3. SRF Cavity Design

3.1 CW Operation with SRF RF Cavity
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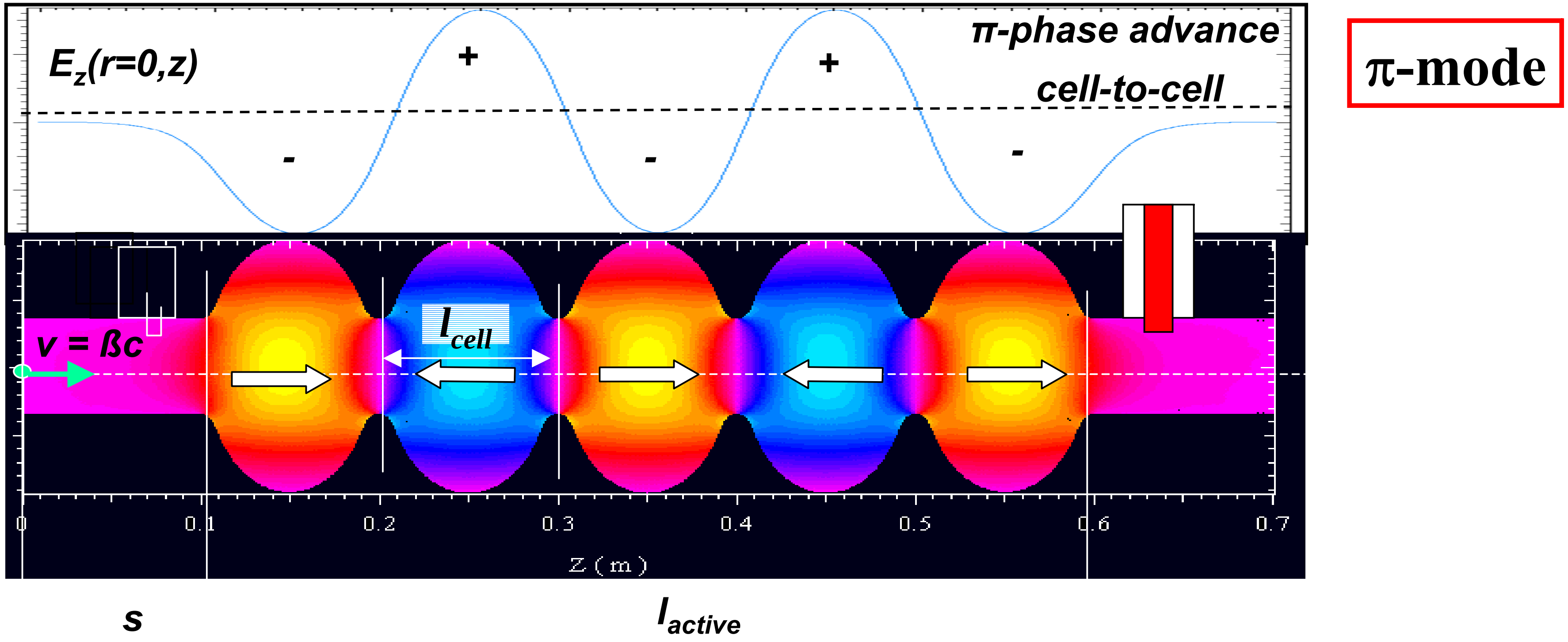
3.1 Sanding Wave (SW) Operation in SRF Cavity

SW Scheme in SRF Cavity Operation

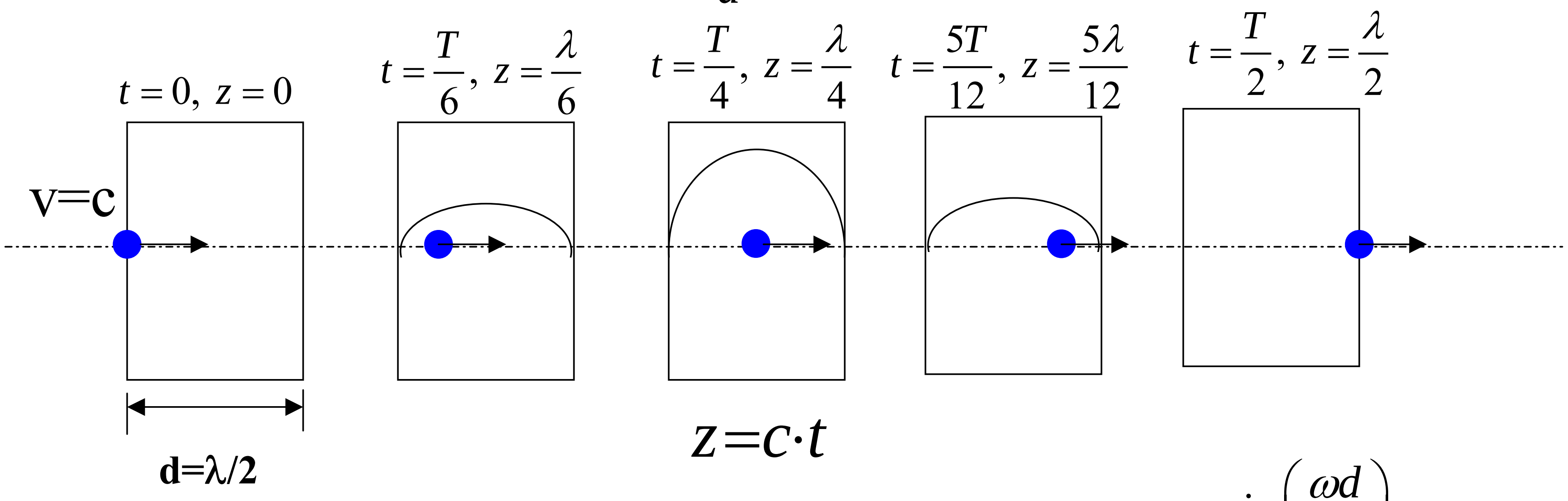


Standing wave (CW) is used in SRF cavity acceleration !

Synchronic acceleration and max of $(R/Q)_{acc} \leftrightarrow l_{active} = Nl_{cell} = Nc\beta/(2f)$ and the injection takes place at an optimum phase ϕ_{opt} which ensures that particles will arrive at the mid-plane of the first cell when E_{acc} reaches its maximum (+q passing to the right) or minimum (-q passing to the right).

Transit Time Factor Due to SW Operation

$$E_z(r=0,z,t)=E_0 J_0\left(\frac{\rho_{0,1}}{a}r\right)\cdot\exp(-i\omega t)$$



$$V=\left|\int_0^d E_z(r=0,z)e^{i\omega t}dz\right|=\left|\int_0^d E_z(r=0,z)e^{i\omega\frac{z}{c}}dz\right|=E_0\left|\int_0^d e^{i\omega\frac{z}{c}}dz\right|=E_0 d\,\frac{\sin\left(\frac{\omega d}{2c}\right)}{\frac{\omega d}{2c}}=E_0 d\cdot T$$

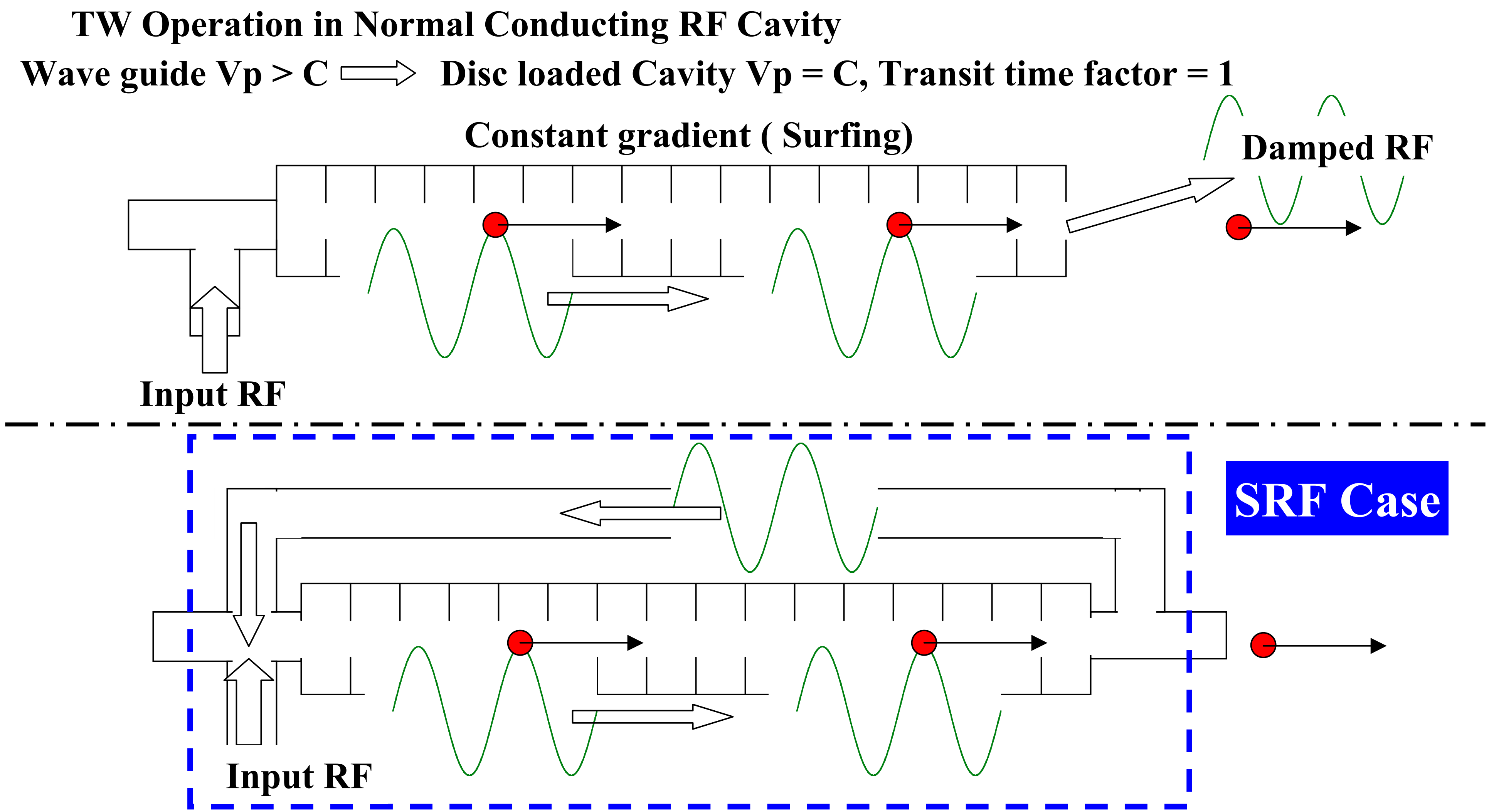
T : Transit time factor

$$T=\frac{2}{\pi}=0.637\quad\text{(for Pill Box Cavity)}$$

$$Eacc\equiv\frac{V}{d}=E_0 T$$

Acceleration efficiency is automatically reduced by ~ 40% in the SW scheme.

Why TW Operation is not used with SRF Cavities ?



The merit of SRF is that RF consumption is very small ! The damped RF has to be reused.

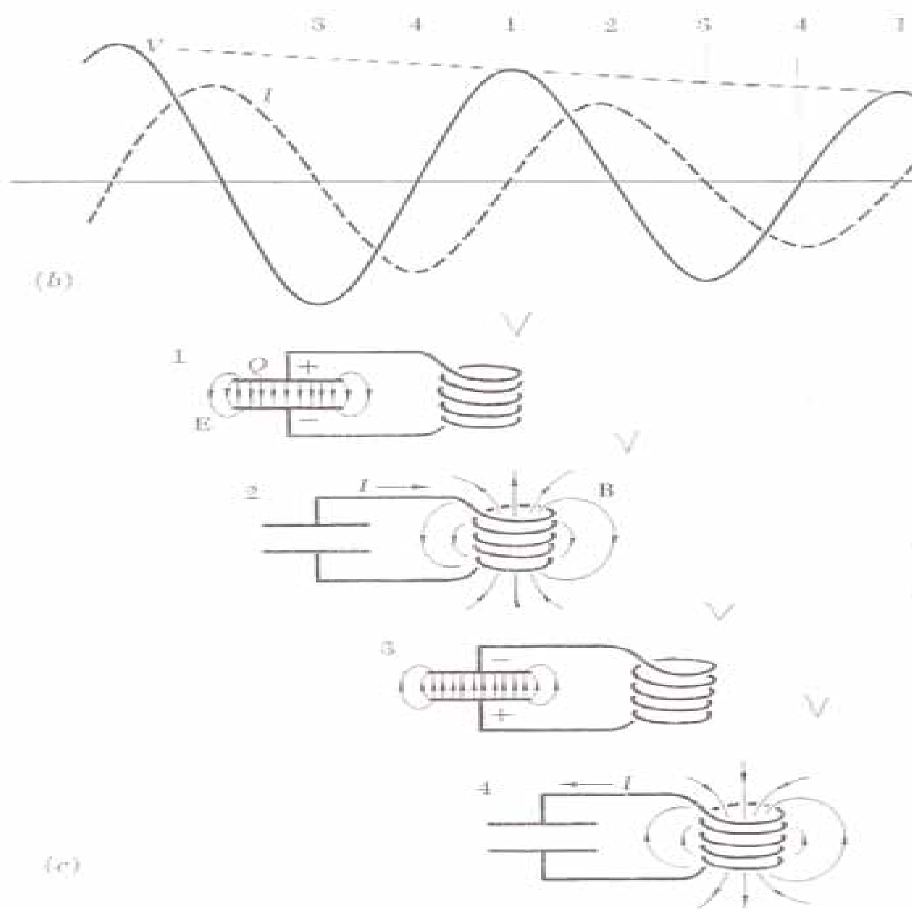
The wave-guide returned RF should be superconduting, which makes very complex cryogenic system.

3.2 LC Circuit Model

LC Circuit Model for RF Cavity

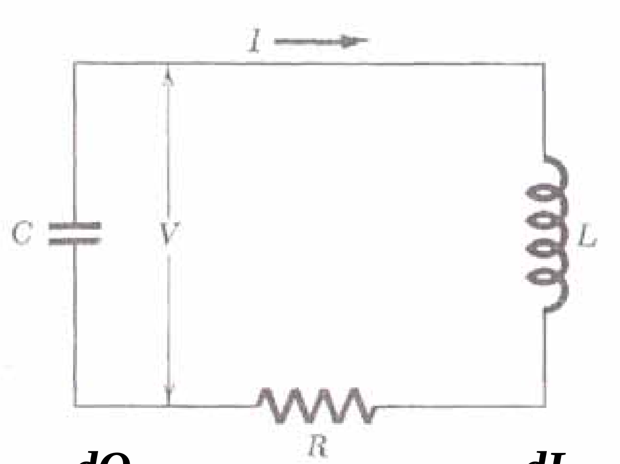
$$V(t) = V_O \exp\left(-\frac{\omega}{2Q} t\right), \qquad W(t) = W_O \exp\left(-\frac{\omega t}{Q}\right)$$

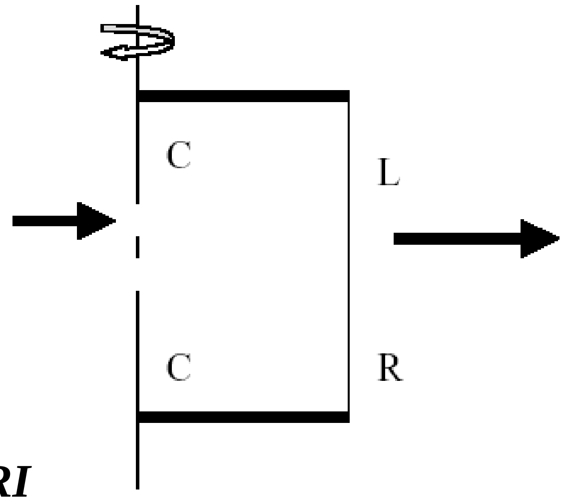
Q : Damping factor of the stored energy



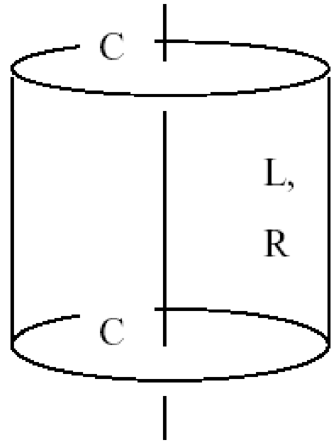
(*a*) The damped sinusoidal oscillation of voltage in the *RLC* circuit.
(*b*) A portion of (*a*) with the time scale expanded and the graph of the *current I included*.
(*c*) The periodic transfer of energy from electric field to magnetic field, and back again. Each picture represents the condition at times marked by the corresponding number in (*b*).

LC Equivalent Circuit of Cavity





Pill Box Cavity



$$I = -\frac{dQ}{dt}, \quad Q = CV, \quad V = L\frac{dI}{dt} + RI$$

$$\frac{d^2V}{dt} + \left(\frac{R}{L}\right)\frac{dV}{dt} + \left(\frac{1}{LC}\right) = 0, \quad V(t) = V_O \exp(-\alpha + i\omega)t$$

$$(-\alpha + i\omega)^2 + (-\alpha + i\omega)\left(\frac{R}{L}\right) + \left(\frac{1}{LC}\right) = 0,$$

$$\alpha = \frac{R}{2L}, \quad \omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$$

$$R << L, \quad \omega_O^2 = \frac{1}{LC} \Rightarrow \boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

Q-value of the circuit

$$Q \equiv \omega \cdot \frac{\text{stored energy}}{\text{power loss / sec}} = \omega \cdot \frac{P}{dP/dt} = \boxed{\omega \cdot \frac{L}{R}}$$

$$= \frac{\omega}{2\alpha}$$

Q : proportional to 1/R

When resistance in not zero, the resonance frequency has a band width.

Resonance Spectrum and Band Width

Cavity wall loss: P_{loss} $$P_{loss} = \frac{1}{2} R_s \int i^2 dS, i = H$$

Damping of Stored Energy

$$\frac{dU}{dt} = -P_{loss} = -\frac{\omega U}{Q} \Rightarrow U = U_0 e^{-\omega t/Q} \qquad Q \equiv \frac{\omega U}{P_{loss}} = \frac{Const}{R_s}$$

$$U_0(t) = \frac{\varepsilon_0}{2} E_0^2(t) \longrightarrow E(t) = E_0 e^{-\omega_0 t/2Q} e^{-i\omega t} = \int_{-\infty}^{\infty} E(\omega) e^{-i\omega t} d\omega$$

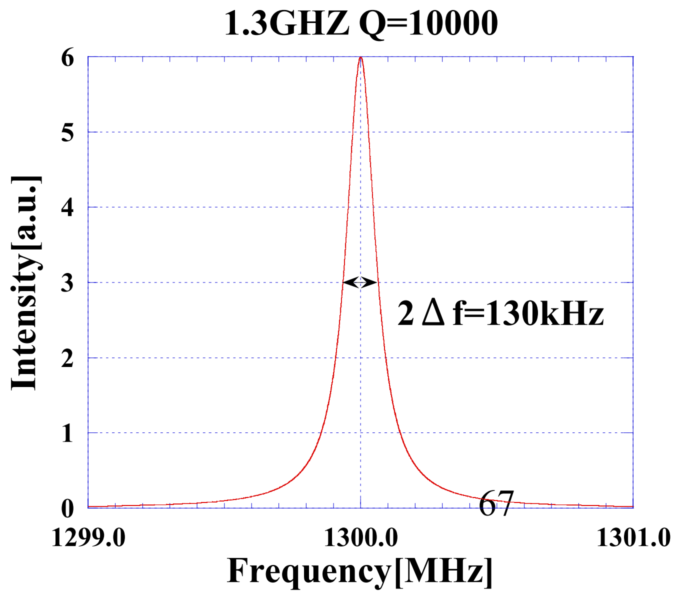
Furrier Transformation

$$E(\omega) = \frac{1}{2\pi} \int_0^\infty E_0 e^{-\frac{\omega_0 t}{2Q}} e^{-i\omega_0 t} e^{i\omega t} dt = \frac{1}{2\pi} \int_0^\infty E_0 \exp(-\frac{\omega_0 t}{2Q} - i\omega_0 t + i\omega t) dt = \frac{1}{2\pi} \frac{E_0}{-\frac{\omega_0}{2Q} + i(\omega - \omega_0)}$$

Resonance Spectrum of RF power in Frequency domain

$$P|E(\omega)|^2 = \left| \frac{A}{-\frac{\omega_0}{2Q} + i(\omega - \omega_0)} \right|^2 = \frac{A}{-\left(\frac{\omega_0}{2Q}\right)^2 + (\omega - \omega_0)^2}$$

$$Q = \frac{f_0}{2\Delta f} \; (\omega_0 = 2\pi f_0) \Rightarrow \boxed{\Delta f = \frac{f_0}{2Q} \propto \frac{f_0}{2} R_S}$$

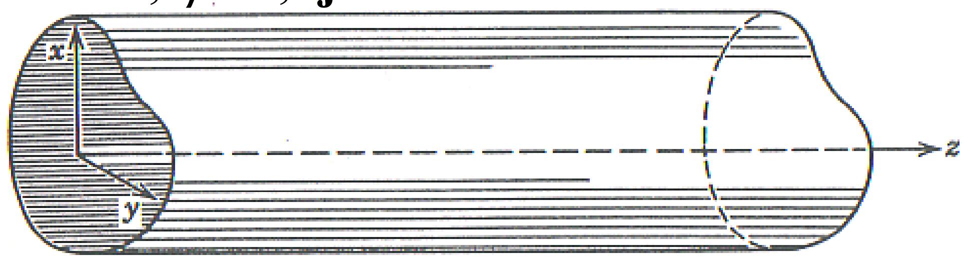


3.3 Pill Box Cavity

Electro-magnetic field in a waveguide

Maxwell equations in a waveguide

$$\nabla\times\mathbf{E}=i\frac{\omega}{c}\mathbf{B},\ \nabla\cdot\mathbf{B}=0,\ \nabla\times\mathbf{B}=-i\mu\varepsilon\frac{\omega}{c}\mathbf{E},\ \nabla\cdot\mathbf{E}=0,\ \rho=0,\ \mathbf{j}=0$$



$$\left(\nabla^2+\mu\varepsilon\frac{\omega^2}{c^2}\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{B}\end{Bmatrix}=0,$$

$$\mathbf{E}(x,y,z,t)=\mathbf{E}(x,y)\exp(\pm ikz-i\omega t),\ k\text{: wavevector},$$

$$\mathbf{B}(x,y,x,t)=\mathbf{B}(x,y)\exp(\pm ikz-i\omega t),$$

$$\left[\nabla_t^2+(\varepsilon\mu\frac{\omega^2}{c^2}-k^2\right]\begin{Bmatrix}\mathbf{E}\\ \mathbf{B}\end{Bmatrix}=0,\quad \nabla_t^2\equiv\nabla^2-\frac{\partial^2}{\partial^2 z},\quad \mathbf{E}=E_z\mathbf{e}_z+\mathbf{E}_t,\quad \mathbf{B}=B_z\mathbf{e}_z+\mathbf{B}_t$$

$$\mathbf{B}_t=\frac{1}{\left(\varepsilon\mu\frac{\omega^2}{c^2}-k^2\right)}\left[\nabla_t\left(\frac{\partial B_z}{\partial z}\right)+i\varepsilon\mu\frac{\omega}{c}\mathbf{e}_z\times\nabla_t E_z\right],$$

$$\mathbf{E}_t=\frac{1}{\left(\varepsilon\mu\frac{\omega^2}{c^2}-k^2\right)}\left[\nabla_t\left(\frac{\partial E_z}{\partial z}\right)-i\frac{\omega}{c}\mathbf{e}_z\times\nabla_t B_z\right]$$

Exercise IV.
Get this formula.

