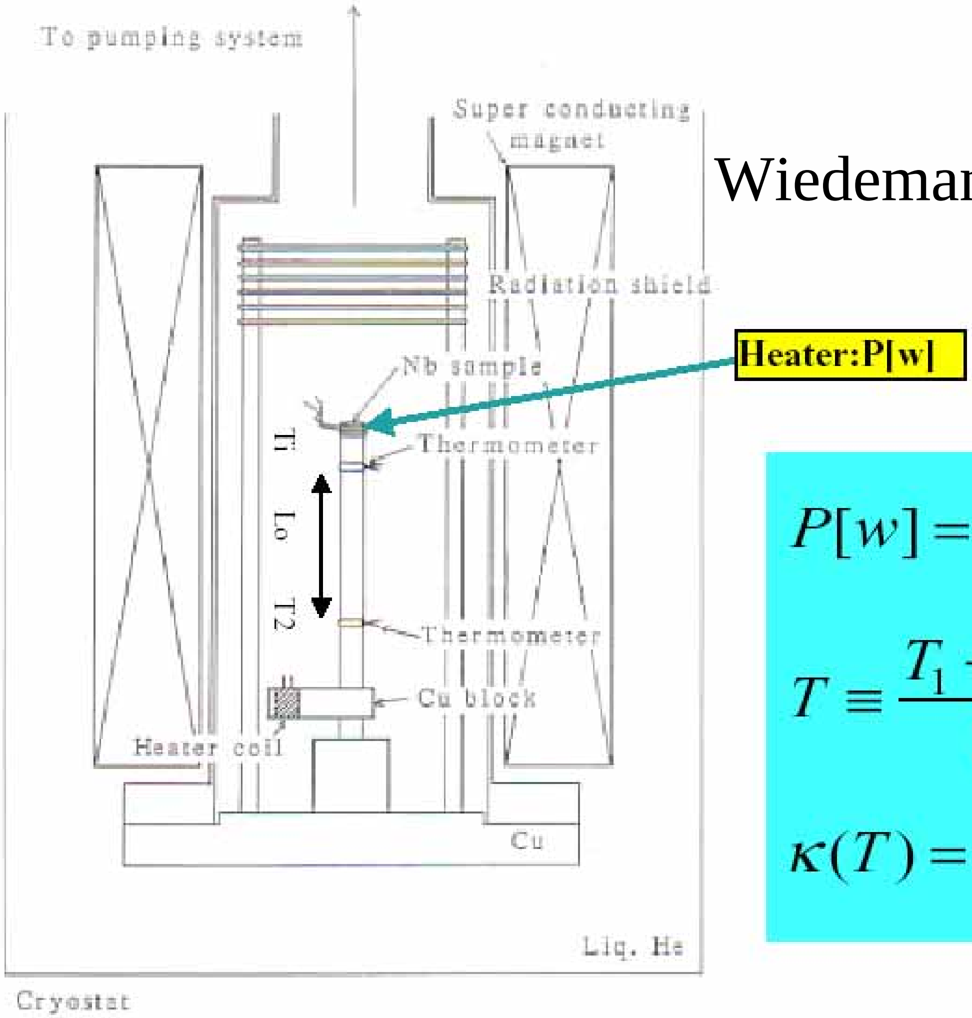
Normal conductor : $\kappa_{en} = \frac{1}{W_{en}} = \left[\frac{\rho}{L_O T} + aT^2 \right]^{-1}$

$\rho = \frac{\rho_{300K}}{RRR}$ e-impurities scatt.

e-lattices scatt.

Wiedemann-Franz low:

$\kappa_e = \frac{\pi^2 n k_B^2 \tau}{3m} \cdot T, \quad \frac{\kappa_e}{\sigma} = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right)^2 \cdot T = L_O T$

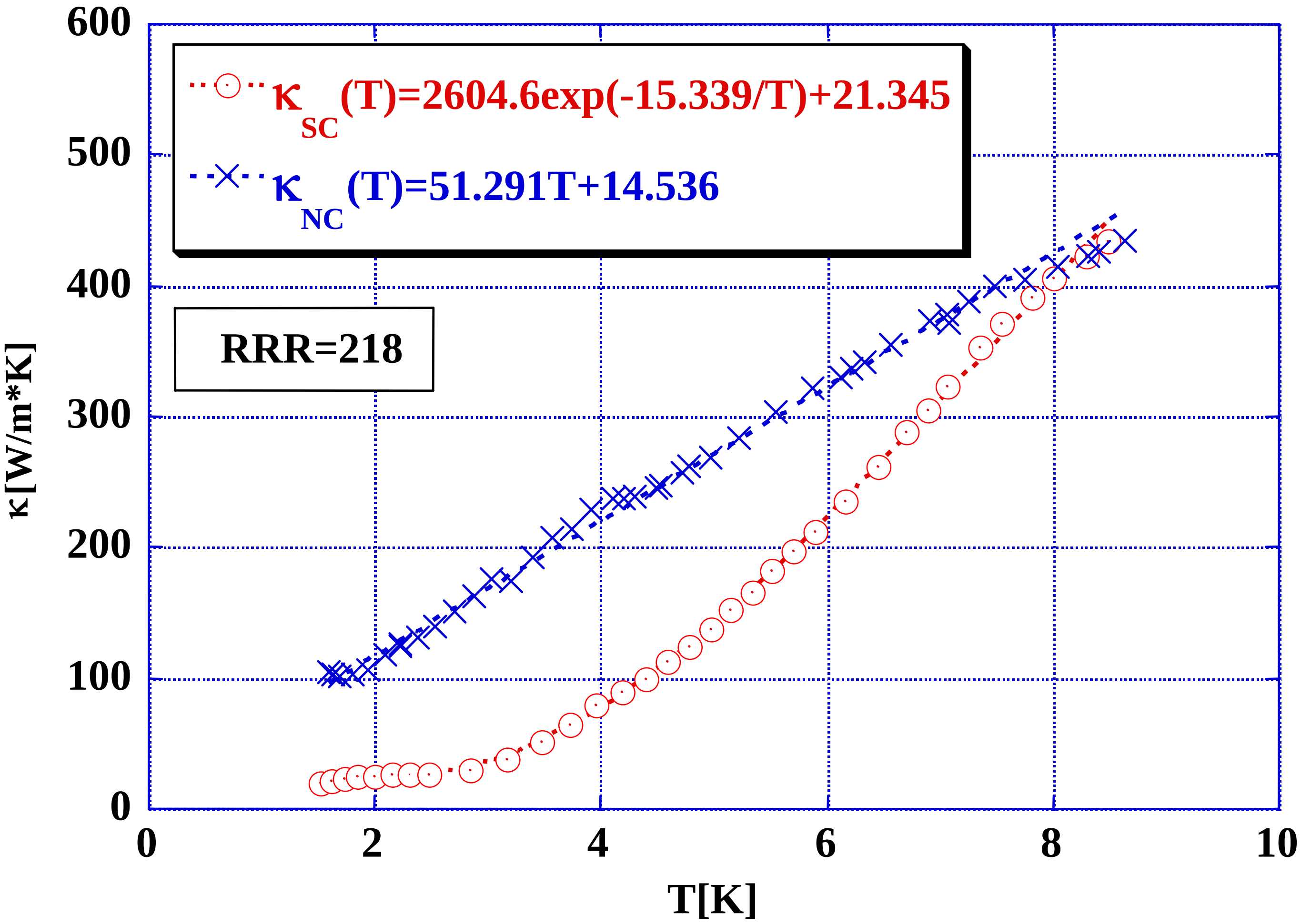


$$P[w] = S(m^2) \cdot \kappa(T) \cdot \frac{T_1(K) - T_2(K)}{L_O(m)}$$

$$T \equiv \frac{T_1 + T_2}{2}, \ S\text{: area of cross-section}$$

$$\kappa(T) = \frac{P}{S} \cdot \frac{L_O}{T_1 - T_2} \qquad [\frac{w}{m \cdot K}]$$