TM- Mode Assign

TM-mode : $\boldsymbol{B_z} = 0,\ \boldsymbol{E_z} \neq 0$ → Can accelerate beam Beam

$$\mathbf{B}_t = \frac{i\varepsilon\mu\frac{\omega}{c}}{\left(\varepsilon\mu\frac{\omega^2}{c^2} - k^2\right)}\left[\mathbf{e}_z \times \nabla_t E_z\right],$$

$$\mathbf{E}_t = \frac{1}{\left(\varepsilon\mu\frac{\omega^2}{c^2} - k^2\right)}\nabla_t\left(\frac{\partial E_z}{\partial z}\right),$$

$$\left[\nabla_t^2 E_z + (\varepsilon\mu\frac{\omega^2}{c^2} - k^2)\right]E_z = 0,$$

→ **Solve the eigenvalue problem, get k and Ez**

Boundary condition $E_z|_S = 0$ ($\because$ $\mathbf{n}\times\mathbf{E} = 0$ **on the surface of perfect conductor)**

$\frac{\partial B_z}{\partial n}|_S = 0$ ($\because$ $\mathbf{n}\cdot\mathbf{B} = 0$ **on the surface,**

but automatically satisfied by the TM - mode condition)

TE-Mode Assign

TE-mode : $E_z = 0$, $B_z \neq 0$ ⇒ **Can not accelerate beam,**

Kicks the beam.

$$\mathbf{B}_t = \frac{i\varepsilon\mu\frac{\omega}{c}}{(\varepsilon\mu\frac{\omega^2}{c^2} - k^2)} \nabla_t \left(\frac{\partial B_z}{\partial z} \right),$$

$$\mathbf{E}_t = \frac{-i\frac{\omega}{c}}{\left(\varepsilon\mu\frac{\omega^2}{c^2} - k^2\right)} \mathbf{e}_z \times \nabla_t B_z,$$

$$\left[\nabla_t^2 B_z + (\varepsilon\mu\frac{\omega^2}{c^2} - k^2) \right] B_z = 0,$$

Boundary condition $E_z|_S = 0$ ($\because$ $\mathbf{n} \times \mathbf{E} = 0$ **on the surface of perfect conductor**

but automatically satisfied by the TE- mode condition)

$$\frac{\partial B_z}{\partial n}|_S = 0 \; (\because \; \mathbf{n} \cdot \mathbf{B} = 0 \text{ on the surface})$$

Eigevalue problem

$$\psi(x,y) = E_z(x,y) \text{ for TM- mode or } B_z(x,y) \text{ for TE- mode}$$

$$\left(\vec{\nabla}_t^2 + \gamma^2\right)\psi = 0, \quad \psi\big|_S = 0 \text{ (for TM - mode) or } \frac{\partial}{\partial n}\psi\big|_S = 0 \text{ (for TE - mode)}$$

$$\gamma^2 = \varepsilon\mu\frac{\omega^2}{c^2} - k^2 \geq 0$$

From the boundary condition,

$$\gamma^2 = \gamma_\lambda^2, \ \psi = \psi_\lambda \quad (\lambda = 1, 2, \cdots)$$

$$\Rightarrow \quad k_\lambda^2 = \varepsilon\mu\frac{\omega^2}{c^2} - \gamma_\lambda^2$$

If $\omega < c\dfrac{\gamma_\lambda}{\sqrt{\varepsilon\mu}}$, then k_λ is an imaginary number. The wave is damped in the waveguide.

$\omega_\lambda = c\dfrac{\gamma_\lambda}{\sqrt{\varepsilon\mu}} \cdots$cutoff frequency

When $\omega \geq \omega_\lambda$, wave number k_λ is a real number,

then the wave can propagate into the waveguide.

TM-Modes in a Pill Box Cavity

TM-modes

$$\mathbf{E}(x,y,z,t)=\mathbf{E}(x,y)\exp(ikz-i\omega t)$$

When shorted at $z=0$ and $z=d$, then the wave makes a standing wave.

$$\therefore\ \mathbf{E}(x,y,z,t)=\left[\mathbf{A}(x,y)\cos(kz)+\mathbf{B}(x,y)\sin(kz)\right]\exp(-i\omega t)$$

If the cavity is made from perfect conductor, $\mathrm{E_t}=0\ at\ z=0$ and d.

$$\therefore\ \mathbf{E}(x,y,z)=\mathbf{B}(x,y)\sin(kz)\ \text{and}\ \sin(kd)=0\Rightarrow kd=p\pi(p=0,1,2,\cdots)\Rightarrow k=\frac{p\pi}{d}$$

$$\mathbf{E}_z(x,y,z)=\Psi(x,y,z)\mathbf{e}_z=\left[\mathbf{A}_z(x,y)\cos(kz)+\mathbf{B}_z(x,y)\sin(kz)\right]\mathbf{e}_z$$

$$\mathbf{E}_t(x,y,z,)=\frac{1}{\gamma^2}\nabla_t\left(\frac{\partial\Psi}{\partial z}\right),$$ and the boundary condition: $\mathrm{E_t}=0$ at $z=0$.

$$\Rightarrow\Psi=B_z(x,y)\cos(kz)=B_z(x,y)\cos(\frac{p\pi}{d}z)$$

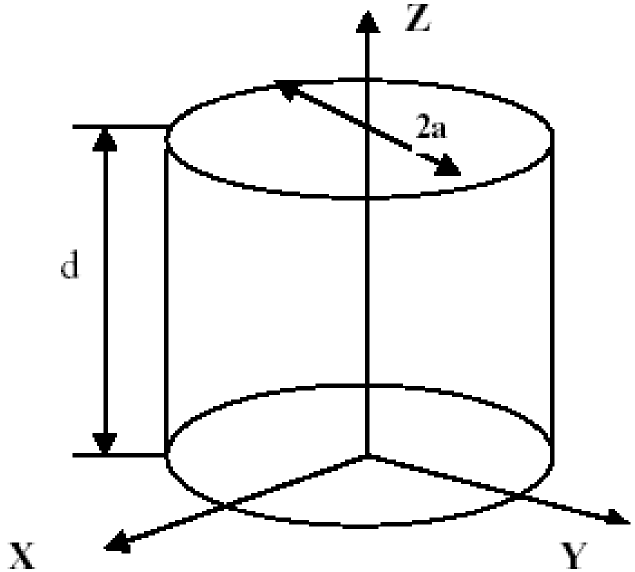
Now one can solve the eigenvalue problem.

$$\left(\nabla_t^2+\gamma^2\right)\Psi=0,\ \gamma^2=\varepsilon\mu\frac{\omega^2}{c^2}-k^2=\varepsilon\mu\frac{\omega^2}{c^2}-\left(\frac{p\pi}{d}\right)^2$$

Cylindorical cordinate (r,θ,z), $\Psi\rightarrow\Psi{=}B_z(r,\theta)$

$$\left(\nabla_t^2+\gamma^2\right)\Psi=\left(\frac{\partial^2}{\partial^2 r}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial^2\theta}\right)\Psi+\gamma^2\Psi=0$$

$$\Psi(r,\theta)=R(r)\cdot\Theta(\theta)$$



$$r^2\frac{\partial^2 R(r)}{\partial^2 r}+\frac{r}{R(r)}\frac{\partial R(r)}{\partial r}+\gamma^2 r^2=-\frac{1}{\Theta(\theta)}\frac{\partial^2\Theta(\theta)}{\partial^2\theta}$$

$$-\frac{1}{\Theta(\theta)}\frac{\partial^2\Theta(\theta)}{\partial^2\theta}=m^2\Rightarrow\Theta(\theta)=\Theta_0\exp(\pm im\theta),m=0,1,2,\cdots$$

Θ is for a single-value function at θ=0 ~ 2π.

ρ=γr,

$$\frac{\partial^2 R}{\partial^2\rho}+\frac{1}{\rho}\frac{\partial R}{\partial\rho}+(1-\frac{m^2}{\rho^2})R=0\Rightarrow R: mthBesselfunction(J_m)$$

For no divergence at ρ=0 $\Rightarrow R(\rho)=J_m(\rho)$

Boundary condition: $E_z(r,\theta)=0$ at $r=a\Rightarrow J_m(\gamma a)=0\Rightarrow\gamma a=\rho_{m,n}$: nth solution of J_m

$\rho_{m,n}$	n=1	n=2	n=3
m=0	$\rho_{0,1}=2.405$	$\rho_{0,2}=5.520$	$\rho_{0,3}=8.654$
m=1	$\rho_{1,1}=3.832$	$\rho_{1,2}=7.016$	$\rho_{1,3}=10.173$
m=2	$\rho_{2,1}=5.136$	$\rho_{2,2}=8.417$	$\rho_{2,3}=11.620$

$$\gamma_{m,n}=\frac{\rho_{m,n}}{a},\quad\text{thus }\Psi(r,\theta)=J_m(\frac{\rho_{m,n}}{a}\cdot r)\cdot\exp(\pm im\theta),$$

Resonance frequency ($\text{TM}_{m,n,p}$ – mode)

$$\omega_{m,n,p}=\frac{c}{\sqrt{\varepsilon\mu}}\sqrt{\frac{\rho_{m,n}}{a^2}+\frac{p^2\pi^2}{d^2}}$$

For E_t and B_t, calculate

$$\mathbf{B}_t = \frac{i\varepsilon\mu\frac{\omega}{c}}{\left(\varepsilon\mu\frac{\omega^2}{c^2} - k^2\right)}\left[\mathbf{e}_z \times \nabla_t E_z\right],$$

$$\mathbf{E}_t = \frac{1}{\left(\varepsilon\mu\frac{\omega^2}{c^2} - k^2\right)}\nabla_t\left(\frac{\partial E_z}{\partial z}\right),$$

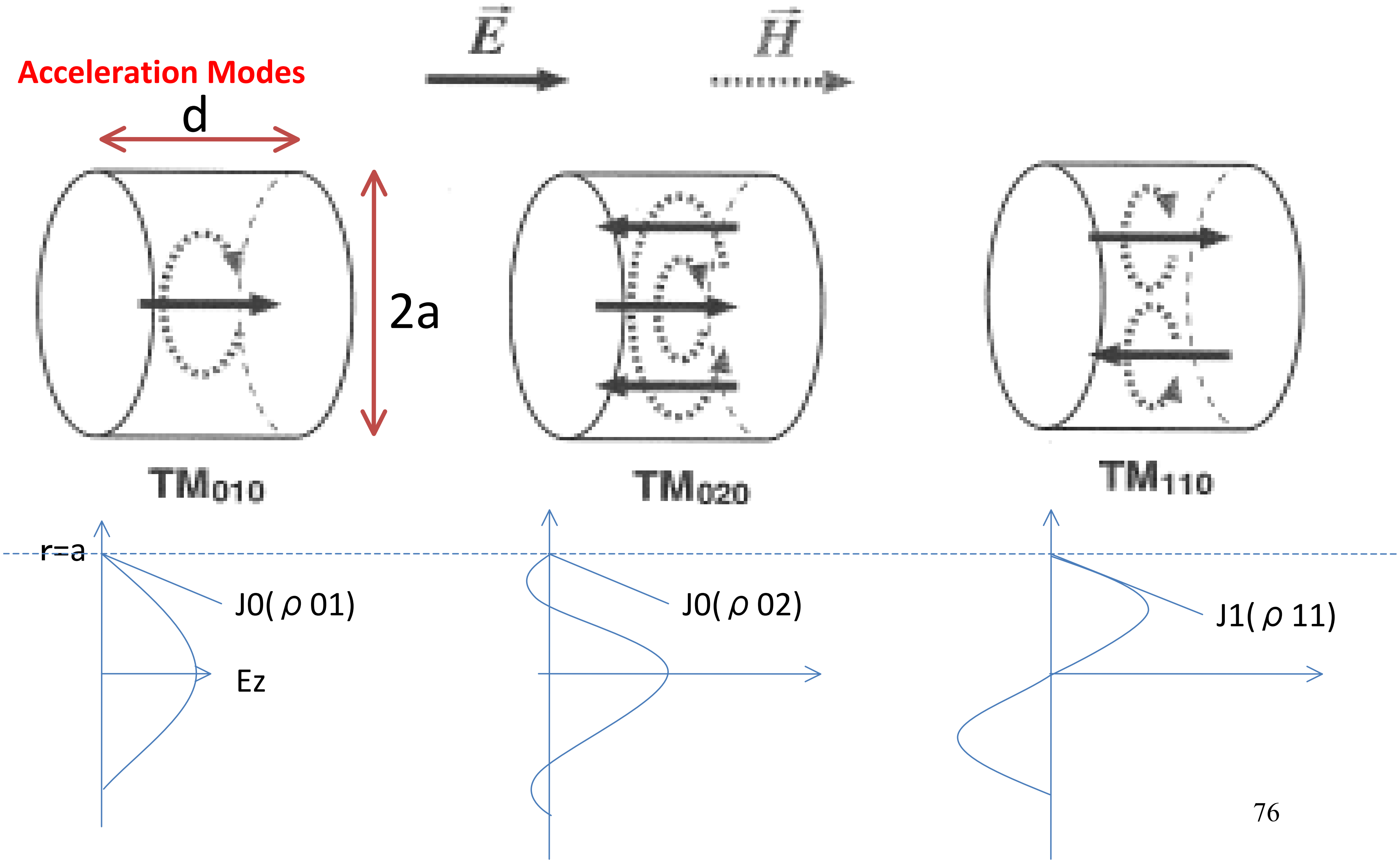
$TM_{m,n,p}$ – mode

$$E_z = E_o \cos(kz) J_m\left(\frac{\rho_{m,n}}{a} r\right)\exp(-im\theta), \qquad B_z = 0$$

$$E_r = \frac{iE_0 p\pi}{\gamma_{m,n,p}} \cos\left(\frac{p\pi}{d} z\right)\frac{\partial J_m(\rho)}{\partial \rho}\exp(-im\theta), \qquad B_r = -\frac{E_0 m\varepsilon\mu\omega_{m,n,p}}{} \cos(kz) J_m\left(\frac{\rho_{m,n}}{a} r\right)\exp(-im\theta)$$

$$E_\theta = \frac{E_0 m p\pi}{\gamma^2_{m,n,p} dc} \cos\left(\frac{p\pi}{d} z\right) J_m\left(\frac{\rho_{m,n}}{a} r\right)\exp(-im\theta), \quad B_\theta = \frac{iE_0 \varepsilon\mu\omega_{m,n,p}}{\gamma_{m,n,p} c}\cos(kz)\exp(-im\theta)\frac{\partial J_m(\rho)}{\partial \rho}$$

TM_{mnp} Modes



Acceleration Modes
d
2a
TM010
TM020
TM110
r=a
J0(ρ 01)
Ez
J0(ρ 02)
J1(ρ 11)

TE_{mnp} Modes

TE_{mnp} Modes

$$E_r = \frac{i\omega\varepsilon}{k^2}\frac{m}{r}E_0 J_m\left(\frac{\rho'_{mn}}{a}r\right)\sin(m\theta)\sin(\frac{p\pi z}{d})$$

$$H_r = \frac{1}{k^2}\frac{p\pi}{d}E_0 J_m\left(\frac{\rho'_{mn}}{a}r\right)\cos(m\theta)\cos(\frac{p\pi z}{d})$$

$$E_\theta = \frac{i\omega\varepsilon}{k^2}\frac{\rho'_{mn}}{a}E_0 J_m\left(\frac{\rho'_{mn}}{a}r\right)\cos(m\theta)\sin(\frac{p\pi z}{d})$$

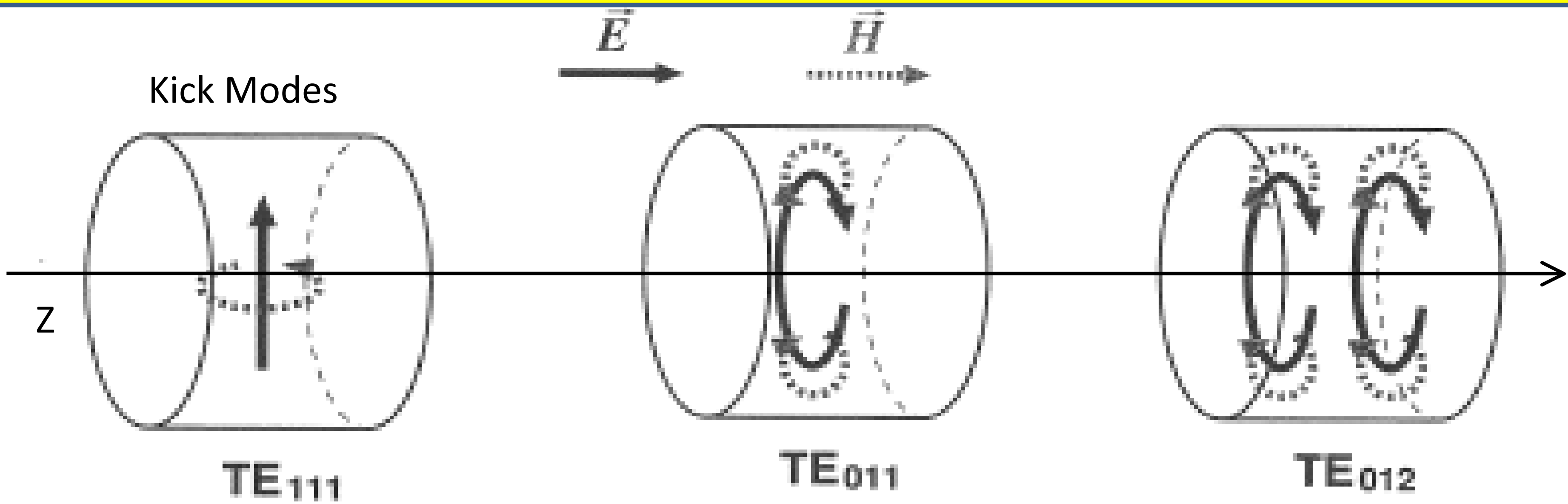
$$H_\theta = -\frac{1}{k^2}\frac{p\pi}{d}\frac{m}{r}E_0 J_m\left(\frac{\rho'_{mn}}{a}r\right)\sin(m\theta)\cos(\frac{p\pi z}{d})$$

$$E_z = 0$$

$$H_z = E_0 J_m\left(\frac{\rho'_{mn}}{a}r\right)\cos(m\theta)\sin(\frac{p\pi z}{d})$$

Resonance frequency

$$\omega^2\varepsilon_0\mu_0 = \left(\frac{\rho'_{mn}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \Rightarrow f = \frac{c}{2\pi}\sqrt{\left(\frac{\rho'_{mn}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$



Pill Box TM_{010}-Mode Single Cell Cavity Design

This is a very much instructive example for the RF cavity design.