Thermal conductivity comparison with NC and SC



$$\frac{\Delta}{k_B} = 15.339$$

$$\Downarrow$$

$$\frac{2\Delta}{k_B T_c} = \frac{2 \times 15.339}{k_B\, 9.25}$$

$$2\Delta = 3.317 k_B T_c$$

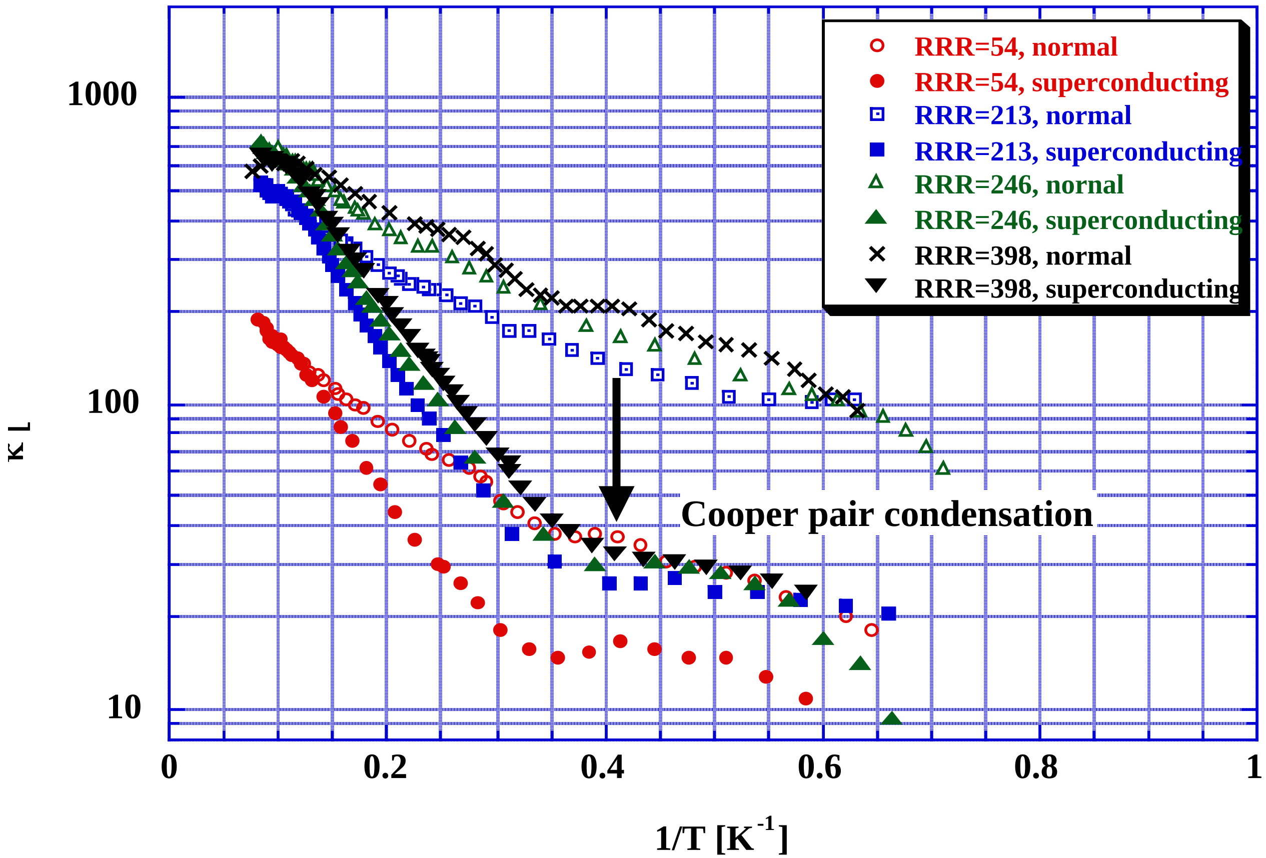
BCS theory

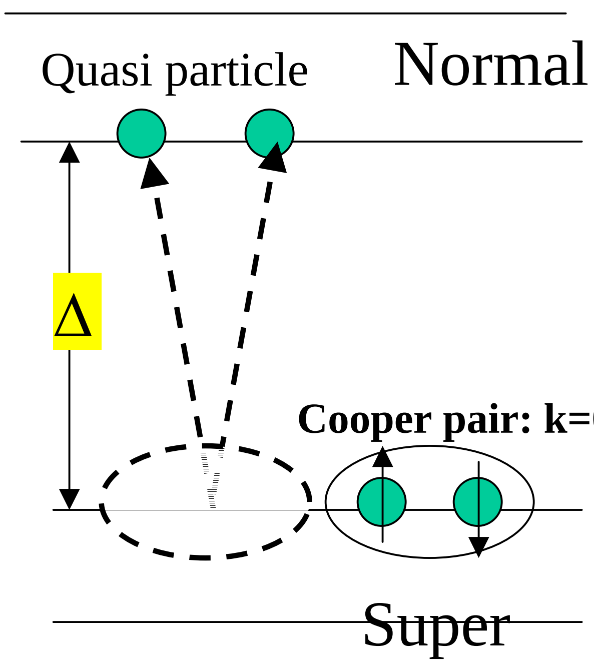
$$2\Delta = 3.52 k_B T_c$$

Thermal conductivity of Nb material at low temperature

Boltzmann statistics : Existing probability at energy Δ, and Temp. T

$$\exp\left(-\frac{\Delta}{k_B \cdot T}\right)$$





Calculation of thermal conductivity based on Quantum mechanics

$$\kappa_s(T) = R(y)\cdot\left[\frac{\rho_{295K}}{L\cdot RRR\cdot T} + a\cdot T^2\right]^{-1} + \left[\frac{1}{D\cdot\exp(y)\cdot T^2} + \frac{1}{BlT^3}\right]^{-1}$$

e - impurities scatt.   e - phonons scatt.   lattice - phonnons scatt.   lattice - grain boundaries scatt.

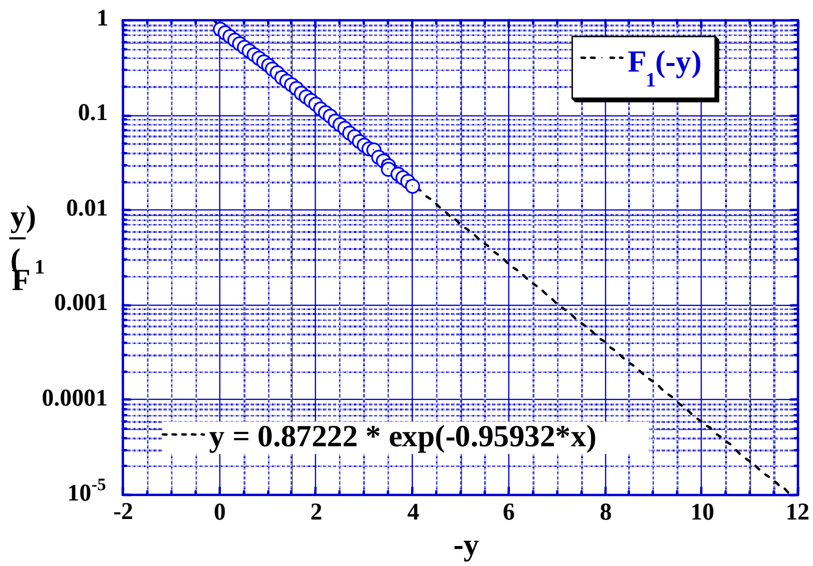
$L = 2.05E-8,\ RRR = 200,\ \rho_{295K} = 14.5E-8\ \Omega m,\ a = 7.52E-7$

$-y = \alpha\cdot\frac{T_c}{T},\ \alpha = 1.53,\ T_c = 9.25K,\ T \le 0.6\cdot T_c$

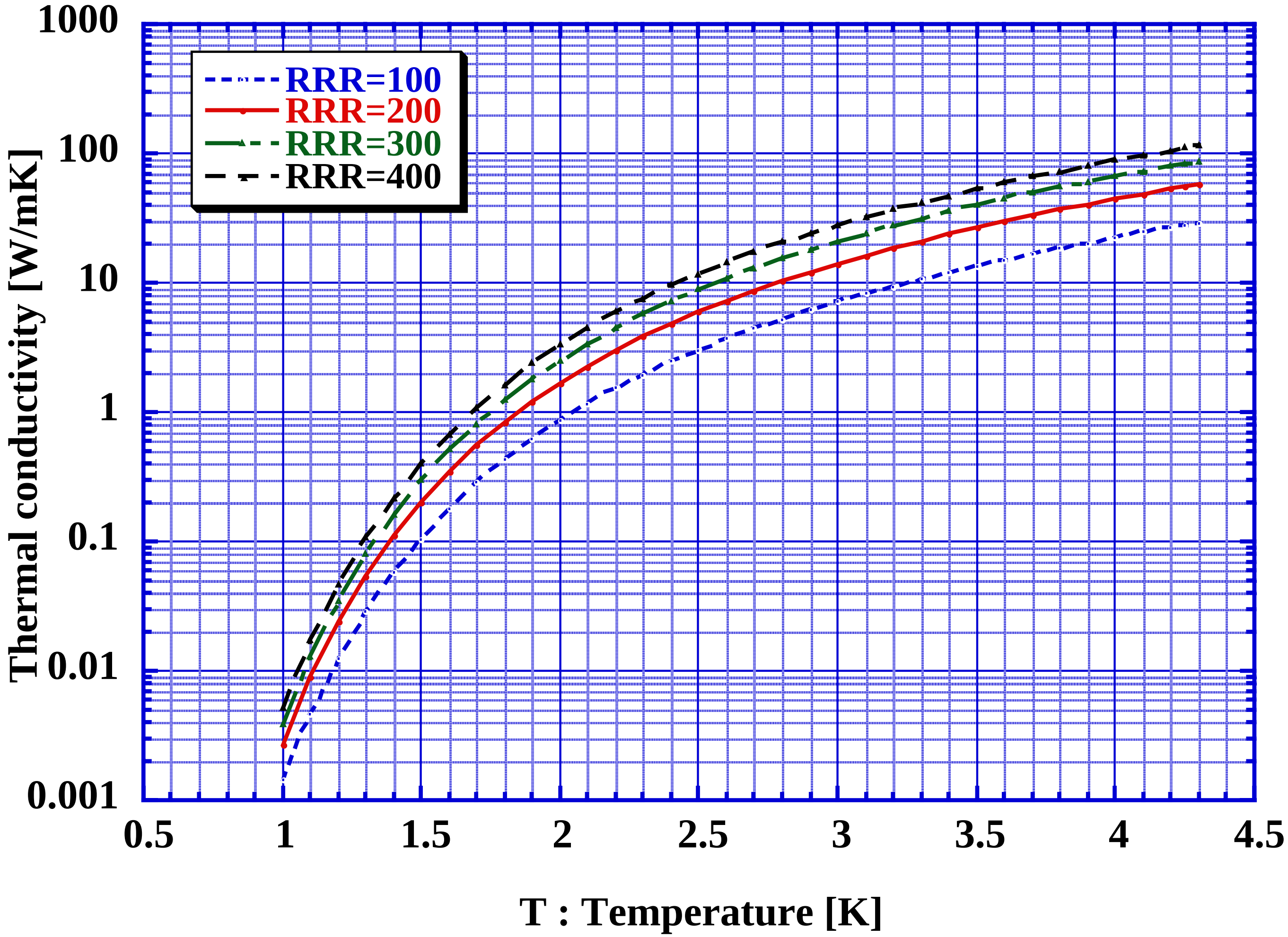
$D = 4.27E-3,\ B = 4.34E3,\ l = 50\mu m$

$$R(y) = \frac{\kappa_{es}}{\kappa_{en}} = \frac{2F_1(-y) + 2y\ln(1+e^{-y}) + \dfrac{y^2}{(1+e^y)}}{2F_1(0)},$$