The essential is included in this example.

Exercise V.
Make design a 1300MHz single cell Pill Box cavity
1.What is the diameter of the cell?
2. What is the cell length?

Summaries of Characteristic Parameters of RF cavity

Surface Impedance Z[Ω]: $\boldsymbol{Z} \equiv \frac{\mathbf{E}_{//}}{\mathbf{H}_{//}} = \mathbf{R_S} + i\boldsymbol{X}, \quad \mathbf{R_S} = \frac{1}{\sigma\delta} = \sqrt{\frac{\mu\omega}{2\sigma}},$

Skin depth δ [m] : $\delta = \sqrt{\frac{2}{\mu\omega\sigma}}$

Wall loss P_{loss} [W]: $P_{loss} = \frac{1}{2} R_S \int_S H_s^2 ds \quad (= \frac{\pi R_S E_o^2}{(\mu/\varepsilon)} J_1^2(2.405) \cdot a \cdot (a+d)$ for pill box cavity)

Transit time factor T : $T = \frac{\int_0^d E_z e^{i(\omega \times \frac{z}{c})} dz}{\int_0^d E_z dz} \quad (= \frac{2}{\pi}$ for pill box cavity)

Accelerating Voltage V : $V = \int_0^d E_o(\rho = 0, z)\, e^{i(\omega \frac{z}{c})} dz \quad (= dE_o T$ for pill box)

Accelerating gradient E_{acc} : $E_{acc} = \frac{V}{d} \quad (= E_o T = 2\frac{E_o}{\pi}$ for pill box cavity)

Stored energy U: $U = \frac{1}{2}\mu \int_V H^2 dv = \frac{1}{2}\varepsilon \int_V E^2 dv \quad (= \frac{\pi\varepsilon E_0^2}{2} \cdot J_1^2(2.405) \cdot d \cdot a^2$ for pill box cavity

Unloaded Q-value Q_O: $Q_O = \frac{\omega \cdot U}{P_{loss}} \quad (= \omega \cdot \frac{\mu \cdot a^2 d}{2 \cdot a(a+d)} \cdot \frac{1}{R_S}$ for pill box cavity)

Summaries of Characteristic parameters of RF cavity

Shunt impedance R_{sh} **[Ω]:** $R_{sh} = \dfrac{V^2}{P_{loss}}$ $\left(= \dfrac{4(\varepsilon/\mu)d^2}{\pi^3 R_S J_1^2(2.405)a(a+d)} \right.$ **for pill box cavity**

Geometrical factor Γ: $\Gamma = Q_O \cdot R_S = \dfrac{\omega\mu\int_V H^2 dv}{\int_S H_S^2 ds}$ $\left(= \dfrac{\omega\mu d a^2}{2(a^2+ad)}\right.$ for pill box cavity) $\Rightarrow$ $R_S = \dfrac{\Gamma}{Q_O}$

R/Q : $\left(R/Q\right) = \dfrac{R_{sh}}{Q_O} = \dfrac{V^2}{\omega U}$ Goodness of the cavity shape, No dependent on materia

E_{SP}/E_{acc} $\left(= \dfrac{\pi}{2} = 1.57\right.$ for pill box cavity), H_{SP}/E_{acc} $\left(= 30.5 \dfrac{O_e}{MV/m}\right.$ for pill box cavity)

Smaller value is better from field emission problem point of view

Smaller value is better from high gradient point of view

Pill-box cavity maximum Eacc = 1750/30.5 =57.4MV/m

Frequency Dependence of Cavity Parameters

Characteristic Parameter	ω dependence Normal conducting	ω dependence Super conducting
R_S	$\omega^{\frac{1}{2}}$	ω^{2}
P_{loss}	$\omega^{-\frac{3}{2}}$	No dependence
U	ω^{-3}	ω^{-3}
Q_O	$\omega^{-\frac{1}{2}}$	ω^{-2}
R_{sh}	$\omega^{-\frac{1}{2}}$	ω^{-2}
R_{sh}/L	$\omega^{\frac{1}{2}}$	ω^{-1}
Γ	No dependence	No dependence
R/Q	No dependence	No dependence

Rsh per length linearly increases to $\sqrt{\omega}$, so normal conducting choose higher frequency, for example 11.4GHz @ warm LC.

3.4 Realistic Cavity Cell Design Criteria

Real Cavity Cell Design

Diameter of BP	→	bigger
Ep/Eacc	→	larger
Hp/Eacc	→	larger
R/Q	→	larger
Cell to cell coupling	→	smaller
HOM issue	→	more serious

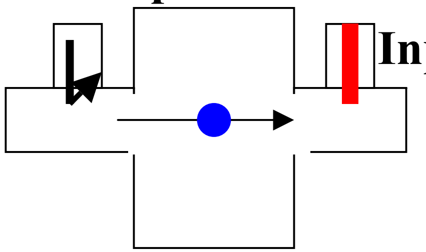
1) Need a hole on the cavity for electron to pass the cavity
2) Need RF input port
3) HOM coupler port

→ Need Beam pipes on both Ends

4) Higher acceleration efficiency
5) Higher gradient
   Smaller Ep/Eacc : Field emission
   Smaller Hp/Eacc : Multipaction

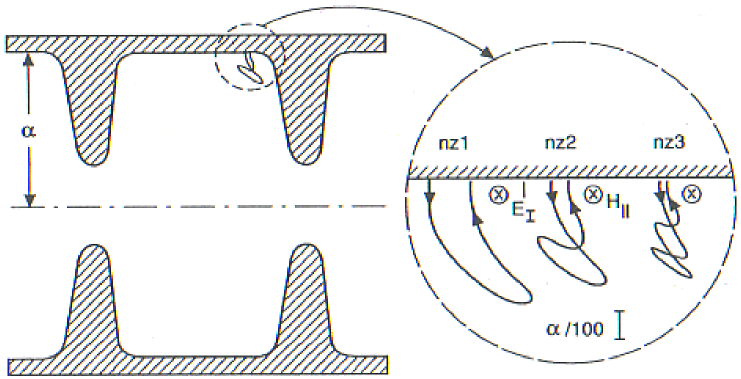
→ Optimization of cell shape
Multi-cell cavity

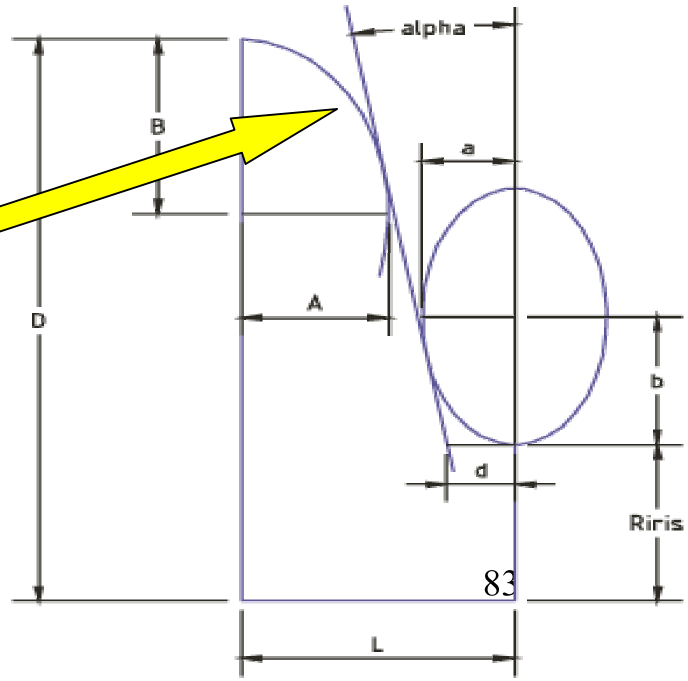
HOM coupler



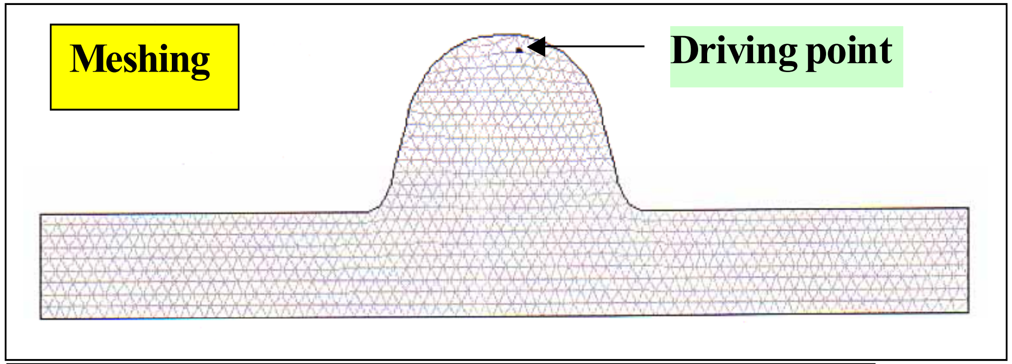
Input coupler

Choose spherical shape to reduce multipacting



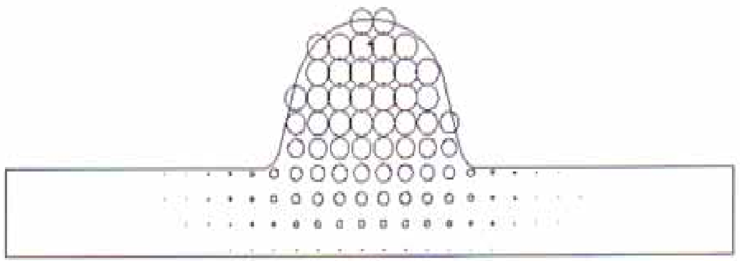


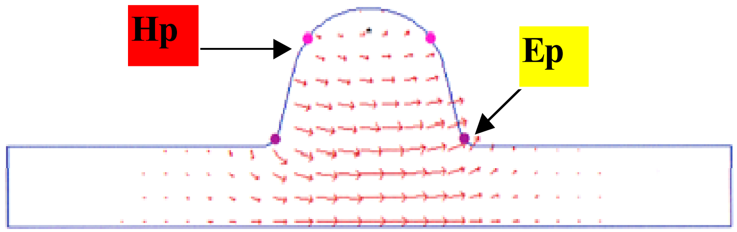
Cavity Design (single cell cavity)

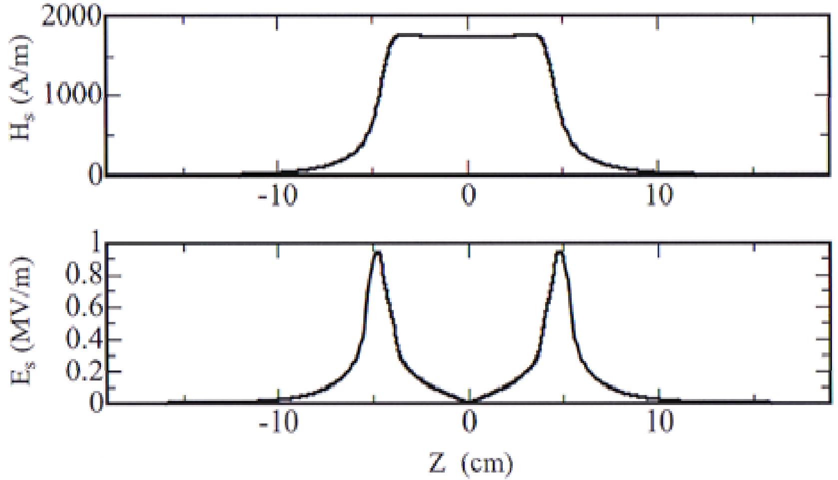


All calculated values below refer to the mesh geometry only.
Field normalization (NORM = 0): EZERO = 1.00000 MV/m
Length used for E0 normalization = 10.76000 cm
Frequency (starting value = 1300.000) = 1293.77430 MHz
Particle rest mass energy = 0.510999 MeV
Beta = 1.0000000
Normalization factor for E0 = 1.000 MV/m = 7048.913
Transit-time factor Abs(T+iS) = 0.5454664
Stored energy = 0.0038869 Joules
Using standard room-temperature copper.
Surface resistance = 9.38405 milliOhm
Normal-conductor resistivity = 1.72410 microOhm-cm
Operating temperature = 20.0000 C
Power dissipation = 1118.1551 W
Q = 28257.6 Shunt impedance = 96.230 MOhm/m
Rs*Q = 265.171 Ohm Z*T*T = 28.632 MOhm/m
r/Q = 109.024 Ohm Wake loss parameter = 0.22157 V/pC
Average magnetic field on the outer wall = 1729.9 A/m, 1.40411 W/cm^2
Maximum H (at Z,R = 3.32643,8.55466) = 1753.44 A/m, 1.44258 W/cm^2
Maximum E (at Z,R = 4.75232,4.24425) = 0.946176 MV/m, 0.02953 Kilp.
Ratio of peak fields Bmax/Emax = 2.3288 mT/(MV/m)
Peak-to-average ratio Emax/E0 = 0.9462

Superfish







Exercise VI.

Superfish outputs

f_0=1293.77430MHz
Ploss=118.1551W
RsQ=265.171 Ω
Qo=28257.6
(Rsh/Q)=109.24 Ω
Hp=1753.44 A/m
Ep=0.946176 MV/m

Calculate the following cavity RF parameters from the Superfish outputs.

Rsh [Ω] =

Accelerating Voltage V [MV]=

RF wave length λ[m] =

Gradient Eacc = V/L_{eff} [MV/m]= ,defined as $L_{eff} = \lambda/2$

Hp/Eacc[Oe/(MV/m)] = , use 1A/m= $4\pi 10^{-3}$ Oe

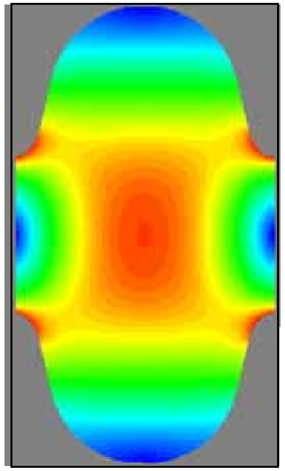
Ep/Eacc =

Eacc [MV/m] = $Z \cdot \sqrt{P_{loss} \cdot Q_O}$, Z=

Geometrical factor Γ [Ω] =

What are figures of merit for a cavity storing E-H energy?

J.Sekutwitz's Slide



W_n ≡ stored energy of a mode n : {ω_n, E_n, H_n}.

$$W_n \equiv \mu \int_V \frac{H_n^2}{2} dV = \varepsilon \int_V \frac{E_n^2}{2} dV$$

Quality Factors

The measure of the energy loss in the metal wall and due to the radiation via open ports:

Intrinsic Q ≡ Qo

$$Q_{O,n} \equiv \frac{\omega_n \cdot W_n}{P_n} = \frac{\omega_n \cdot W_n}{\frac{R_{s,n}}{2} \int_S H_n^2 ds}$$

n : n-th mode

External Q ≡ Q_{ext}

$$Q_{ext,n} \equiv \frac{\omega_n \cdot W_n}{P_{rad,n}} = \frac{\omega_n \cdot W_n}{\frac{1}{2} \int_{S_{allports}} E_n \times H_n ds}$$

What are figures of merit for a cavity storing E-H energy? Continued

J.Sekutwitz's Slide

Geometric Factor

The measure of the energy loss in the metal wall for the ***unit*** *surface resistance* $R_{s,n}$

$$G_n \equiv Q_{O,n} \cdot R_{s,n} = \frac{\omega_n \cdot W_n \cdot R_{s,n}}{P_n} = \frac{\omega_n \cdot W_n}{\frac{1}{2}\int_S H_n^2 ds}$$

It is the ratio of the stored energy to the integral of $(\mathbf{H}_n)^2$ *on the metal surface.*
It is independent of cavity material and depends on cavity shape.

What are figures of merit for the beam-cavity interaction?

J.Sekutwitz's Slide

This interaction which is:

- ***Acceleration***
- ***HOMs excitation***

*can be described in **Frequency Domain (FD)** or/and in **Time Domain (TD)**.*

Very Important RF Parameter (R/Q)

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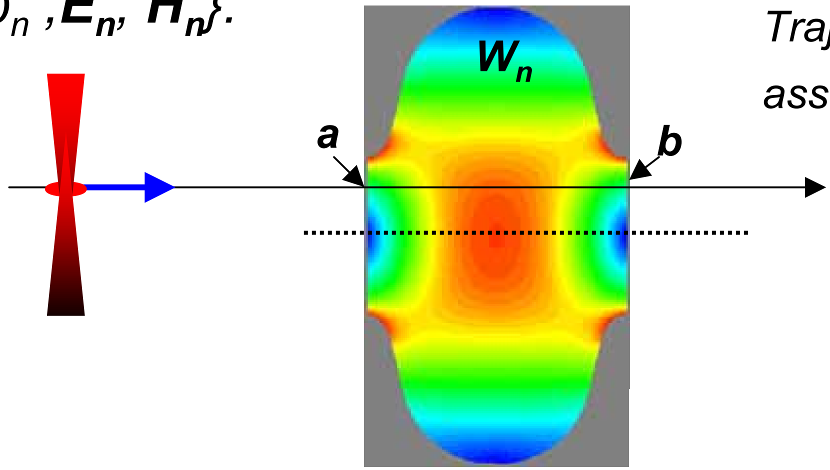
$(R/Q)_n$, a "measure" of the energy exchange between point charge and mode n (FD).

mode n : {ω_n , $\mathbf{E}_n$, $\mathbf{H}_n$}.



Trajectory of the point charge q, assumed here to be a straight line.

$$V_n = \sqrt{\left(\int_{z_a}^{z_b} E_{n,z} \sin\left(\frac{\omega_n}{\beta c}(z - z_a)\right) dz\right)^2 + \left(\int_{z_a}^{z_b} E_{n,z} \cos\left(\frac{\omega_n}{\beta c}(z - z_a)\right) dz\right)^2}$$

$$(R/Q)_n \equiv \frac{V_n^2}{\omega_n W_n}$$

(R/Q) for Accelerating Mode

J.Sekutwitz's Slide

For acceleration modes, V_n is calculated on the beam axis. (R/Q) means efficiency of the acceleration, which is independent on material. It means the goodness of cavity shape for beam acceleration.

For the accelerating mode we often use the product of $\boldsymbol{G_{acc} \cdot (R/Q)_{acc}}$ *, as a **"measure"** of the power **P** dissipated in the metal wall at the given accelerating voltage* $\boldsymbol{V_{acc}}$ *and the given surface resistance* $\boldsymbol{R_s}$*.*

$$\frac{P_{dissipated}}{V_{acc}^{2}} \equiv \frac{R_s}{G_{acc} \cdot (R/Q)_{acc}}$$

This is due to the surface quality; Big improvement possible. (refers to R_s)

This is due to the geometry of cells; Moderate improvement possible by cavity shape (refers to $G_{acc} \cdot (R/Q)_{acc}$)

Longitudinal and Transverse Loss Factors (TD)

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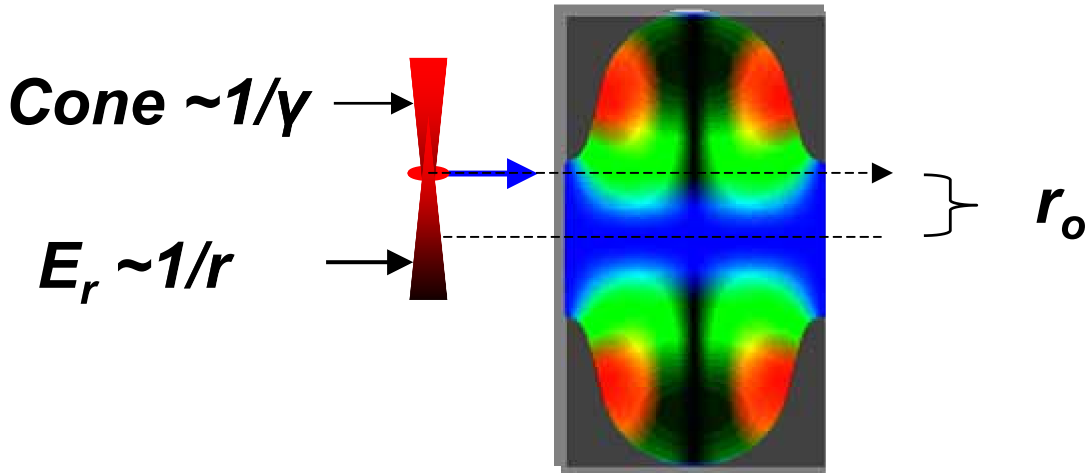
Ultra relativistic point charge ***q*** *passes* ***empty cavity***



a. Density of the inducted charge on the wall depends on the distance to the beam trajectory.

b. The non uniform charge density on the metal wall causes the current flow on the surface.