$$F_n(-y) = \int_0^\infty \frac{z^n}{1+e^{z+y}}dz$$

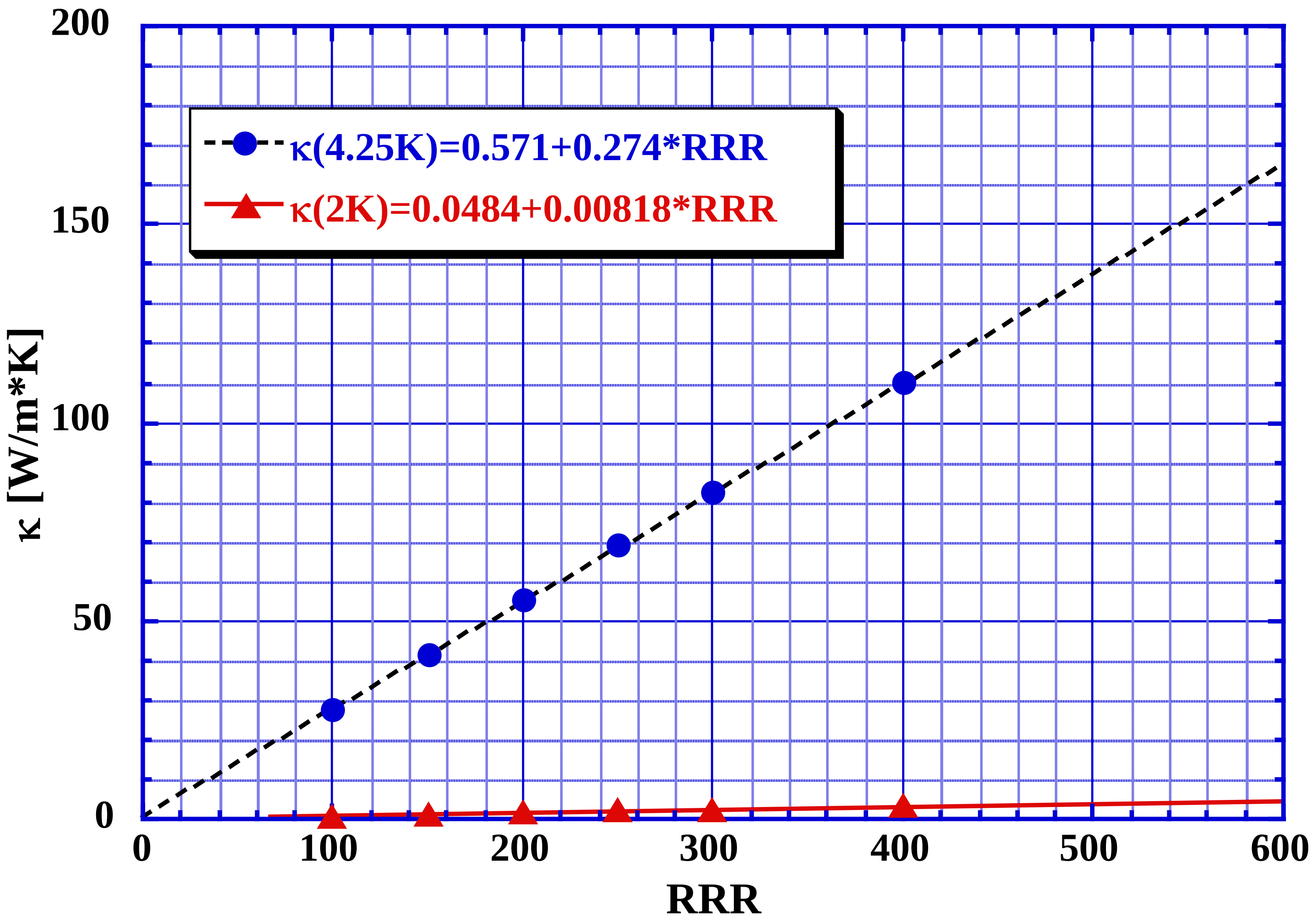


Calculated $\kappa_{sc}(T)$



Thermal conductivity of niobium in superconductivity @ 2K is 1/15 that of stainless at R.T. (15W/(m•K)) and 1/6800 of pure cooper at 4.2K

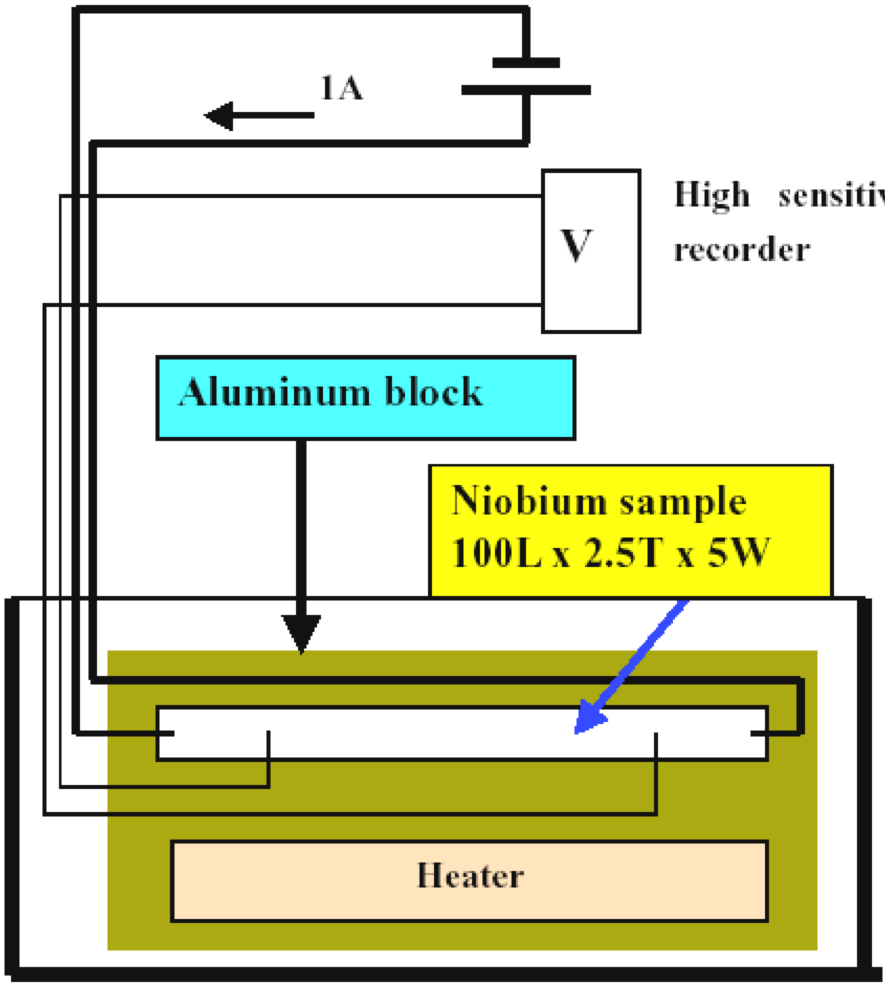
Linear relationship between κ_{sc} (2K, 4.25K) and RRR

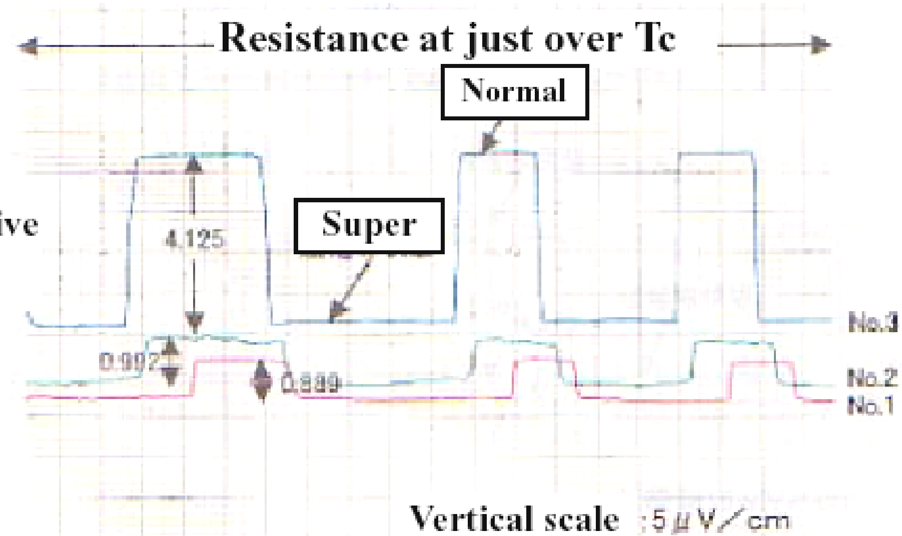


RRR measurement

$$RRR \equiv \frac{R_{300K}}{R_{9.5K}}$$

Very simple measurement!!
RRR is linearly proportional to thermal conductivity.





High RRR Material to Suppress the Thermal Instability

$$\text{Quench Field : } Hq=\sqrt{\frac{4\kappa(Tc-Tb)}{r_D\cdot R_s(Tb)}}\propto RRR^{\frac{3}{4}}\cdot\sqrt{\frac{(Tc-Tb)}{r_D\cdot Rs(300K)}}$$

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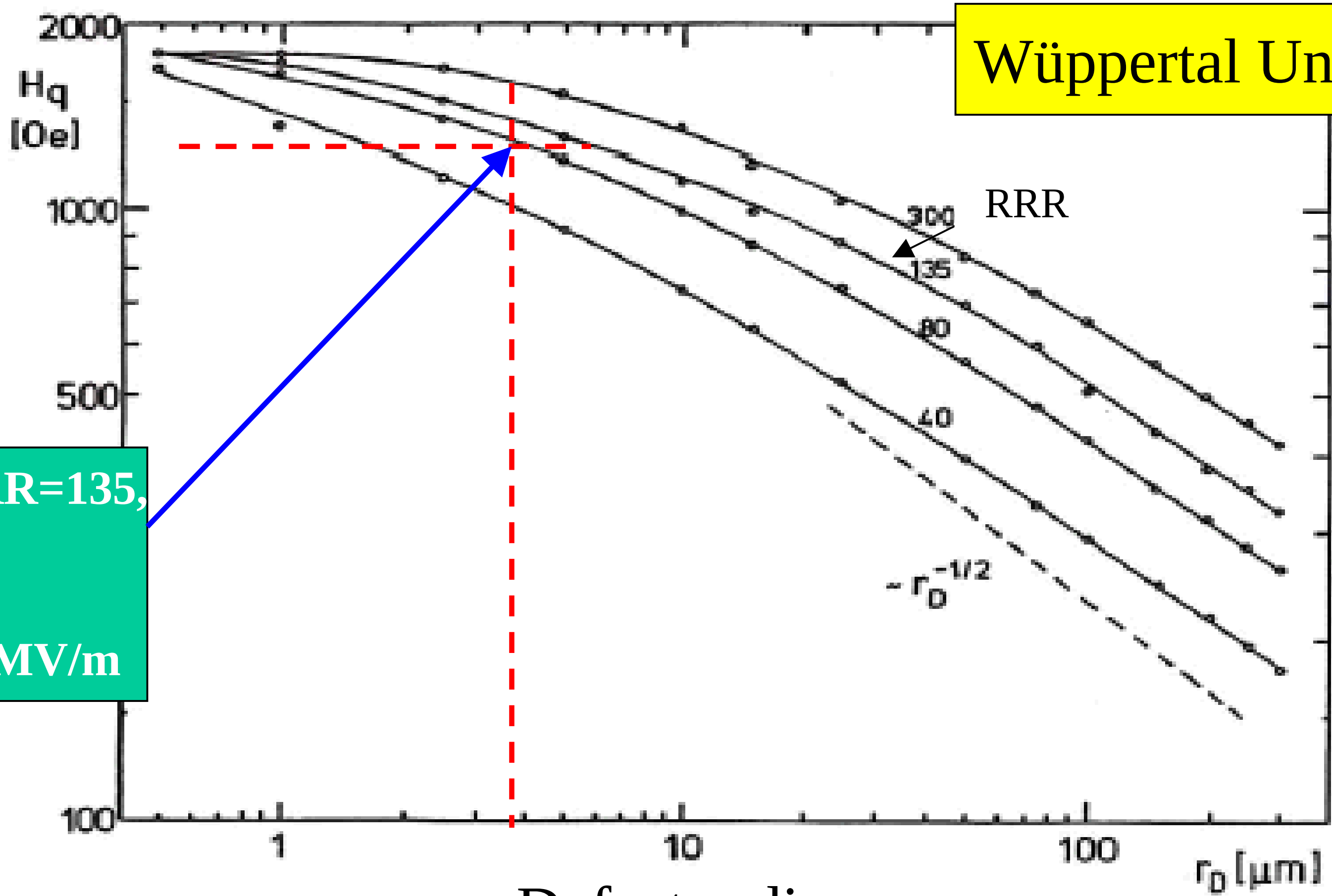
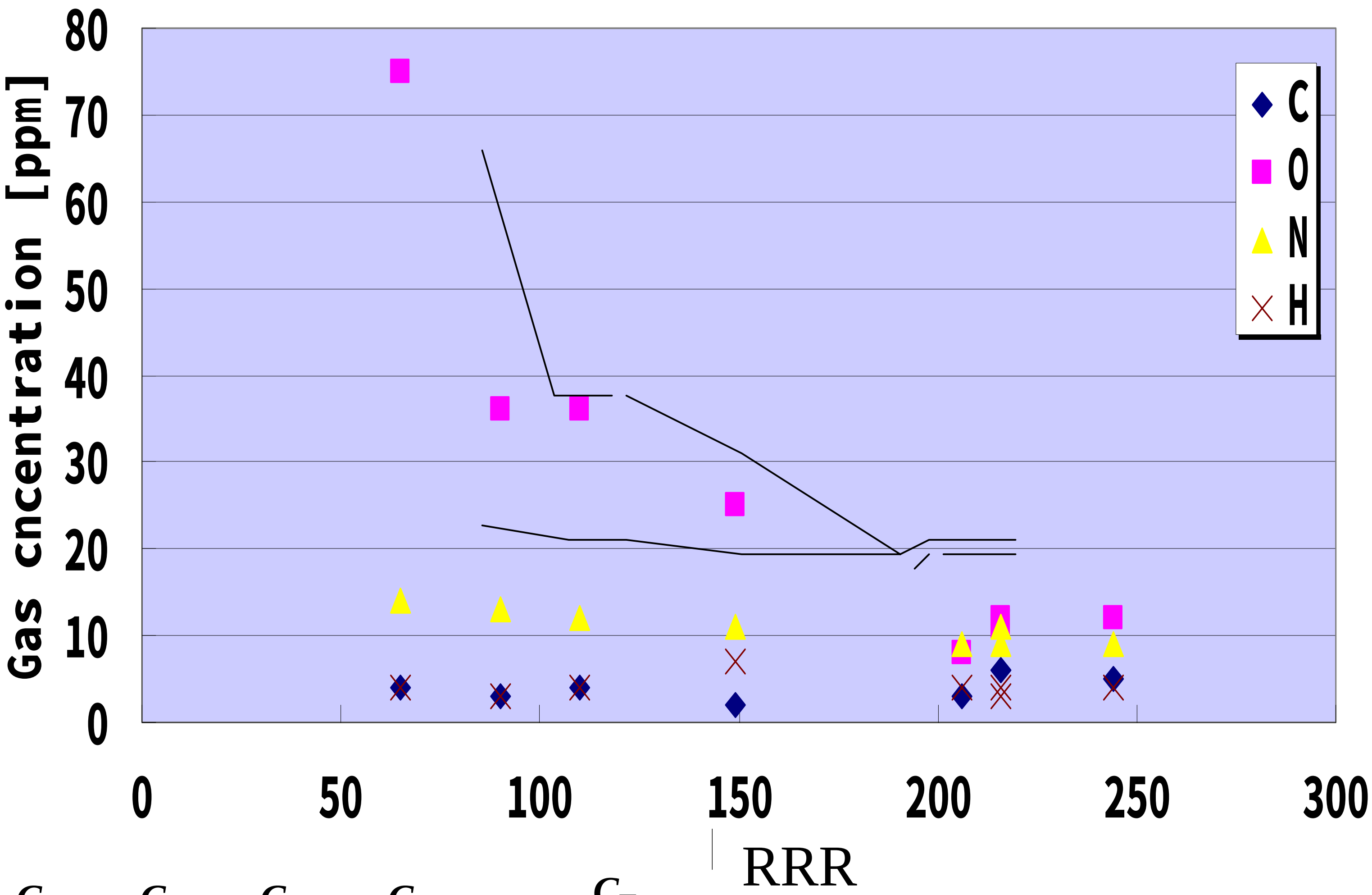