Longitudinal and Transverse Loss Factors (TD), Continued

J.Sekutwitz's Slide

The amount of energy lost by charge q to the cavity is:

$\Delta U_q = k_{\|} \cdot q^2$ ***for monopole modes (max. on axis)***

$\Delta U_q = k_{\perp} \cdot q^2$ ***for non monopole modes (off axis)***

where $\mathbf{k}_{\|}$ *and* $\mathbf{k}_{\perp}\mathbf{(r)}$ *are loss factors for the monopole and transverse modes respectively.*

The induced ***E-H field (wake)*** *is a superposition of* ***cavity eigenmodes (monopoles and others)*** *having the* $\mathbf{E_n(r,\varphi,z)}$ *field* ***along the trajectory.***

Both description methods FD and TD are equivalent.

For individual mode n and point-like charge:

$$k^{\mathbf{p}}_{\|,n} = \frac{\omega_n \cdot (R/Q)_n}{4}$$

Note please the linac convention of (R/Q) definition.

Similar for other loss factors…….

RF parameters of the accelerating mode more practical

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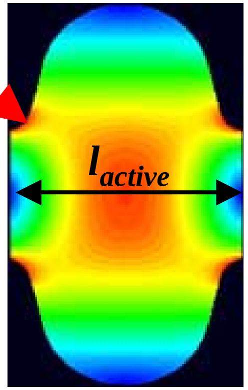
At stored energy W_{acc} the mean value of the accelerating gradient is:

$$E_{acc} = \frac{\sqrt{\omega_{acc} \cdot W_{acc} \cdot (R/Q)_{acc}}}{l_{active}}$$

E_{peak} on the surface

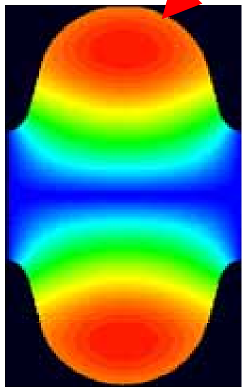
$$\frac{E_{peak}}{E_{acc}}$$



Contour plot of /E/

B_{peak} on the surface

$$\frac{B_{peak}}{E_{acc}}$$



Contour plot of /B/

Ratio shows sensitivity of the shape tothe field electron emission phenomenon.

Ratio shows limit in E_{acc} due to the break-down of superconduc-tivity (Nb ~180mT).

Cell to Cell Coupling K_{cc}

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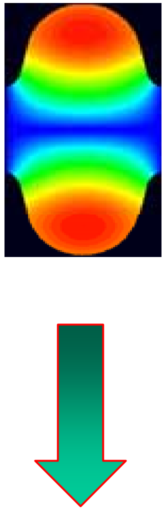
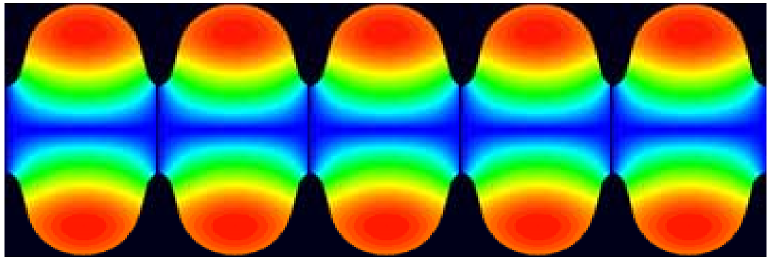
The last parameter, relevant for multi-cell accelerating structures, is the coupling $\boldsymbol{k_{cc}}$ between cells for the accelerating mode passband (Fundamental Mode passband).



Single-cell is attractive from the RF-point of view:

- ***Easier to manage HOM damping***
- ***No field flatness problem.***
- ***Input coupler transfers less power***
- ***Easy for cleaning and preparation***
- ***But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.***



A multi-cell structure is less expensive and offers higher real-estate gradient but:

- ***Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells***
- ***Other problems arise: HOM trapping…***

Cell to Cell Coupling $\kappa_{cc,}$ Continued

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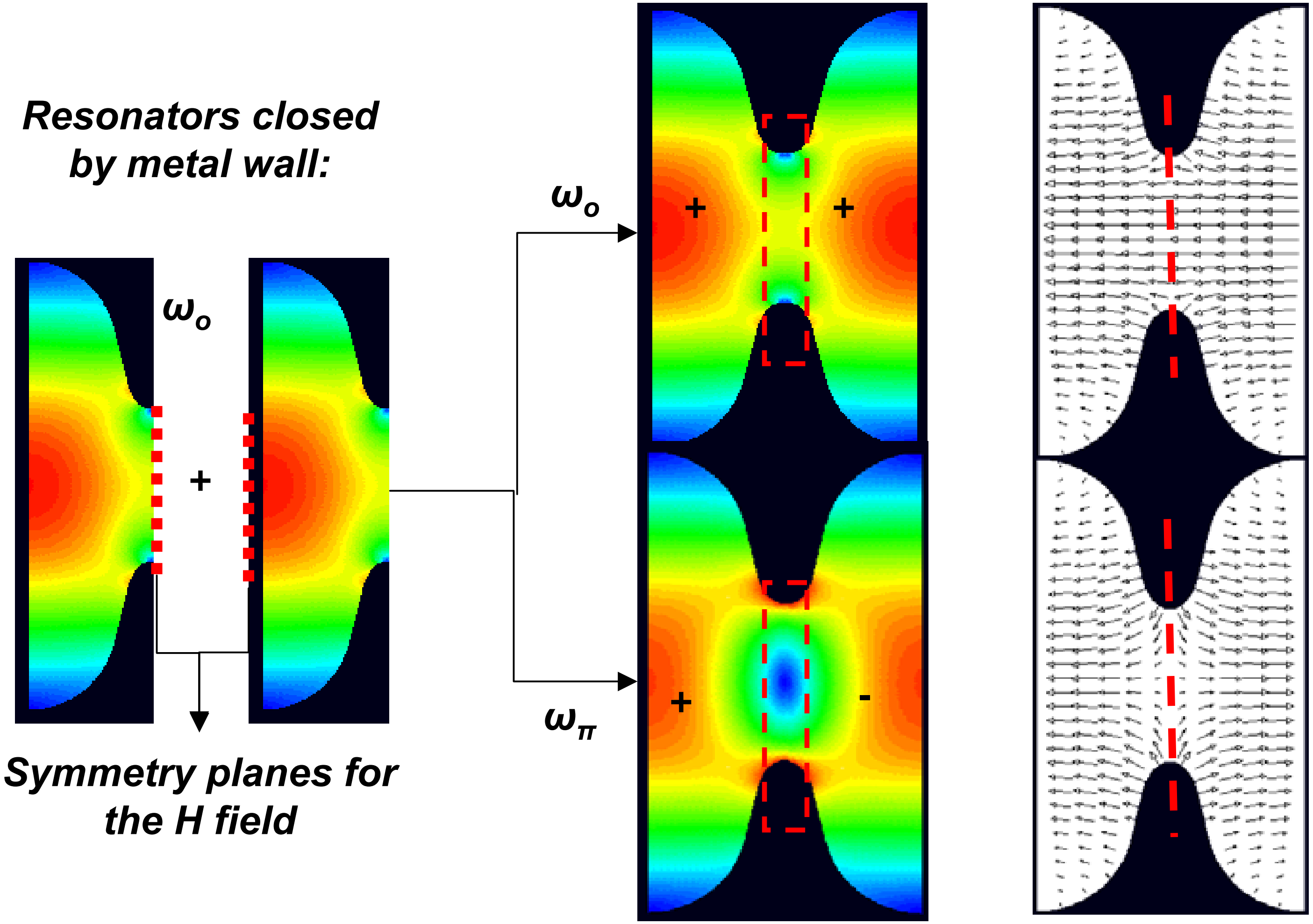

Symmetry plane for the H field

Symmetry plane for the E field

which is an additional solution

Cell to Cell Coupling κ_{cc}, Continued

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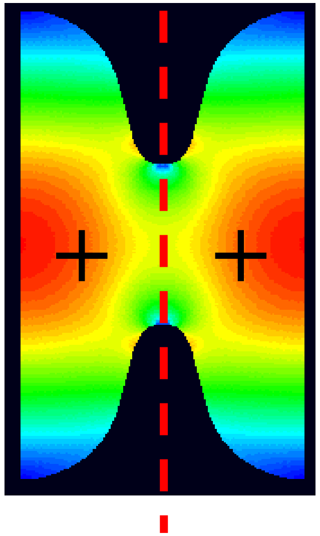
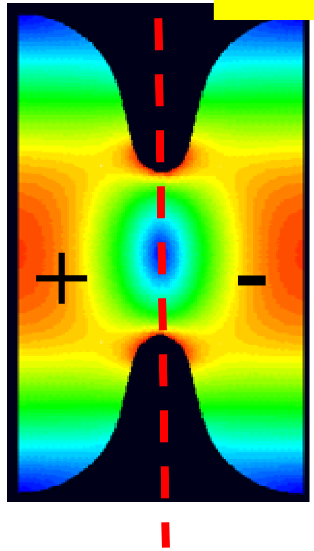
ω_o



ω_π



no E_r (in general transverse E field) component at the symmetry plane

no H_φ (in general transverse H field) component at the symmetry plane

The normalized difference between these frequencies is a measure of the Pointing vector (energy flow via the coupling region)

$$k_{cc} = \frac{\omega_\pi - \omega_0}{\dfrac{\omega_\pi + \omega_0}{2}}$$

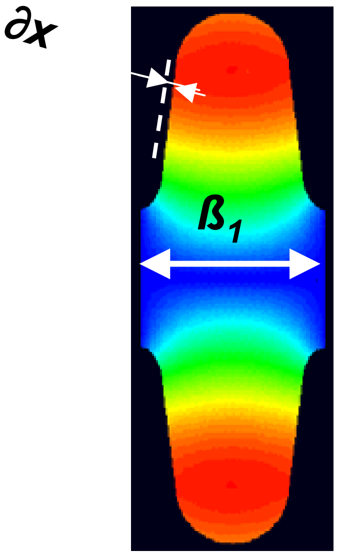
Field Flatness Factor a_{ff}

J.Sekutwitz's Slide

Field flatness factor for elliptical cavities with arbitrary ß=v/c

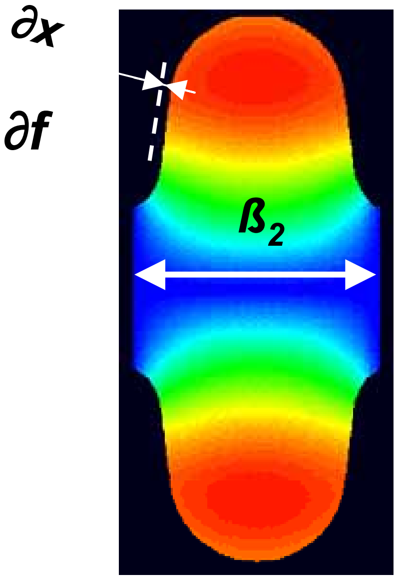
$$a_{ff} = \frac{N^2}{k_{cc} \cdot ß}$$

This is an empirical correction, based on intuition.



The same error ∂x causes bigger ∂f when a cell is thinner

$$\frac{\partial f_1}{\partial f_2} = \frac{\partial x \cdot ß_2}{\partial x \cdot ß_1}$$



Cells which geometric ß <1 are more sensitive to shape errors

Optimization of Cell Shape Against Multipacting

J.Sekutwitz's Slide

Before we will look for the correlation between the RF-parameter set and the Geometry of a cavity we need to look at the Multipacting phenomenon.

Multipacting ***≡ a resonant bombardment of the metal wall (ceramics) synchronous with E-H fields, which may develop an avalanche of electrons "consuming" stored energy (cavities) or transmitted energy (waveguides, couplers) in RF devices.***

How does this process go?

1-phase: electron is accelerated by the orthogonal to the wall electric field

2-phase: further acceleration and bending of its trajectory towards the wall

3-phase: electron bombs the wall and if impact energy is in a certain region more then 1 electron is emitted from the surface.

Phases 1, 2, 3 repeat which leads to the avalanche of electrons bombarding the wall and dissipation of the E-H energy.

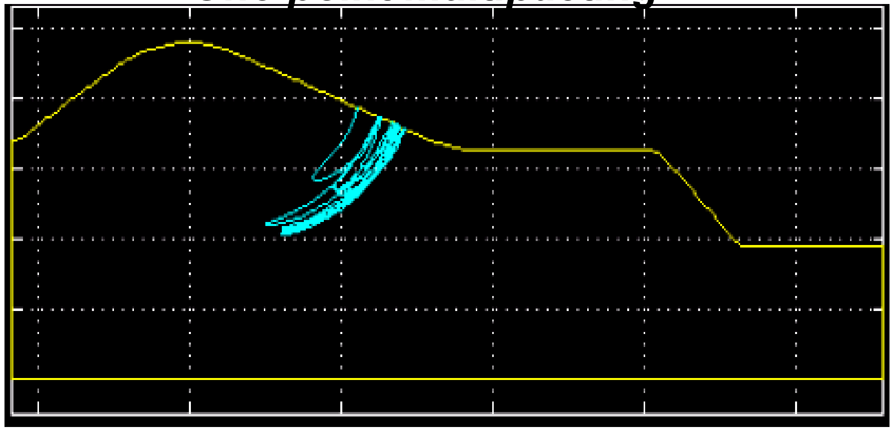
Development of the avalanche is possible if :

1. ***Geometry + power level fulfills resonant condition.***
2. ***Secondary electron emission coefficient is > 1.***

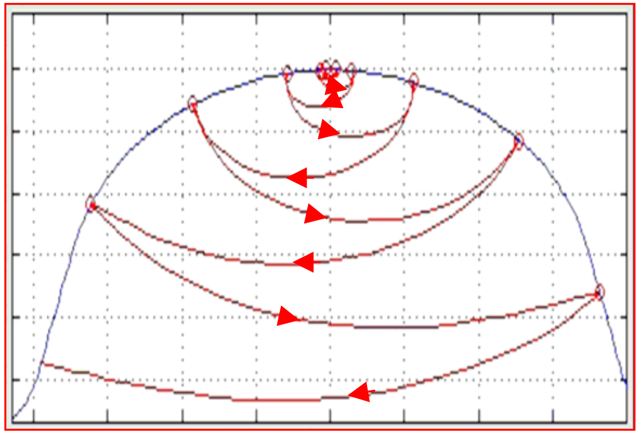
Multipacting on Beam Pipe or Cell

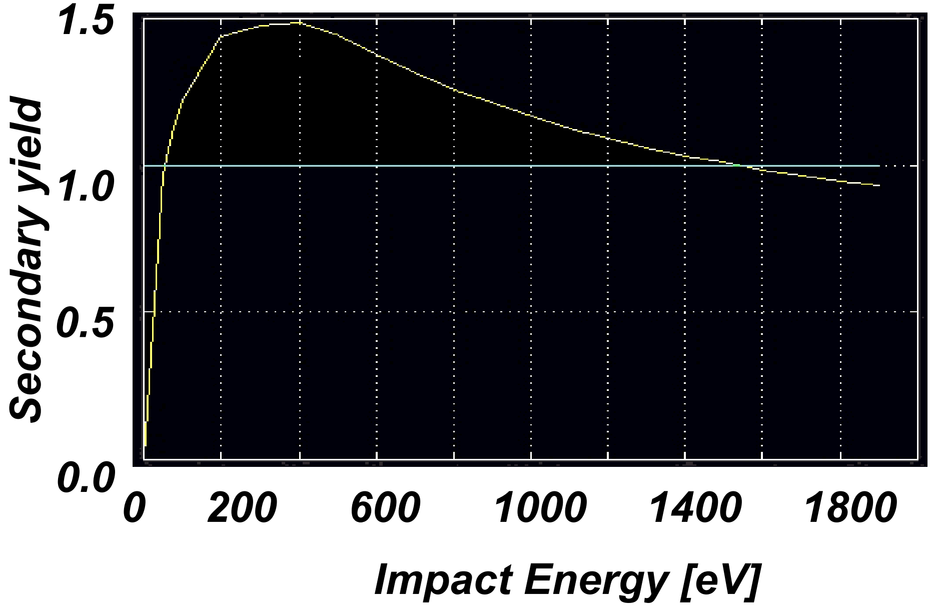
J.Sekutwitz's Slide

One-point multipacting



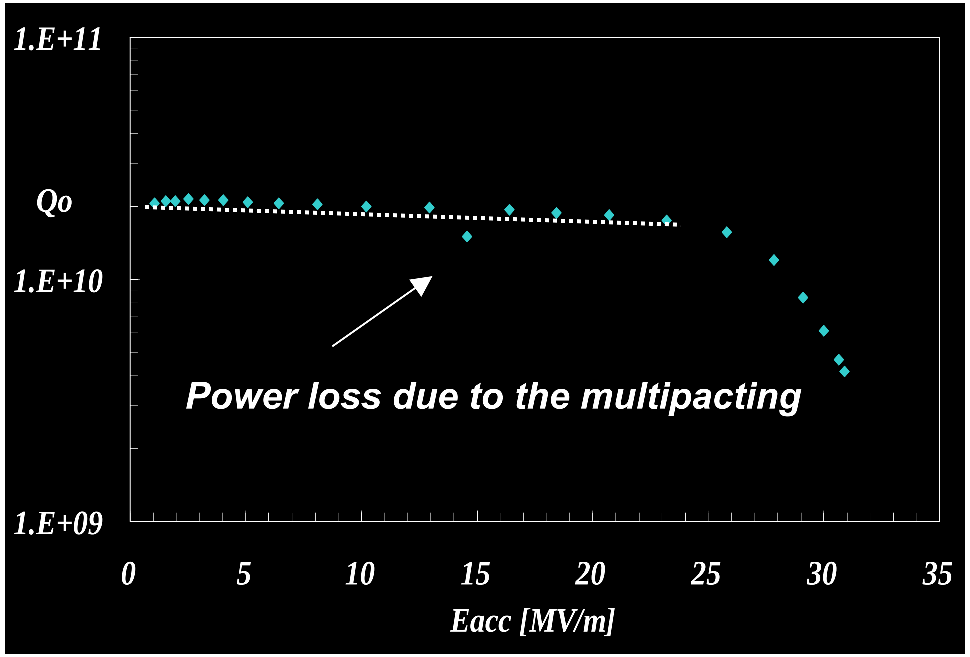
Two-point multipacting





Secondary yield of clean Nb surface. Condense gases on the surface may increase secondary yield up to 3 !!!

Cell Shaped Suppressed Multipacting



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The phenomena can be very often cured by processing which leads to change of the secondary yield below 1.

R. Parodi ***(1979)*** *presented first* ***spherical*** *C-band cavity with much less multipacting barrier than other cavities at that time.*

P. Kneisel ***(early 80's)*** *proposed for the DESY experiment the* ***elliptical shape*** *of 1 GHz cavity preserving good performance of the spherical one and stiffer mechanically.*

Optimization of Cell Shape

We begin with inner cells design because these cells "dominate" parameters of a multi-cell superconducting accelerating structure.

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RF parameters summary:

FM : (R/Q), G, E_{peak}/E_{acc}, B_{peak}/E_{acc}, k_{cc}

HOM : $k_{\perp}$, $k_{\parallel}$.

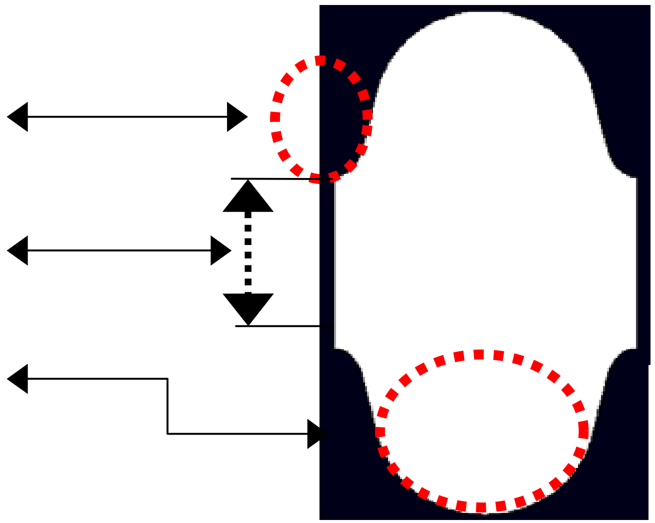
There are 7 parameters we want to optimize for a inner cell

Geometry :

iris ellipsis : half-axis h_r, h_z

iris radius : r_i

equator ellipsis : half-axis h_r, h_z



There is some kind of conflict <u>7 parameters</u> and only <u>5 variables</u> to "tune"

General Trends of Cavity Optimization on RF Geometrical Parameters

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Criteria	*RF-parameter*	*Improves when*	*Cavity examples*
Operation at high gradient	E_{peak}/E_{acc} B_{peak}/E_{acc} ↓	r_i ↓ Iris, Equator shape	*TESLA, HG CEBAF-12 GeV*
Low cryogenic losses	$(R/Q) \cdot G$ ↑	r_i ↓ Equator shape	*LL CEBAF-12 GeV LL- ILC cavity*
High I_{beam} ↔ *Low HOM impedance*	$k_{\perp}, k_{//}$ ↓	r_i ↑	*B-Factory RHIC cooling*

We see here that r_i is a very "powerful variable" to trim the RF-parameters of a cavity.