Thae case :RRR=135,
and r_D=3μm
Hq~1500Oe
Eacc,max ~34MV/m

Defect radius

One has to control the defects smaller than 1μm radius. Use the material with RRR >200.

Impurities in Nb Material



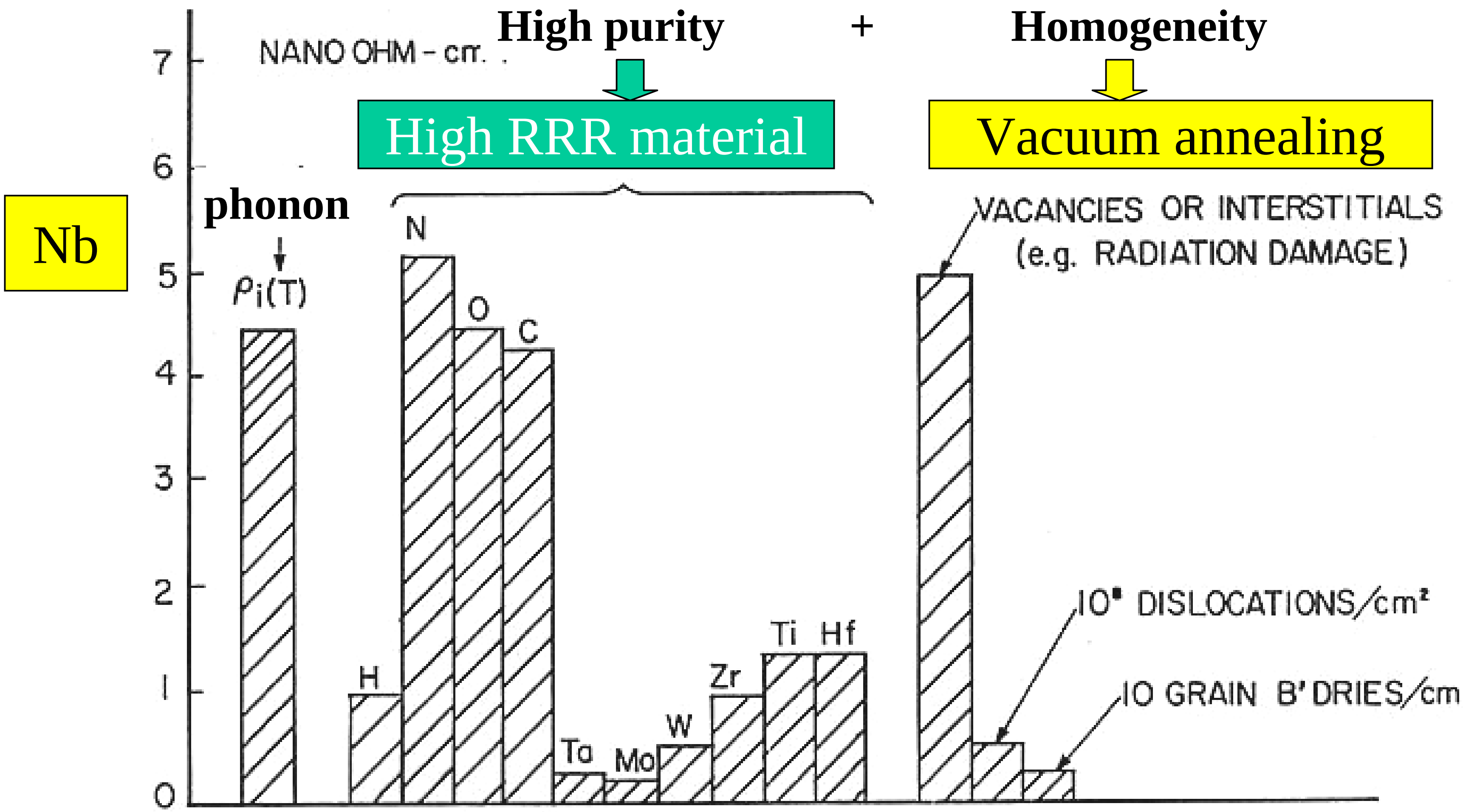
$$\frac{1}{RRR} = \frac{C_H}{1500} + \frac{C_C}{4100} + \frac{C_N}{3900} + \frac{C_O}{5000} + \cdots + \frac{C_{Ta}}{550000} + \cdots$$