Effect of Cavity Aperture

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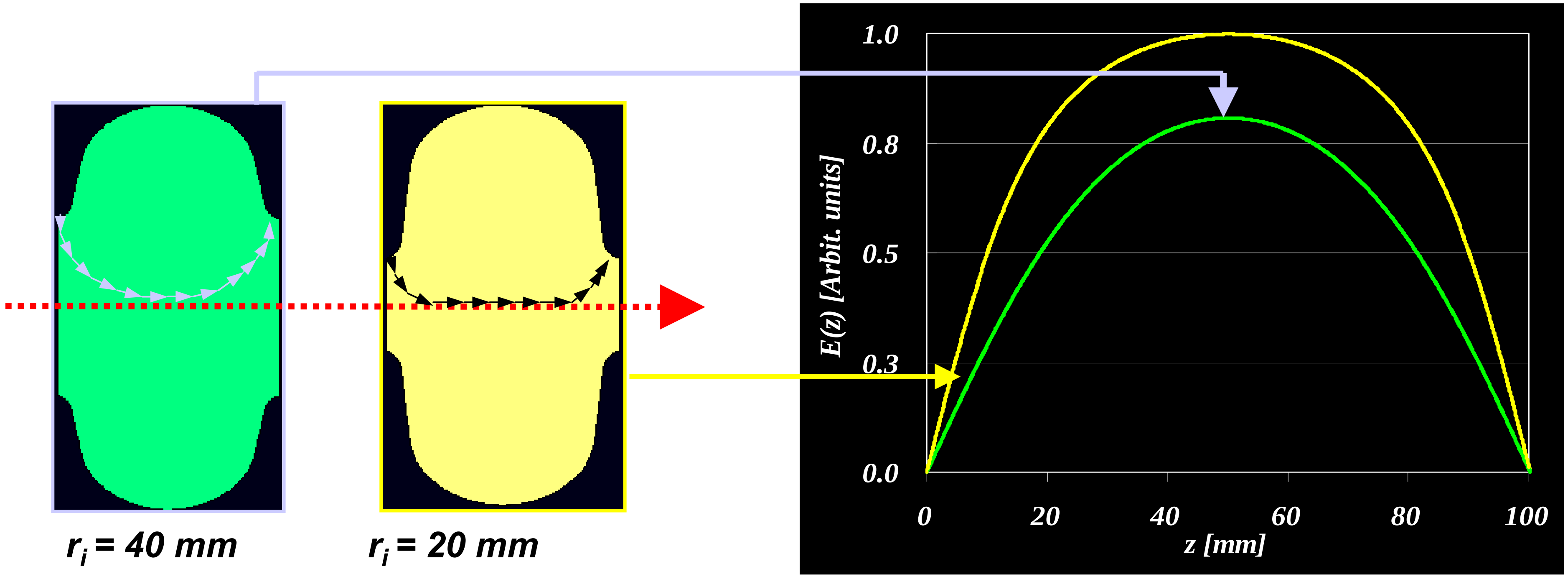
Why for a smaller aperture (r_i)

- ***(R/Q) is bigger***
- ***E_{peak}/E_{acc}, B_{peak}/E_{acc} is lower ?***

E_{acc} is higher at the same stored energy in the cell



E_z (z) for small and big iris radius

Effect of Cavity Aperture on RF Parameters

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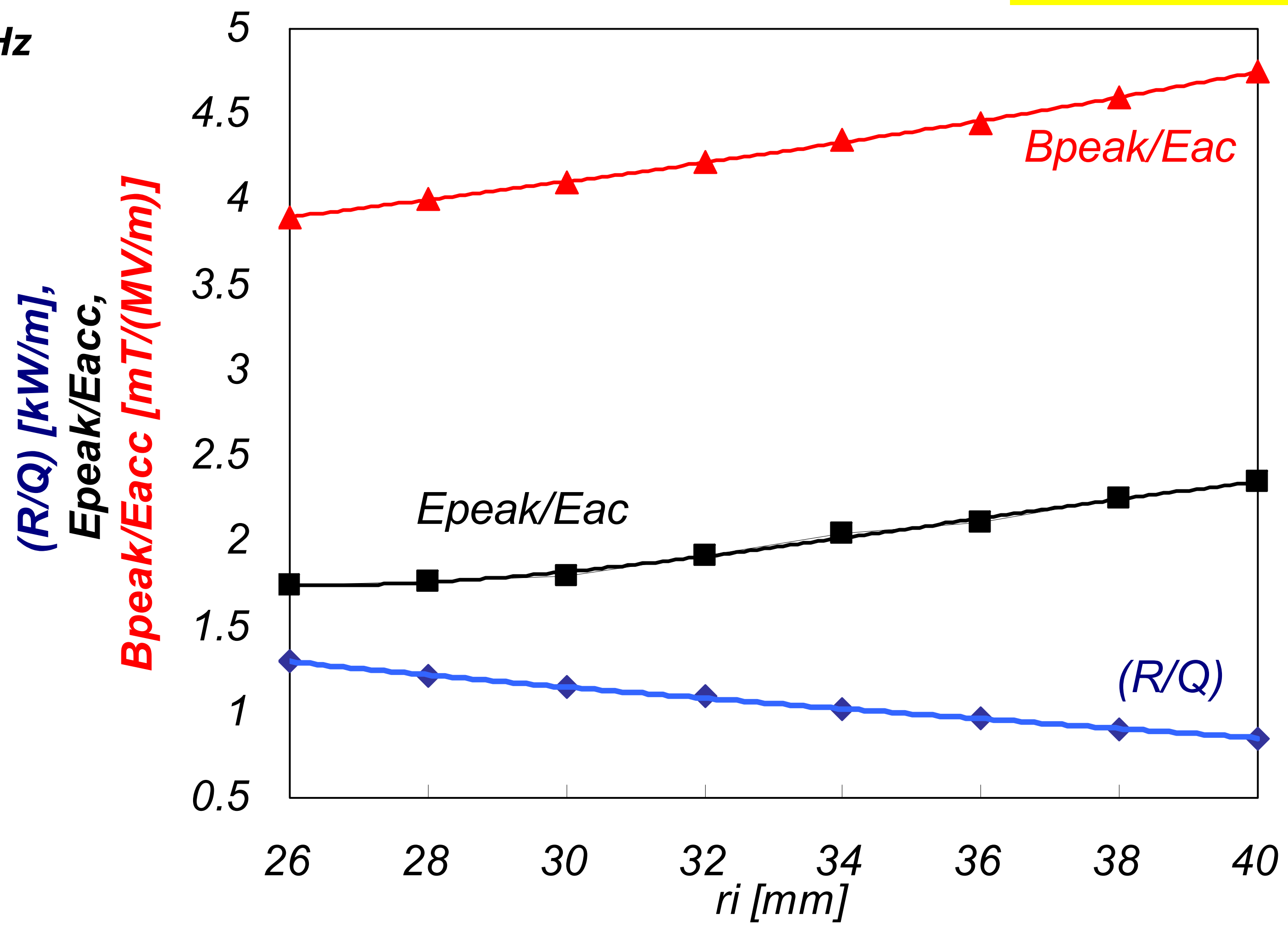
Example:
f = 1.5 GHz



A. Mosnier, E. Haebel, SRF Workshop 1991

Effect of Cavity Aperture on B_{peak}/E_{acc}

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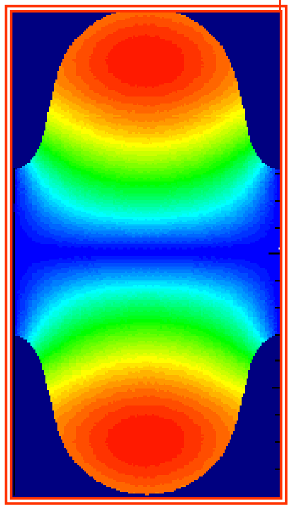
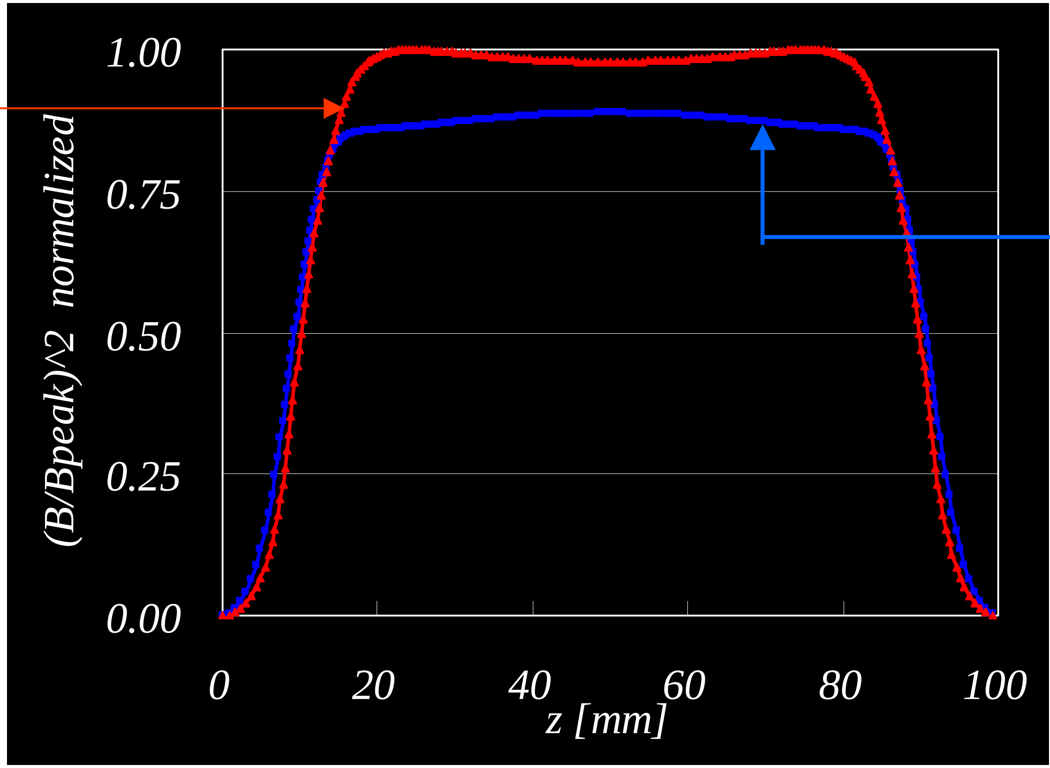
In addition to the iris radius :

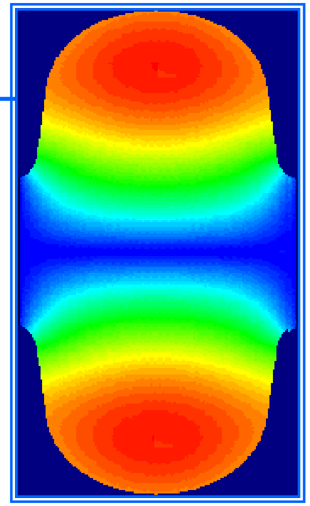
- ***B_{peak}/E_{acc} (and G) changes vs. Equator shape***

1 Joule





1 Joule



Effect of Cavity Aperture on E_{peak}/E_{acc}

Similar : ***E_{peak}/E_{acc} changes vs. Iris shape***

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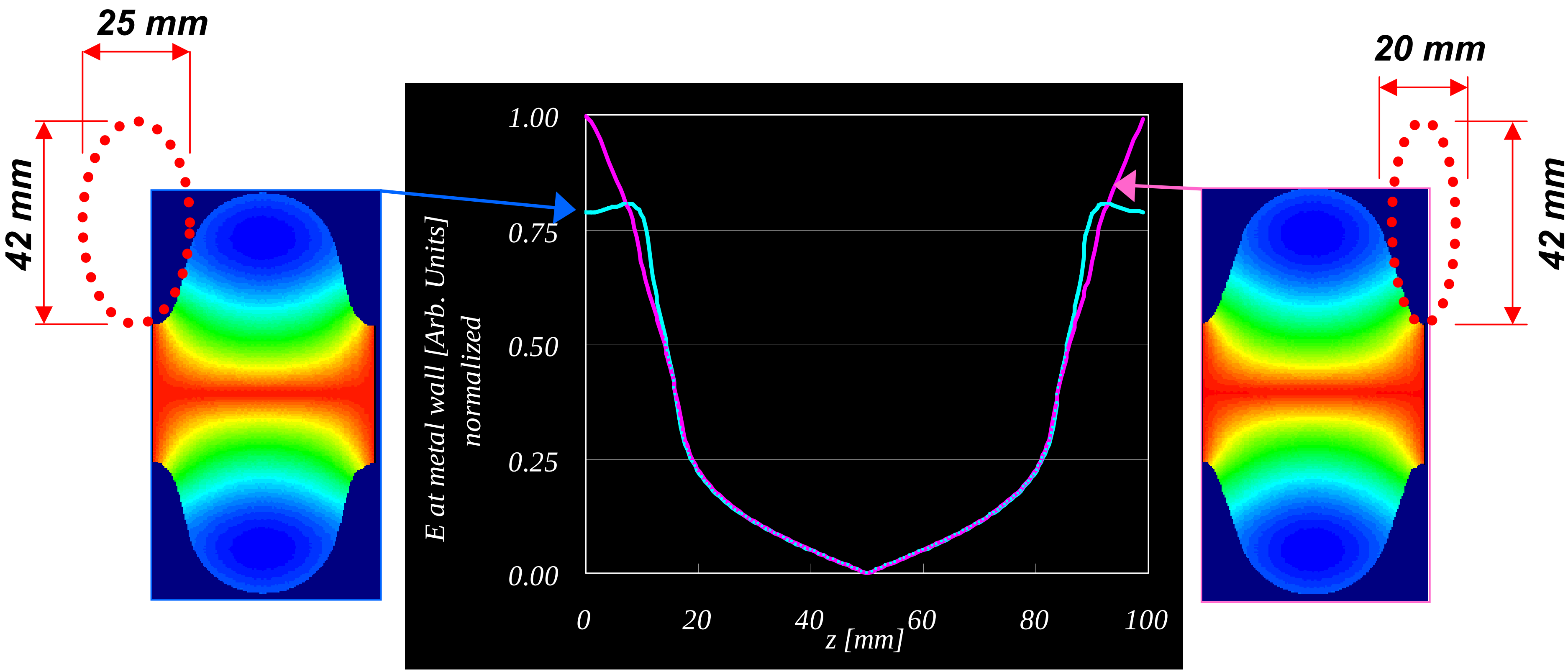

Both cells have the same: f, (R/Q), and iris radius

Prons and Cons of Aperture Effect

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We know that a smaller aperture makes FM :

- *(R/Q) higher*
- B_{peak}/E_{acc} , E_{peak}/E_{acc} *lower*

(+)

but unfortunately a smaller aperture makes:

- *HOMs impedances* $(k_{\perp}, k_{\parallel})$ *higher*
- *cell-to-cell coupling* (k_{cc}) *weaker*

(-)

Aperture Effects on $\kappa_{//}$ and $\kappa_{\perp}$)

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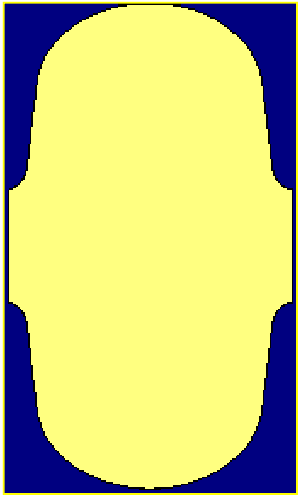

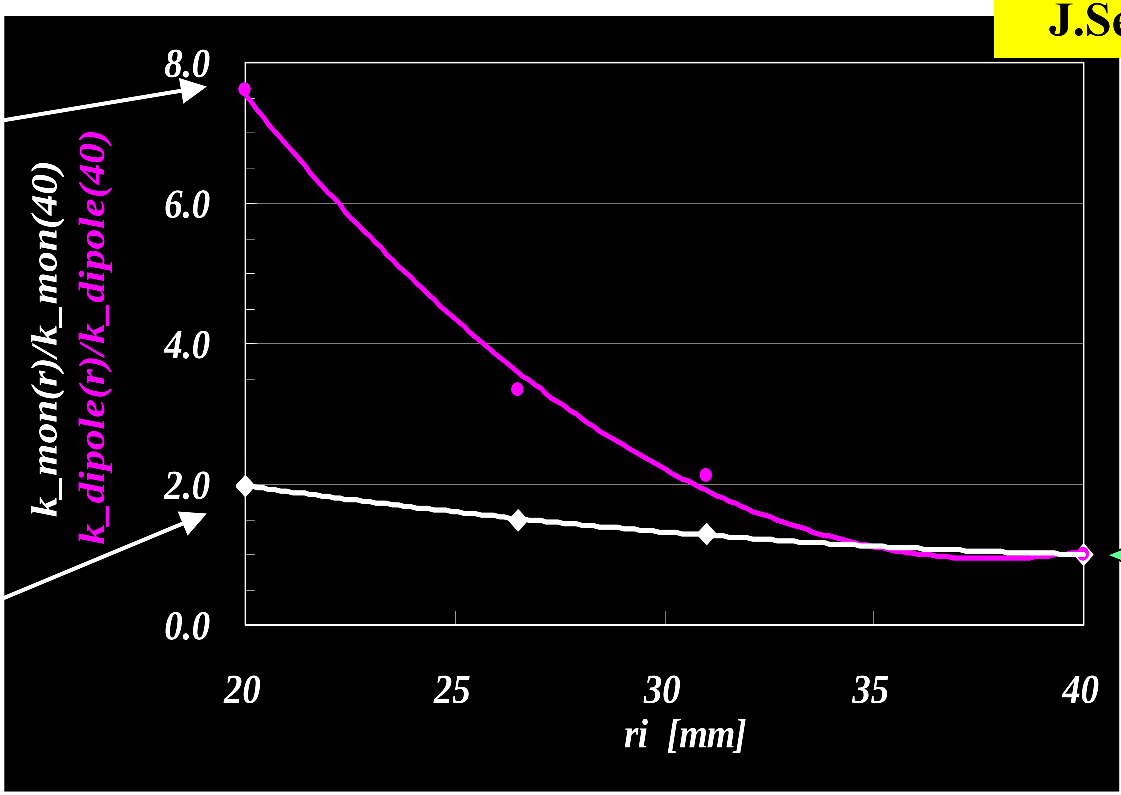

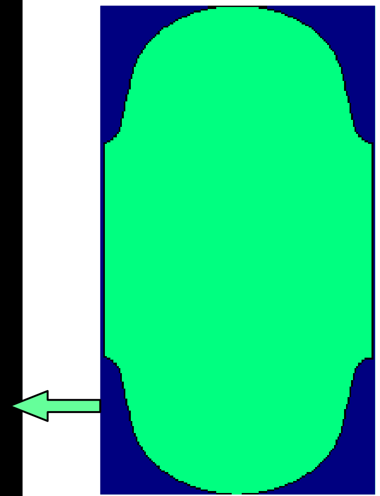

(R/Q) = 152 Ω

B_{peak} / E_{acc} = 3.5 mT/(MV/m)

E_{peak} / E_{acc} = 1.9

(R/Q) = 86 Ω

B_{peak} / E_{acc} = 4.6 mT/(MV/m)

E_{peak} / E_{acc} = 3.2

Aperture Effect on Cell to Cell Coupling (κ_{CC})

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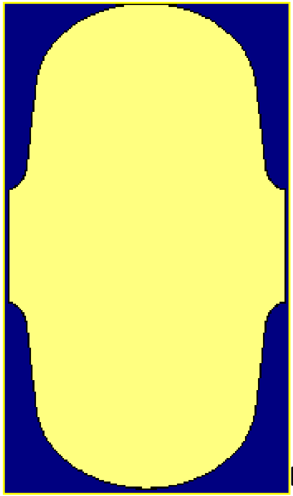

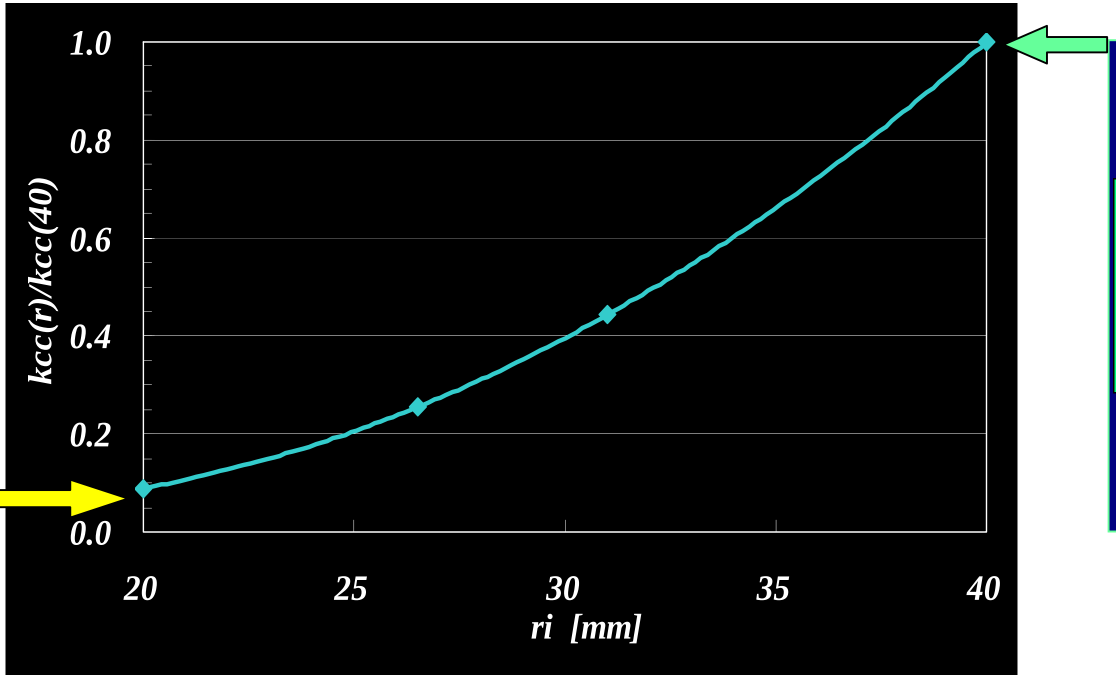

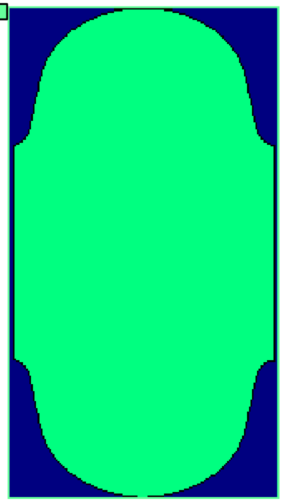

(R/Q) = 152 Ω

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(R/Q) = 86 Ω

B_{peak} / E_{acc} = 4.6 mT/(MV/m)

E_{peak} / E_{acc} = 3.2

Choice of the RF Frequency

What about accelerating mode frequency of a superconducting cavity?

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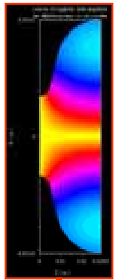

x 2 =

f_π	[MHz]	2600
R/Q	[Ω]	57
r/q=(R/Q)/l	[Ω/m]	2000
G	[Ω]	271

f_π	[MHz]	1300
R/Q	[Ω]	57
r/q=(R/Q)/l	[Ω/m]	1000
G	[Ω]	271

r/q=(R/Q)/l ~ f

Frequency Dependence of SRF Surface Resistance

From the formula, we learned before:

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$$\frac{P_{dissipated}}{V_{acc}^2} \equiv \frac{R_s}{G_{acc} \cdot (R/Q)_{acc}}$$

one obtains:

$$P_{dissipated} = \frac{R_s \cdot V_{acc}^2}{G_{acc} \cdot (r/q)_{acc} \cdot l_{active}}$$

A higher frequencies would be a good choice to minimize power dissipation in the metal wall when the length l_{active} and the final energy V_{acc} are fixed.

Unfortunately this applies only to room temperature conductors, which R_s ~ $(f)^{1/2}$.

For superconductors like Nb:

$$R_s(f) = R_{res} + R_{BCS} = R_{res} + 0.0002 \cdot \frac{1}{T} \cdot (\frac{f[GHz]}{1.5})^2 \cdot exp(-\frac{17.67}{T})$$

and increase of R_s for higher f must be compensated with lower temperature T.

This is why ILC (1.3GHz) will operate at 2K, and HERA (0.5GHz) and LEP (0.352GHz) can (could) operate at 4.2 K

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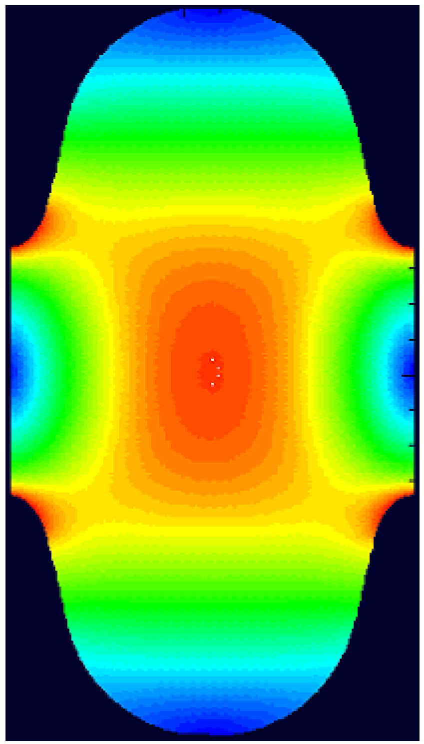
The inner cell geometry *was optimize with respect to:* ***low*** E_{peak}/E_{acc} *and coupling* k_{cc}.

At that time (1992) the field emission phenomenon and field flatness were of concern, no one was thinking about reaching the magnetic limit.

f_{π}	[MHz]	1300.0
r_{iris}	[mm]	35
k_{cc}	[%]	1.9
E_{peak}/E_{acc}	-	1.98
B_{peak}/E_{acc}	[mT/(MV/m)]	4.15
R/Q	[Ω]	113.8
G	[Ω]	271
R/Q*G	[Ω*Ω]	30840



Inner cell; Contour of E field

Overview of Cavities

J.Sekutwitz's Slide

Examples of Inner cells

			new	new				new	new
		CEBAF Original Cornell ß=1	*CEBAF -12 High Gradient ß=1*	*CEBAF -12 Low Loss ß=1*	*TESLA ß=1*	*SNS ß=0.61*	*SNS ß=0.81*	*RIA ß=0.47*	*RHIC Cooler ß=1*
f_o	[MHz]	1448.3	1468.9	1475.1	1278.0	792.8	792.8	793.0	683.0
f_π	[MHz]	1497.0	1497.0	1497.0	1300.0	805.0	805.0	805.0	703.7
k_{cc}	[%]	3.29	1.89	1.49	1.9	1.52	1.52	1.52	2.94
E_{peak}/E_{acc}	-	2.56	1.96	2.17	1.98	2.66	2.14	3.28	1.98
B_{peak}/E_{acc}	[mT/(MV/m)]	4.56	4.15	3.74	4.15	5.44	4.58	6.51	5.78
R/Q	[Ω]	96.5	112	128.8	113.8	49.2	83.8	28.5	80.2
G	[Ω]	273.8	266	280	271	176	226	136	225
R/Q*G	[Ω*Ω]	26421	29792	36064	30840	8659	18939	3876	18045
$k_\perp$ (σ_z=1mm)	[V/pC/cm²]	0.22	0.32	0.53	0.23	0.13	0.11	0.15	0.02
$k_{/}$ (σ_z=1mm)	[V/pC]	1.36	1.53	1.71	1.46	1.25	1.27	1.19	0.85

3.5 High Gradient Cavity Shape

- Suppressed Fied Emission and Multipacting
- Lower Surface Electric field
- Lower Surface Magnetic field
- High Efficient
- High Gradient

} incorporate

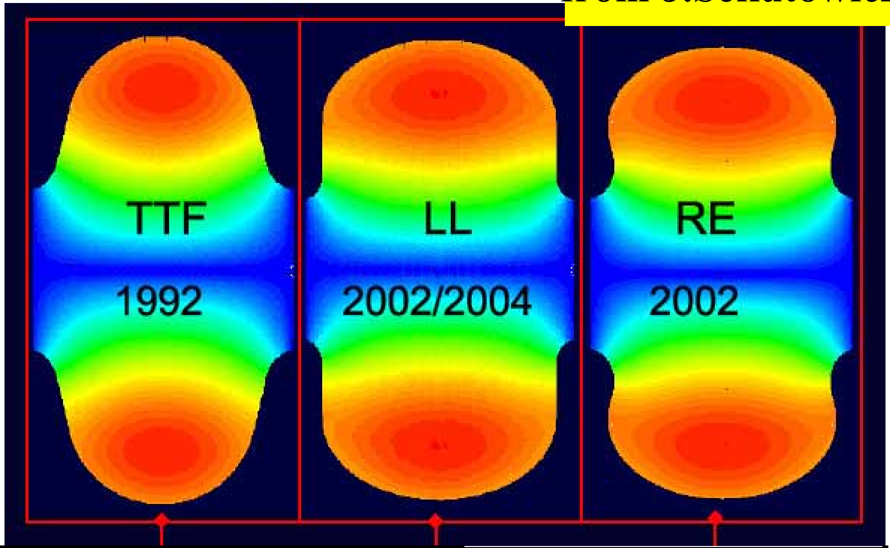
2001-2004

High Gradient Shapes

Cavity shape designs with low Hp/Eacc

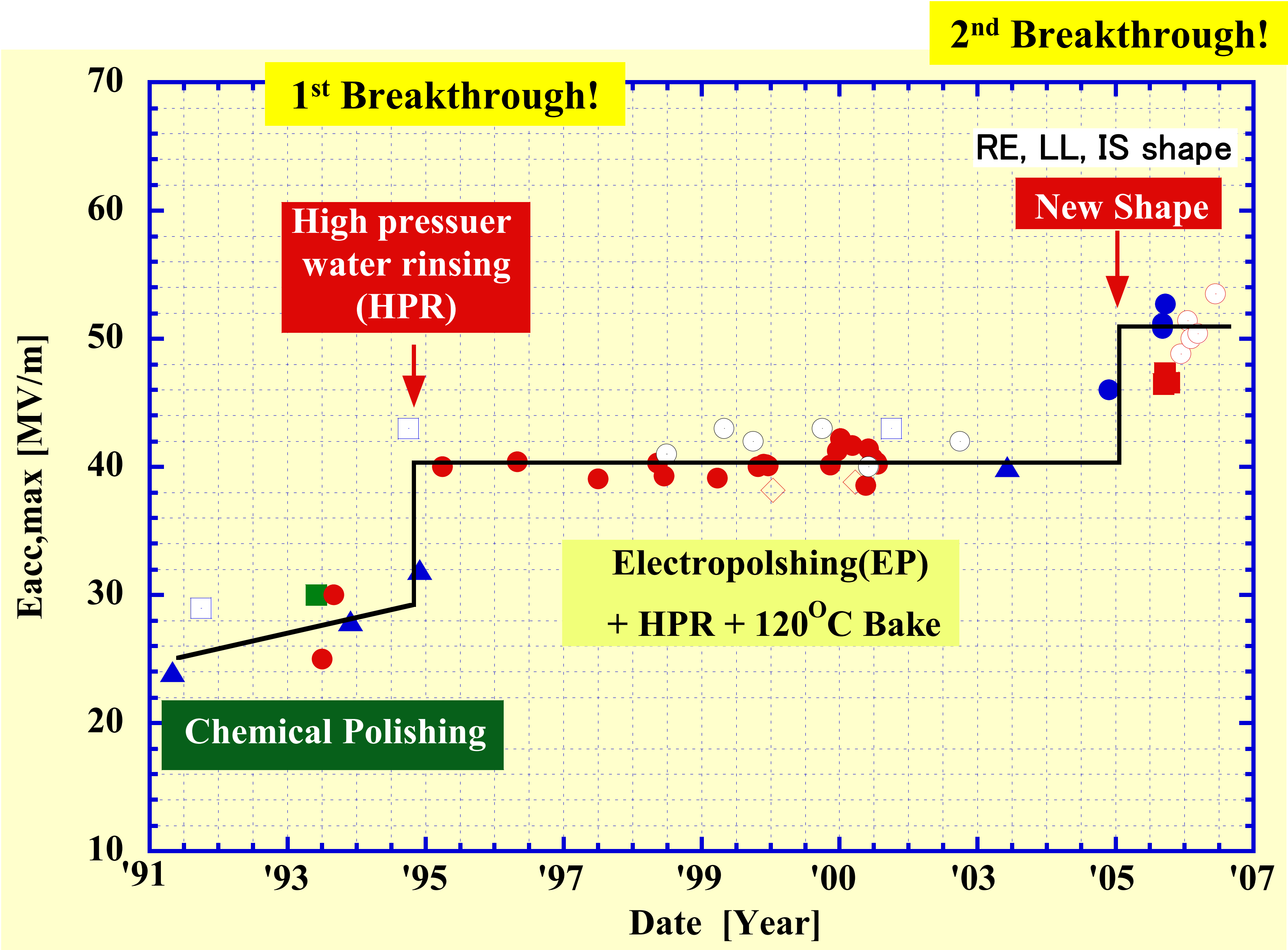
from J.Sekutowicz lecture Note

TTF: TESLA shape
Reentrant (RE): Cornell Univ.
Low Loss(LL): JLAB/DESY
Ichiro-Single(IS): KEK



	TESLA	LL	RE	IS
Diameter [mm]	**70**	**60**	**66**	**61**
Ep/Eacc	**2.0**	**2.36**	**2.21**	**2.02**
Hp/Eacc [Oe/MV/m]	**42.6**	**36.1**	**37.6**	**35.6**
R/Q [W]	**113.8**	**133.7**	**126.8**	**138**
G[W]	**271**	**284**	**277**	**285**
Eacc max	**41.1**	**48.5**	**46.5**	**49.2**

Eacc vs. Year



2nd Breakthrough!
1st Breakthrough!
RE, LL, IS shape
New Shape
High pressuer water rinsing (HPR)
Electropolshing(EP) + HPR + 120OC Bake
Chemical Polishing
Eacc,max [MV/m]
70
60
50
40
30
20
10
'91
'93
'95
'97
'99
'00
'03
'05
'07
Date [Year]

Comparison of DESY and KEK single results

<u>KEK recipe</u>
CBP+CP+Anneal+EP+HPR+Baking

Eacc=43.5+-4.8MV/m for ICHIRO

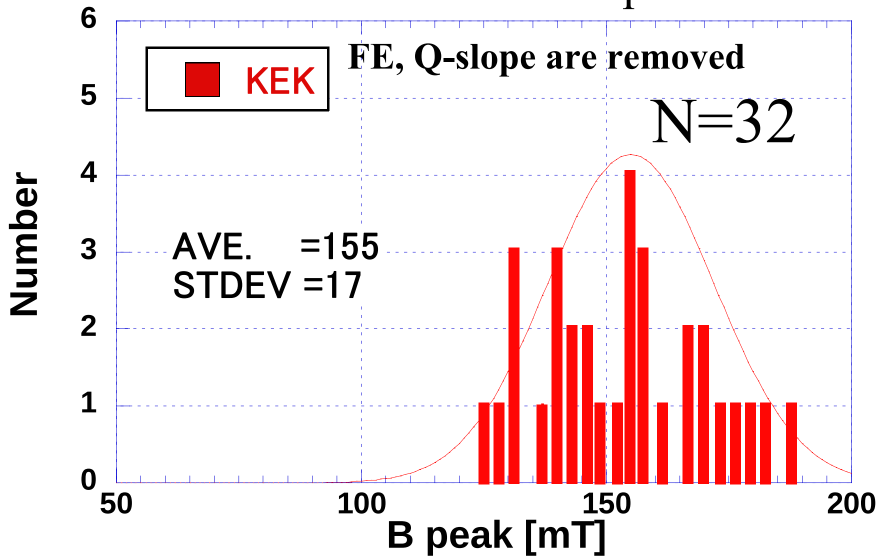
Eacc=37.3+-4.1MV/m for TESLA

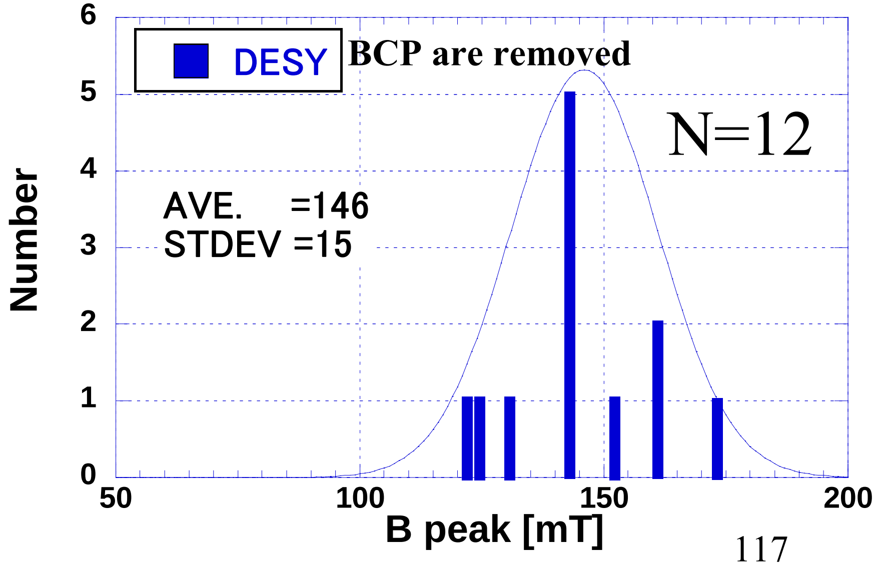
<u>DESY recipe</u>
EP+Anneal+EP+HPR+Baking

Eacc=35.2+-3.6MV/m for TESLA

EP technology is close between KEK and DESY. But still there is 6% margin for DESY, if they use KEK recipe.
DESY can be push up the gradient about 6%, which guaranteed the BCD acceptance performance with high yield(>90%).

Included pilot run





Using DESY/ Detlef Reschke's data.

3.6 Criteria for Multi-cell Structures

J.Sekutwitz's Slide

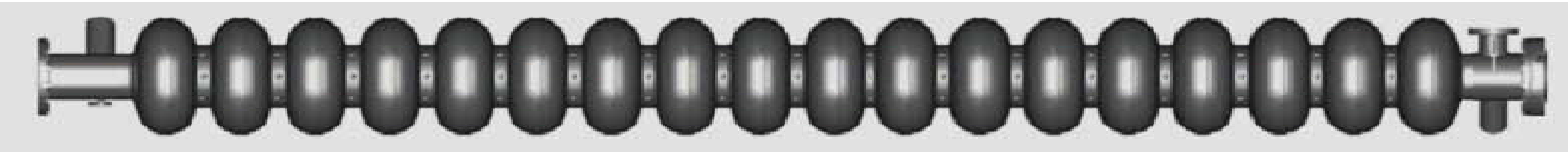
Single-cell is attractive from the RF-point of view:

- ***Easier to manage HOM damping***
- ***No field flatness problem.***
- ***Input coupler transfers less power***
- ***Easy for cleaning and preparation***
- ***But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.***

A multi-cell structure is less expensive and offers higher real-estate gradient but:

- ***Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells***
- ***Other problems arise: HOM trapping…***

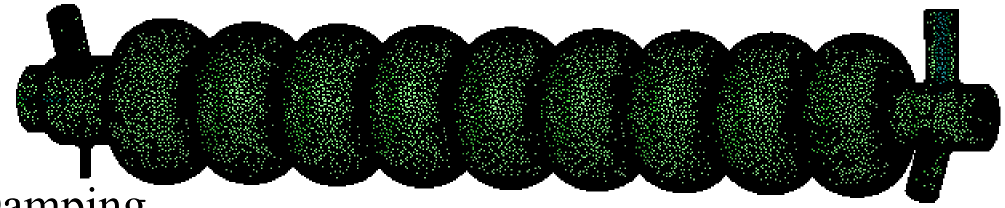
How to decide the number of cells ?