Example **:** $C_H = 1ppm, C_C = 5ppm, C_N = 5ppm, C_O = 7ppm$, ***Ta = 400ppm*** **(purity:99.9582%)**

$$\frac{1}{RRR} = \frac{1}{1500} + \frac{5}{4100} + \frac{5}{3900} + \frac{7}{5000} + \frac{400}{550000}, \qquad RRR = 188.8$$

Theoretical limit : Ta = 100ppm, RRR = 5500

Scattering mechanism limits both thermal conductivity and electric conductivity



$$R = \text{e-phonon scat.} + \text{e-imprity scat.} + \text{e-inhomogenety scat.} + \cdots$$

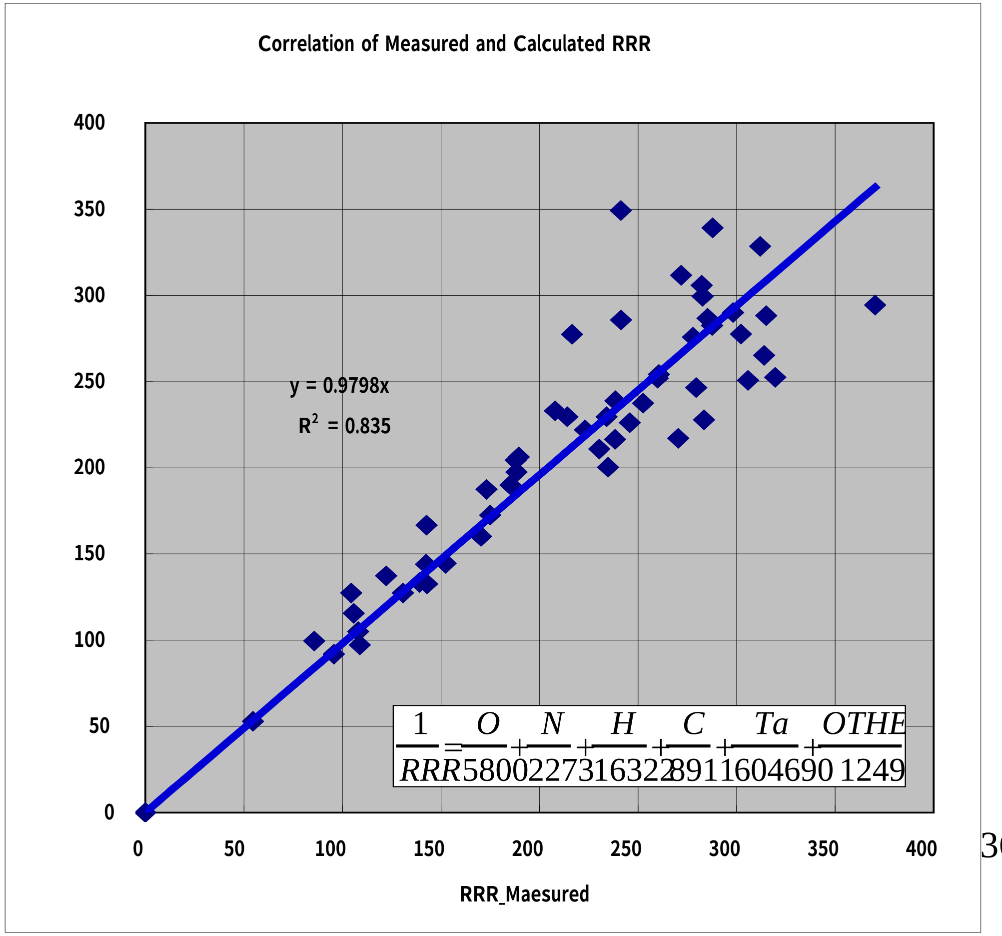
$$\frac{1}{\kappa} = \text{e-phonon scat.} + \text{e-imprity scat.} + \text{e-inhomogenety scat.} + \cdots$$

RRR vs. Impurities

Umezawa's calculation (Tokyo Denkai).

$$\frac{1}{RRR} = \frac{O}{5800} + \frac{N}{2273} + \frac{H}{16322} + \frac{C}{8911} + \frac{Ta}{604690} + \frac{1}{1249}$$

$$RRR \equiv \frac{R(300K)}{R(9.5K)}$$



History of RRR improvement in Tokyo Denkai