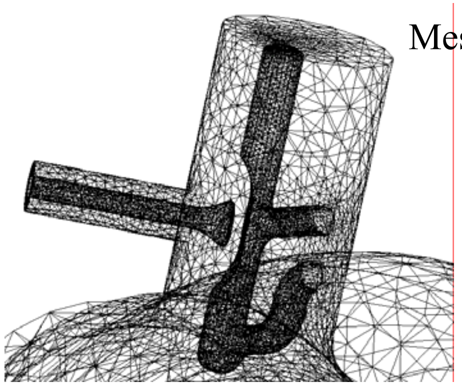
RF Structure Simulation

Currently full 3D analysis is possible using cords Omega or ANALIS, example SLAC, KEK

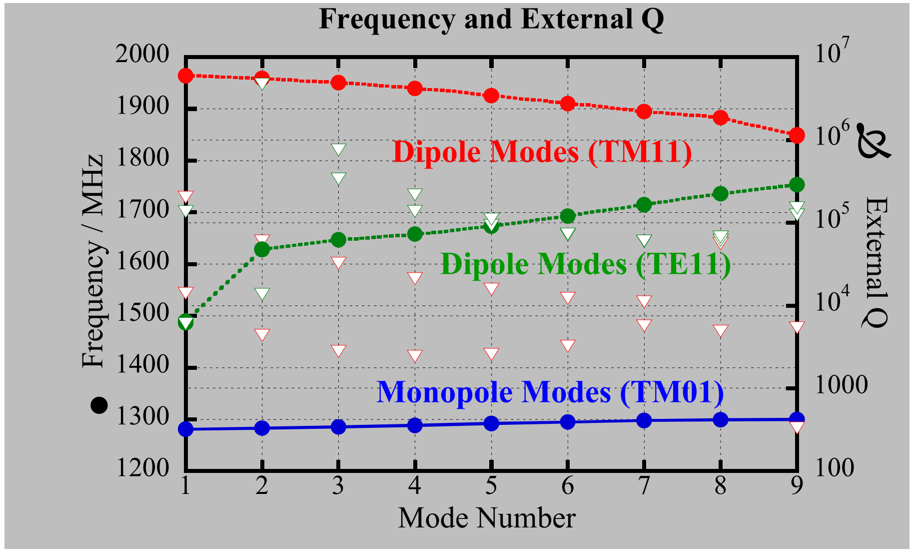
Element Model (Mesh)



Simulation of Higher Order Mode Damping



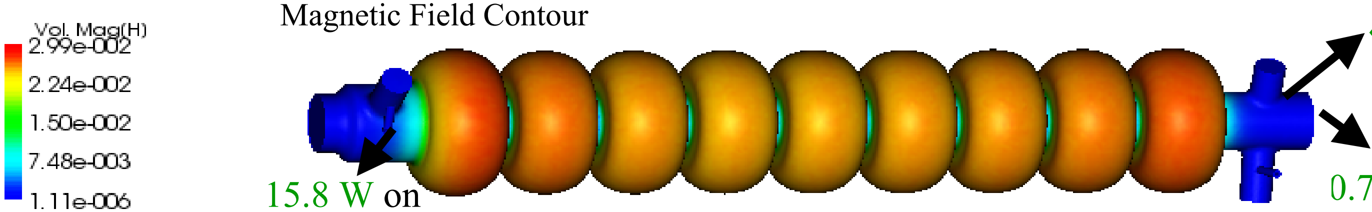
Mesh of HOM Damper



Analysis of Wall Loss in Vertical Testing

Measured Low Q of 1.1×10^{10} at 21 MV/m for 38 W ----- Reproduced by Simulation

Magnetic Field Contour



21.8 W on Niobium Walls

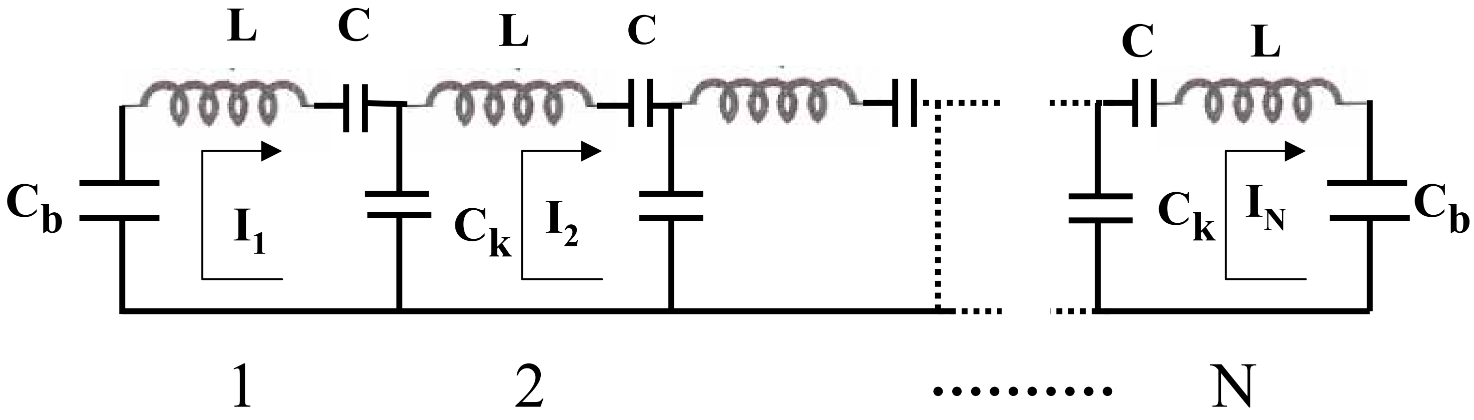
Pros and Cons for Multi-cell Structure

J.Sekutwitz's Slide

Cost of accelerators is lower (less auxiliaries: LHe vessels, tuners, fundamental power couplers, control electronics)

- ***Higher real-estate gradient (better fill factor)***
- ***Field flatness vs. N***
- ***HOM trapping vs. N***
- ***Power capability of fundamental power couplers vs. N***
- ***Chemical treatment and final preparation become more complicated***
- ***The worst performing cell limits whole multi-cell structure***

Equivalent Circuit Model for Multi-Cell Cavity



$$\left(\frac{1}{i\omega C_b}+i\omega L\right)I_1+\left(\frac{1}{i\omega C}\right)I_1+\left(\frac{1}{i\omega C_k}\right)(I_1-I_2)=0$$

$$\frac{1}{i\omega C_k}(I_j-I_{j-1})+\left(i\omega L+\frac{1}{i\omega C}\right)I_j+\left(\frac{1}{i\omega C_k}\right)(I_j-I_{j+1})=0$$

$$1<j<N$$

$$\left(\frac{1}{i\omega C_k}\right)(I_N-I_{N-1})+\left(i\omega L+\frac{1}{i\omega C}\right)I_N+\left(\frac{1}{i\omega C_b}\right)I_N=0$$

$$(1+k+\gamma)I_1-kI_2=\Omega I_1$$

$$-kI_{j-1}+(1+2k)I_j-kI_{j+1}=\Omega I_j,\quad 1<j<N$$

$$-kI_{N-1}+(1+k+\gamma)I_N=\Omega I_N$$

$$\omega_0^2=\frac{1}{LC},\ k=\frac{C}{C_k},\ \gamma=\frac{C}{C_b},\ \text{and}\ \Omega=\frac{\omega^2}{\omega_0^2}$$

$$\begin{bmatrix}1+k+\gamma & -k & 0 & \cdots & 0\\ -k & 1+2k & -k & \cdots & 0\\ 0 & & \ddots & & 0\\ \vdots & & -k & 1+2k & -k\\ 0 & \cdots & 0 & -k & 1+k+\gamma\end{bmatrix}\begin{bmatrix}I_1\\ I_2\\ \vdots\\ I_{N-1}\\ I_N\end{bmatrix}=\Omega\begin{bmatrix}I_1\\ I_2\\ \vdots\\ I_{N-1}\\ I_N\end{bmatrix}$$

Solution for j-th cell in m-th mode

π-mode

$$\mathrm{v} = \frac{1}{\sqrt{N}}[1,\ -1,\ 1,\ -1,\ \cdots]$$

$$\begin{bmatrix} 1+k+\gamma & -k & 0 & \cdots & 0 \\ -k & 1+2k & -k & \cdots & 0 \\ 0 & & \ddots & & 0 \\ \vdots & & -k & 1+2k & -k \\ 0 & \cdots & 0 & -k & 1+k+\gamma \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ \vdots \\ \\ \\ \end{bmatrix} = \Omega^{(N)} \begin{bmatrix} 1 \\ -1 \\ \vdots \\ \\ \\ \end{bmatrix}$$

General equation

$$\begin{bmatrix} 1+3k & -k & 0 & \cdots & 0 \\ -k & 1+2k & -k & \cdots & 0 \\ 0 & & \ddots & & 0 \\ \vdots & & -k & 1+2k & -k \\ 0 & \cdots & 0 & -k & 1+3k \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{N-1} \\ I_N \end{bmatrix} = \Omega \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{N-1} \\ I_N \end{bmatrix}$$

$$1+2k+\gamma = \Omega^{(N)}$$

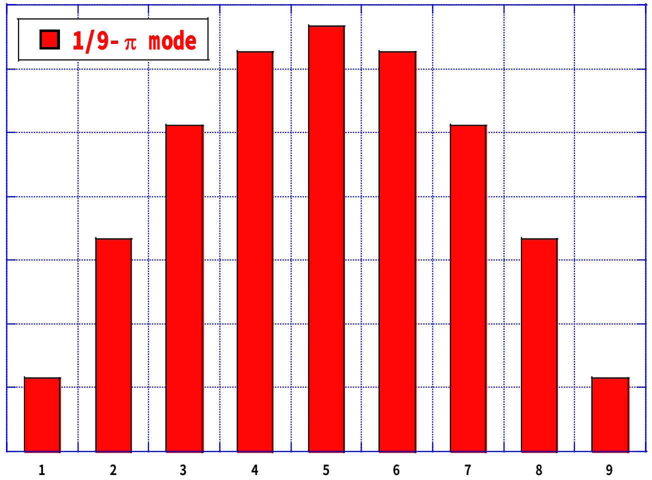
$$-1-4k = -\Omega^{(N)} \rightarrow \quad \gamma = 2k$$

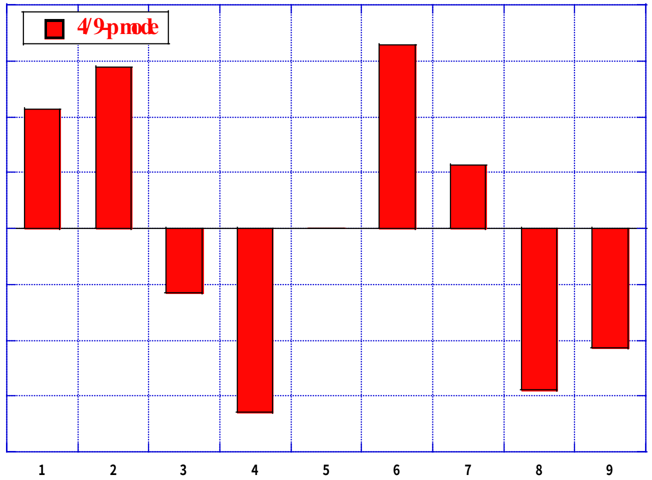
$$v_j^{(m)} = B^{(m)} \sin\left[m\pi \left(\frac{2j-1}{2N} \right) \right], \quad B^{(m)} = \sqrt{\frac{2-\delta_{m,N}}{N}}$$

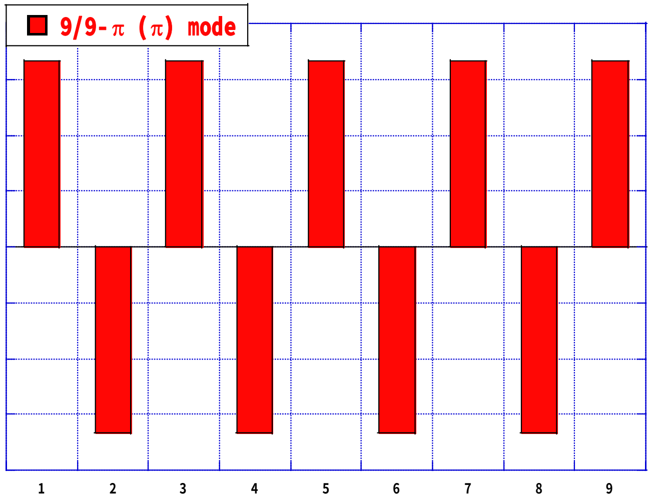
m = m-th mode,

j = j-th cell in mode m-th

Field distribution in pass-bands

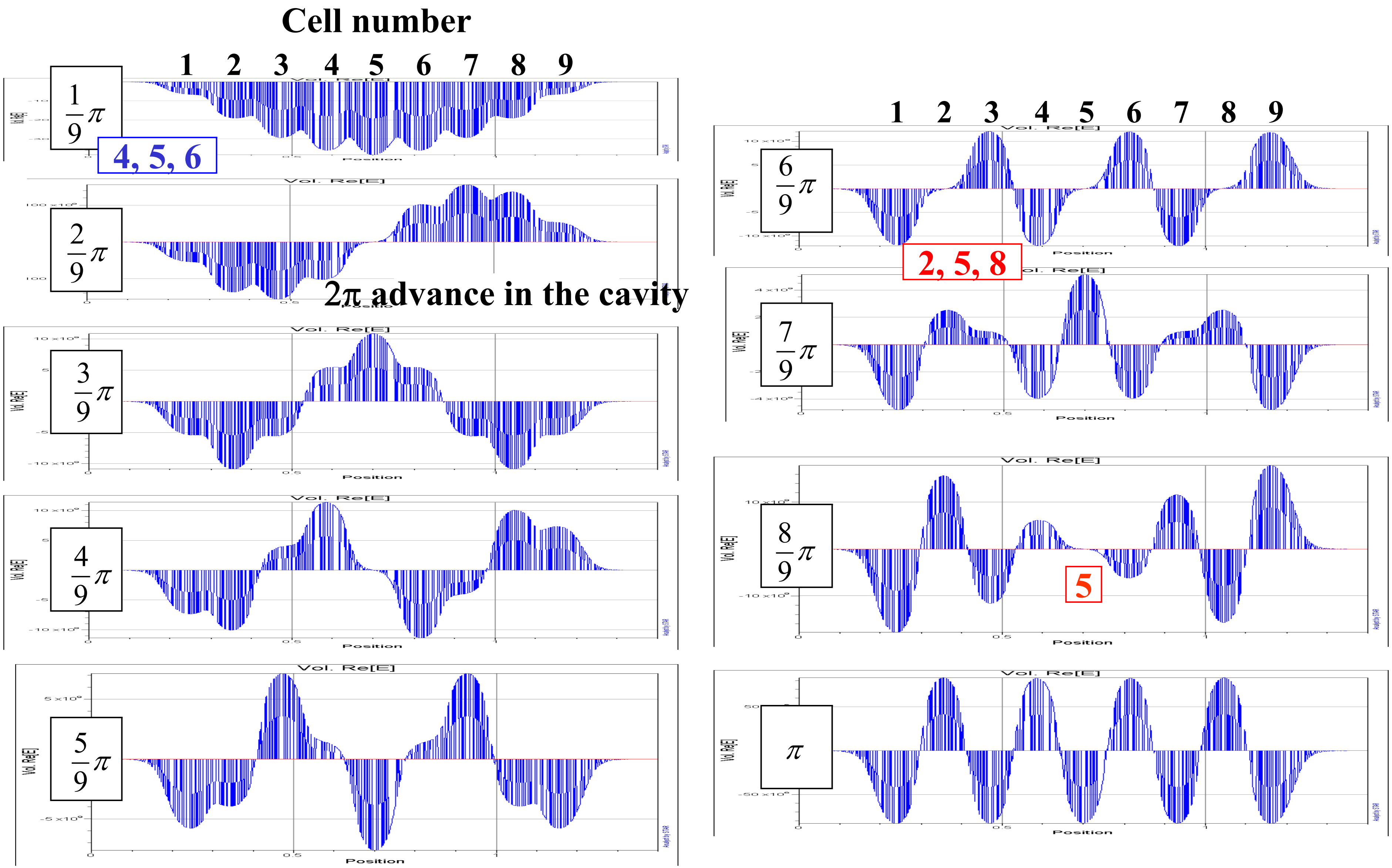




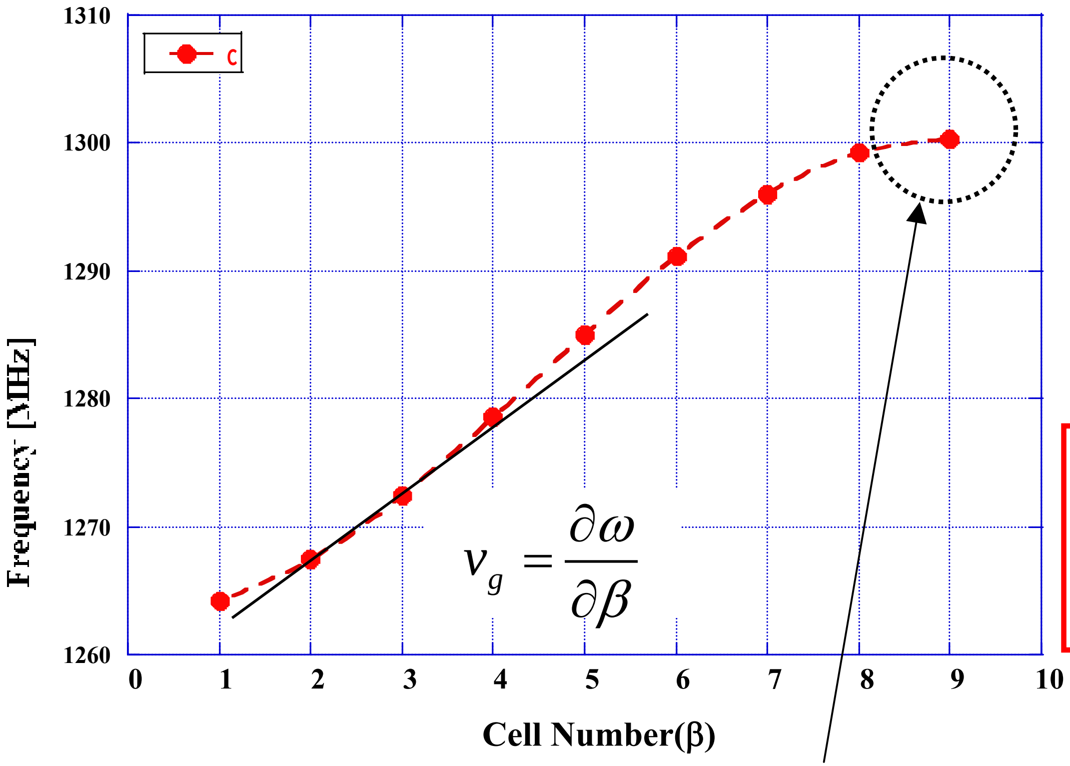


m-th mode: mπ /9 phase advance per cell

Phase Advance per 9-Cell Cavity in Pass-Band Modes of TM_{010}



Pass-Band Modes Frequency in 9-Cell Cavity

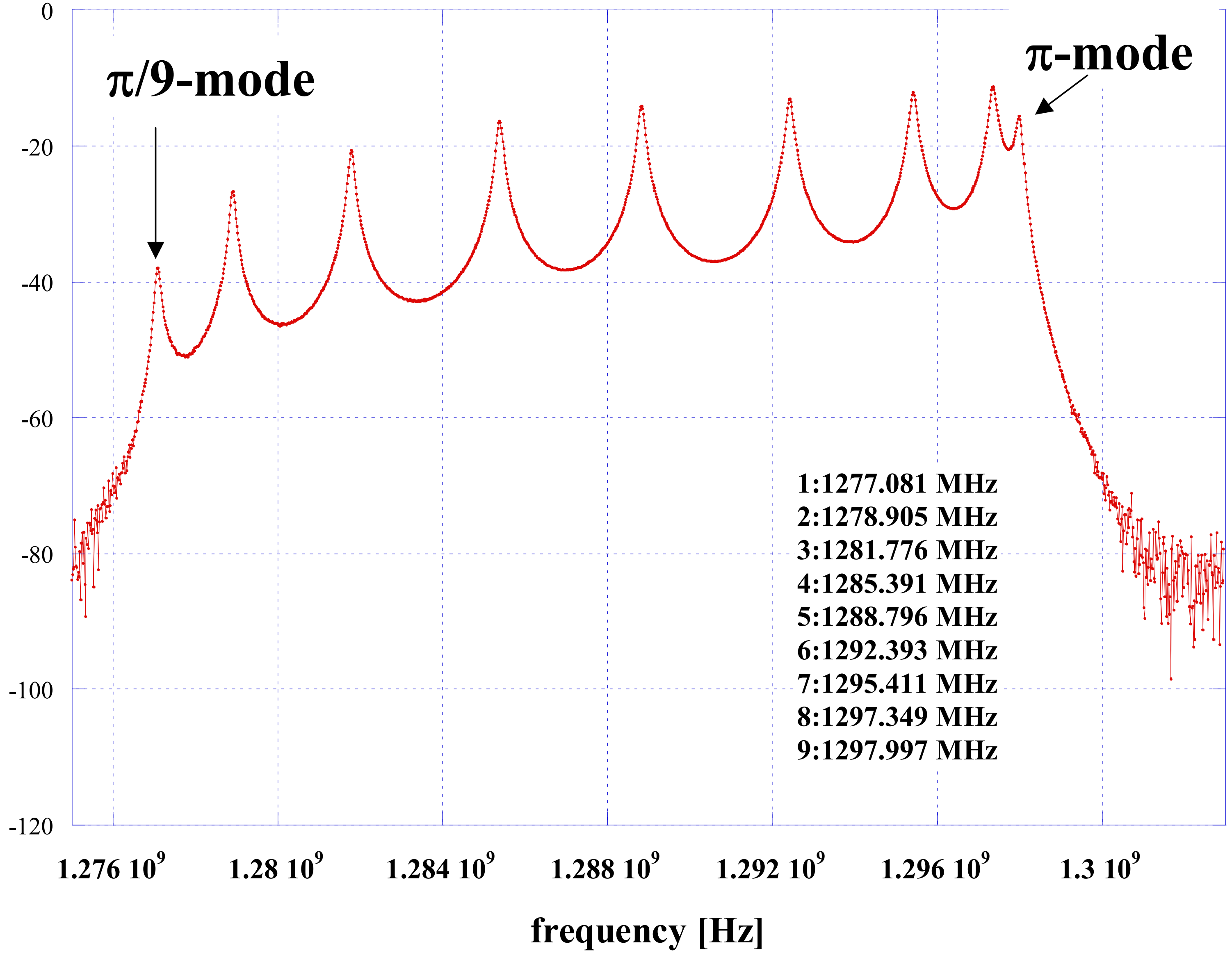


$$v_{j-1}^{(m)} + v_{j+1}^{(m)} = B^{(m)} \sin\left[\left(m\pi \frac{(2j-1)}{2N}\right) - \frac{m\pi}{N}\right] + B^{(m)} \sin\left[\left(m\pi \frac{(2j-1)}{2N}\right) + \frac{m\pi}{N}\right] = 2v_j^{(m)} \cos\left(\frac{m\pi}{N}\right)$$

$$\Omega^{(m)} = \left(\frac{f_m}{f_0}\right)^2 = 1 + 2k\left[1 - \cos\left(\frac{m\pi}{N}\right)\right]$$

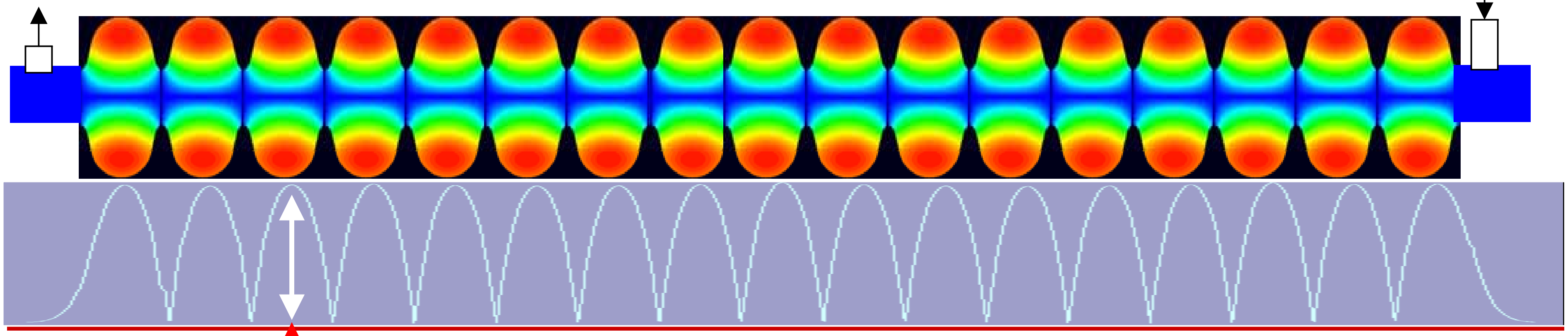
When N is infinite, V_g=0 @ π-mode. It means no energy flow between cells.
At large N, beam acceleration becomes unstable because less energy flow between cells.

TM_{010} Pass-Band Modes Frequency in 9-Cell Cavity



Field Flatness Sensitivity Factor vs. N

J.Sekutwitz's Slide



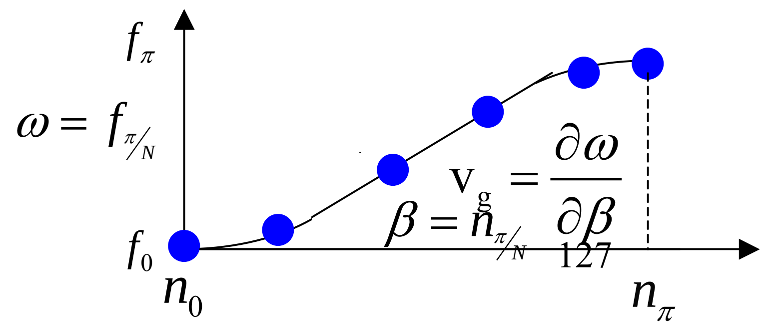
$$\frac{\Delta A_i}{A_i} = a_{ff}\,\frac{\Delta f_i}{f_i} = \frac{N^2}{k_{cc}} \cdot \frac{\Delta f_i}{f_i}$$

N: number of cells

Beam pipe has no acceleration beam.
BP reduce the efficiency.
Multi-cell is more efficient.

Field flatness factor : $a_{ff} = \dfrac{N^2}{k_{cc}}$

Cell to cell coupling : $\mathrm{k}_{cc} = 2 \cdot \dfrac{f_\pi - f_0}{f_\pi + f_0}$



Effect of N on Field Flatness Sensitivity Factor

J.Sekutwitz's Slide

- *Field flatness vs. N*

	Original Cornell N = 5	High Gradient N =7	Low Loss N =7	TESLA N=9	SNS ß=0.61 N=6	SNS ß=0.81 N=6	RIA ß=0.47 N=6	RHIC N=5
year	*1982*	*2001*	*2002*	*1992*	*2000*	*2000*	*2003*	*2003*
a_{ff}	*1489*	*2592*	*3288*	*4091*	*3883*	*2924*	*5040*	*850*

Many years of experience with: heat treatment, chemical treatment, handling and assembly allows one to preserve tuning of cavities, even for those with bigger N and weaker k_{cc}

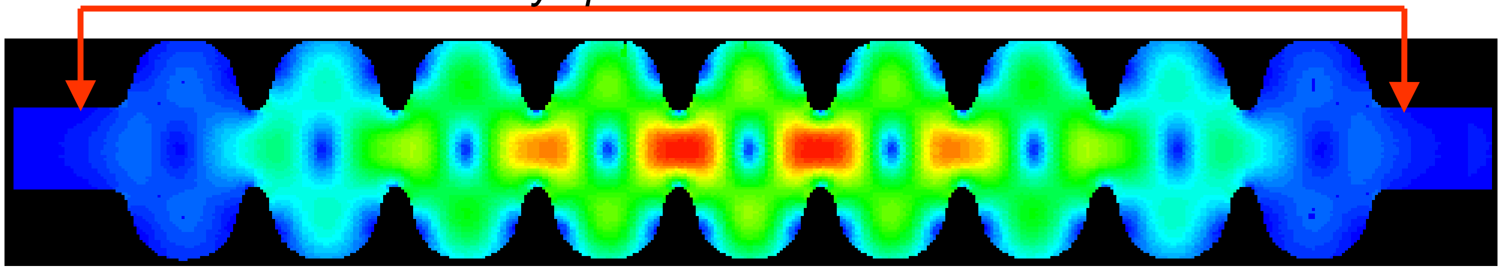
For the TESLA cavities: field flatness is better than 95 %

HOM Issue with Multi-Cell Structure

HOM couplers limit RF-performance of sc cavities when they are placed on cells

J.Sekutwitz's Slide

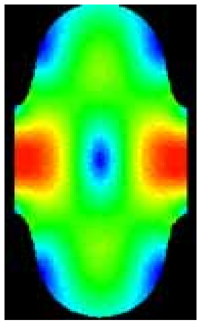
no E-H fields at HOM couplers positions, which are always placed at end beam tubes



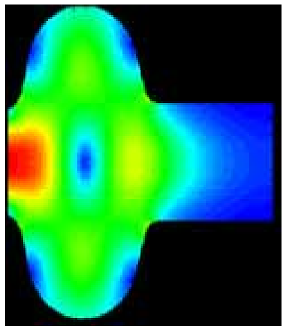
The HOM trapping mechanism is similar to the FM field profile unflatness mechanism:

- *weak coupling HOM cell-to-cell,* $\boldsymbol{k}_{cc,HOM}$
- *difference in HOM frequency of end-cell and inner-cell*

f = 2385 MHz

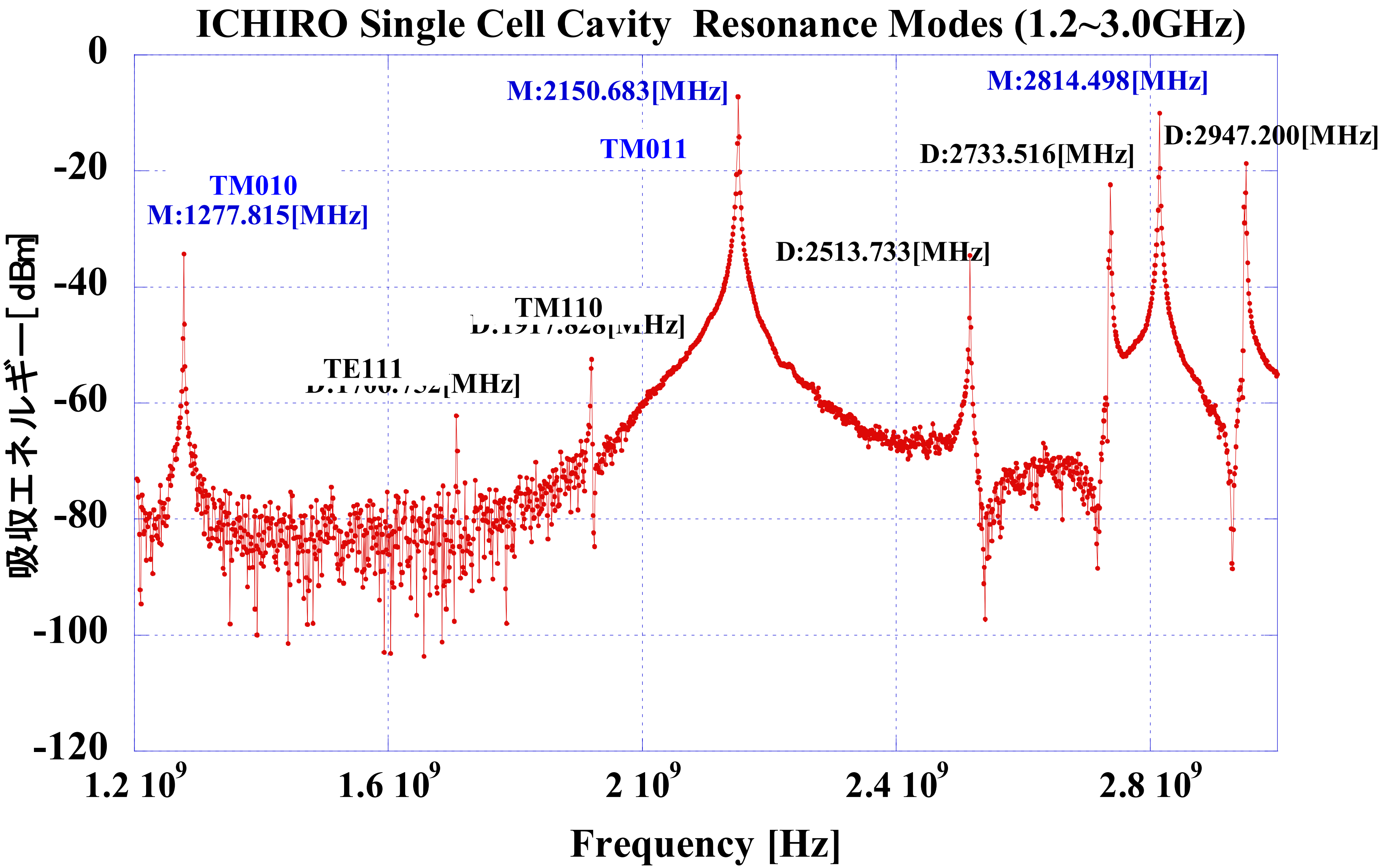


That is why they hardly resonate together



f = 2415 MHz

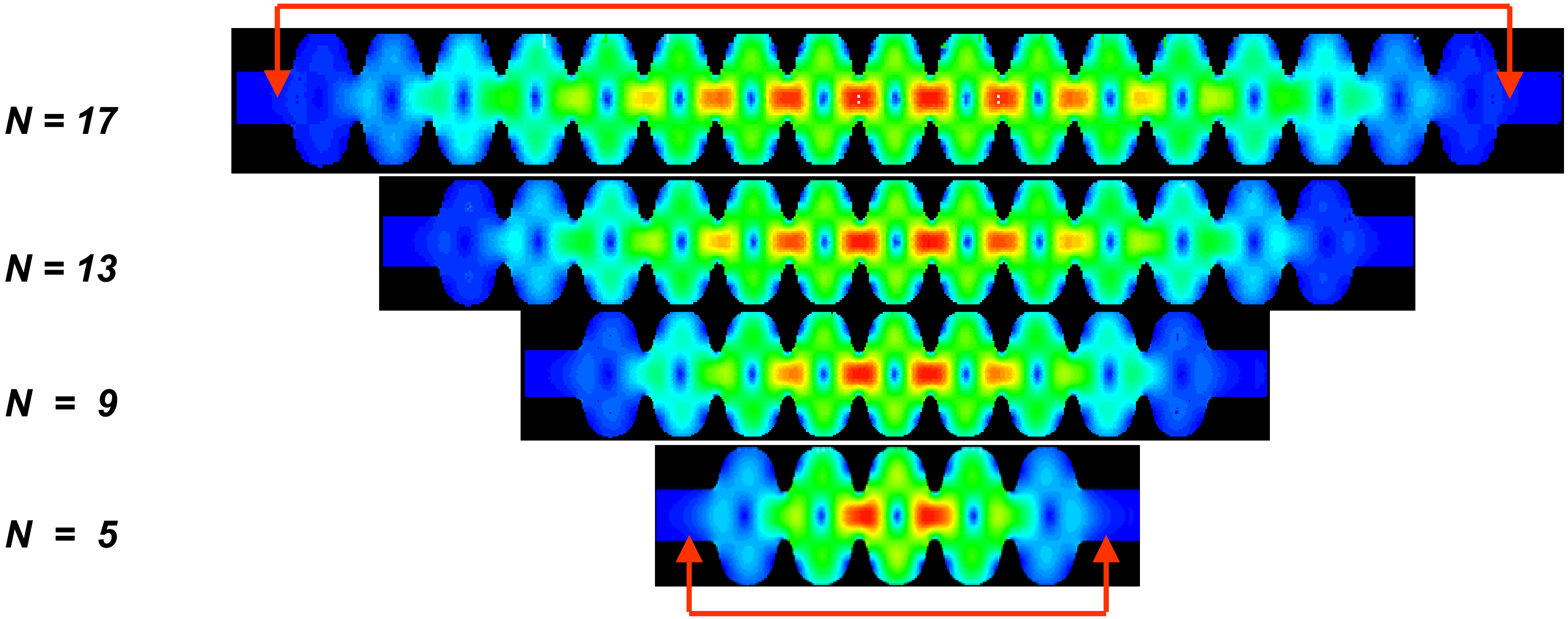
HOM Modes in Single Cell Cavity



HOM trapping vs. N

J.Sekutwitz's Slide

No fields at HOM couplers positions, which are always placed at end beam tubes



e-m fields at HOM couplers positions

Smaller number of cells is easy to take out HOMs.

Capable Input Power Dependence on N

J.Sekutwitz's Slide

- *Power capability of fundamental power couplers vs. N*

When I_{beam} and E_{acc} are specified and a superconducting multi-cell structure does not operate in the energy recovery mode:

$$P_{in} \sim N$$

The Q_{ext} of the FPC, which usually is << than intrinsic Qo, is:

$$Q_{ext} \cong \frac{E_{acc} \cdot \beta \cdot \lambda \cdot N}{I_{beam} \cdot (R/Q)_{cell} \cdot N} = \frac{E_{acc} \cdot \beta \cdot \lambda}{I_{beam} \cdot (R/Q)_{cell}} = \frac{\omega_{acc} \cdot W_{onecell} \cdot N}{\frac{1}{2} \int_{S_{inputport}} E_{acc} \times H_{acc} \, ds}$$

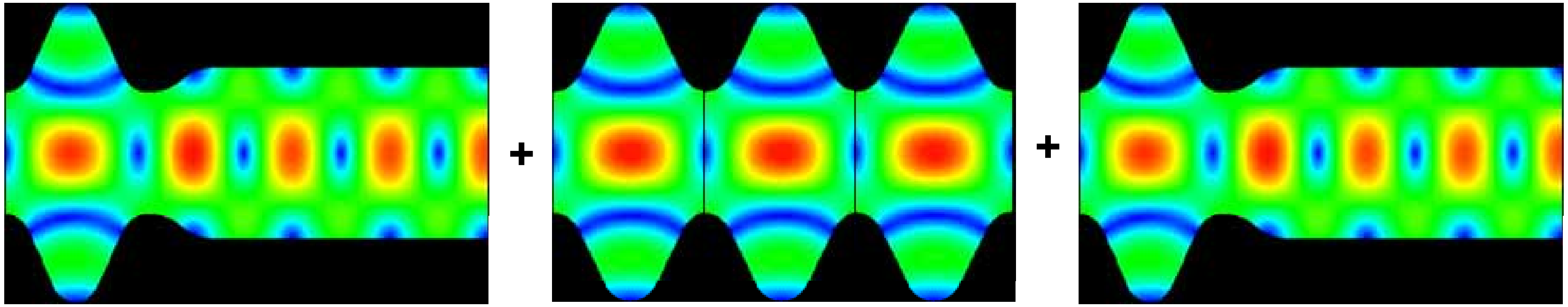
Independent of N

It must be ~ N to keep the ratio constant

Adjustment of End-Cells

J.Sekutwitz's Slide

The geometry of end-cells differs from the geometry of inner cells due to the attached beam tubes



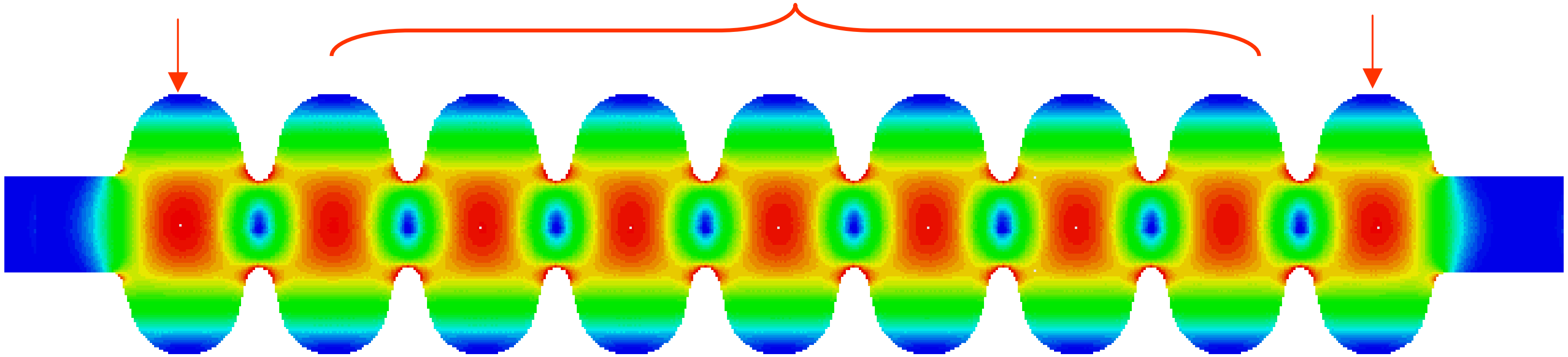
Their function is multi-folded and their geometry must fulfill three requirements:

- ***field flatness and frequency of the accelerating mode***
- ***field strength of the accelerating mode at FPC location enabling operation with matched Qext***
- ***fields strength of dangerous HOMs ensuring their required damping by means of HOM couplers or/and beam line absorbers.***

All three make design of the end-cells more difficult than inner cells.

The cavity was designed in 1992 (A. Mosnier, D. Proch and J.S.).

J.Sekutwitz's Slide



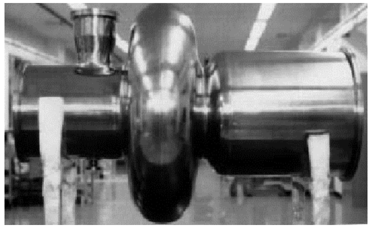
TTF 9-cells; Contour of E field

f_{π}	[MHz]	1300.00
$f_{\pi\text{-}1}$	[MHz]	1299.24
R/Q	[Ω]	1012
G	[Ω]	271
Active length	[mm]	1038

3.7 Example of SRF Cavities

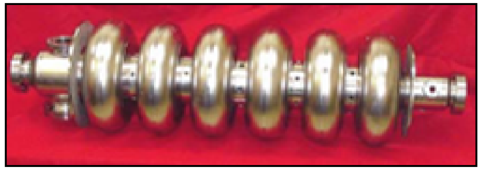
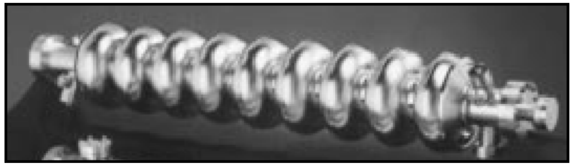
Cavities operating with highest I_{beam} or E_{acc}

J.Sekutwitz's Slide

Type /No. of cavities			P_{beam}/cavity [kW]	P_{HOM} /cavity [kW]
KEK-B 0.5 GHz  x 8	Single-cell with max I_{beam}	I_{beam}= 1.34 A 1389 bunches cw	350	16
HERA 0.5 GHz x 16	Multi-cell with max I_{beam}	I_{beam} ≤40 mA 180 bunches cw	60	0.13
TTF-I , 1.3 GHz  x 1	Multi-cell with max E_{acc}	E_{acc}= 35 MV/m 1.3ms/pulse 1Hz PRF	~100 Almost no beam loading	0

J.Sekutwitz's Slide

Cavities which will operate with high I_{beam} in the near future

Type /No. cavities			*P_{beam}/cavity [kW]*	*P_{HOM} /cavity [kW]*
SNS ß= 0.61, 0.805 GHz  *x 33*	*Multi-cell with max I_{beam}*	*I_{beam}=38 (59) mA 1.3ms/pulse DF = 6 %*	*240 (366)*	*0.06 peak*
SNS ß= 0.805 GHz  *x 48*			*482*	*0.06 peak*
TTF-II ep , 1.3 GHz  *x 8*	*Multi-cell with max E_{acc}*	*E_{acc}= 35 MV/m 1.3ms/pulse 10Hz PRF*	*146*	*< 0.02>*